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ESTIMATING SINGLE NUTRIENT YIELD RESPONSE SURFACES FOR NEW VARIETIES WITH LIMITED DATA

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INTRODUCTION

The relationship between crop yields, irrigation water, fertilizers and new varieties is a crucial part of the green revolution and of agricultural development generally. Quantitative estimates of this relationship are useful in planning at the farm and regional level and for use in simulation models designed for projecting economic activity and policy analysis. In this paper we report the results of a statistical analysis of crop yield response to fertilizer for traditional varieties under irrigated conditions.[1] Further, we develop a method for estimating single nutrient response functions for new varieties when data are available for traditional varieties and very limited data are available for new varieties.

At the time of this study, appropriate experimental data existed only for traditional varieties grown under irrigated conditions. This is often the case where extensive experimental data on new high-yielding varieties are not available. At best a few controlled experiments on new varieties have been undertaken at experimental stations or universities. On the basis of these, a package of recommendations for nutrient use on new varieties is developed. However, the need to know a wider range of nutrient response than available in the point estimates of the recommended package often arises. In this case, response functions from experimental data for traditional varieties can be combined with the point estimates available from recommended packages for nutrient use on new varieties to develop response function estimates for new varieties. The purpose of this paper is to show how and under what restrictive conditions this is possible.

Using experimental data for some major field crops in the Punjab, response functions are first estimated for traditional varieties using conventional methodology. These results are reported in section II. The response effects for new high-yielding varieties, had to be derived from these "objective relations" using some specific economic assumptions and data fragments. Our methodology, which we report in section III together with the empirical estimates, is novel and may be useful to others who are forced to piece together the best estimates they can when complete data are unavailable.

In the concluding section of the paper, we suggest adjustment of the estimated yield response functions to allow for average weather conditions.

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1. This paper includes material originally reported in Inderjit Singh (1972, pp. 112-141, 357-397).

II

TRADITIONAL CROPS UNDER IRRIGATION

In a given agronomic setting, the yield of a crop using standard irrigation practices may be regarded as a function of the amount of nutrients added. Let the yield per acre be Y and the amount of nitrogen, phosphorus and potash be N, P, and K, respectively measured in kilograms (kgs.) per acre. Then we may write

(1) $Y = f(N, P, K)$

A functional form widely used to approximate this relation is the quadratic function

(2) $Y = a_0 + a_{n_1} N + a_{n_2} N^2 + a_{p_1} P + a_{p_2} P^2 + a_{k_1} K$
$+ a_{k_2} K^2 + a_{np} NP + a_{nk} NK + a_{pk} PK + a_{npk} NPK.$

The first term, a_0, represents all unaccounted for yield producing factors. The next six terms represent the independent effects of N, P, and K while the last four terms represent the interaction effects.[2]

In most cases, a soil is most deficient in one or the other of these three nutrients. If we fix all but this one, we arrive at a single input relation shown in Figure 1. This curve assumes diminishing return to a single nutrient
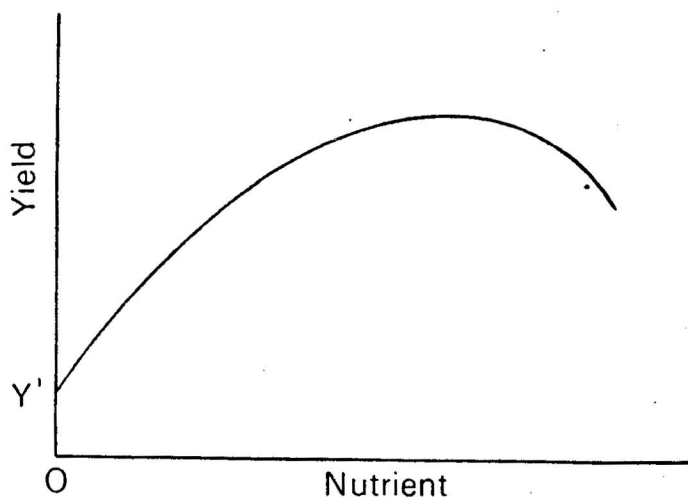

Figure 1 - Yield Response for a Single Nutrient

2. Basic material on yield response and functional form is in Heady and Dillon (1964), OECD (1966) and Tisdale and Nelson (1966). See also Brown, *et. al.* (1957), Baum, *et. al.* (1957), Heady, *et. al.* (1955) and Heady (1957).

which, in terms of equation (2) means that the coefficients of the squared terms are negative ($^{\alpha}n_2$, $^{\alpha}p_2$, $^{\alpha}k_2$, < 0). The effect of changing the application of the other nutrients, is to shift this curve. If the interaction terms are unimportant, then the curves for the given nutrient would merely shift upward. When the interaction terms are important, then the curves change shape as well, with both the slope and biological maximum changing.

Interaction terms are frequently found to be relatively unimportant and can be safely ignored. This possibility was explored by estimating equation (2) and comparing it with a second estimate of equation (2) assuming that the four interaction terms could be left out. It was found that the interaction effects could indeed be safely ignored in most cases.

Experiments were carried out in 1964-65 at various Punjab Agricultural University Research Stations and on a number of cultivators' fields at different locations in different districts throughout the State under the direct supervision of the personnel from the Department of Soils. These carefully designed experiments included several levels, depending on the crop, of nitrogen, phosphorus, and potash.

It was decided to limit our use of this data to the field trials.[3] These presumably came closer than the research station experiments to contemporary operating conditions of interest to us.[4] Twenty-four observations were available for each of the two districts. Functions were fitted by least squares to each district data set with and without the interaction terms. The results are shown in Table I. As can be seen, interdependence terms were insignificant (at the 5 per cent level) except in the case of rice.[5] Since the last (NPK)

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3. The data used here were compiled by Shri Tilak Raj of Punjab Agricultural University and were made available through the courtesy of Professor S. S. Johl, Chairman, Department of Economics and Rural Sociology. They were originally reported in the 1965-66 Annual Report of the Department of Soils, Punjab Agricultural University, Hissar.

4. Even here, however, an upward bias over average yields might be expected. It is likely, for example, that farmers who co-operated in such experiments possessed greater managerial abilities and had more frequent contacts with the extension personnel. Aggregate regional analysis using their data would therefore tend to over-estimate production.

5. To test the hypotheses that the interaction terms are insignificant an F test was used. The statistic in this case is:

$$F^* = \frac{RSS_{NI}^2 - RSS_I^2}{(N-k_2) - (N-k_1)} \quad \frac{RSS_I^2}{(N-k_1)}$$

where RSS_{NI}^2 = the residual sum of squares from the equation with no interaction (equation II);

RSS_I^2 = the residual sum of squares from the equation with interaction (equation I); N = the number of observations in equations; k_1 = the number of independent variables in equation I; and k_2 = the number of independent variables in equation II.

For all estimated equations, there are 24 observations, and 13 degrees of freedom for equation I and 17 degrees for equation II.

With regard to the "t" statistic for the test of the significance, ** indicates a 5 per cent level of significance, and + a 1 per cent level of significance with the appropriate degrees of freedom for the equation under consideration. The "t" statistic is given in parentheses under each coefficient.

F^* statistic testing the significance of interdependence has to be greater than 3.18 to reject the null hypothesis that there is no interdependence among the nutrient inputs. This is the value of the F distribution at a 5 per cent level of significance with 4 and 13 degrees of freedom.

TABLE I—COEFFICIENTS OF QUADRATIC YIELD-FERTILIZER RESPONSE FUNCTIONS

District	Constant	N	P	K	N^2	P^2	K^2	NP	NK	PK	NPK	R^2	F*
WHEAT													
Ludhiana ..	18.85	.161	.1157	.0584	—.0008	—.0008	—.0002	0.0001	—.0001	—.0025	.0000	.9712	
	(35.13)+	(6.07)+	(3.61)+	(1.51)	(2.12)*	(1.91)*	(.3048)	(.1955)	(.4187)	(1.18)	(1.62)		
Ludhiana ..	18.17	.1659	.1325	.0776	—.0009	—.001	—.0005					·9314	0.47
	(47.34)+	(7.06)+	(5.08)+	(2.67)**	(2.72)**	(2.76)**	(1.11)						
Patiala ..	17.45	.2087	—.0152	—.2273	—.0017	.0005	.0034	0.0012	.001	.0144	—.0002	.8766	
	(12.47)+	(3.02)+	(.1946)	(2.26)**	(1.72)	(.4138)	(2.45)**	(1.4)	(1.53)	(2.59)**	(2.79)**		
Patiala ..	15.98	.2689	.0928	—.0852	—.002	—.005	.0018					.7894	2.29
	(12.67)+	(3.59)+	(1.12)	(.9197)	(1.94)+	(4131)	(1.39)						
AMERICAN COTTON													
Sangrur ..	9.44	.0543	.0214	—.0046	—.0002	.0002	.0003	—.0001	.0001	.0018	—.0000	.9542	2.15
	(33.02)+	(6.63)+	(1.24)	(.294)	(3.21)+	(.7822)	(.9132)	(1.23)	(1.7798)	(1.55)	(1.68)		
Sangrur ..	9.51	.0574	.0294	.0041	.0003	—.0001	.0002					.9238	
	(37.41)+	(6.56)+	(1.74)*	(.2173)	(3.76)+	(.2699)	(.719)						
Patiala ..	12.97	.0692	.0292	—.0105	—.0005	.0004	.0005	—.0003	—.0001	—.0022	.0000	.6699	
	(17.07)+	(3.18)+	(.6389)	(.1906)	(2.53)**	(.5626)	(.6757)	(1.01)	(.1868)	(.7117)	(.7599)		
Patiala ..	13.36	.0634	.0084	—.0385	—.0005	.0004	.0009					.632	0.359
	(24.19)+	(3.33)+	(.2295)	(.9364)	(2.99)+	(.7343)	(1.42)						
MAIZE													
Ambala ..	11.91	.0867	.0933	—.0713	—.0003	—.001	.0012	.0002	—.0000	—.0003	.0000	.9496	
	(20.85)+	(5.31)+	(2.71)**	(1.72)	(2.41)**	(2.08)**	(2.18)**	(.9506)	(.9149)	(.1438)	(.2234)		
Ambala ..	(11.60)	·0871	.0978	—.0604	—.0003	—.0008	.0011					.9404	0.59
	(27.1)+	(5.91)+	(3.43)+	(1.89)*	(2.27)**	(2.01)*	(2.39)**						
Gurdaspur ..	10.74	.0403	.0227	—.0637	.0000	.0001	.0013	.0002	.0006	.0028	—.0000	.8919	
	(13.97)+	(1.66)	(.4972)	(1.13)	(.0169)	(.1269)	(1.729)	(.6464)	(1.26)	(.9126)	(1.26)		
Gurdaspur ..	10.39	.0611	.0413	—.0269	—.0001	—.0001	.0009					.8635	0.86
	(17.73)+	(3.0)+	(1.06)	(.616)	(.7608)	(.1496)	(1.47)						
Ludhiana ..	22.46	0.388	.1831	.0037	.0004	—.0021	.0001					.8388	0.71
	(18.24)+	(.9157)	(2.23)**	(.0404)	(1.17)	(1.70)*	(.0447)						
Patiala ..	25.87	—.0061	.2677	—.079	.0011	—.0039	—.0003					.5705	0.17
	(8.98)+	(.0614)	(1.39)	(.3686)	(1.32)	(1.43)	(.001)						
RICE													
Ambala ..	10.12	.1023	.0228	.0062	—.0006	.0002	.0004	.0005	.0004	.0066	.001	.9743	
	(22.76)+	(4.66)+	(0.8591)	(.1947)	(1.83)*	(.5103)	(.8954)	(1.97)*	(1.39)	(3.75)+	(3.89)+		
Ambala ..	9.35	.1275	.0745	.0769	—.0007	—.0002	—·0004					.9413	4.138**
	(20.21)+	(4.65)+	(2.45)**	(2.26)**	(1.816)*	(.5215)	(.7183)						
Ambala ..	10.63	.1145	.0505		—.001	—·003		.0006	—.001	—.003		.9065	
	(17.71)+	(3.18)+	(1.20)		(1.89)*		(1·47)	(3.13)+	(.5838)				

(Contd.)

TABLE I—(Concld.)

District	Constant	N	P	K	N²	P²	K²	NP	NK	PK	NPK	R²	F*
RICE													
Gurdaspur ..	15.6018	.1253	.0489	—.0349	—.0006	—.0001	.0005	.0003	.0002	.0007	—.0000	.96	
	(28.48)+	(4.63)+	(1.49)	(.8856)	(1.55)	(.8712)	(.8912)	(.7477)	(.3457)	(.3031)	(.2792)		
Gurdaspur ..	15.298	.1295	.0583	—.0185	—.0005	—.0000	.0003					.9569	0.255
	(38.97)+	(5.56)+	(2.25)**	(.6425)	(1.59)	(.0461)	(.7778)						
Gurdaspur ..	15.43	.1249	.0545		—.0006	—.0001		.0002	.0001	.0000		.9575	
	(38.57)+	(5.20)+	(2.06)*		(1.69)	(.205)		(.6301)	(.3693)	(.0955)			
GROUNDNUT													
Ludhiana ..	19.25	.2904	.1103	.1008	—.0042	.0001	—.0004	—.0015	—.0009	.0023	—.0001	.9288	
	(29.92)+	(3.74)+	(2.46)**	(1.86)*	(1.67)	(.1467)	(.5223)	(1.388)	(.7622)	(.6499)			
Ludhiana ..	19.93	.2897	.1072	.0792	—.0061	—.0004	—.0004					.8847	2.02
	(33.94)+	(3.53)+	(2.48)**	(1.64)	(2.29)**	(.5607)	(.5422)						
Patiala ..	9.8	.1291	.1168	.0549	.002	—.0013	—.0009	—.0007	—.0002	—.0004	.0000	.9168	
	(17.13)+	(1.86)*	(3.03)+	(1.18)	(.9066)	(2.09)*	(1.22)	(.7211)	(.1527)	(.3105)	(.2689)		
Patiala ..	9.89	.1216	.1183	.0567	.0014	—.0015	—.0009					.912	0.186
	(24.27)+	(2.14)**	(3.95)+	(1.69)	(.7874)	(2.93)+	(1.58)						
BAJRA													
Sangrur ..	17.49	.3988	.1255	—.108	—.0048	.0003	.0015	.0015	.0005	—.0016	.0000	.7916	
	(9.67)+	(4.46)+	(1.16)	(.8293)	(3.75)+	(.1705)	(.8934)	(1.33)	(.4226)	(.2185)	(.4351)		
Sangrur ..	17.43	.382	.1076	—.1015	—.0048	.0000	.0021					.6964	1.48
	(11.57)+	(4.28)+	(1.09)	(.9173)	(3.84)+	(.0315)	(1.267)						
Rohtak ..	7.31	.0557	.0286	—.0039	—.0006	—.0002	.0001	.0001	.0002	.0004	—.0000	.8459	
	(20.44)+	(3.15)+	(1.39)	(.1497)	(2.22)**	(.6119)	(.1767)	(.6595)	(.9712)	(2.767)	(.3463)		
Rohtak ..	7.11	.0604	.034	.0061	—.0005	—0002	.0000					.8332	0.26
	(27.69)+	(3.97)+	(2.01)*	(.3252)	(2.43)**	(.7448)	(.1162)						
SUGARCANE													
Ambala ..	329.8	1.4586	1.1877	.2807	—.004	—.0035	—.0022	.0026	.0002	—.0136	.0001	.8929	
	(14.0)+	(2.66)**	(1.68)	(.3289)	(.9333)	(.6918)	(.3772)	(.7148)	(.0595)	(.5715)	(.6062)		
Ambala ..	322.9	1.4157	1.1179	.2544	—.0024	—.0013	—.0018					.8862	0.37
	(18.69)+	(2.72)**	(1.94)*	(.3953)	(.6455)	(.3072)	(.3731)						
Ludhiana ..	602.3	2.4967	.715	—.5942	—.0117	.0034	—.0073	—.0002	.0019	.0001	—.0000	.8957	
	(23.8)+	(3.92)+	(.9425)	(.6481)	(2.55)**	(.6232)	(1.1366)	(.0509)	(.4459)	(.0054)	(.0434)		
Ludhiana ..	600.4	2.5439	.7167	—.5584	—.0017	.0031	.0078					.8934	0.071
	(33.78)+	(4.76)+	(1.21)	(.8436)	(3.080)+	(.7237)	(1.59)						
Patiala ..	417.2	1.9776	3.1931	.2723	—.0045	—.0218	—.0016	.0008	—.0031	—.0362	.001	.7125	
	(10.3)+	(1.94)*	(2.63)	(.1856)	(.6229)	(2.5236)	(.1525)	(.1310)	(.467)	(.8873)	(.8944)		
Patiala ..	428.4	1.7325	2.7402	—.4128	—.003	—.0183	.0013					.6865	0.294
	(14.58)+	(1.96)*	(2.8)**	(.3772)	(.4741)	(2.578)**	(.1544)						

coefficient for rice in equation (2) was very small and significantly different from zero only for one of the two districts for which data were available it was dropped and a third equation estimated with the three other interaction terms present. This function was used in the further analysis. In most cases, the amount of variation explained by the quadratic function was very high. This can be seen in the R^2 column of Table I.

There are several ways of proceeding at this point and the choice depends upon the unit of analysis. All, however, proceed from a knowledge of the coefficients of nutrient response for traditional varieties. If the unit of analysis is the district, the coefficients from Table I can be used directly. However, more often the unit of analysis is a group of districts or the State as a whole. In this case, estimates for yield response for the State as a whole should be obtained.

One way to proceed in these cases is to *pool* the data for the various districts and fit the nutrient response functions to the pooled data. An alternative is to take a weighted average of the estimated coefficients obtained for each district separately. The weights in this case should reflect the proportion of total area sown to the crop in each district. In fact, the pooling of district data also requires the use of weighted least squares but in this case the coefficients are "averaged" over the districts without reflecting the relative importance of each district for the crop under study.[6]

The choice of method depends upon the purpose for which the nutrient response functions are required. If they are to be used to estimate the average total response of a specific crop to nutrient use for the State, given a constant inter-district cropping pattern, then a weighted average of district coefficients should be used. If, however, the purpose is to use the nutrient response coefficients in simulation or programming models, especially where projections under alternative cropping patterns are desired then pooled estimates are better.

Since our concern here is with the *method* of obtaining new variety response surfaces from estimates of traditional variety response coefficients, we have proceeded as simply as possible. Lacking data on *all* the districts and being unable to estimate coefficients for more than one or two, we refrained from either pooling or using weighted averages. Instead we have simply averaged all the coefficients with equal weights attached to the districts for which we have fitted functions in Table I for each crop. This gave the figures shown in Table II. The coefficients for P^2 and K^2 are positive in several cases, a result that may be interpreted as meaning that over the range of field trial nutrient levels, increasing returns were observed for these nutrients. Since in practice, only two levels for P and K were used, zero and experimentation

6. We are grateful to the referee for bringing these points to our attention.

station recommended levels, this did not cause any troubles in the further analysis. Again we wish to emphasize that ideally "average" functions for a region should be obtained from data for *each* of the districts in the area, using either pooled or weighted average estimates. The results in Table II are only illustrative, allowing us to proceed with the method and should not be construed as average response functions for the Punjab. While better estimates could not be obtained because of lack of data a comparison of the expected yield at zero level of fertilization with actual average yield in 1964-65 indicated that these estimates are well within the range of experience.

TABLE II—COEFFICIENTS FOR CENTRAL PUNJAB FERTILIZER-YIELD RESPONSE FUNCTIONS FOR LOCAL, IRRIGATED CROPS

Crop	Constant	N	P	K	N^2	P^2	K^2
Wheat	.. 17.4	.2174	.1127	.0038	—.0015	—.0008	.0007
Cotton	.. 11.43	.0604	.0189	—.0172	—.0004	.00015	.0006
Rice[b]	.. 13.03	.1197	.052	a	—.0008	—.0002	a
Groundnut	.. 14.9	.0256	.1126	·0677	—.0024	—.0009	—.0007
Bajra	.. 12.3	.2212	.0708	—.0477	— .0027	—.00015	.0016
Sugarcane	.. 450.5	1.8973	1.5249	—.2389	—.0048	—.0055	.0016

a A "very small" number.

b The average interaction terms for rice are .004 (NP), .005 (NK) and —.0015 (PK).

Setting P and K at zero levels for each crop, we obtained the one dimensional yield-nitrogen functions shown in Figure 2.

III

NEW VARIETIES

Yield-fertilizer response functions for new varieties could be obtained in the manner just described if experimental data were available. However, no reported results for experiments conducted either at agronomic stations or on cultivators' fields were available. The main reason for this deficiency was lack of time to initiate controlled experiments due to the recent development and introduction of the new varieties. This is often the case in many less developed countries where the development and adaptation of new varieties to local conditions are still in its early phase. However, a few field trials had been conducted for new varieties using levels of fertilization recommended by the Directorate of Extension Education, Punjab Agricultural University, Ludhiana. These fertilizer recommendations and the expected yields associated with them by the Directorate are shown in Table III. This table contains similar data for the unimproved, local varieties.
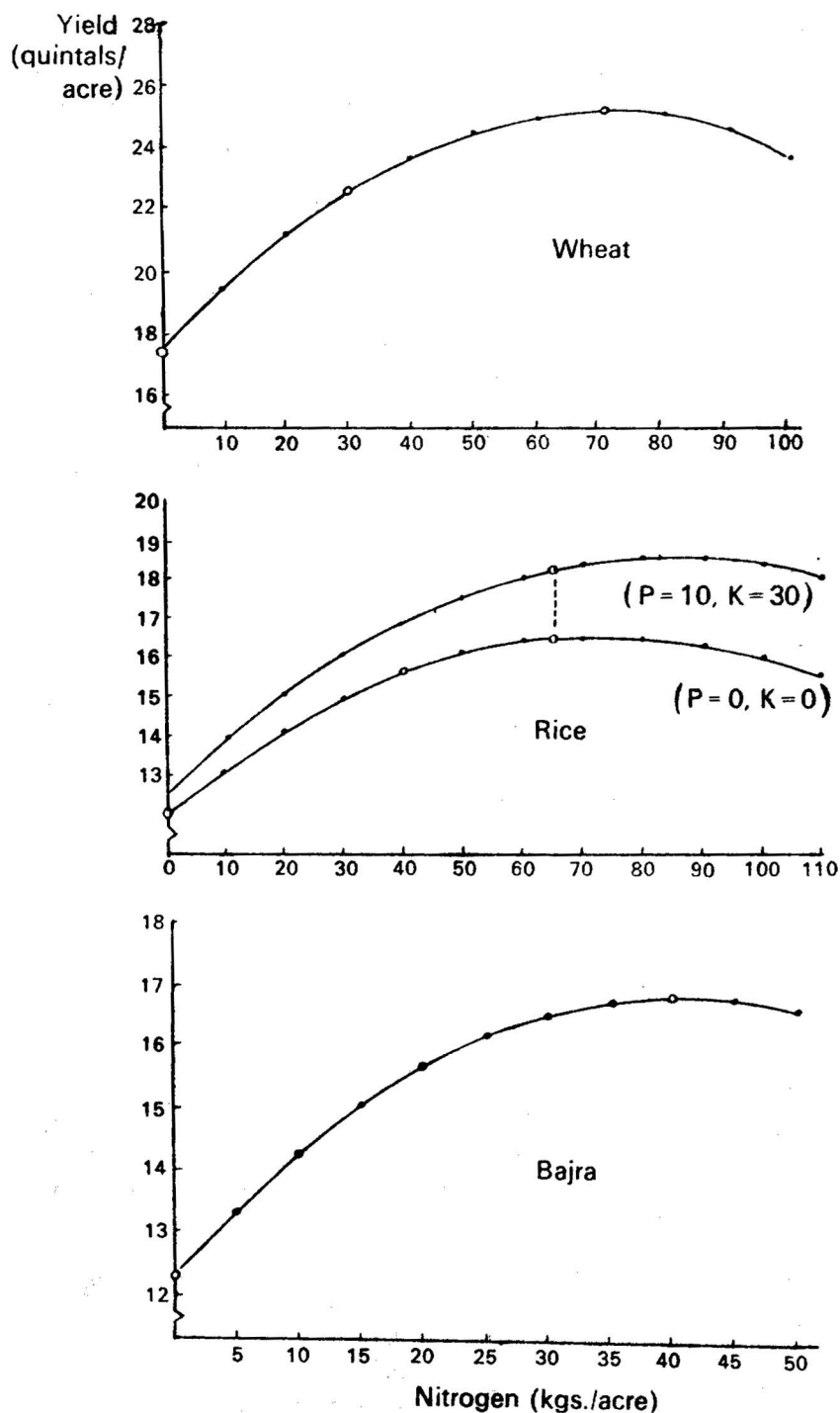
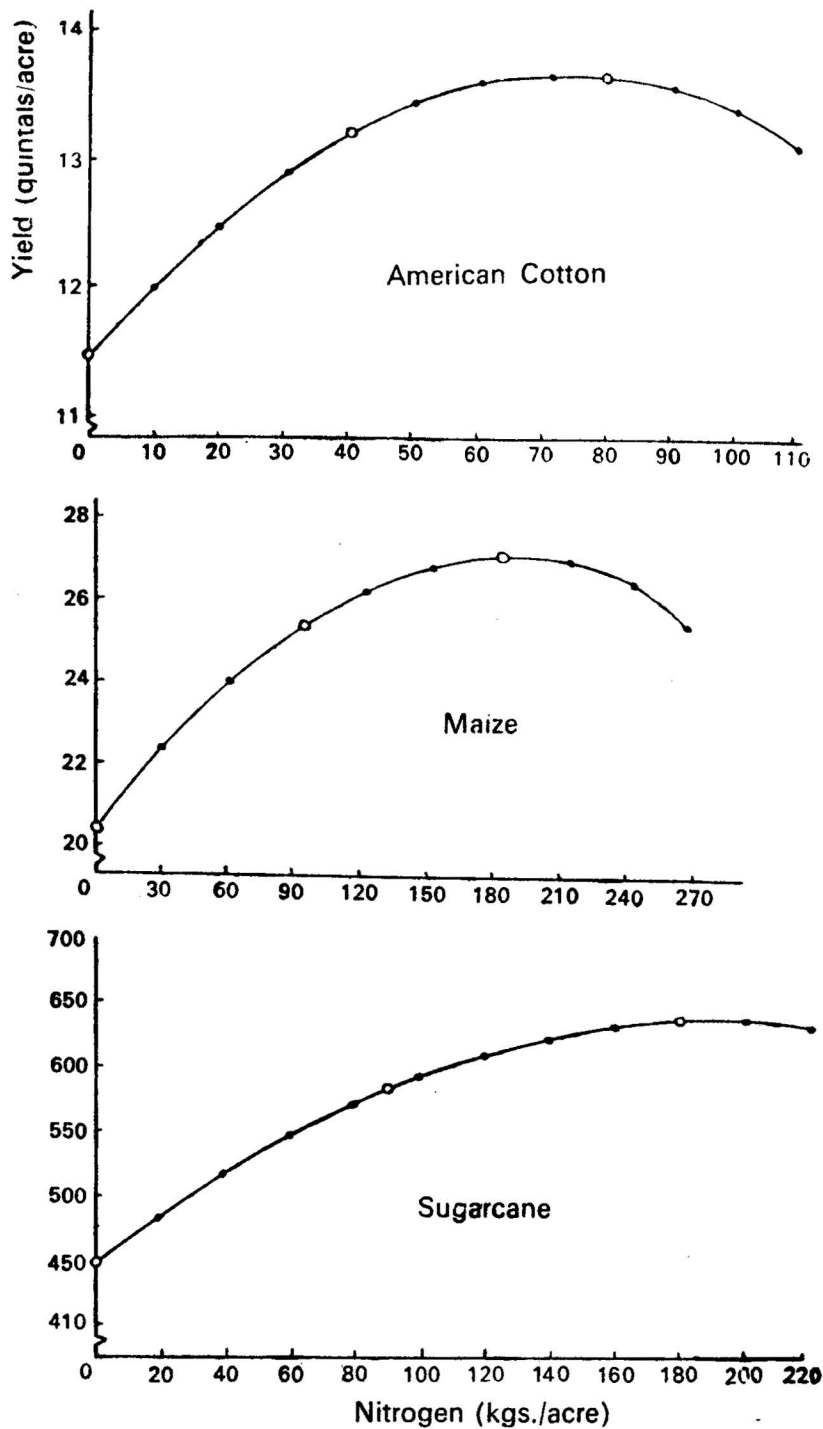Figure 2 - Yield-Fertilizer Response of Irrigated Local Crops (contd)

Figure 2 - Yield-Fertilizer Response of Irrigated Local Crops.

TABLE III—FERTILIZER RECOMMENDATIONS AND EXPECTED YIELDS FOR CENTRAL PUNJAB

Crop	Variety	Recommended fertilization[a]			
		N	P	K	Yield[b]
Wheat (local)	.. C 273	44.5	22.0	26.7	29.7
Wheat (high yield)	.. PV 18	138.4	67.2	51.9	54.5
Maize (local)	.. Local	61.3	15.6	37.1	29.7
Maize (hybrid)	.. Ganga 101	113.9	36.3	44.5	44.5
Rice (local)	.. Jhona	57.3	10.4	29.6	37.1
Rice (high yield)	.. TN 1	74.1	20.8·	58.2	49.4
Bajra (local)	.. Local	49.4	23.7	29.6	24.7
Bajra (hybrid)	.. Hybrid No. 1	123.5	19.8	74.1	44.5

Source : Directorate of Extension Education (1967-68, 1968-69).
a. kg/acre. b. quintals/acre.

For some purpose it might be adequate to use these data directly. For others, it would be quite useful to have functions of the form estimated in section II. Using a few assumptions the data of Table III can be combined with the estimates of Table II to obtain average yield-fertilizer response functions for new varieties at least for a single nutrient at a time. Let us see how this can be done.

Concentrating on a single nutrient—nitrogen—, we adopt the following assumptions :

I. Varietal differences affect only the constant (a_0) and nitrogen response coefficients (a_{n_1}, a_{n_2}). The phosphorus, potash and interaction terms are unaffected by varietal improvement.

II. Recommended nutrient levels as shown in Table III are economic optima for yield response with "average" weather, for N at given levels of P and K.

The first assumption is clearly not true but the relative economic importance of nitrogen justifies special attention while subsuming less important distinctions.[7] This assumption reduces the number of new parameters to be estimated for each new variety to three. The second limiting assumption becomes necessary because we have only one point estimate on the nutrient response surface for new varieties to work with and this estimate is *conditional*

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7. In 1964-65 there were some 95,000 metric tons of N distributed compared to some 4,000 metric tons of phosphorus in Punjab and Haryana. See Statistical Abstract of Punjab, 1965, Economic and Statistical Organisation, Punjab and D. R. Bhumbla, N. S. Randhawa and B. Das (1966).

at recommended levels of P and K. Much more additional information would be required if all ten parameters of the nutrient response functions for new varieties had to be estimated.

We have now in addition to equation (2) a quadratic response equation for new varieties

(3) $\quad Y^* = \beta_0 + \beta n_1\, N + \beta n_2\, N^2 + \alpha p_1\, P + \ldots$

where the remaining terms are the same as in equation (2).

Using field trial data for new varieties when no fertilizers are added, we obtain the estimates given in Table IV[8] for the constant coefficient β_0

TABLE IV—ESTIMATE OF Bo FOR NEW VARIETIES

Wheat	..	..	..	..	17.4
Maize	..	..	..	..	22.4
Rice	..	..	..	..	15.0
Bajra	..	..	..	..	15.0

This leaves the nitrogen coefficients βn_1 and βn_2 for estimation. These can now be obtained from Table III by exploiting assumption II.[9]

The effect of this assumption is to define an equation between the parameters of equation (2) and those of equation (3). Let

(4) $\quad \pi = pY - q_n N - q_p P - q_k K$

be the gross profit per acre for a given crop. Since Y is a function of N, P, and K and holding P and K fixed, we get for the first order condition of a maximum :

(5) $\quad \dfrac{\partial \pi}{\partial N} = p[\alpha n_1 + 2\alpha n_2 N^r + (\alpha_{np} P^r + \alpha_{nk} K^r + \alpha_{npk} P^r K^r)] - q_n = 0$

for traditional varieties and

(6) $\quad \dfrac{\partial \pi}{\partial N_*} = p[\beta n_1 + 2\beta n_2 N_*^r + (\alpha_{np} P_*^r + \alpha_{nk} K_*^r + \alpha_{npk} P_*^r K_*^r)] - q_n = 0$

for new varieties. These equations are illustrated in Figure 3 which shows the points at which the slope of the yield response function, that is, the mar-

8. Raghbir Singh, Punjab Agricultural University assisted in these estimates.
9. According to agronomists at the Punjab Agricultural University, the recommendations are thought to satisfy assumption II.

ginal product of nitrogen, equals the price ratio q_n/p.[10]   In equation (6), we have used the first assumption to set the coefficients associated with potash, phosphorus and interaction terms for new varieties equal to those for traditional varieties.
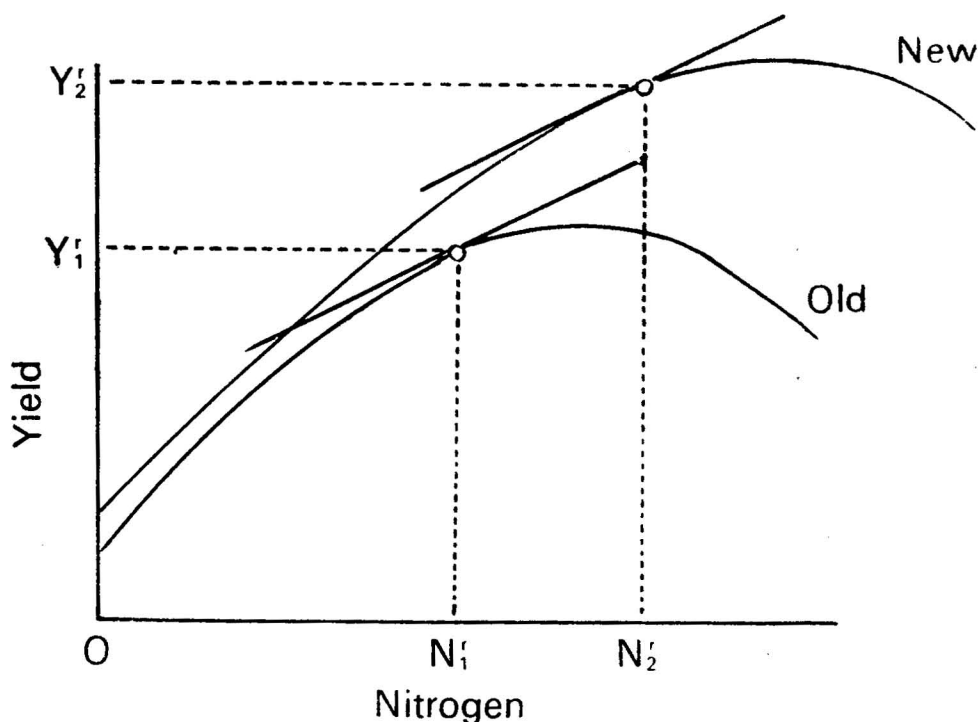

Figure 3 - Economic Optima for New and Old Varieties

Equating (5) and (6) and eliminating p and q_n, we get a single equation in the unknown βn_1 and βn_2.

$$(7) \quad \beta n_1 + 2\beta n_2 N^r_* = {}^\alpha n_1 + 2{}^\alpha n_2 N^r + [{}^\alpha_{np} (P^r - P^r_*) + {}^\alpha_{nk} (K^r - K^r_*) + {}^\alpha_{npk} (P^r K^r - P^r_* K^r_*)] = A_n$$

Table III implies the equation

$$(8) \quad Y^r_* = \beta_0 + \beta n_1 N^r_* + \beta n_2 (N^r_*)^2 + {}^\alpha_{p1} P^r_* + \ldots + {}^\alpha_{npk} N^r_* P^r_* K^r_* = \Upsilon_n + \beta n_1 N^r_* + \beta n_2 (N^r_*)^2$$

from which we obtain

$$(9) \quad \beta n_1 N^r_* + \beta n_2 (N^r_*)^2 = Y^r_* - \Upsilon_n = \beta_*$$

where

$$\Upsilon_n = {}^\alpha_{p1} P^r_* + {}^\alpha_{p2} (P^r_*)^2 + \ldots + {}^\alpha_{npk} N^r_* P^r_* K^r_*$$

10. For economic analysis of this kind, see the references of footnote 2. In addition, see also Seth and Abraham (1965), Baum, Heady and Blackmore (1956) and Heady and Pesek (1960).

In equation (8) Υ_n is a constant which includes the effects of P and K at fixed levels. Furthermore, the fixed levels are recommended levels of P and K, because of the conditional nature of the point estimate available. Although this single nutrient response function has limited uses, as it cannot be used to give responses at levels of P and K other than the fixed levels, it can still be used where responses to the most important single nutrient are desirable.

Solving equations (7) and (9) for the unknown parameters βn_1 and βn_2 we get

(10) $\quad \beta n_1 = (2B_n - A_n N_*^r)/N_*^r$

(11) $\quad \beta n_2 = (A_n N_*^r - B_n)/(N_*^r)^2$.

Because the interaction terms are assumed zero for each crop but rice, equations (10) and (11) are quite simple in these cases. Estimates for β_{1n} and β_{2n} obtained in this way are shown in Table V.

TABLE V—ESTIMATED NITROGEN RESPONSE COEFFICIENTS FOR NEW VARIETIES

Crop			β_{1n}	β_{2n}
Wheat	..	..	0.364077	—.0010122
Maize	..	..	0.307402	—.00096274
Rice	..	..	0.81132[a]	—.00521797
Bajra	..	..	(0.83874) 0.3981	—.0017785

a. Adjusted for interaction effects. The figure in brackets is the figure for P=20.8 and K=58.2.

The yield response functions for new varieties using equation (3) are illustrated in Figure 4 and yield response functions for local varieties from Figure 2 are included for comparison.

We have outlined a method for estimating single nutrient response functions for new varieties from estimated coefficients of response functions for traditional varieties and conditional point estimates for new varieties from recommended fertilizer practices. We have illustrated how to do this in the case of nitrogen, given fixed levels of P and K.

It is possible in the case of some other crops that the partial response to phosphorus or potash is of critical importance. In this case, similar single response surfaces can be estimated by taking the partials $\partial \pi/\partial P$ or $\partial \pi/\partial K$ and following the same procedures. However, similar assumptions to those used for estimating the single nutrient nitrogen response functions have to be made.
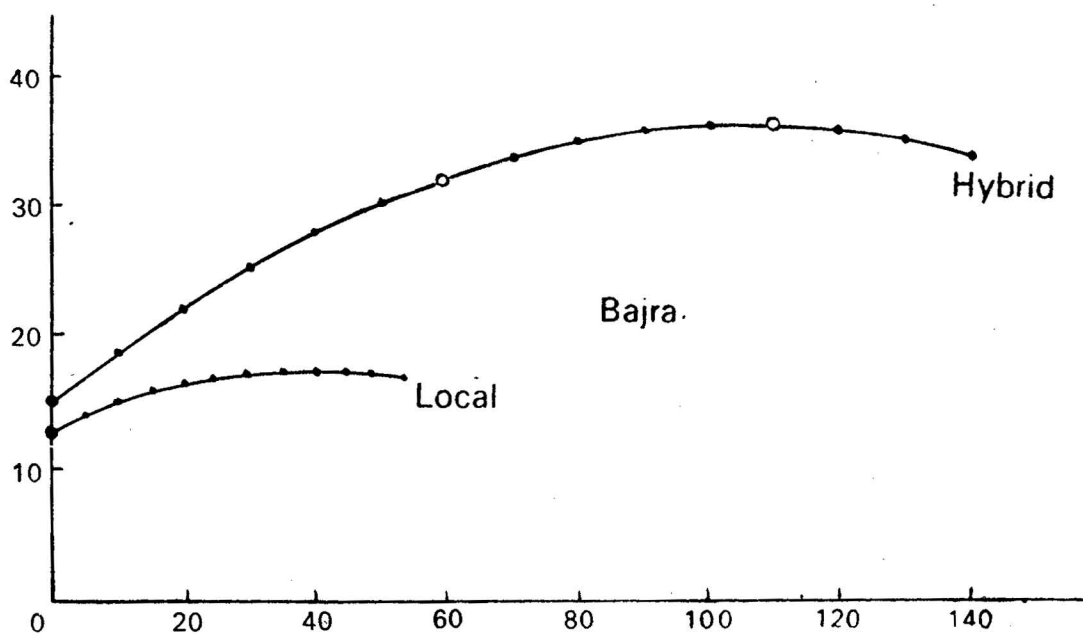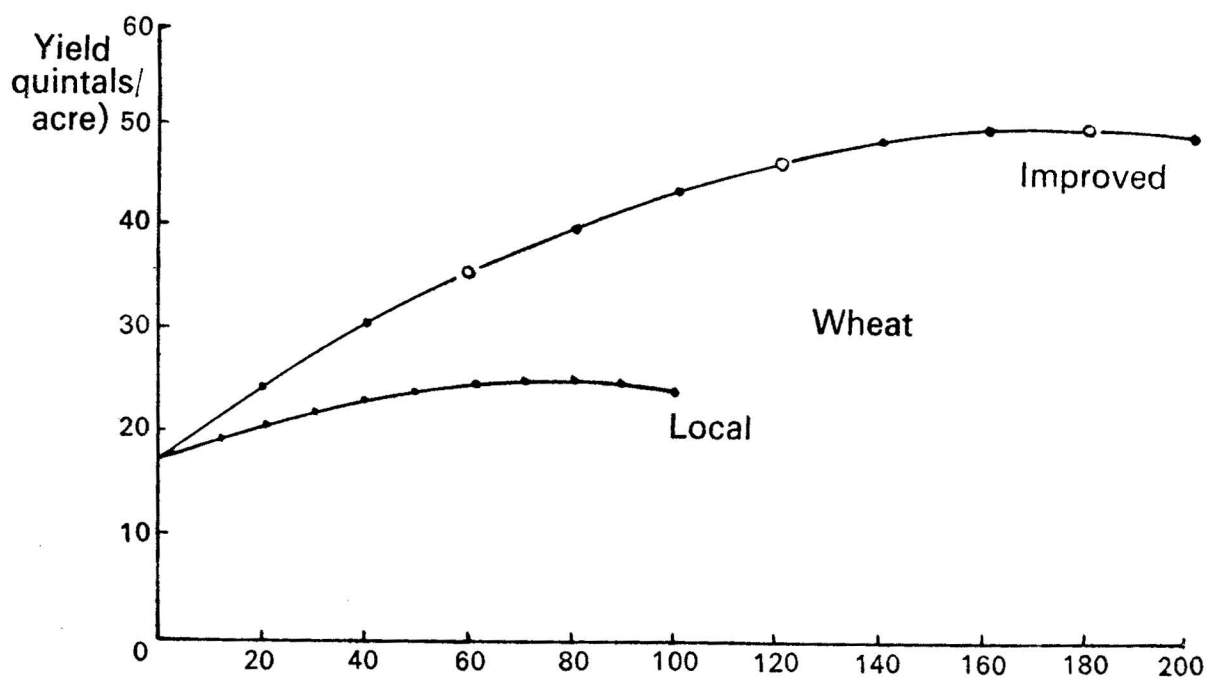
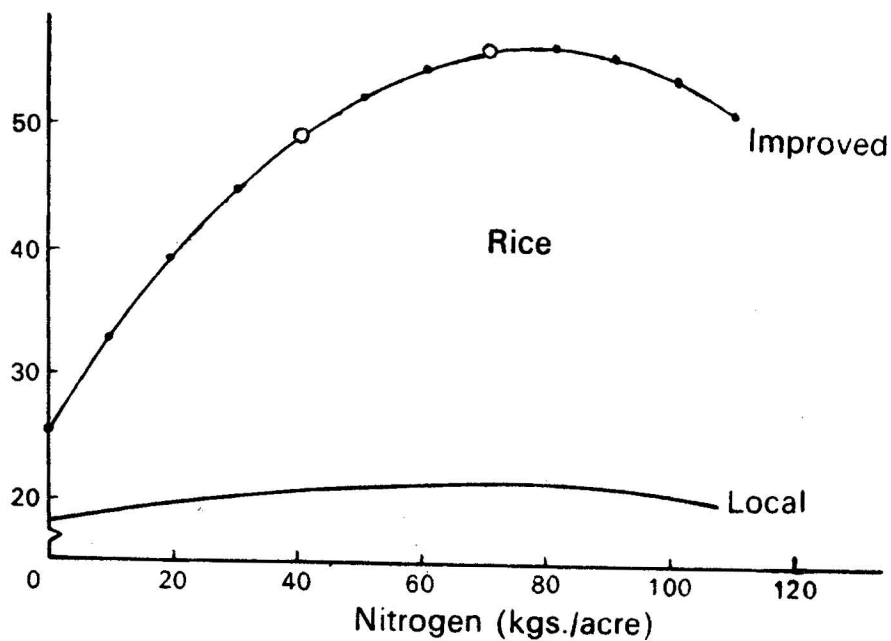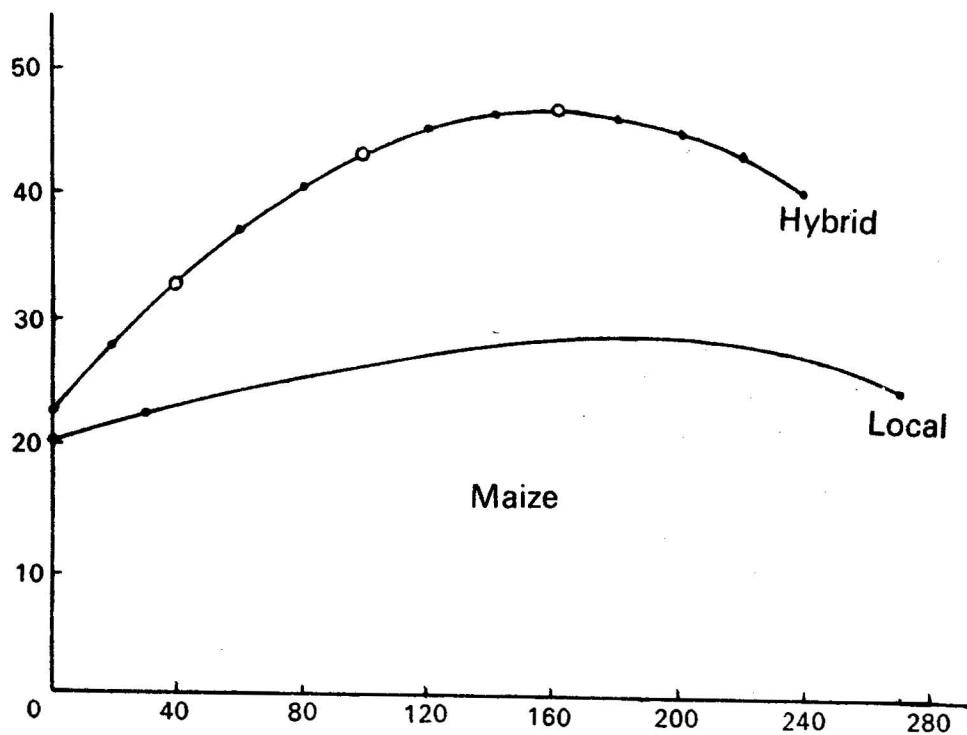Figure 4 - Yield-Response for Irrigated New Varieties (contd.)

Figure 4 - Yield-Response for Irrigated New Varieties

The single nutrient response functions for phosphorus and potash can be written as follows :

$$\text{(8a)} \quad Y_*^r = \beta_0 + \beta p_1 P_*^r + \beta p_2 (P_*^r)^2 + {}^\alpha n_1 N_*^r + {}^\alpha n_2 (N_*^r)^2$$
$$+ {}^\alpha k_1 K_*^r + {}^\alpha k_2 (K_*^r)^2 + {}^\alpha{}_{np} N_*^r P_*^r + \ldots + {}^\alpha{}_{npk} N_*^r P_*^r K_*^r$$
$$= \Upsilon_p + \beta p_1 P_*^r + \beta p_2 (P_*^r)^2 = B_p$$

and

$$\text{(8b)} \quad Y_*^r = \beta_0 + \beta k_1 K_*^r + \beta k_2 (K_*^r)^2 + {}^\alpha n_1 N_*^r + {}^\alpha n_2 (N_*^r)^2$$
$$+ {}^\alpha p_1 P_*^r + {}^\alpha p_2 (P_*^r)^2 + {}^\alpha{}_{np} N_*^r P_*^r + \ldots + {}^\alpha{}_{npk} N_*^r P_*^r K_*^r$$
$$= \Upsilon_k + \beta k_1 K_*^r + \beta k_2 (K_*^r)^2 = B_k$$

Estimates for βp_1, βp_2 and βk_1, βk_2 can be obtained from

$$\text{(9a)} \quad \beta p_1 = (2 B_p - A_p P_*^r)/P_*^r$$

$$\text{(10a)} \quad \beta p_2 = (A_p P_*^r - B_p)/(P_*^r)^2$$

$$\text{(9b)} \quad \beta k_1 = (2 B_k - A_k K_*^r)/K_*^r$$

$$\text{(10b)} \quad \beta k_2 = (A_k K_*^r - B_k)/(K_*^r)^2$$

where B_p and B_k are defined above and

$$A_p = {}^\alpha p_1 + 2 {}^\alpha p_2 P + [{}^\alpha{}_{np} (N - N_*)$$

$$+ {}^\alpha p_k (K - K_*) + {}^\alpha{}_{npk} (NK - N_* K_*)]$$

$$A_k = {}^\alpha k_1 + 2 {}^\alpha k_2 K + [{}^\alpha n_k (N - N_*)$$

$$+ {}^\alpha p_k (P - P_*) + {}^\alpha{}_{npk} (PK - P_* K_*)]$$

As pointed out earlier, additional information would be necessary in order to solve for all the three nutrient response surfaces simultaneously.

The procedure outlined above seems limited in that it allows a solution only in terms of a single nutrient, assuming a response to other nutrients that does not vary with varietal change. It is nonetheless very useful especially in cases where large yield responses are confined mainly to one nutrient. It cannot, of course, replace estimates obtained from field trial data on new varieties, but in the absence of such data it may serve many purposes, especially those related to macro-economic sectoral planning.

IV

ADJUSTMENT FOR AVERAGE WEATHER

All the yield response functions obtained above were derived from field experiments for the cropping year 1964-65, and include implicitly the weather effects peculiar to that year.   Even if the systematic variations in yields due to variety, water use and fertilizer level are unaffected by weather and were to remain constant, yields will still vary from year to year, due to the effects of weather.   In order to account for this, base yields coefficients representing "average weather" were obtained.   These are reported in Table VI.

TABLE VI—BASE YIELD COEFFICIENTS FOR "AVERAGE" WEATHER IN CENTRAL PUNJAB

Activity	Estimated base yield (β_0) (quintals/acre)
Wheat (local) unirrigated	2.82
Wheat (local) irrigated	5.43
Wheat (high yield) irrigated	5.43
Gram (local) unirrigated	4.40
Gram (local) irrigated	5.58
Barley (local) unirrigated	2.75
Cotton (*desi*) irrigated	2.0
Cotton (American) irrigated	3.0
Maize (local) unirrigated	3.09
Maize (local) irrigated	6.32
Maize (high yield) irrigated	7.08
Rice (local) irrigated	4.93
Rice (high yield) irrigated	5.67
Groundnut (local) unirrigated	2.39
Groundnut (local) irrigated	3.19
Bajra (local) unirrigated	1.27
Bajra (local) irrigated	2.54
Bajra (high yield) irrigated	3.10
Sugarcane (local) irrigated	137.8

The method by which these figures were obtained is described elsewhere. It is suggested that these figures replace the constant term β_0 of the estimated functions in Tables I and IV.   The reader will note that according to these "average" figures, 1964-65 must have been an extraordinarily good year, or yield response on the field trial plots was greatly above what one can expect to be attainable in the region as a whole.[11]

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11. See footnote 4 above.

This is a crude but effective way of incorporating weather effects into yield nutrient response functions. More sophisticated methods that allow detailed interactions between such weather factors as humidity, temperature and seasonal rainfall can also be used,[12] but these require detailed and often farm specific weather data over many years. These data are not readily available, limiting the ability to incorporate weather effects fully.

APPENDIX

An alternative method of arriving at estimates for βn_1 and βn_2 is to use the equations :

$$(1) \quad \frac{\partial \pi}{\partial N_*} = q_n/P = \beta n_1 + 2\beta n_2 \, N_* + \alpha_n P_* + \alpha_{kn} K_* + \alpha_{npk} \, P_* \, K_*$$

and

$$(2) \quad Y_* = \Upsilon_n + \beta n_1 \, N_* + \beta n_2 N_*^2$$

to solve for the two unknown βn_1 and βn_2.

This gives us the two equations.

$$(1a) \quad \beta n_1 + 2\beta n_2 N_* = q/P - \left\{ \frac{\alpha_{pn} P_* + \alpha_{kn} K_* + \alpha_{npk} P_* \, K_*}{C_n} \right\}$$
$$= q_n - C_n$$

$$(2a) \quad \beta n_1 + \beta n_2 N_*^2 = Y_* - \Upsilon_n = B_n$$

and solving for the unknowns we get

$$(3) \quad \beta n_1 = (q/P - C + B/N_*)$$

$$(4) \quad \beta n_2 = [q_n/P - A]\frac{1}{N_*} - \left(\frac{B}{N_*^2} \right)$$

as solutions, where

$$(5) \quad \Upsilon_n = \beta_0 + \alpha p_1 P_*^r + \alpha \, p_2 \, (P_*^r)^2 + \alpha k_1 K_*^r + \alpha k_2 \, (K_*^r)^2 + \alpha_{nk} N_*^r \, K_*^r + \alpha_{pk} \, P_*^r \, K_*^r + \alpha_{np} \, N_*^r \, P_*^r + \alpha_{npk} N_*^r \, P_*^r \, K_*^r$$

and is defined as before.

The problems with this method is that it gives a solution that is *not* invariant with respect to nutrient and product prices since it includes the ratio q_n/P. It is in order to avoid this problem and to get solutions that do not depend upon spatially varying nutrient and product prices that the method presented in the text, that eliminates these from the solution, is adopted.

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12. There is an extensive literature available on incorporating weather effects on yields. An extensive survey is available in B. Ovry (1965).

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