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Ann Maharaj

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Ann Maharaj

Department of Econometrics and Business Statistics

Monash University

Melbourne, Australia

ABSTRACT

In this paper we propose a test statistic to compare two or more stationary time series that are not necessarily independent. The test is based on the difference between estimated parameters of the autoregressive models that are fitted to the series.

1. INTRODUCTION

The comparison of time series has applications in various fields including economics, geology, engineering and climatology. Hypothesis tests designed to compare two stationary independent time series involving the use of fitted parameter estimates were considered by De Souza and Thomson (1982) and Maharaj (1996). Other tests in the literature for the comparison of two independent stationary series involve the use of the estimated spectra of the series. Some relevant papers are by Jenkins (1961), Swanepoel and Van Wyk (1986), Coates and Diggle (1986) and Diggle and Fisher (1991). In practice the application of these tests to real time series is limited since comparisons are often made between logically connected series and in some instances, one may wish to make comparisons between more than two series.

We will consider the comparison of two or more stationary time series that are not necessarily independent. We will assume that if the series are not stationary, then the same order of differencing will be needed to make each one stationary. Truncated AR(∞) models of order k, are fitted to each series. and the test statistic is based on the difference between the AR(k) estimates. These estimates are generalised least squares estimates. It will be assumed that the disturbances of the models are correlated for series that are not independent and uncorrelated for series that are independent. In Section 2 we present the test statistic and in Section 3 a simulation study is carried out to investigate the distributional properties, size and power of this test statistic, which has an asymptotic chi-square distribution. In Section 4 we make power comparisons with some of the existing tests for independent series in the literature and in Section 5 we apply the test based on this test statistic to a set of economic time series and to a set of time series in climatology.

2. HYPOTHESIS TESTING PROCEDURE

Let Z_t be a zero mean univariate stochastic process such that $Z_t \in L$ where L, is the class of stationary and invertible ARMA models. Using the standard notation of Box and Jenkins (1976), such a model is defined as

$$\phi(B)Z_t = \theta(B)a_t$$

where a_t is a univariate white noise process with mean 0 and variance, σ_a^2 and where

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$$

with the usual stationarity and invertibility restrictions on the roots of $\phi(B)$ and $\theta(B)$.

$$Z_t = \sum_{j=1}^{\infty} \pi_j Z_{t-j} + a_t$$

where

$$\Pi(B) = \phi(B)\,\theta^{-1}(B) = 1 - \pi_1 B - \pi_2 B^2 - \ldots.$$

Let $\{x_{ti}\}$, $t = 1,2,\ldots,T$, $i = 1, 2, \ldots, q$, be q stationary correlated series. Then using a definite criterion such as Schwartz's BIC for modelling AR structures, truncated AR(∞) models of order k_i, can be fitted to each corresponding $\{x_{ti}\}$, i=1, 2, . . . , q. Define the vector of the AR(k_i) parameters of the i *th* generating processes X_{ti} as

$$\Pi_i' = \left[\pi_{1i}\ \pi_{2i}\ \ldots\ \pi_{k_i i}\right], \qquad i = 1, 2, \ldots, q.$$

and the corresponding AR(k_i) parameter estimates of the series $\{x_{ti}\}$ as

$$\hat{\pi}_{ji}\ , \quad j = 1, 2, \ldots, k_i, \qquad i=1, 2, \ldots, q.$$

Let $k = \max(k_1,k_2, \ldots, k_q)$. In constructing the test statistic the maximum order k is assumed to be fitted to all series. Then define

$$\hat{\Pi}_{ki}' = \left[\hat{\pi}_{1i},\ \hat{\pi}_{2i},\ \ldots,\ \hat{\pi}_{ki}\right],\ i = 1, 2, \ldots, q.$$

The hypotheses to be tested are:

H_0: There is no significant difference between the generating processes of q stationary series (i.e. $\Pi_{k1} = \Pi_{k2} = \ldots = \Pi_{kq} = \Pi_k$).

H_1: There is a significant difference between the generating processes of at least two stationary series.

The model to be fitted is of the form of 'the seemingly unrelated regressions model' as proposed by Zellner (1962). The T-k observations of the models fitted to the q series $\{x_{ti}\}$, $i = 1, 2, \ldots, q$, can be expressed collectively as

$$x_i = W_i\, \Pi_{ki} + a_i \qquad (2.1)$$

where

$$x_i' = [x_{k+1\,i} \;\dots\; x_{T-1\,i} \;\; x_{Ti}]$$

$$W_i = \begin{bmatrix} x_{ki} & x_{k-1i} & \cdot & \cdot & \cdot & x_{1i} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{T-2i} & x_{T-3\,i} & \cdot & \cdot & \cdot & x_{T-k-1i} \\ x_{T-1i} & x_{T-2\,i} & \cdot & \cdot & \cdot & x_{T-ki} \end{bmatrix}$$

$$\Pi_{ki}' = [\pi_{1i} \quad \pi_{2i} \quad \dots \quad \pi_{ki}]$$

$$a_i' = [a_{k+1\,i} \quad \dots \quad a_{T-1\,i} \quad a_{T\,i}]$$

and

$$E[a_i] = 0 \qquad E[a_i a_i'] = \sigma_i^2 I_{T-k}$$

and where I_{T-k} is a (T-k) x (T-k) identity matrix. We will assume that the disturbances of the q models are correlated at the same points *in time but uncorrelated across* observations. That is

$$E(a_i a_j') = \sigma_{ij} I_{T-k} \qquad i, j = 1, 2, \dots, q$$

Then assuming that a *total of (T-k)*q *observations* are *used in estimating the* parameters of the q equations in (2.1), the combined model may be expressed as

$$Z = W\Pi + a \tag{2.2}$$

where

$$Z = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_q \end{bmatrix} \qquad W = \begin{bmatrix} W_1 & 0 & . & . & . & 0 \\ 0 & W_2 & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & W_q \end{bmatrix}$$

$$\Pi = \begin{bmatrix} \Pi_{k1} \\ \Pi_{k2} \\ \cdot \\ \cdot \\ \cdot \\ \Pi_{kq} \end{bmatrix} \qquad a = \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_q \end{bmatrix}$$

and

$$E(a) = 0$$

$$E(aa') = V = \Sigma \otimes I_{T-k}$$

where

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & . & . & . & \sigma_{1q} \\ \sigma_{12} & \sigma_2^2 & . & . & . & \sigma_{2q} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \sigma_{1q} & \sigma_{2q} & . & . & . & \sigma_q^2 \end{bmatrix}.$$

The generalised least squares estimator is

$$\hat{\Pi} = \left[W'V^{-1}W\right]^{-1} W'V^{-1}Z. \tag{2.3}$$

Assuming that disturbances are normally distributed, then by results in Anderson (1971) amongst other authors, $\hat{\Pi}$ has been shown to be asymptotically normally distributed with mean Π and covariance matrix

$$\underset{T\to\infty}{lim}\,\mathrm{Var}\left(\sqrt{T}\hat{\Pi}\right) = plim\left(\frac{W'V^{-1}W}{T}\right)^{-1}.$$

Now $\qquad H_0 \; : \Pi_{kx1} = \Pi_{kx2} = \ldots = \Pi_{kxq}$

may be expressed as

$$H_0 : \; R\Pi = 0$$

where R is a k(q-1) x kq matrix where each row consists of a one, (kq -2) zeros and a minus one, namely

$$R = \begin{bmatrix} 1 & 0 & -1 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & -1 & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & 1 & 0 & 0 & -1 \end{bmatrix}.$$

Hence $R\hat{\Pi}$ is asymptotically normally distributed with mean $R\Pi$ and covariance matrix

$$\underset{T\to\infty}{lim}\,\mathrm{Var}\left(\sqrt{T}\,R\hat{\Pi}\right) = plim\frac{R\left(W'V^{-1}W\right)^{-1}R'}{T}.$$

Let

$$F = \left(R\left(W'V^{-1}W\right)^{-1}R'\right)^{-1/2}\left(R\hat{\Pi} - R\Pi\right). \qquad (2.4)$$

Then by substituting (2.3) into (2.4), F becomes

$$F = \left[R\left(W'V^{-1}W\right)^{-1}R'\right]^{-1/2} R\left(\left(W'V^{-1}W\right)^{-1}W'V^{-1}\left(W\Pi + a\right) - \Pi\right).$$

Under H_0

$$F = \left[R\left(W'V^{-1}W\right)^{-1}R'\right]^{-1/2} R\left(W'V^{-1}W\right)^{-1}W'V^{-1}a.$$

Under the assumption that

$$a \sim N(0, V)$$

$$E(F) = 0 \quad \text{and} \quad E(FF') = I_k,$$

$$F \overset{A}{\sim} N(0, I_k).$$

Therefore

$$F'F = \left(R\hat{\Pi}\right)'\left[R\left(W'V^{-1}W\right)^{-1}R'\right]^{-1}\left(R\hat{\Pi}\right) \overset{A}{\sim} \chi^2\left(k(q-1)\right).$$

Since Σ is unknown, a feasible generalised least squares estimator of Π will have to be used. By Zellner (1962), least squares residuals may be used to estimate consistently the elements of Σ with $\hat{\sigma}_i^2 = \dfrac{\hat{a}_i'\hat{a}_i}{T-k}$, and $\hat{\sigma}_{ij} = \dfrac{\hat{a}_i'\hat{a}_j}{T-k}$.

Hence the feasible generalised least squares estimator is

$$\hat{\Pi} = \left[W'\hat{V}^{-1}W\right]^{-1}W'\hat{V}^{-1}Z \tag{2.5}$$

with

$$\lim_{T\to\infty} \mathrm{Var}\left(\sqrt{T}\hat{\Pi}\right) = plim\left(\frac{W'\hat{V}^{-1}W}{T}\right)^{-1}$$

where

$$\hat{V} = \hat{\Sigma} \otimes I_{T\text{-}k} \quad \text{and} \quad \hat{\Sigma} = \begin{bmatrix} \hat{\sigma}_1^2 & \hat{\sigma}_{12} & . & . & . & \hat{\sigma}_{1q} \\ \hat{\sigma}_{12} & \hat{\sigma}_2^2 & . & . & . & \hat{\sigma}_{2q} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \hat{\sigma}_{1q} & \hat{\sigma}_{2q} & . & . & . & \hat{\sigma}_q^2 \end{bmatrix}.$$

Since $\hat{V}$ is nonsingular and $plim\hat{V} = V$, it is easily seen that

then under H_0

$$D = F'F = \left(R\hat{\Pi}\right)'\left[R\left(W'\hat{V}^{-1}W\right)^{-1}R'\right]^{-1}\left(R\hat{\Pi}\right) \overset{A}{\sim} \chi^2\left(k(q-1)\right). \qquad (2.6)$$

3 *SIMULATION STUDY*

3.1 *Assessment of the Test for q=2*

To investigate the finite sample behaviour of the test statistic D , series of lengths 50 and 200 are simulated from a number of ARMA processes. For q=2 distributional properties of the test based on D are checked by obtaining estimates of the mean, variance, skewness of the test statistic and size of the test procedure This is done by applying the test to pairs of series simulated from AR(1) processes for $\phi = 0$, 0.1, 0.5, 0.9, MA(1) processes for $\theta = 0.1$, 0.5, 0.9, AR(2) processes for $\phi_1 = 0.6$, $\phi_2 = 0.2$, MA(2) processes for $\theta_1 = 0.8$, $\theta_2 = -0.6$ and ARMA(1,1) processes for $\phi = 0.8$, $\theta = 0.2$. It is assumed that the correlation between disturbances of each pair of processes from which the series were generated are in turn 0, 0.5 and 0.9. Estimates of size are obtained for the 5% and 1% significance levels. Estimates of power for the 5% and 1% significance levels are obtained by applying the test to series generated from the following processes: AR(1) $\phi = 0$ versus AR(1), $\phi > 0$, AR(1) $\phi = 0$ versus AR(2) $\phi_1 = 0$, $\phi_2 > 0$ and AR(1) $\phi = 0.5$ versus AR(1) $\phi \neq 0.5$. This is again done by assuming that the correlation between disturbances of each pair of processes from which the series were generated are in turn 0, 0.5 and 0.9.

The order (up to 10) of the truncated AR model to be fitted to each series is determined by Schwartz's BIC. However in estimating the model in (2.1), the maximum order k is fitted to both the series in each pair. The test statistic D is then obtained. This is repeated 2000 times. As well as obtaining size and power estimates for the various degrees of freedom, overall estimates of power and size are also obtained.

For series of length 50, size is considerably overestimated and estimates of the mean, variance and skewness of the test statistic do not correspond closely to the respective theoretical values. The overall size estimates are shown in Table A3.1. Since the implication here is that the test does not perform well for series of this length, no further analysis is carried out on series of length 50.

For T=200, the results for which there are at least 100 test statistics corresponding to a particular degree of freedom are shown in Tables A3.2 to A3.4. We observe that the size estimates for the series simulated from the AR models are fairly close to the predetermined significance levels when the correct order k is fitted but size is often overestimated for other values of k. For the MA and ARMA models, for some values of k, the size estimates are fairly close to the predetermined significance levels but in other cases size is overestimated. Hence this often causes the overall estimates of size to be slightly overestimated. These overall size estimates are shown in Table A3.5. It is clear from the size estimates that size improves (i.e. gets closer to the nominal 5% and 1% levels) as the correlation between disturbances of processes from which the series are generated gets larger. It can be seen from Tables A3.2 to A3.4 that for those values of k for which reasonably good size estimates are obtained, the estimates of the means, variances and skewness of the test statistic are very often fairly close to the theoretical means, variances and skewness respectively.

Power estimates based on at least 100 test statistics corresponding to particular degrees of freedom and overall power estimates based on all 2000 test statistics are given in Tables A3.6.1 to A3.8.2. From these results it appears that the test has reasonably good power for the series length T = 200. It can also been seen that the power of the test improves as the correlation between disturbances of processes from which the series are generated gets larger.

3.2 Assessment of the Test for q=3

The same simulation scenario which is considered in Section 3.1 for obtaining estimates of the mean, variance, skewness of the test statistic and size of the test procedure is again considered but this time for q=3 (i.e. testing for significant differences between the generating processes of 3 series) and only for T=200. Estimates of mean, variance and skewness of the test statistic and the size of the test are shown in Tables A3.9.1 to A3.9.3 and estimates of overall size are shown in Table A3.10. Just as for the case q=2 in Section 3.1, size estimates for the series simulated from the AR models are fairly close to the predetermined significance levels when the correct order is fitted but size is often overestimated for other values of k. For the MA and ARMA models, for some values of k, the size estimates are fairly close to the predetermined significance levels but in other cases size is overestimated. Hence again, this often causes the overall estimates of size to be slightly overestimated. Observation of the overall size estimates in Table A3.10 reveal that size generally improves (i.e. gets closer to the nominal 5% and 1% levels) as the correlation between disturbances of processes from which the series are generated gets larger. However it can be seen that the overall size estimates are slightly larger than for the case q = 2. In those cases for which reasonably good size estimates are obtained, the estimates of the means, variances and skewness of the test statistic are very often fairly close to the theoretical means, variances and measures of skewness respectively.

4 POWER COMPARISONS

In this section we compare the power of our test for the case of two independent series with tests proposed by Jenkins (1961), Diggle and Fisher (1991) and Swanpoel and Van Wyk (1986). There is no evidence in the literature that Jenkins test was previously simulated whereas Diggle and Fisher and Swanepoel and Van Wyk simulated their tests and obtained estimates of size and power. All of these tests compare two independent stationary time series by comparing their estimated spectra.

Jenkins test requires one to obtain windowed periodogram ordinates and then use the equivalent number of independent windowed periodogram ordinates to construct a test statistic which has an approximate standard normal distribution. In obtaining estimates of size and power for T = 200 we use a rectangular window with every tenth ordinate assumed to be independent. The choice of the number of equivalent number of independent ordinates follows from guides in Jenkins (1961) and Chatfield (1975). The results are shown in Table A4.1. Diggle and Fisher's test which we replicate for T = 200 uses normalised cumulative periodograms ordinates and a randomozation test based on the Kolgomorov-Smirnov type test statistic. The results are shown in Table A4.2. Swanpoel and Van Wyk use bootstrap methods and three test statistics, namely a Chi-square type (CS), Kolmogorov -Smirnov type (KS) and a Kullback-Leiber type (KL) test statistic. We replicate these tests for T = 200 and the results are shown in Table A4.3 from where it is clear that there is very little power difference between the three tests

The power curves for the various cases mentioned in Section 3, for our test (AR), Jenkins test (J) , Diggle and Fisher's test (DF) and the Kullback-Leiber type test of Swanapoel and Van Wyk (SW), at the 5% of significance are shown in Figures 4.1 - 4.3.

Figure 4.1 Power Curves for White noise versus AR(1) $\phi > 0$ (5% level of Significance)

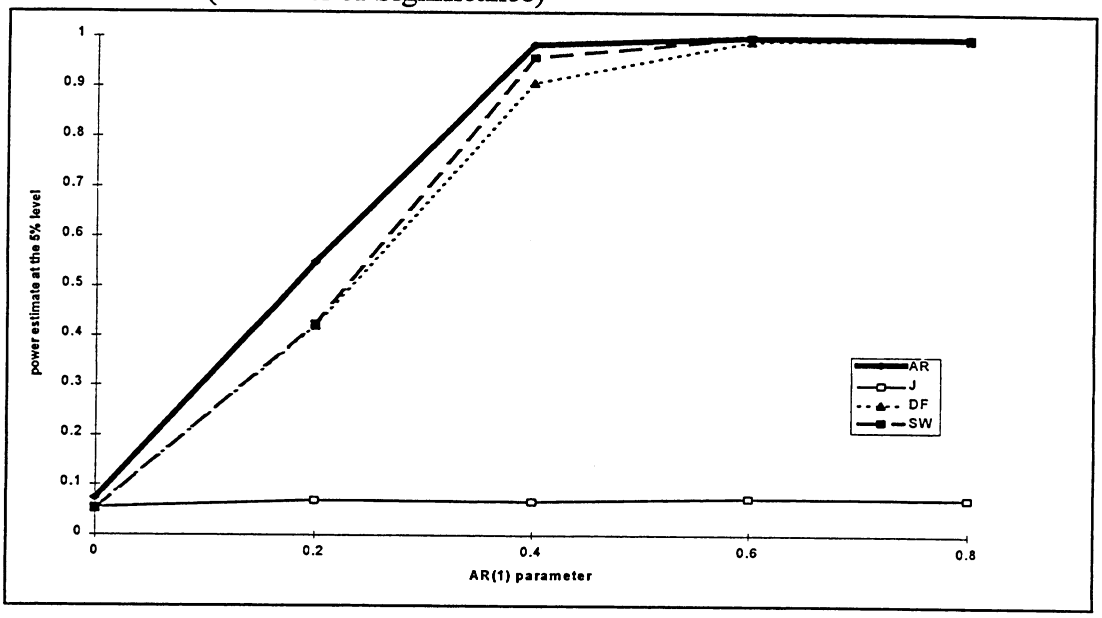


Figure 4.2 Power Curves for White noise versus AR(2) $\phi_1 = 0$, $\phi_2 > 0$ (5% level of Significance)

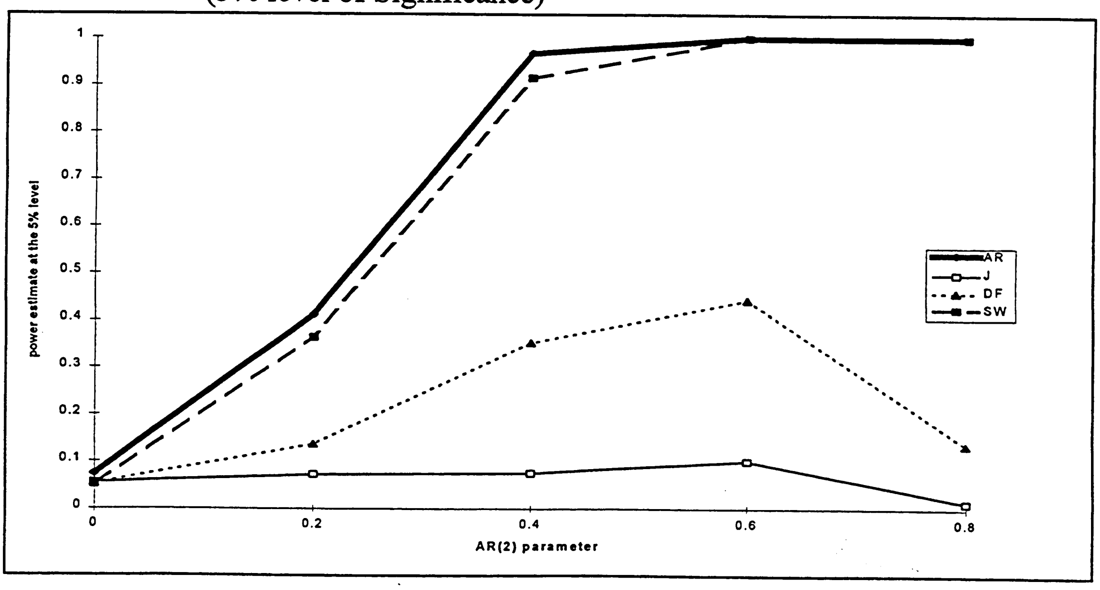
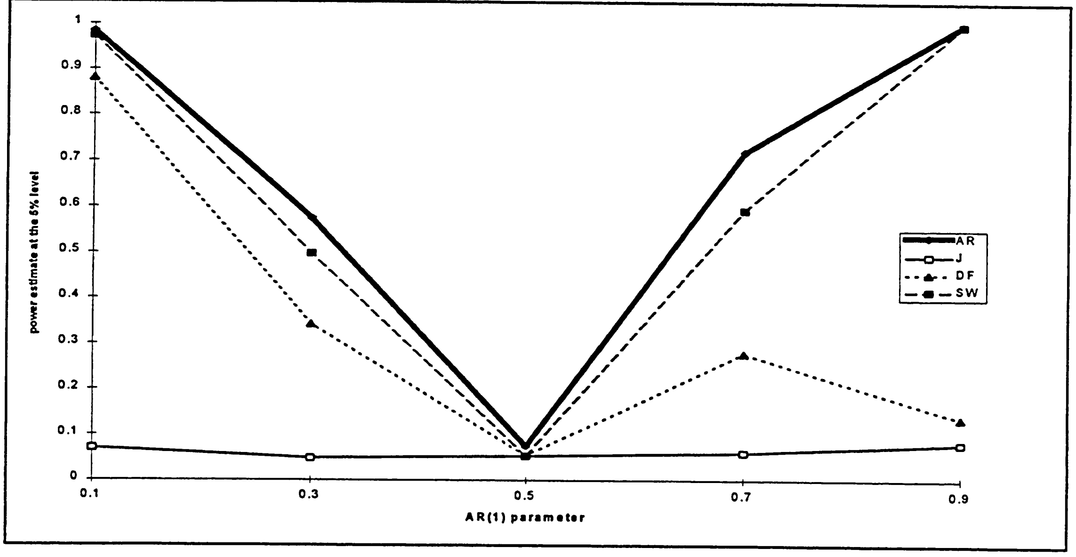


Figure 4.3 Power Curves for White noise versus AR(1) $\phi=0.5$, $\phi\neq0.5$ (5% level of Significance)



In all the above cases, the overall size estimates of our are slightly overestimated. As explained in Section 3, this is due to the overestimation of size when orders of the autoregressive model other than the correct orders were fitted. It can be seen from Tables A3.6.1, A3.6.2, A3.7.1, A3.7.2, A3.8.1 and A3.8.2 that even though the specific (i.e. when the correct AR order is fitted) and overall size estimates differ slightly, there is very little difference in the specific and overall power estimates. So even though size estimates of the other tests under consideration are closer to the nominal significance levels than the overall size estimates of the our test, it can still be concluded from an examination of the power curves above that our test has slightly better power than Swanepoel and Van Wyk's test in all cases. With exception of the case in Figure 4.1, it has much better power than of Diggle and Fisher's test and in all cases it has considerably better power than Jenkin's test . In fact the power of Diggle and Fisher's test with exception of the case in Figure 4.1 tends to decrease instead of increasing at the one end and Jenkin's test has almost no power. Simulations of Jenkins test using other windows, namely the Parzen and Bartlett windows, reveal similar estimates for power to those shown in Table 4.1.

Since is all of the above cases the series were simulated from autoregressive models and since our test involves fitting autoregressive models, it would be expected that the this test would generally perform better than the other tests. To remove this apparent unfair advantage our test has over the others, it was decided at this point to make the power comparisons for series simulated from moving average processes. Comparisons are made for the following situations: MA(1) $\theta = 0$ versus MA(1) $\theta > 0$ and MA(1) $\theta = 0.5$ versus MA(1) $\theta \neq 0.5$. The results of these simulations for $T = 200$ for the tests six tests are shown in Tables A4.4 to A4.6. The power curves at the 5% of significance are shown in Figures 4.4 to 4.5.

Figure 4.4 Power Curve for White Noise versus MA(1) θ=0 versus θ>0 (5% level of significance)

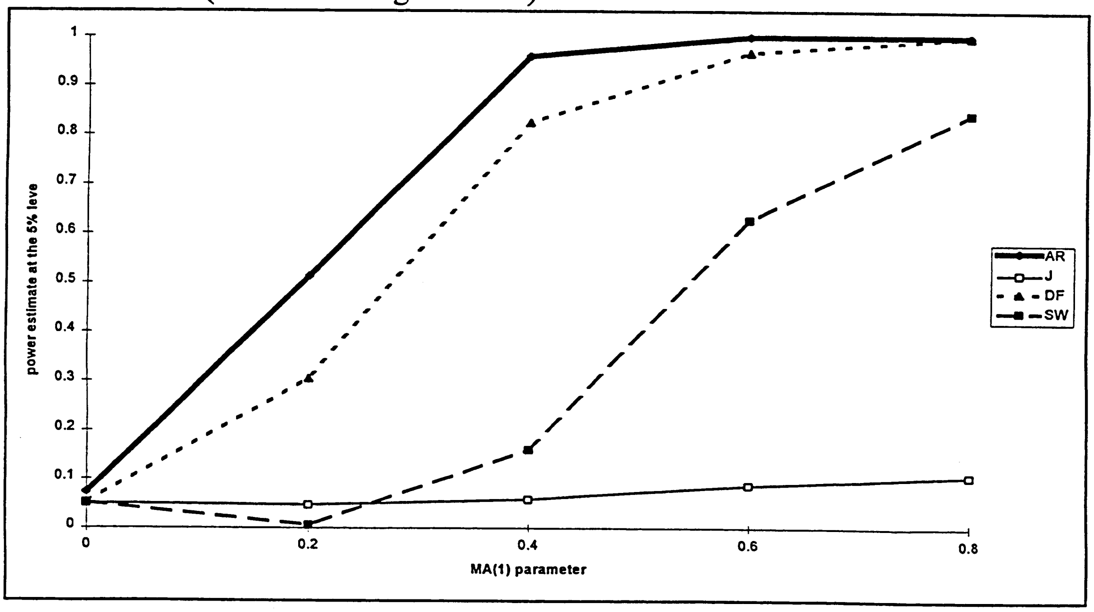
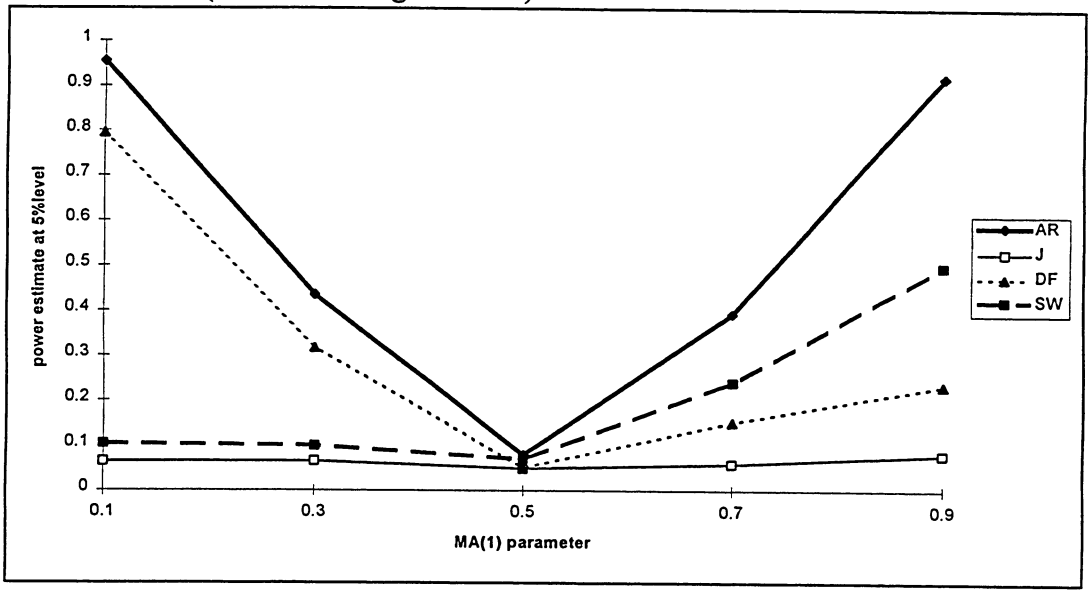


Figure 4.5 Power Curve for MA(1) θ=0.5 versus $\theta \neq 0.5$
(5% level of significance)



It can be seen from these power curves, that even though overall size is slightly overestimated, our test still performs better that the other tests. The test of Swanepoel and Van Wyk which performed nearly as well as our test in some cases earlier on, now performs quite poorly.

5 APPLICATIONS

5.1 Loans Data

Total fixed loan commitments in thousands of dollars of all banks, finance companies and credit co-operative in Australia for the period January 1985 to November 1995 are examined. Of interest is whether there are significant differences in the lending patterns between the institutions. The natural log transformation of these series are shown in Figure 5.1. It can be seen that while lending is on different levels for the three institutions, the lending patterns over the given time period are similar for the banks and finance companies, but differ for the banks and credit co-operatives and for the finance companies and credit co-operatives. Because the series are nonstationary, the series were transformed and differenced in an attempt to make them stationary. If was assumed that the first difference of the natural log transformation of each series was stationary. All further analysis was carried out on these series. Each of these series has 130 observations.

The test derived in Section 2 was first applied to all three series. The results are shown in Table 5.1 from where it can be seen that there is some residual correlation between the two series in each pair. Tests for correlation reveal that there is a significant correlation between the disturbances of the underlying the generating processes of the two series in each pair. This is to be expected since the same economic factors are expected to affect lending commitments from each type of institutions. From Table 5.1 it can be seen that there is a significant difference between the generating processes of the series since the p-value of the test is 0.0005.

Multiple comparisons are then considered by performing the test for $q = 2$ for every pair of series. The results of these multiple comparisons are shown in Table 5.2. from which, we make the following observations: The residual correlation between each pair of series is very similar to those obtained when the test was applied simultaneously to all three series. There is not enough evidence to conclude that the level of lending patterns between the banks and finance companies are significantly different but there is strong evidence to conclude that the level of lending patterns between the banks and credit cooperatives and between finance companies and credit cooperatives are significantly different. Since the levels of the undifferenced bank and finance companies series are clearly different, it is clear from the result of no significant difference between the underlying generating processes of the corresponding differenced series, that the test can distinguish between the underlying stochastic nature of the two series but not the underlying deterministic nature of the two series.

The results of the multiple comparisons tests correspond with casual observations one could make from the examination of the series in Figure 5.1.

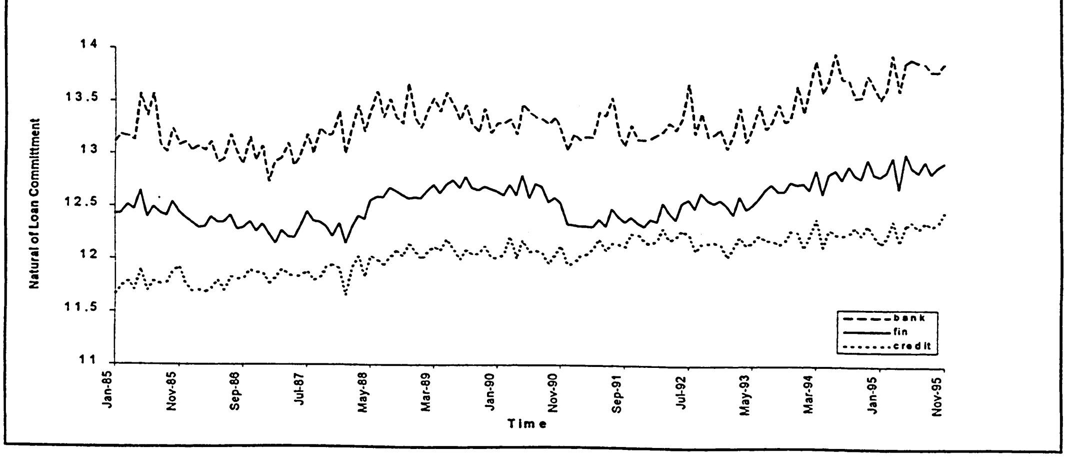
Table 5.1 **Results of Loans Series Application for q=3**

		Banks	Financial Companies	Credit Co-operatives
AR(k) fit		AR(9)	AR(5)	AR(2)
Residual Correlation	Banks		0.4480	0.5568
	Financial Companies			0.5728
p-value	0.0005			

Table 5.2 **Results of Loans Series Application for q=2**

Pair	**AR(k) fit**	**Residual Correlation**	**p-value**
Banks vs Financial Co.	AR(9), AR(5)	0.4256	0.4107
Bank vs Credit Corp.	AR(9), AR(2)	0.5503	0.0034
Credit Corp. vs Financial Co.	AR(2), AR(5)	0.5931	0.0002

Figure 5.1 Total Loan Commitments of the Banks, Finance Companies and Credit Co-operatives from January 1985 to November 1995



5.2 Tree Ring Data

In order to reconstruct historical climates based on information from trees, one type of measurement that climatologists use is distances between the consecutive rings of trees. Figures 5.2 to 5.4 show tree ring data series for three separate sites about 10 km. apart at about the same altitude on Mount Egmont on the North Island of New Zealand. Each series consists of 352 observations which are standardised distances between rings, averaged over a number of trees in a particular site. Standardisation allows samples with large differences in growth rates to be combined. It is also used to remove any undesired growth trends present. The residual correlations between the series of sites 1 and 2 and between the series of sites 1 and 3 are very low and tests for correlation reveal that there is no significant correlation between the disturbances of the generating processes of the two series in each pair. The residual correlation between the series of sites 2 and 3 is higher than the other two residual correlations and a test for correlation reveals that there is significant correlation between the disturbances of the underlying generating processes of these two series. Of interest is whether there are any significant differences between the growth pattern at the three sites given that climatic conditions would be assumed to be the same at the three sites.

The test is first applied to all three series. The results are shown in Table 5.3. The test gives a p-value of 0.4517 thus leading to the conclusion that there are no significant differences between the underlying processes of the three series. Even though there is no need to perform multiple comparisons tests, we nevertheless perform the test for two series at a time. These results are shown in Table 5.4. It can be seen that the residual correlations are similar to those obtained when the test is applied simultaneously to the three series. Furthermore the results of the test for two series at a time reveal no significant differences between the generating processes of the series although the results for the comparison of sites 2 and 3 are fairly close to being significant at the 5% level of significance.

Figure 5.2 Standardised Distance between Tree Rings at Site 1 over 352 years

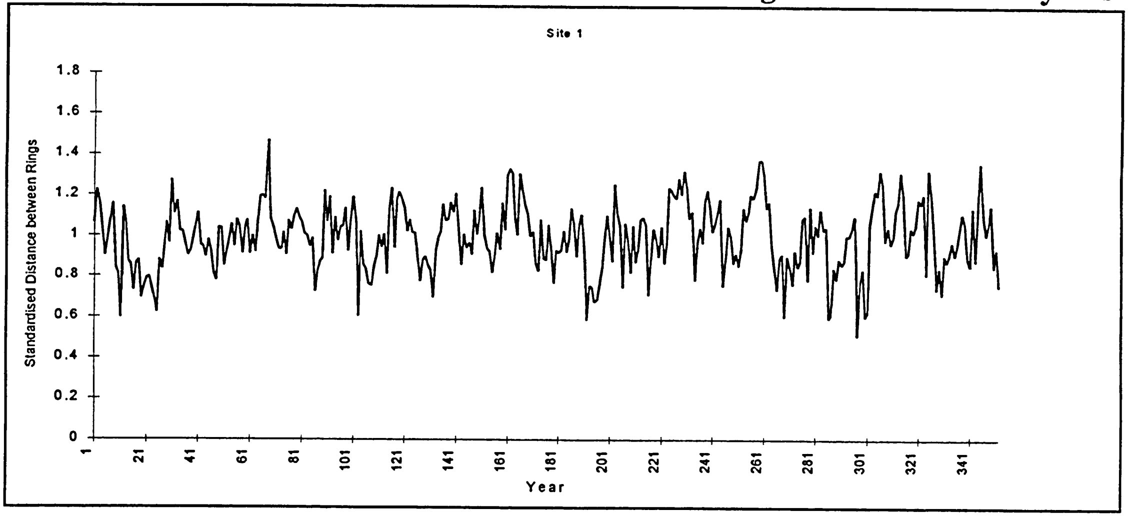


Figure 5.3 Standardised Distance between Tree Rings at Site 2 over 352 years

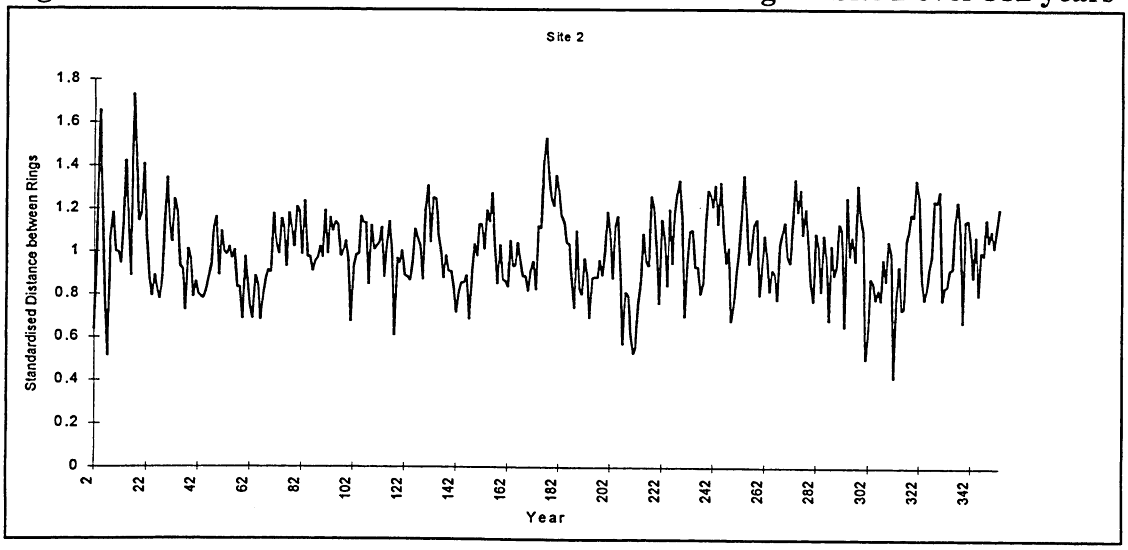


Figure 5.4 Standardised Distance between Tree Rings at Site 3 over 352 years

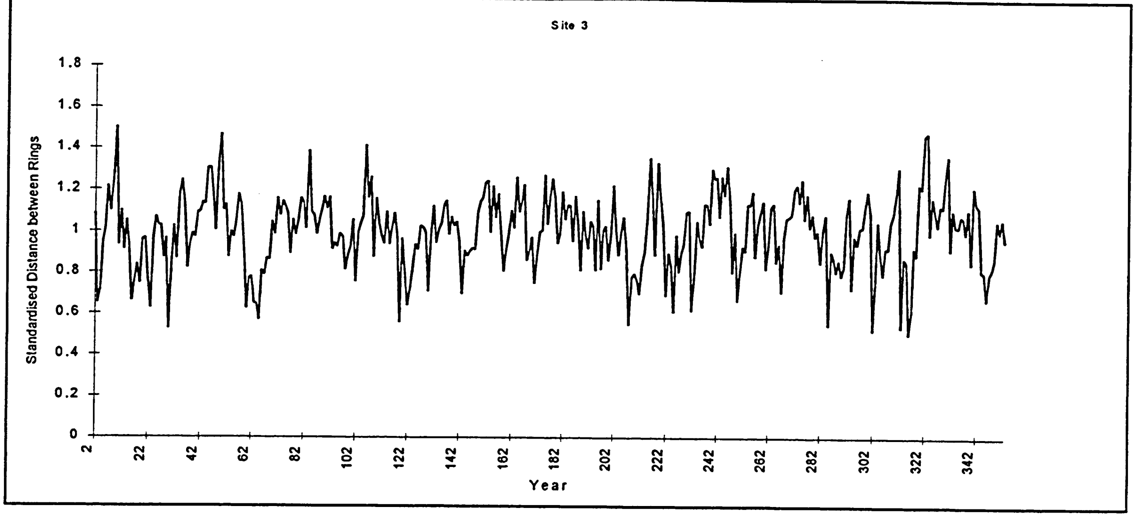


Table 5.3 Results of Tree Ring Series Application for q=3

		Site 1	Site 2	Site 3
AR(k) fit		AR(3)	AR(8)	AR(3)
Residual Correlation	Site 1		0.0500	-0.0323
	Site 2			-0.1742
p-value	0.4517			

Table 5.4 Results of Loans Series Application for q=2

Pair	**AR(k) fit**	**Residual Correlation**	**p-value**
Site 1 vs Site 2.	AR(3), AR(8)	0.0484	0.8814
Site 1 vs Site 3	AR(3), AR(3)	-0.0259	0.8783
Site 2 vs Site 3	AR(8), AR(3)	-0.1743	0.0698

6 *CONCLUDING REMARKS*

From the simulation study is clear that for series of reasonable length, distributional approximations of our proposed test statistic to the chi-square distribution are reasonably adequate for both q=2 and 3. The size of the test reasonably approximates the nominal size. The test has reasonably good power and can be seen from the power comparisons in Section 4 it has better power for the case of two independent stationary series than the other tests under consideration. From the results in Section 5, it appears that the test can be quite successfully applied. Furthermore the advantage that our test has over the existing tests in the literature is that it can be applied to independent as well as related time series and it can also be applied to testing for significant differences between more than two stationary time series.

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APPENDIX

Table A3.1 Overall Estimates of Size for T = 50 for q=2

Generating Process	Level of Significance	Correlation 0	0.5	0.9
AR(1)				
ϕ=0.0	5%	0.1470*	0.1305*	0.0820*
	1%	0.0595*	0.0515*	0.0270*
ϕ=0.1	5%	0.1455*	0.1225*	0.0710*
	1%	0.0515*	0.0440*	0.0235*
ϕ=0.5	5%	0.1475*	0.1245*	0.0975*
	1%	0.0545*	0.0470*	0.0350*
ϕ=0.9	5%	0.1560*	0.1290*	0.1045*
	1%	0.0615*	0.0445*	0.0360*
MA(1)				
θ=0.1	5%	0.1545*	0.1365*	0.0930*
	1%	0.0545*	0.0565*	0.0300*
θ=0.5	5%	0.1550*	0.1420*	0.0940*
	1%	0.0560*	0.0520	0.0325*
θ=0.9	5%	0.1800*	0.1795*	0.1365*
	1%	0.0755*	0.0755*	0.0505*
AR(2)	5%	0.1615*	0.1560*	0.1030*
ϕ_1=0.6 ϕ_2=0.2	1%	0.0715*	0.0550*	0.0345*
MA(2)	5%	0.1800*	0.1660*	0.1255*
θ_1= 0.8 θ_2=-0.6	1%	0.0710*	0.0675*	0.0425*
ARMA(1,1)				
ϕ=0.8 θ=0.2	5%	0.1570*	0.1390*	0.0706*
	1%	0.0350*	0.0480*	0.0165*

* size differs from nominal size by a significant amount (5% level)

Table A3.2-A3.4 Estimates of Mean, Variances, Skewness and Size for T = 200 for q=2

Table A3.2 Correlation = 0

Generating Process	Degrees of freedom	Number of Test Statistics	Mean	Variance	Skewness	Size 5%	Size 1%
AR(1) ϕ=0	1	1619	1.1064	2.1911	0.3900	0.0531	0.0124
	2	255	3.1882	5.5245	0.2137	0.1255*	0.0196*
AR(1) ϕ=0.1	1	1662	1.0214	2.0306	0.3770	0.0511	0.0123
	2	240	3.4933	8.0018	0.2271	0.1750*	0.0417*
AR(1) ϕ=0.5	1	1623	1.0161	2.0533	0.3903	0.0462	0.0148
	2	252	3.5020	7.5468	0.1813	0.1706*	0.0357*
AR(1) ϕ=0.9	1	1648	1.0579	2.3850	0.3787	0.0564	0.0146
	2	249	3.3558	8.6883	0.2515	0.1888*	0.0522*
MA(1) θ=0.1	1	1618	1.0381	2.2504	0.3704	0.0544	0.0148
	2	269	3.7089	8.8782	0.3160	0.1970*	0.0595*
MA(1) θ=0.5	2	1037	2.2041	5.3012	0.3019	0.0665	0.0154
	3	627	3.4591	6.1579	0.2100	0.0686	0.0080
	4	184	5.6847	12.0599	0.1351	0.1630*	0.0272
MA(1) θ=0.9	4	111	4.7068	11.7943	0.0952	0.0541	0.0360
	5	327	5.4548	11.8254	0.2387	0.0581	0.0092
	6	462	6.5769	13.0084	0.1732	0.0800*	0.0108
	7	408	7.6273	18.0725	0.2292	0.0882*	0.0196
	8	344	8.9412	20.5206	0.1997	0.0930*	0.0262*
	9	192	10.7266	22.4665	0.1780	0.1094*	0.0643*
	10	140	12.7247	35.3618	0.2707	0.1714*	0.0643*
AR(2)	1	113	1.2847	2.6521	0.3474	0.0531	0.0265
ϕ_1=0.6	2	1572	2.0913	4.0517	0.2873	0.0541	0.0115
ϕ_2=0.2	3	208	3.9970	9.4500	0.2679	0.1106*	0.0337*
MA(2)	4	814	4.1453	8.1814	0.1929	0.0541	0.0074
θ_1= 0.8	5	469	5.7609	12.8913	0.1910	0.0918*	0.0171
θ_2=-0.6	6	258	7.5594	14.5066	0.0654	0.0930*	0.0310*
	7	297	8.3352	21.8085	0.2475	0.1111*	0.0337*
ARMA(1,1)	1	602	1.2775	3.2159	0.3935	0.0797*	0.0249*
ϕ=0.8	2	1145	2.2906	4.7083	0.3037	0.0655	0.0131
θ=0.2	3	176	4.5763	10.8567	0.2505	0.1705*	0.0450*

* size differs from nominal size by a significant amount (5% level)

Table A3.3 Correlation=0.5

Generating Process	Degrees of freedom	Number of Test Statistics	Mean	Variance	Skewness	Size 5%	Size 1%
AR(1) ϕ=0	1	1669	1.0320	2.3441	0.4014	0.0593	0.0144
	2	229	2.9672	8.0086	0.3347	0.1528*	0.0306*
AR(1) ϕ=0.1	1	1641	1.0466	2.1346	0.3887	0.0609	0.0197
	2	259	3.7052	9.0065	0.2853	0.1313*	0.0579*
AR(1) ϕ=0.5	1	1650	1.0713	2.3751	0.3832	0.0582	0.0133
	2	227	2.8520	6.7121	0.3348	0.1322*	0.0308*
AR(1) ϕ=0.9	1	1640	1.0204	2.0791	0.3877	0.0573	0.0098
	2	280	2.6904	6.1286	0.2872	0.1120*	0.0200
MA(1) θ=0.1	1	1621	0.9616	1.8854	0.3900	0.0432*	0.0093*
	2	262	2.9045	7.4221	0.3175	0.1260*	0.0496*
MA(1) θ=0.5	1	112	1.0469	2.0740	0.4471	0.0538	0.0089
	2	1037	2.1237	4.8950	0.2780	0.0601	0.0155
	3	567	3.4209	7.4025	0.2173	0.0723	0.0176
	4	203	4.6443	12.0947	0.2297	0.0837*	0.0246
MA(1) θ=0.9	4	165	4.5195	10.1064	0.3129	0.0909*	0.0242
	5	373	5.7407	14.9021	0.2414	0.0965*	0.0348*
	6	411	6.6279	13.6251	0.1027	0.0803*	0.0097
	7	381	7.9259	18.8673	0.2068	0.0866*	0.0262*
	8	307	9.4861	26.0783	0.2158	0.1433*	0.0325*
	9	196	10.6881	21.8468	0.1511	0.0765	0.0352*
	10	149	12.2770	25.1335	0.0682	0.1392*	0.0070
AR(2)	1	172	1.5739	5.0712	0.6384	0.0930*	0.0465*
ϕ_1=0.6	2	1508	2.1354	4.5089	0.2910	0.0517	0.0146
ϕ_2=0.2	3	212	3.8088	7.4842	0.3192	0.1038*	0.0235
MA(2)	4	815	4.0574	8.7903	0.2182	0.0541	0.0147
θ_1= 0.8	5	457	5.6743	11.9856	0.1991	0.0656	0.0175
θ_2=-0.6	6	242	7.1842	16.9086	0.1743	0.0785	0.0289*
	7	297	8.3022	18.3549	0.1975	0.1111*	0.0237*
ARMA(1,1)	1	680	1.3028	2.8338	0.3820	0.0838	0.0191
ϕ=0.8	2	1051	2.1556	4.2158	0.2668	0.0533	0.0124
θ=0.2	3	187	3.9038	8.2476	0.2849	0.0963*	0.0214

* size differs from nominal size by a significant amount (5% level)

Table A3.4 Correlation = 0.9

Generating Process	Degrees of freedom	Number of Test Statistics	Mean	Variance	Skewness	Size 5%	Size 1%
AR(1) ϕ=0	1	1708	1.0027	2.0285	0.3971	0.0480	0.0111
	2	178	2.4257	4.6159	0.3345	0.0730	0.0112
AR(1) ϕ=0.1	1	1721	1.0542	2.3089	0.3730	0.0546	0.0110
	2	181	2.0860	3.0161	0.2786	0.0387	0.0000
AR(1) ϕ=0.5	1	1721	0.9875	2.0501	0.3950	0.0465	0.0178
	2	181	2.1479	4.6068	0.3063	0.0387	0.0110
AR(1) ϕ=0.9	1	1728	1.1496	2.8594	0.3845	0.0700*	0.0197*
	2	185	2.4680	5.2420	0.2682	0.0649	0.0108
MA(1) θ=0.1	1	1702	1.0273	2.1623	0.3874	0.0546	0.0106
	2	197	2.5126	5.8476	0.3197	0.0863*	0.0254*
MA(1) θ=0.5	1	249	0.9327	1.6290	0.4371	0.0361	0.0040
	2	1043	2.1257	4.5600	0.3163	0.0575	0.0144
	3	500	3.3144	6.8186	0.2255	0.0660	0.0100
	4	140	4.4116	10.2635	0.3009	0.0857	0.0200*
MA(1) θ=0.9	4	273	4.4535	10.7046	0.2420	0.0659	0.0256*
	5	401	5.5335	12.6025	0.1849	0.0898*	0.0174
	6	428	6.6741	17.8894	0.1915	0.0818*	0.0327*
	7	334	7.5475	15.0322	0.1534	0.0629	0.0210*
	8	236	8.8853	24.2061	0.2244	0.1059*	0.0254*
	9	149	10.1462	27.1070	0.2906	0.1074*	0.0336*
	10	106	11.9456	29.0685	0.1614	0.1226*	0.0377*
AR(2)	1	299	1.1693	2.1391	0.4055	0.0836*	0.0000
ϕ_1=0.6	2	1430	2.1245	4.3668	0.3060	0.0587	0.0104
ϕ_2=0.2	3	183	3.3645	6.1012	0.2337	0.0656	0.0111
MA(2)	4	985	4.3069	9.5422	0.2617	0.0680*	0.0193*
θ_1= 0.8	5	368	5.4096	11.3894	0.2615	0.0897*	0.0054
θ_2=-0.6	6	199	6.0084	14.2029	0.1813	0.0754	0.0100
	7	230	7.6441	14.6101	0.1326	0.0696	0.0040
ARMA(1,1)	1	911	1.1735	3.1296	0.3858	0.0790*	0.0209*
ϕ=0.8	2	886	2.1728	4.6367	0.3504	0.0632	0.0147
θ=0.2	3	142	2.9639	5.2179	0.2972	0.0634	0.0000

* size differs from nominal size by a significant amount (5% level)

Table A3.5 Overall Estimates of Size for T = 200 for q=2

Generating Process	Level of Significance	Correlation 0	0.5	0.9
AR(1) ϕ=0	5%	0.0730*	0.0730*	0.0505
	1%	0.0175*	0.0190*	0.0110
ϕ=0.1	5%	0.0740*	0.0770*	0.0525
	1%	0.0175	0.0195*	0.1000
ϕ=0.5	5%	0.0740*	0.0735*	0.0455
	1%	0.0215*	0.0200*	0.0125
ϕ=0.9	5%	0.0830*	0.0680*	0.0695*
	1%	0.0225*	0.0130	0.0195*
MA(1) θ=0.1	5%	0.0835*	0.0600	0.0610*
	1%	0.0270*	0.0165	0.0125*
θ=0.5	5%	0.0795*	0.0690*	0.0575
	1%	0.0145	0.0185*	0.0125
θ=0.9	5%	0.0880*	0.0985*	0.0860*
	1%	0.0220*	0.0245*	0.0260*
AR(2) ϕ_1=0.6 ϕ_2=0.2	5%	0.0700*	0.0680*	0.0630*
	1%	0.0185	0.0210*	0.0095*
MA(2) θ_1= 0.8 θ_2=-0.6	5%	0.0880*	0.0770*	0.0750*
	1%	0.0215*	0.0220*	0.0145*
ARMA(1,1) ϕ=0.8 θ=0.2	5%	0.0850*	0.0715*	0.0715*
	1%	0.0240*	0.0160	0.0180*

* size differs from nominal size by a significant amount (5% level)

Table A3.6.1 Power Estimates for T=200 (AR(1) ϕ=0 vs AR(1) ϕ>0) for q=2

Correlation			0		0.5		0.9	
Generating Process	Degrees of Freedom	Level of Sign.	Number of Test Statistics	Power	Number of Test Statistics	Power	Number of Test Statistic s	Power
AR(1) 0	1	5%	1619	0.0531	1669	0.0593	1708	0.0480
		1%		0.0124		0.0144		0.0111
	2	5%	255	0.1244	229	0.1528	178	0.0730
		1%		0.0196		0.0306		0.0112
0.1	1	5%	1649	0.1825	1650	0.2655	1736	0.8669
		1%		0.0612		0.1188		0.6959
	2	5%	215	0.3070	249	0.2681	177	0.8301
		1%		0.1302		0.1084		0.6271
0.2	1	5%	1658	0.5434	1624	0.7241	1716	1.0000
		1%		0.3070		0.5006		1.0000
	2	5%	242	0.5579	273	0.6960	196	1.0000
		1%		0.3471		0.4652		1.0000
0.3	1	5%	1619	0.8678	1655	0.9758		
		1%		0.6835		0.9124		
	2	5%	262	0.8282	240	0.9583		
		1%		0.6183		0.8833		
0.4	1	5%	1686	0.9864	1655	0.9758		
		1%		0.9407		0.9124		
	2	5%	218	0.9725	262	0.9583		
		1%		0.9083		0.8833		
0.5	1	5%	1635	0.9982	1686	1.0000		
		1%		0.9933		0.9940		
	2	5%	256	0.9961	235	1.0000		
		1%		0.8867		0.9960		

Table A3.6.2 Overall Power Estimates for T=200 (AR(1) ϕ=0 vs AR(1) ϕ>0) for q=2

Generating Process	Level of Significance	Correlation 0	0.5	0.9
AR(1) ϕ				
0	5%	0.0730	0.0730	0.0505
	1%	0.0175	0.0144	0.0111
0.1	5%	0.2030	0.2635	0.8535
	1%	0.0740	0.1180	0.6805
0.2	5%	0.5485	0.7210	1.0000
	1%	0.3120	0.4950	1.0000
0.3	5%	0.8575	0.9715	
	1%	0.6720	0.9030	
0.4	5%	0.9845	1.0000	
	1%	0.9365	0.9930	
0.5	5%	0.9980		
	1%	0.9935		

Table A3.7.1 Power Estimates for T=200 for q=2 (AR(1) ϕ=0 vs AR(2) ϕ_1=0 $\phi_2 > 0$)

Correlation			**0**		**0.5**		**0.9**	
Generating Process	Degrees of Freedom	Level of Sign.	Number of Test Statistics	Power	Number of Test Statistics	Power	Number of Test Statistics	Power
AR(1) 0	1	5%	1619	0.0531	1669	0.0593	1708	0.0480
		1%		0.0124		0.0144		0.0111
	2	5%	255	0.1244	229	0.1528	178	0.0730
		1%		0.0196		0.0306		0.0112
0.1	1	5%	1295	0.0618	1286	0.0610	1376	0.0477
		1%		0.0154		0.0163		0.0116
	2	5%	557	0.3070	579	0.3316	503	0.8529
		1%		0.0898		0.1451		0.6779
0.2	1	5%	465	0.0839	433	0.0790	442	0.0633
		1%		0.0237		0.0113		0.0181
	2	5%	1303	0.5112	1339	0.7326	1359	1.0000
		1%		0.2640		0.4937		0.9990
	3	5%	160	0.5186	139	0.6906	133	1.0000
		1%		0.3063		0.4173		1.0000
0.3	2	5%	1761	0.7956	1709	0.9549	1765	1.0000
		1%		0.5837		0.8455		1.0000
	3	5%	136	0.8161	167	0.9641	145	1.0000
		1%		0.6175		0.8922		1.0000
0.4	2	5%	1764	0.9688	1747	0.9966		
		1%		0.8872		0.9880		
	3	5%	156	0.9615	164	0.9939		
		1%		0.8634		0.9878		
0.5	2	5%	1764	0.9977				
		1%		0.9870				
	3	5%	151	0.9870				
		1%		0.9805				

Table A3.7.2 Overall Power Estimates for T=200 for q=2 (AR(1) ϕ=0 vs AR(2) ϕ_1=0 ϕ_2 > 0)

Generating Process	Level of Significance	Correlation 0	0.5	0.9
AR(2) ϕ_2				
0	5%	0.0730	0.0730	0.0505
	1%	0.0175	0.0190	0.0100
0.1	5%	0.1520	0.1585	0.2970
	1%	0.0460	0.0610	0.2120
0.2	5%	0.4125	0.5830	0.7930
	1%	0.2105	0.3780	0.7820
0.3	5%	0.7860	0.8245	1.0000
	1%	0.5795	0.8245	0.9925
0.4	5%	0.9680	0.9965	1.0000
	1%	0.8860	0.9870	1.0000
0.5	5%	0.9970	1.0000	1.0000
	1%	0.9855	1.0000	1.0000

Table A3.8.1 Power Estimates for T=200 for q=2 (AR(1) ϕ=0.5 vs AR(1) $\phi \neq 0.5$)

Correlation			**0**		**0.5**		**0.9**	
Generating Process	Degrees of Freedom	Level of Sign.	Number of Test Statistics	Power	Number of Test Statistics	Power	Number of Test Statistics	Power
AR(1)	1	5%	1608	0.9857	1653	0.9994		
0.1		1%		0.9464		0.9976		
	2	5%	269	0.9740	237	0.9958		
		1%		0.9182		0.9873		
0.2	1	5%	1662	0.8881	1683	0.9792		
		1%		0.7383		0.8627		
	2	5%	241	0.8838		0.9764		
		1%		0.7261		0.8915		
0.3	1	5%	1636	0.5807	1673	0.8111	1717	1.0000
		1%		0.3545		0.4204		1.0000
	2	5%	242	0.5331	222	0.7793	187	1.0000
		1%		0.3388		0.5450		1.0000

Table A3.8.1 (contd.)

Correlation			0		0.5		0.9	
Generating Process	Degrees of Freedom	Level of Sign.	Number of Test Statistics	Power	Number of Test Statistics	Power	Number of Test Statistics	Power
AR(1) ϕ								
0.4	1	5%	1639	0.2038	1628	0.3157	1709	0.9233
		1%		0.0769		0.1364		0.7975
	2	5%	259	0.2239	266	0.3195	205	0.8634
		1%		0.1081		0.1278		0.7220
0.5	1	5%	1632	0.0462	1650	0.0582	1721	0.0465
		1%		0.0148		0.0133		0.0178
	2	5%	252	0.1706	227	0.1322	181	0.0387
		1%		0.0357		0.0308		0.1100
0.6	1	5%	1635	0.2300	1673	0.3282	1713	0.9515
		1%		0.0850		0.1434		0.8569
	2	5%	250	0.3080	204	0.3480	197	0.9137
		1%		0.1120		0.1268		0.7665
0.7	1	5%	1663	0.7216	1664	0.8894	1739	1.0000
		1%		0.4848		0.7428		1.0000
	2	5%	233	0.7082	240	0.8958	176	1.0000
		1%		0.4549		0.7250		1.0000
0.8	1	5%	1646	0.9775	1646	0.9775		
		1%		0.9228		0.9228		
					238	0.9706		
	2	5%	238	0.9706		0.8992		
		1%		0.8990				
0.9	1	5%	1652	1.0000	1652	1.0000		
		1%		0.9958		1.0000		
	2	5%	232	1.0000	232	1.0000		
		1%		0.9985		0.9957		

Table A3.8.2 Overall Power Estimates for T=200 for q=2 (AR(1) ϕ=0.5 vs AR(1) $\phi \neq$0.5)

Generating Process	Level of Significance	Correlation 0	0.5	0.9
AR(1) ϕ				
0.1	5%	0.9835	0.9990	1.0000
	1%	0.9445	0.9955	1.0000
0.2	5%	0.8835	0.9800	1.0000
	1%	0.7295	0.9255	1.0000
0.3	5%	0.5740	0.7975	1.0000
	1%	0.3530	0.5860	1.0000
0.4	5%	0.2130	0.3140	0.9130
	1%	0.0885	0.1325	0.7775
0.5	5%	0.0740	0.0735	0.0455
	1%	0.0215	0.0200	0.0125
0.6	5%	0.2470	0.3305	0.9435
	1%	0.0915	0.1425	0.8410
0.7	5%	0.7200	0.8875	1.0000
	1%	0.4615	0.7315	1.0000
0.8	5%	0.9745	0.9747	1.0000
	1%	0.9165	0.9165	1.0000
0.9	5%	1.0000	1.0000	1.0000
	1%	0.9985	0.9985	1.0000

Table A3.9.1-A3.9.3 Estimates of Mean,Variance, Skewness, Size for T=200 q=3

Table A3.9.1 Correlation = 0

Generating Process	Order k	df.	No. Test Statistics	Mean	Variance	Skewness	Size 5%	Size 1%
AR(1) ϕ=0	1	2	1464	2.1536	4.3601	0.2839	0.0622	0.0110
	2	4	353	5.7093	15.6382	0.2176	0.1558*	0.0360*
	3	6	119	9.9939	31.0292	0.1659	0.3109*	0.0924*
AR(1) ϕ=0.1	1	2	1488	2.0582	4.5671	0.3256	0.0598	0.0128
	2	4	343	5.9373	13.0724	0.2310	0.1720*	0.0350*
	3	6	103	9.4728	29.4055	0.2079	0.2427*	0.0971*
AR(1) ϕ=0.5	1	2	1480	2.0437	3.8786	0.3213	0.0527	0.0090
	2	4	338	5.8358	14.1082	0.1574	0.1509*	0.0414
	3	6	108	9.9380	25.1544	0.0798	0.2778*	0.1759*
AR(1) ϕ=0.9	1	2	1499	2.0909	4.5760	0.3252	0.0607	0.0120
	2	4	316	5.9423	14.2252	0.1994	0.1835*	0.0506*
	3	6	119	9.5988	28.0726	0.0689	0.2101*	0.0750*
MA(1)θ=0.1	1	2	1437	1.9361	3.9563	0.2935	0.0431	0.0070
	2	4	386	5.6014	13.4475	0.1990	0.1451*	0.0440*
	3	6	110	9.5756	19.8818	0.1828	0.2727*	0.0636*
MA(1)θ=0.5	2	4	778	4.1441	8.8187	0.2341	0.0655	0.0128
	3	6	807	6.5793	12.2808	0.1407	0.0718*	0.0099
	4	8	283	9.9908	23.5453	0.1341	0.1378*	0.0318
MA(1)θ=0.9	5	10	211	10.4608	18.2307	0.1076	0.0664	0.0047
	6	12	414	12.8240	24.3385	0.2182	0.0797*	0.0217*
	7	14	445	15.0691	32.5675	0.1084	0.0674	0.0247*
	8	16	416	18.0408	37.9889	0.1313	0.1010*	0.0240*
	9	18	256	20.3595	41.0488	0.0549	0.1133*	0.0156
	10	20	223	24.1985	51.2715	0.0207	0.1570*	0.0403*
AR(2)ϕ_1=0.6	2	4	1521	4.2328	8.5597	0.2377	0.0585	0.0120
ϕ_2=0.2	3	6	305	7.9925	17.9659	0.1333	0.1213*	0.0328*
MA(2)	4	8	497	8.3236	17.5144	0.1269	0.0604	0.0121
θ_1=0.8	5	10	500	10.8697	23.1179	0.1901	0.0720*	0.0260*
θ_2=-0.6	6	12	292	14.0335	32.8761	0.1768	0.1027*	0.0445*
	7	14	468	16.0355	34.2007	0.1015	0.1154*	0.0340*
	8	16	143	20.1388	38.2732	0.1021	0.1608*	0.0280*
ARMA(1,1)	1	2	315	3.1133	9.6033	0.2638	0.1397*	0.0254*
ϕ=0.8	2	4	1278	4.5164	9.4308	0.2154	0.0704*	0.0219*
θ=0.2	3	6	273	7.3369	16.7235	0.1746	0.0952*	0.0403*

* size differs from nominal size by a significant amount (5% level)

Table A3.9.2 Correlation=0.5

Generating Process	Order k	df	No. Test Statistics	Mean	Variance	Skewness	Size 5%	Size 1%
AR(1) ϕ=0	1	2	1517	2.0356	4.1444	0.3251	0.0547	0.0119
	2	4	346	5.1092	10.8383	0.2059	0.1213*	0.0202*
AR(1) ϕ=0.1	1	2	1532	2.0793	4.3351	0.3364	0.0555	0.0117
	2	4	313	5.2158	12.5528	0.2254	0.1022*	0.0319*
AR(1) ϕ=0.5	1	2	1524	2.1650	4.0763	0.2865	0.0525	0.0080
	2	4	326	5.4626	11.1717	0.2000	0.1380*	0.0245*
AR(1) ϕ=0.9	1	2	1524	2.0052	4.0272	0.2716	0.0486	0.0070
	2	4	329	5.1695	11.9728	0.1919	0.1094*	0.0304*
MA(1) θ=0.1	1	2	1507	2.0551	4.1736	0.2887	0.0504	0.0162
	2	4	324	5.1869	12.0633	0.3236	0.1173*	0.0401*
	3	6	100	8.2593	24.1085	0.1236	0.0150*	0.0600*
MA(1) θ=0.5	2	4	905	4.2845	8.6050	0.1982	0.0653	0.0144
	3	6	684	6.6008	14.5181	0.2013	0.0833*	0.0190*
	4	8	249	9.6445	22.9420	0.2607	0.1365*	0.0241*
MA(1) θ=0.9	5	10	251	11.2863	28.3483	0.1550	0.0916*	0.0239*
	6	12	427	13.3160	30.1757	0.1734	0.0867*	0.0234*
	7	14	438	15.4526	40.5049	001587	0.0913*	0.0365*
	8	16	390	18.1986	46.9576	0.1171	0.1103*	0.0385*
	9	18	228	19.7813	45.1942	0.0994	0.0921*	0.0307*
	10	20	189	23.0506	52.0827	0.1607	0.1375*	0.0423*
AR(2) ϕ_1=0.6	2	4	1504	4.1066	8.8446	0.2159	0.0552	0.0146
ϕ_2=0.2	3	6	273	7.1949	13.7443	0.1489	0.0879*	0.0182
MA(2)	4	8	580	8.3596	17.9577	0.1826	0.0741*	0.0121
θ_1= 0.8	5	10	501	11.1510	27.8689	0.1939	0.1058*	0.0299*
θ_2=-0.6	6	12	270	13.4514	36.0416	0.0822	0.1037*	0.0444*
	7	14	400	15.0550	28.3617	0.1394	0.0675	0.0150
	8	16	155	18.3429	52.4222	0.1554	0.1290*	0.0581*
ARMA(1,1)	1	2	412	2.6247	8.0032	0.3191	0.0995*	0.0485*
ϕ=0.8 θ=0.2	2	4	1210	4.2287	9.0253	0.2180	0.0545	0.0174*

* size differs from nominal size by a significant amount (5% level)

Table A3.9.3 Correlation = 0.9

Generating Process	Order k	df	No. Test Statistics	Mean	Variance	Skewness	Size 5%	Size 1%
AR(1) ϕ=0	1	2	1647	2.0693	4.1043	0.3099	0.0577	0.0134
	2	4	240	4.6532	13.7232	0.2633	0.0958*	0.0458*
AR(1) ϕ=0.1	1	2	1651	2.0581	4.4665	0.3116	0.0533	0.0115
	2	4	225	4.0859	8.6811	0.3220	0.0622	0.0222
AR(1) ϕ=0.5	1	2	1656	2.1139	4.5390	0.3008	0.0519	0.0133
	2	4	227	4.4024	9.2642	0..1754	0.0661	0.0132
AR(1) ϕ=0.9	1	2	1652	2.0337	4.3430	0.2948	0.0508	0.0109
	2	4	237	4.3802	8.8555	0.2292	0.0591	0.0211
MA(1) θ=0.1	1	2	1646	2.0076	3.8747	0.3308	0.0468	0.0070
	2	4	233	4.2134	8.5742	0.2372	0.0601	0.0172
MA(1) θ=0.5	1	2	171	1.9832	3.5110	0.3367	0.0526	0.0000
	2	4	1017	4.3143	9.0959	0.2456	0.0708*	0.0128
	3	6	539	6.3550	14.2028	0.1913	0.0779*	0.0113
	4	8	172	9.0087	23.6376	0.1267	0.0930*	0.0233*
MA(1) θ=0.9	4	8	202	8.8701	21.3018	0.1639	0.0743	0.0248
	5	10	247	11.1131	22.0282	0.1328	0.0720	0.0231*
	6	12	440	13.3687	31.1968	0.1233	0.1000*	0.0273*
	7	14	358	15.9041	37.5100	0.1423	0.1061*	0.0251*
	8	16	278	16.9891	37.1933	0.1938	0.0791*	0.0216*
	9	18	177	20.9981	48.6536	0.0619	0.1469*	0.0338*
	10	20	143	21.8172	53.5329	0.1908	0.1189*	0.0209*
AR(2)	1	2	250	2.2647	4.4684	0.2673	0.0640	0.0240*
ϕ_1=0.6	2	4	1451	4.0476	15.1995	0.2572	0.0551	0.0214
ϕ_2=0.2	3	6	200	6.5257	15.1995	0.2570	0.0650	0.0300*
MA(2)	4	8	905	8.4618	20.1365	0.1955	0.0718*	0.0232*
θ_1= 0.8	5	10	409	11.2202	24.8740	0.1969	0.0961*	0.0318*
θ_2=-0.6	6	12	218	13.1552	23.4891	0.0688	0.0688	0.0138*
	7	14	273	15.4774	32.0733	0.0763	0.0989*	0.0183*
ARMA(1,1)	1	2	836	2.2479	4.7138	0.3290	0.6460	0.0144
ϕ=0.8	2	4	924	4.4393	9.1784	0.2309	0.0790*	0.0141
θ=0.2	3	6	171	6.7993	15.0262	0.1703	0.0760	0.0175

* size differs from nominal size by a significant amount (5% level)

Table A3.10 Overall Estimates of Size for T = 200 q=3

Generating Process	Level of Significance	Correlation 0	0.5	0.9
AR(1)	5%	0.1030*	0.0770*	0.0645*
ϕ=0	1%	0.0250	0.0170*	0.0165*
ϕ=0.1	5%	0.0965*	0.0775*	0.0550
	1%	0.0240	0.0215*	0.0125
ϕ=0.5	5%	0.0880*	0.0765*	0.0535
	1%	0.0210	0.0155	0.0125
ϕ=0.9	5%	0.0980*	0.0625*	0.0560
	1%	0.0285*	0.0125	0.0125
MA(1)				
θ=0.1	5%	0.0825*	0.0740*	0.0495
	1%	0.0020*	0.0195*	0.0095
θ=0.5	5%	0.0870*	0.0805*	0.0725*
	1%	0.0205*	0.0170*	0.0120
θ=0.9	5%	0.0940*	0.0960*	0.0960*
	1%	0.0220*	0.0310*	0.0255*
AR(2)				
ϕ_1=0.6 ϕ_2=0.2	5%	0.0830*	0.0690*	0.0585
	1%	0.0190*	0.0185*	0.0165*
MA(2)				
θ_1= 0.8 θ_2=-0.6	5%	0.0960*	0.0950*	0.0840*
	1%	0.0305*	0.0027*	0.0230*
ARMA(1,1)				
ϕ=0.8 θ=0.2	5%	0.0945*	0.0770*	0.0730*
	1%	0.0320*	0.0315*	0.0140

*** size differs from nominal size by a significant amount (5% level)**

Table A4.1 Estimates of Power for Jenkins Test for T=200

	Significance Level 5%	1%
White Noise vs. AR(1) $\phi > 0$		
ϕ		
0.0	0.0550	0.0080
0.2	0.0690	0.0190
0.4	0.0670	0.0160
0.6	0.0740	0.0180
0.8	0.0730	0.0160
White Noise vs. AR(2) $\phi_1=0$ $\phi_2>0$		
ϕ_2		
0.0	0.0550	0.0080
0.2	0.0720	0.0210
0.4	0.0750	0.0160
0.6	0.1000	0.0250
0.8	0.1260	0.0450
AR(1) $\phi=0.5$ vs. $\phi>0.5$		
ϕ		
0.1	0.0700	0.0210
0.3	0.0480	0.0140
0.5	0.0530	0.0090
0.7	0.0600	0.0140
0.9	0.0800	0.0210

Table A4.2 Estimates of Power for Diggle and Fisher's T=200

	Significance Level 5%	1%
White Noise vs. AR(1) $\phi > 0$		
ϕ		
0.0	0.0530	0.0090
0.2	0.4200	0.1810
0.4	0.9080	0.7070
0.6	0.9930	0.9120
0.8	0.9970	0.7970
White Noise vs. AR(2) $\phi_1=0$ $\phi_2>0$		
ϕ_2		
0.0	0.0530	0.0090
0.2	0.1370	0.0390
0.4	0.3530	0.1010
0.6	0.4430	0.1280
0.8	0.1360	0.0220
AR(1) $\phi=0.5$ vs. $\phi>0.5$		
ϕ		
0.1	0.8880	0.6340
0.3	0.3420	0.1200
0.5	0.0540	0.0090
0.7	0.2780	0.0950
0.9	0.1350	0.0290

Table A4.3 Estimates of Power for Swanepoel and Van Wyk's Test for T=200

Test Statistic	KS		CS		KL	
Significance level	5%	1%	5%	1%	5%	1%
White Noise vs. AR(1) $\phi > 0$						
ϕ						
0.0	0.0450	0.0110	0.0500	0.0140	0.0510	0.0160
0.2	0.3920	0.2010	0.4170	0.2400	0.4240	0.2370
0.4	0.9400	0.8290	0.0950	0.8870	0.9590	0.8890
0.6	0.9990	0.9830	1.0000	1.0000	1.0000	1.0000
White Noise vs. AR(2) ϕ_1=0 ϕ_2>0						
ϕ_2						
0.0	0.0450	0.0110	0.0500	0.0140	0.0510	0.0160
0.2	0.3660	0.2820	0.3430	0.1750	0.3640	0.2160
0.4	0.8230	0.6050	0.8960	0.6870	0.9150	0.7240
0.6	0.9990	0.9750	1.0000	0.9990	1.0000	0.9980
AR(1) ϕ=0.5 vs. ϕ>0.5						
ϕ						
0.1	0.9630	0.8780	0.9770	0.9230	0.9740	0.9240
0.3	0.4860	0.2580	0.4790	0.3040	0.4970	0.3060
0.5	0.0390	0.0100	0.0500	0.0120	0.0530	0.0140
0.7	0.5450	0.2940	0.5880	0.3730	0.5920	0.3730
0.9	0.9830	0.8750	0.9980	0.9960	0.9980	0.9910

Table A4.4 Jenkins's Test

	Significance Level 5%	1%
White Noise vs. MA(1)		
θ=0 $\theta \neq 0$		
0.0	0.0550	0.0080
0.2	0.0470	0.0120
0.4	0.0580	0.0180
0.6	0.0870	0.0250
0.8	0.1060	0.0280
MA(1) θ=0.5 vs. θ>0.5		
0.1	0.0650	0.0160
0.3	0.0660	0.0200
0.5	0.0490	0.0100
0.7	0.0590	0.0170
0.9	0.0800	0.0250

Table A4.5 Diggle and Fisher's Test

	Significance Level 5%	1%
White Noise vs. MA(1) θ=0 θ≠0		
0.0	0.0530	0.0090
0.2	0.3150	0.1100
0.4	0.8250	0.5130
0.6	0.9680	0.8150
0.8	0.9900	0.8800
MA(1) θ=0.5 vs. θ>0.5		
0.1	0.7950	0.4810
0.3	0.3170	0.1180
0.5	0.0510	0.0110
0.7	0.1530	0.0580
0.9	0.2330	0.0860

Table A4.6 Swanepoel and Van Wyk's Test

Test Statistic	KS		CS		KL	
Significance level	5%	1%	5%	1%	5%	1%
White Noise vs. MA) θ > 0						
0.0	0.0450	0.0110	0.0500	0.0140	0.0510	0.0160
0.2	0.0130	0.0040	0.0010	0.0000	0.0060	0.0020
0.4	0.1760	0.0340	0.0920	0.0150	0.1600	0.0390
0.6	0.3230	0.0800	0.6280	0.3100	0.6280	0.2610
0.8	0.4020	0.1380	0.9560	0.7250	0.8420	0.5390
MA) θ=0.5 vs. θ≠0.5						
0.1	0.0680	0.0600	0.1010	0.0210	0.1050	0.0210
0.3	0.1000	0.0900	0.1000	0.0900	0.1000	0.0900
0.5	0.0700	0.0200	0.0210	0.0200	0.0700	0.0200
0.7	0.2400	0.1370	0.2390	0.1380	0.2400	0.1380
0.9	0.1950	0.4910	0.5200	0.4990	0.4970	0.4910