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RESEARCH NOTES

AN APPLICATION OF GENERALISED LEAST SQUARES ESTIMATION OF LINEAR REGRESSION MODEL WITH RANDOM COEFFICIENTS TO PADDY PRODUCTION FUNCTION FOR SAMBALPUR DISTRICT (ORISSA)*

INTRODUCTION

The linear model most frequently used in economic analysis may be indicated as :

$$Y_i = \sum_{j=1}^{K} B_j X_{ij} + U_i, \quad i = 1, 2, \ldots\ldots\ldots\ldots\ldots, N$$

where Y_i is an observed random variable, the X_{ij} are known non-random magnitudes, the B_j are unknown constants to be investigated and the U_i is independently and identically distributed variate with mean zero and finite variance.

In some empirical studies, specially in the agricultural fields, the constancy of the coefficients, B_j, in successive observation may reasonably be questioned. For example, a particular B_j represents the response of fertilizer on yield, and it it well-known that this response is strongly influenced by the method and time of fertilizer application, temperature, method of irrigation, quantity and timing of water and management, etc. If these can be held constant, B_j might be expected to be constant to a tolerable approximation. If the above-mentioned factors vary from farmer to farmer but can be observed, then it is desirable to explicitly incorporate their influences into the model. If they vary and are unobserved, then it will not be realistic to assume that the response of the input on output in a regression model is the same for each observation. In this situation the mean of a random response rate may be better than assuming the rate to be constant.

Considerable progress [1, 2, 4, 6] has been made by econometricians in the use of multiple regression model with random coefficients for estimating the means and variances of the random coefficients. But the application of the model is very limited particularly in agricultural research due to difficulties such as convergence of the non-linear equations being very slow in the case of maximum likelihood method of estimation, and the possibilities of negative estimates of one or more variances of random coefficients. To overcome these problems suitable methods suggested by earlier researchers have been examined for the application of random coefficient model to agricultural data. In this paper, the linear regression model with random coefficients and the generalised least squares procedure for estimating the means of the random coefficients have been described and the model has been used as an empirical tool to study the input-output relationships for paddy crops grown in two sets of villages with and without field channels of Sambalpur district (Orissa).

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THE RANDOM COEFFICIENT MODEL

The relationship between the dependent variable Y and the explanatory variables X_j ($j = 1, 2, \ldots\ldots\ldots K,$) is assumed to be of the form,

$$Y_i = \sum_{j=i}^{k} B_{ij} X_{ij} + U_i, \; i = 1, 2, \ldots\ldots N(N > 2\,k) \quad \ldots\ldots\ldots\ldots (1)$$

where the subscript 'i' refers to observation and 'j' to explanatory variable. U is the disturbance term. In addition to the common assumptions as in the general linear regression model, the assumptions of random coefficients are :

(*i*) For any given j, the coefficients B_{ij} $(i = 1, 2 \ldots\ldots\ldots\ldots, N)$ are random variables with

$E(B_{ij}) = a_j$; $Var\ (B_{ij}) = \sigma_{jj} \geqslant 0$

(*ii*) $Cov\ (B_{ij}, B_{i'j'}) = 0 \qquad j \neq j'$

(*iii*) $Cov\ (U_i, B_{ij}) = 0 \qquad i = 1, 2, \ldots\ldots\ldots\ldots, N$

$\qquad j = 1, 2, \ldots\ldots\ldots\ldots, k$

With the above assumptions, model (1) can be written as :

$$Y_i = \sum_j a_j x_{ij} + W_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where

$$W_i = \sum_j (B_{ij} - a_j)\ X_{ij} + U_i.$$

It can be shown that $E\ (W_i) = 0$

$$Var\ (W_i) = \sigma^2 + \sum_{j=1}^{k} \sigma_{jj}\ X_{ij}^2$$

$$Cov\ (W_i\ W'_{i'}) = 0,\ i \neq i' \quad i, i' = 1, 2, \ldots\ldots\ldots\ldots, N$$

Thus the linear regression model with random coefficients and homoscedastic disturbances (1) is reduced to a linear regression model with constant coefficients and heteroscedastic disturbances (2). Let us revert to model (2) expressed in matrix notation and write

$$Y = Xa + W$$

$$E(W) = 0 ; \quad E(WW') = V ;$$

where V is a non-singular positive definite diagonal matrix of order NXN. The N diagonal elements are linear functions of the variances, σ^2 and σ_{jj} $(j=1, 2, \ldots\ldots, k)$. If $x_{i1}=1$, for all i then separate estimates of σ^2 and σ_{11} cannot be obtained. However, $(\sigma^2 + \sigma_{11})$ can be estimated jointly where $\sigma_{11}^{1} = \sigma^2 + \sigma_{11}$. Thus, the knowledge of the k parameters σ_{11}^{1}, σ_{22}, $\ldots\ldots$, σ_{kk} enables us to obtain the generalised least square estimators of the means of random coefficients.

ESTIMATION OF VARIANCE OF THE RANDOM COEFFICIENTS

Let us consider the model

$$\hat{\dot{W}} = ZS + V$$

where, $\hat{\dot{W}}$ is the estimated square of the least square residual,
Z is the square of the explanatory variables,
S is the variance of random coefficients, and
V is the disturbance term and satisfy the assumptions of Gauss-Markov theorem.

The least square estimates of the variances of the random coefficients are given by

$$\hat{S} = (Z'Z)^{-1} \; Z'\hat{\dot{W}}.$$

These are the estimates suggested by Nagar and Tiwari [3].

Sometimes this procedure provides the negative estimates of the variance of random coefficients.

To avoid negative estimates of the variances

Minimize $Q(S) = V'V = (\hat{\dot{W}} - ZS)'(\hat{\dot{W}} - ZS)$ subject to $S \geqslant 0$.

It is the problem of minimizing a quadratic function of the parameters subject to linear inequalities. This quadratic programming problem can be solved by the method suggested by Theil and Van de Panne[5].

ESTIMATION OF VARIANCES SUBJECT TO LINEAR INEQUALITIES CONSTRAINT

Now in the model

$$\hat{\dot{W}} = ZS + V$$

The quantity to be minimized

$Q(S) = V'V = (\hat{\dot{W}} - ZS)'(\hat{\dot{W}} - ZS)$ is a scalar quantity.

This can be written as

$$Q(S) = \hat{\dot{W}}'\hat{\dot{W}} - 2(S'Z'\hat{\dot{W}} - \tfrac{1}{2}S'Z'ZS)$$

Minimize $Q(S)$ subject to $S \geqslant 0$
or maximize $[-Q(S)]$ subject to $(-S) \leqslant 0$.

Now the problem is reduced to maximizing a quadratic function of vector S,

$$[-Q(S)] = -\dot{\hat{W}}'\dot{\hat{W}} + 2(S'Z'\dot{\hat{W}} - \tfrac{1}{2}S'Z'ZS)$$

subject to the condition that $(-S) \leqslant 0$.

As $\dot{\hat{W}}'\dot{\hat{W}}$ is a scalar constant, maximization of $[-Q(S)]$ is equal to maximizing :

$$S'Z'\dot{\hat{W}} - \tfrac{1}{2}S'Z'ZS \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\text{subject to } C'S \leqslant d \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$\text{where } C = \begin{bmatrix} -1 & & 0 \\ & -1 & \\ 0 & & -1 \end{bmatrix} \qquad d = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

The vector which maximizes $-Q(S)$ without regarding the constraints is $\hat{S}^\circ = (Z'Z)^{-1}\, Z'\dot{\hat{W}}$ as is easily verified by straight forward differentiation. Clearly, if S° satisfies the constraints (*i.e.*, if $C'S^\circ \leqslant d$), then S° is the required solution because a constrained maximum can never exceed unconstrained maximum. The interesting possibility is therefore the one in which S° violates one or more constraints.

Here Theil and Van de Panne[5] suggest that finding the solution vector S amounts to the problem of finding the subset P out of the k constraints (4) such that, when (3) is maximized with the constraints belonging to P binding (*i.e.*, in equality form), a vector S^p results which is feasible ($C'S^p = d$) and optimal (S^p maximizes (3) subject to (4); hence $S^p = S$). Quite generally, we define S^p as the vector maximizing (3) with all constraints of P binding, where P is any subset of the K constraints. Such a vector exists, provided the constraints belonging to P are not inconsistent when written in equational form. Sets P for which S^p does not exist can be disregarded.

As this particular P is not known in general, the main task will be to find it. Even so, it is important to observe at this stage that one can easily derive for any P the vector S^p which maximizes $-Q(S)$ subject to P in equational form. Let us therefore arrange the K constraints such that those of P are the first ones and let us denote T the set of constraints not in P. Then the coefficient matrices of (4) can be partitioned according to :

$$C = (C_p \quad C_T) \qquad d = \begin{bmatrix} d_p \\ d_T \end{bmatrix} \qquad \dots\dots\dots\dots\dots \quad (5)$$

Consider the well-known langrangian expression

$$S'Z'\hat{\dot{W}} - \tfrac{1}{2}S'Z'ZS - \lambda_p' (C_p' \; S - d_p) \qquad \dots\dots\dots\dots\dots \quad (6)$$

where λ_p is a vector of langrange multipliers and C_p' $S—d_p = 0$ are the constraints in P written in equational form. Differentiating (6) with respect to S we obtain

$$S^p = (Z'Z^{-1} \; Z'\hat{\dot{W}} - (Z'Z)^{-1} \; C_p \; \lambda_p \qquad \dots\dots\dots\dots\dots \quad (7)$$

Premultiplying (7) by C_p' gives

$$C_p' \; S^p = C_p' \; S^\circ - C_p' \; (Z'Z)^{-1} \; C_p \; \lambda_p$$

because C_p' $S^p = d_p$, therefore,

$$C_p' \; (Z'Z)^{-1} \; C_p \; \lambda_p = C_p' \; S^\circ - d_p \qquad \dots\dots\dots\dots\dots \quad (8)$$

If we now define

$$E = C' \; (Z'Z)^{-1} \; C = \begin{bmatrix} C_p' \; (Z'Z)^{-1} \; C_p & C_p' \; (Z'Z)^{-1} \; C_T \\ C_T' \; (Z'Z)^{-1} \; C_p & C_T' \; (Z'Z)^{-1} \; C_T \end{bmatrix}$$

$$E = \begin{bmatrix} E_p & F' \\ F & E_T \end{bmatrix}$$

$$\text{where } E_p = C_p' \; (Z'Z)^{-1} \; C_p \qquad \dots\dots\dots\dots\dots \quad (9)$$

$$F = C_T' \; (Z'Z)^{-1} \; C_p \qquad E_T = C_T' \; (Z'Z)^{-1} \; C_T$$

$$C = C' \; S^\circ—d = \begin{bmatrix} C_p' \; S^\circ & - & d_p \\ C_T' \; S^\circ & - & d_T \end{bmatrix} = \begin{bmatrix} e_p \\ e_T \end{bmatrix} \qquad \dots\dots\dots\dots\dots \quad (10)$$

where $e_p = C_p' \; S^\circ - d_p$
$e_T = C_T' \; S^\circ - d_T$

From (8)

$$\lambda_p = (C_p' \; (Z'Z)^{-1} \; C_p)^{-1} \quad (C_p' \; S^\circ - d_p)$$

$$\lambda_p = E_p^{-1} \; e_p \qquad \dots\dots\dots\dots\dots \quad (11)$$

Which express λ_p in known quantities; the existence of E_p^{-1} is ensured if the rank of C_p equals the number of constraints in P. Furthermore, by premultiplying (7) by C_T' we find,

$$C'_T\, S^p = C'_T\, S^\circ - C'_T\, (Z'Z)^{-1}\, C_p\, \lambda_p \qquad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

and the right hand side should be $\leqslant d'_T$ in order that S^p satisfies the constraints in T. Applying (9), (10), (11), we find that this condition can be written in simple form :

$$-C'_T\, S^p \equiv F\, E_p^{-1}\, e_p - e_T \geqslant 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

The left hand side of (13) is the only thing that needs to be computed for the relevant subsets P of the constraints.

Find the set P such that S^p is the optimum solution that may be heuristically explained. Now, begin by considering the empty set 0 and the vector S°. This S° is simply the vector that maximizes (3) without regard to any constraint.

$$\hat{S}^\circ = (Z'Z)^{-1}\, Z'\hat{\dot{W}}$$

At this point, there may and usually will be the problem that S° is not feasible. That is

$$C'\, S^\circ = C'\, (Z'Z)^{-1}\, Z'\hat{W} \underline{\nless} d$$

In this case S° violates at least one of the constraints (4).

Now estimate the vector S^P by maximizing —Q(S) subject to the constraints in set P written in equational form. The constraints in the equational form implies that the variances of these random coefficients are zero which in turn leads the random coefficients to the constant coefficients in the model. Therefore, the main task is to find a set P consisting of minimum number of constraints in the equational form. To this end, consider all one-element sets of P consisting of constraints violated by S°. For each set of P obtain the estimate $S^P/1$ of S^P from (13), where the notatin S^P/f indicates that P has f elements (constraints) in the equational form. Now the feasible solution among the vectors $S^P/1$ which maximizes—Q(S) will yield the optimal solution for the variances of random coefficients. If none of the resulting vectors $S^P/1$ is feasible, the vectors $S^P/2$ are considered which are computed by maximizing —Q(S) subject to constraints in set P/2. The set P/2 contains two constraints in the equational form, the first constraint of P/2 is one that was violated by S° and the second one violated by $S^P/1$. For all possible sets P/2, compute the corresponding vectors $S^P/2$ and find the optimal vector. In this case also, if none of the resulting vectors is feasible, the vectors of the types $S^P/3$ are to be considered. Thus one continues and finds a set P/f such that S^P/f is feasible and optimal. The optimal vector S^P/f yields the non-negative variances for (k-f) random coefficients under the restrictions that the variances of the f out of k random coefficients are zero. Hence the optimal vector $\hat{S}$ of the order $k \times 1$ can now easily be obtained by incorporating zeros in f appropriate positions in the optimal vector S^P/f.

This method will provide positive estimated variances of the random coefficients and the knowledge of these estimated variances enables us to obtain the variance-covariance matrix of the disturbance term in the generalised linear regression model (2).

ESTIMATION OF MEANS OF THE RANDOM COEFFICIENTS

In the generalised linear regression model the best linear unbiased estimate of means of the random coefficients are given by

$$\hat{a} = (X'V^{-1} X)^{-1} X'V^{-1} Y \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

whose variance matrix is

$$\Sigma_{\hat{a}\hat{a}} = (X'V^{-1} X)^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

First estimate matrix V say, V_1, based on ordinary least squares (OLS) estimates and replace it in (14) and (15) which will provide us the generalised least squares (GLS) estimates of a vector and their covariance matrix, *i.e.*,

$$\hat{a}^1 = (X'V_1^{-1}X)^{-1} X'V_1^{-1} Y.$$

$$\Sigma_{\hat{a}^1\hat{a}^1} = (X'V_1^{-1} X)^{-1}$$

Now use the iterative procedure and estimate the variance-covariance matrix V_2 based on a^1 which will provide the new GLS estimates a^2. The iterative process is continued until in successive iterations m, (m + 1).

$$\left| a_j^{m+1} - a_j^m \right| < \delta \; ; \; j = 1, 2, \ldots\ldots\ldots\ldots\ldots, k$$

where δ is some small number.

Hence the GLS estimates of means of the random coefficients are given by

$$\hat{a}^m = (X'V_m^{-1} X)^{-1} \; X'V_m^{-1} \; Y$$

and their variance-covariance matrix given by

$$\Sigma_{\hat{a}^m \hat{a}^m} = (X'V_m^{-1} X)^{-1}.$$

EMPIRICAL TESTING OF THE MODEL

The Study Area

The present study was undertaken in an area covered by the Hirakud canal system which is situated in Sambalpur district of Orissa. Sambalpur district falls in the paddy growing belt of the country having high rainfall and other favourable climatic conditions. As long as paddy cultivation was dependent on rainfall there was nothing much that the farmers could do either to change the cropping pattern or to control the water. But with the commissioning of the Hirakud canal system, abundant water supply was available to the farmers so that they were no more a slave to the vagaries of the monsoon. But this created a new set of problems. The *Bahal* lands which were most prized lands due to their location (being low lying, water used to accumulate there and even during the lean monsoon season gave good yields) became waterlogged because of the flow of irrigation system from the upper fields to the lower ones. Though the manures and fertilizers that were washed away from the uplands accumulated in *Bahal* lands due to flood irrigation, they did not have any beneficial effect on the yield in *Bahal* lands because of the further adverse effects of pest and disease resulting from excessive waterlogging. This was mainly because while designing the canal system no attention was given to the drainage problem. The other problem was the non-availability or low availability of water at the tail ends.

The district agricultural staff and the progressive farmers were compelled to devise ways and means to overcome the problems they faced; this resulted in the idea of having field channels. A modest beginning was made in the two villages by organizing the farmers and field channels were laid in these villages.

Classification of Land

The cultivated land of this study region has been conveniently classified into four broad types according to their location with respect to watershed. They are *Att* (uplands), *Mal* (the slopes), *Berna* and *Bahal*, the low lands. The *Mal* lands are terraced in nature to catch surface water. But their water holding capacity is relatively lower than *Berna* or *Bahal* lands. The soil fertility of these different types of land varies according to their location. The *Att* and *Mal* lands are regarded as less fertile than *Berna* and *Bahal* lands. This is because the fine clay particles along with the soluble nutrients are washed out from higher levels and tend to increase the fertility of the lower lands.

Sampling and Collection of Data

The programme of providing the field channels was first introduced in the Attabira block of Sambalpur district. The villages of this block were stratified into two categories, (*i*) villages having field channels for several years (improved villages), and (*ii*) villages that do not have field channels (control villages). Two villages of each type were selected randomly. For the selection of the ultimate unit (the holdings) the size of operational holding of each individual household in each selected village was ascertained and recorded. Twenty per cent of the total farms from each set of villages were randomly selected. The total sample consisted of 60 holdings from the improved villages and 63 from the control villages. The data were collected by personal survey method, interviewing the farmers on questionnaires prepared for the purpose of economic analysis of village irrigation system and related to the agricultural year 1970-71.

Specification of Production Function

The logarithmic linear regression model with random coefficients has been used to express the relationship between the input-output for the paddy crop grown in the villages with and without field channels. The form of the function is :

$$\text{Log } Y_i = \sum_{j=1}^{7} B_{ij} \log X_{ij} + U_i$$

where the subscript (i) refers to observation j to explanatory variable, and

Y = yield in quintals,

X_1 = area in acres,

X_2 = human labour in days,

X_3 = plough units,

X_4 = manures and fertilizers in rupees,

X_5 = plant protection in rupees,

X_6 = dummy variables for *Berna* land,

X_7 = dummy variables for *Bahal* land and

U = disturbance term.

RESULTS AND DISCUSSION

Production equations expressing total yield as a function of total farm inputs for three different types of paddy, namely, local *kharif* and *rabi* paddy and high-yielding *rabi* paddy crop were estimated for the improved and control villages of Sambalpur district. The GLS estimates of the means of random coefficients are presented in Table I. The OLS estimates of the

TABLE I—MEANS OF RANDOM COEFFICIENTS, STANDARD ERRORS AND COEFFICIENT OF MULTIPLE DETERMINATION OF THE IMPROVED AND CONTROL VILLAGES OF SAMBALPUR DISTRICT : 1970-71

Item	No. of observations	Constant ($\log_{10} a$)	Land (acres)	Human labour (days)	Plough unit (days)	Manures and fertilizers (Rs.)	Plant protection (Rs.)	Dummy variables for		$R^2 \times 100$
								Berna land	*Bahal* land	
Improved villages										
Local paddy (*kharif*) ..	.. 106	0.1087 (0.1260)	0.4527*** (0.0886)	0.1927** (0.0807)	0.0828* (0.0492)	0.2581*** (0.0323)	0.0314 (0.0222)	0.0201 (0.0383)	—0.0004 (0.0305)	89.28
Local paddy (*rabi*) ..	.. 61	0.7046 (0.1017)	0.7749*** (0.0743)	—0.0440 (0.0860)	0.0206 (0.0429)	0.2054*** (0.0206)	0.0477*** (0.0137)	—0.0298 (0.0208)	—0.0398 (0.0323)	95.57
High-yielding paddy (*rabi*)	.. 82	0.3611 (0.1435)	0.6981*** (0.0933)	0.0427 (0.0610)	0.0686* (0.0364)	0.3348*** (0.0430)	0.0169* (0.0099)	0.0041 (0.0178)	0.0092 (0.0147)	98.06
Control villages										
Local paddy (*kharif*) ..	.. 125	0.1774 (0.1300)	0.5068*** (0.1012)	0.1862* (0.0974)	0.0431 (0.1050)	0.2111*** (0.0193)	0.0589** (0.0245)	0.0581** (0.0254)	0.0875*** (0.0264)	89.02
Local paddy (*rabi*) ..	.. 80	0.5725 (0.1654)	0.7072*** (0.1130)	—0.2032 (0.2023)	0.1935*** (0.0624)	0.3073*** (0.0452)	0.0196 (0.0131)	0.0322 (0.0353)	—0.0115 (0.0090)	88.63
High-yielding paddy (*rabi*)	.. 75	0.8975 (0.1855)	0.8940*** (0.1543)	—0.1279 (0.1245)	0.0505 (0.1848)	0.2066*** (0.0467)	0.0351 (0.0237)	—0.0600 (0.0539)	0.0065 (0.0249)	85.46

Figures in parentheses denote standard errors of respective means of random coefficients.
* Significant at 10 per cent level.
** Significant at 5 per cent level.
*** Significant at 1 per cent level.

logarithmic linear regression model with constant coefficients and the variances of the random coefficients are given in Appendices 1 and 2 respectively. The GLS estimates of the means of random coefficients were used for the economic analysis in this study because these estimates were more efficient as compared to the OLS estimates of the regression model. It may be easily verified by studying Table II where the gain in efficiency of the parameters of the GLS estimates over the OLS estimates has been presented.

TABLE II—GAIN IN EFFICIENCY OF THE GLS ESTIMATES OVER THE OLS ESTIMATES

Item	Land (acres)	Human labour (days)	Plough unit (days)	Manures and fertilizers (Rs.)	Plant protection (Rs.)	Dummy variables for	
						Berna land	*Bahal* land
Improved villages							
Local paddy (*kharif*) ..	5.39	78.78	10.67	48.88	12.29	15.56	11.34
Local paddy (*rabi*) ..	123.03	32.94	155.26	71.61	227.465	129.46	28.81
High-yielding paddy (*rabi*)	20.53	21.39	50.74	12.19	13.12	34.16	7.24
Control villages							
Local paddy (*kharif*) ..	51.02	91.32	22.12	285.25	27.69	84.74	31.76
Local paddy (*rabi*) ..	40.63	27.69	32.48	107.50	571.89	36.18	41.08
High-yielding paddy (*rabi*)	171.88	208.53	201.85	57.277	161.43	2.01	247.56

The means of random response of land, manure and fertilizers were found to be significant in all the estimated equations (Table I). The coefficient of human labour was found significant only in the production equations for local paddy (*kharif*) crop. The probable reason for the non-significant coefficient of this input may be that the use of human labour on paddy farms was nearly uniform or varied within a very narrow range.[1] The means of random response of plough unit and plant protection measures were found to be significant in three out of six estimated equations. Plough unit was not found significant in the case of local paddy (*rabi*) in the improved villages and local paddy (*kharif*) and high-yielding paddy (*rabi*) in the control villages. The effect of plant protection was not found significant in the case of local paddy (*kharif*) in the improved villages and local and high-yielding paddy (*rabi*) in the control villages. The insignificant effect of plant protection measures on the yield can be attributed to the non-adherence of the recommended doses of plant protection measures. The effect of topography was not found significant except in the solitary case of local paddy (*kharif*) in the control villages.

1. Paddy cultivation starts generally with the onset of monsoon or opening of the canals and in order to get a better yield, operations have to be completed within a short period. This has resulted in uniform operations, irrespective of farm size, throughout a particular region. Thus there is very little variation in the use of human labour in paddy growing areas.

An examination of the coefficients of multiple determination (R^2) of production equations for all the paddy crops indicated that land, human labour, plough unit, manures and fertilizers, plant protection use and topography explained together about 85 to 95 per cent of the variation in yield in both the sets of villages *with* and *without* field channels. The multiple correlation coefficients for all the crops were found statistically significant at one per cent probability level, indicating that the form of the production equation gives a good fit.

In the logarithmic linear equation, the means of random regression coefficients of the inputs yield directly the corresponding production elasticities for the inputs concerned. The elasticities of output with respect to inputs of land, human labour, plough unit, manures and fertilizers and plant protection for each irrigation system were positive except for human labour input in three equations. The negative elasticity of output with respect to human labour for local paddy (*rabi*) crop in the improved villages, local and high-yielding paddy (*rabi*) in the control villages, indicates that there is probably an excessive use of this input which, if further increased, would result in a loss in the crop yield. But these negative elasticities were not found statistically significant. The elasticities of production of land input varied from 0.452 to 0.894, being lowest for local paddy (*kharif*) in the improved villages and highest for high-yielding paddy (*rabi*) in the control villages. The elasticity of production of human labour varied from —0.230 to 0.192, that of plough unit from 0.020 to 0.193, that of manures and fertilizers from 0.205 to 0.334 and that of plant protection varied from 0.016 to 0.058.

An examination of elasticities of production of different inputs with reference to both the improved and control villages producing all types of paddy crops indicated that the coefficients of partial elasticity of production of individual inputs were less than unity, implying decreasing marginal productivity of factor inputs.

In both sets of villages *with* and *without* field channels, the elasticities of production were highest for land, followed by manures and fertilizers. These partial elasticities indicate that the output response to a given increase in land input is greater than from a similar increase in the use of other farm inputs, given the level of use of all other inputs. The elasticity of production with respect to land was greater in the control villages compared to the improved villages for the local paddy (*kharif*) and high-yielding paddy (*rabi*) crops whereas the production elasticity with respect to manures and fertilizer input was greater in the improved villages compared to the control villages. This indicates that there is relatively greater scope in the improved villages for intensive cultivation of land through increased application of manures and fertilizers compared to the control villages. This may be due to better utilization of fertilizers in the improved villages as a result of field channels.

APPENDIX 1

OLS Estimates of Regression Coefficients, Standard Errors and Coefficients of Multiple Determination on Farms in the Improved and Control Villages of Sambalpur District: 1970-71

Item	No. of observations	Constant ($\log_{10} a$)	Land (acres)	Human labour (days)	Plough unit (days)	Manures and fertilizers (Rs.)	Plant protection (Rs.)	Dummy variables for *Berna* land	Dummy variables for *Bahal* land	$R^2 \times 100$
Improved villages										
Local paddy (*kharif*) ..	106	0.2005 (0.1351)	0.4023*** (0.0909)	0.2106* (0.1079)	0.0299 (0.0518)	0.2607*** (0.0394)	0.0313 (0.0235)	0.0192 (0.0412)	0.0029 (0.0321)	89.39
Local paddy (*rabi*) ..	61	0.8147 (0.1394)	0.9000*** (0.1115)	—0.0436 (0.0991)	—0.0557 (0.0686)	0.1943*** (0.0270)	0.0394* (0.0249)	0.0163 (0.0315)	—0.0006 (0.0355)	95.92
High-yielding paddy (*rabi*)	82	0.2812 (0.1588)	0.6416*** (0.1025)	0.0411 (0.0672)	0.0427 (0.0447)	0.3805*** (0.0456)	0.0111 (0.0106)	0.0030 (0.0206)	0.0085 (0.0153)	98.09
Control villages										
Local paddy (*kharif*) ..	125	0.1177 (0.1874)	0.5587*** (0.1315)	0.1762 (0.1348)	0.0391 (0.1161)	0.1802*** (0.0379)	0.0649** (0.0276)	0.0567* (0.0345)	0.0961*** (0.0303)	89.02
Local paddy (*rabi*) ..	80	0.6447 (0.2252)	0.7671*** (0.1341)	—0.3712* (0.2396)	0.2236*** (0.0718)	0.3861*** (0.0651)	0.0209 (0.0340)	0.0412 (0.0412)	—0.0107 (0.0107)	88.99
High-yielding paddy (*rabi*)	75	1.2660 (0.3357)	1.0401*** (0.2544)	—0.2333 (0.2187)	0.0203 (0.1474)	0.0987* (0.0585)	0.0203 (0.0383)	—0.0565 (0.0545)	0.0216 (0.0455)	86.74

Figures in parentheses denote standard errors of respective coefficients.
* Significant at 10 per cent level.
** Significant at 5 per cent level.
*** Significant at 1 per cent level.

APPENDIX 2

VARIANCES OF THE RANDOM REGRESSION COEFFICIENTS OF THE PRODUCTION FUNCTION OF PADDY CROP IN THE IMPROVED AND CONTROL VILLAGES OF SAMBALPUR DISTRICT : 1970-71

Item	Constant	Land	Human labour	Plough unit	Manures and fertilizers	Plant protection	Dummy variables for	
							Berna land	*Bahal* land
	$\sigma_{\frac{1}{11}}$	σ_{22}	σ_{33}	σ_{44}	σ_{55}	σ_{66}	σ_{77}	σ_{88}
Improved villages								
Local paddy (*kharif*)	0.075989	0.046291	0.000000	0.014568	0.001207	0.001327	0.000000	0.000000
Local paddy (*rabi*)	0.013994	0.008745	0.000000	0.003857	0.001065	0.001478	0.000000	0.000129
High-yielding paddy (*rabi*)	0.002394	0.000746	0.000000	0.000000	0.001361	0.007989	0.000226	0.000001
Control villages								
Local paddy (*kharif*)	0.099037	0.370301	0.000013	0.060168	0.008880	0.004857	0.000000	0.000310
Local paddy (*rabi*)	0.042478	0.019210	0.000014	0.002000	0.000000	0.004384	0.000109	0.000232
High-yielding paddy (*rabi*)	0.008218	0.013866	0.000000	0.083890	0.018964	0.004078	0.072114	0.003256

SUMMARY AND CONCLUSIONS

In this paper an attempt has been made to provide an efficient and appropriate estimation procedure which can be applied in the field of agriculture. With this in view, a logarithmic linear regression model with random coefficients instead of fixed coefficients was used to express the relationship between the explanatory and dependent variables. Theoretically, it is not possible to estimate all the parameters of random coefficients. Therefore, the means of the random coefficients were estimated by the Generalised Least Squares (GLS) method. For estimating the variance-covariance matrix of the disturbance term, Nagar-Tiwari [3] estimates of the variances of random coefficients have been used. Sometimes this procedure provides negative estimates of the variances of random coefficients. To overcome this problem, quadratic programming technique (minimizing a quadratic function of the parameters subject to linear inequalities) has been adopted. This model has been used for estimating the production equations for paddy crops.

The empirical evidences reveal that the GLS estimates of the means of random coefficients are more efficient than the OLS estimates of the regression model with fixed coefficients. So, it is suggestive that in situations where information on only a few of the important variables are available, the regression model with random coefficients may be used to get better estimates of the regression coefficients.

The examination of production equations for paddy crops indicates that more than 85 per cent of the variation in yield is explained by the explanatory variables, namely, land, human labour, plough unit, manures and fertilizers, plant protection measures and dummy variables for *Berna* and *Bahal* land included in the production equations. The elasticities of production are highest for land followed by manures and fertilizers in both sets of villages. The elasticities of production with respect to manures and fertilizers indicate that the improved irrigation system has increased the scope of utilization of manures and fertilizers. Thus, it is extremely important that more liberal credit facilities should be provided to the farmers who have adopted the improved system of irrigation so that they can increase the investment on manures and fertilizers and maximize the benefit from field channels.

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EFFECTS OF TENANCY ABOLITION ON THE HOLDINGS OF FARMERS : A CASE STUDY IN WESTERN MAHARASHTRA*

INTRODUCTION

The Amendment ("Land to the Tiller") to the Bombay Tenancy and Agricultural Lands Act made in 1955 and its enforcement from 1st August, 1956 and subsequently its implementation from 1st April, 1957, made a lot of changes in the sphere of tenancy cultivation. The main objective of this Amendment was to abolish tenancy cultivation and to bring about owner cultivation. On and with effect from 1st April, 1957, every tenant, whether permanent, protected or otherwise, was deemed to have purchased from the landlord, the land held by him as a tenant, subject to the condition that he cultivated the land personally and his total holding did not exceed the ceiling limit (Section 32 of the Amendment). The Tillers' Day appointed to be on 1st April, 1957, formed the watershed between the pre-and post-Amendment situation. The owners were given the option to reclaim the land for self-cultivation prior to the Tillers' Day and the tenants were also given the option to voluntarily surrender the land to the owners. After a long lapse of a period of twelve years of the implementation of this Amendment, *i.e.*, in the year 1969-70, it was felt essential to assess the effects of this latest agricultural reform on the holding structure of the farmers involved in tenancy cultivation.

THE CASE STUDY

The Village and the Extent of Tenancy

With the above objective in view, an attempt was made to study a single village from Western Maharashtra in its entirety and to get all relevant details regarding tenancy cases for that village. In view of this, one village, namely, Kalas from Indapur block of Poona district was selected for the study. There were 222 tenancy cases as on the Tillers' Day involving 124 tenants and 168 landlords in the village of 500 *Khatedars* in the year 1956-57. As there were

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