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Aggregation and Cointegration

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Abstract

The analysis of economic time series assumes specific economic behaviour of a representative agent. The data used in analysis is generated by aggregating observations of all individuals in a population. This is valid only if all members of a population have the same data generating process, but what happens if their behaviour is heterogeneous? This paper examines the properties of test statistics for cointegration when the aggregate data consists of heterogeneous individuals.

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1. Introduction

Macro-econometric analyses are usually based on aggregate economic time series. Many of these time series are non-stationary. The problem whether they have unit roots is usually extensively studied. Evidence is sometimes inconclusive, but it frequently points towards integratedness. By now, most macro-economists also engaged in empirical work have learnt that time series with unit roots need special care, and the long-run properties of such models can best be represented by a cointegration model. There has been an abundance of papers analysing whether specific clusters of macroeconomic time series are cointegrated.

Such papers frequently fail to spell out the economic and statistical assumptions leading to cointegration. In many cases however, it is possible to show that if, for any reason, economic agents are in a situation where activities generate integrated series then those series should be cointegrated. For example, accepting the life cycle-permanent income hypothesis of consumer behaviour it is pretty straightforward to demonstrate that if the disposable income and consumption series are integrated they should be cointegrated for that consumer (see Molana (1991), for example). However, the derivation concerns the behaviour of an individual consumer, and the heroic assumption is taken that all consumers behave uniformly, thus the aggregate consumption function has the same characteristics as that of a single household. Unfortunately, cross sectional analyses regularly reveal that households do not have a uniform behaviour (see Blundell *et al.* (1993), for example) and thus, the representative agent does not exist. And this is just one example of macroeconomic models derived from microeconomic assumptions and the hypothesis of homogeneous behaviour. The same representative agent, *i.e.,* homogeneity assumption underpins almost all aspects of macro modelling from money demand to international trade. However, economic decisions are made by individual agents, and their behaviour will only be uniform if there are strong economic and/or social forces ensuring homogeneity. If the liquidity preference of the households is not uniform there is no valid aggregate money demand function, etc. The lack of this uniformity may severely hamper any empirical work.[1]

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[1] Let us just give one example: In a very argumentative paper about money demand in the UK, Hendry and Ericsson (1991) strongly argued that real money and income must be cointegrated, and failing to properly set up the cointegration-error correction model any empirical analysis is flawed. However, when they test if the ratio of real money and income is cointegrated with the interest rate they cannot reject the no-cointegration null at the usual

With the emergence of tests for integration and cointegration there have been several studies analysing the effects of temporal aggregation on the properties of these tests, (*e.g.*, Choi (1992), Granger (1990), and Granger (1992)). However, there has not been any interest in cross sectional aggregation, although that is how macroeconomic time series are derived. The reason may be that the effects of this were extensively studied in the traditional econometric modelling framework. However, we believe that the consequences of cross sectional aggregation are non-trivial on the long-run properties of economic time series and relationships. A previous paper studied the effects of aggregation on the integratedness of time series and on the corresponding tests (Körösi, *et al.* (1995)). In this paper we explore the consequences of cross sectional aggregation on cointegration.

There may be several different situations in reality. It may well happen that for some economic agents the time series in question are cointegrated, but others follow very different behavioural rules and for them the time series are not cointegrated. Accepting the usual interpretation that cointegration represents a long-run equilibrium path, a more likely situation may be that all economic agents behave in a way which generates cointegrated time series, thus their characteristics are qualitatively uniform, but the details of the behaviour of the actual parameters, are not. There may be random differences among the parameters, or there may be distinct, clearly distinguishable behavioural patterns. In this paper we study the consequences of such inhomogeneities.

The plan of this paper is as follows. An introduction of cointegration and a discussion of some popular tests for the hypothesis of cointegration can be found in Section 2. The experiment is outlined and the behaviour of the aggregate series generated is discussed in Section 3. Section 4 contains the results of the Monte Carlo study and Section 5 concludes.

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significance levels. They still maintain the hypothesis of cointegration, without much empirical evidence. However, the lack of cointegration may well be the consequence of their use of aggregate time series in a situation where the behaviour of economic agents-*e.g.*, households and firms-is different on the financial markets.

2. Models and Methods

2.1 Cointegration - definition

We begin with some definitions and properties which are specific versions of those found in Körösi, *et al.* (1995).

Definition 1. Two time series x_t and y_t are cointegrated if:

- Both series are I(1);
- a linear combination $z_t = y_t - \beta x_t$ is I(0) for some β.

In this case, the cointegrating vector is $(1, -\beta)$.

Property 1. If x_{it} and y_{it} are cointegrated as in definition 1, with cointegrating vector $(1, \beta_i)$ for $i = 1,\ldots,n$ individuals, and $\beta_i \neq \beta_j$ for any i and j, then the aggregate time series $x_t = \sum_{i=1}^{n} x_{it}$ and $y_t = \sum_{i=1}^{n} y_{it}$ are not cointegrated. This is important since all individuals in the population are cointegrated, the aggregate series is not cointegrated.

In an applied setting, bivariate cointegration analysis may be conducted in two ways. The first, and more simple method is by the construction of the Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) varieties of tests. The other method is to analyse the data as a Vector Autoregression (VAR) with just two variables and then check the rank of the long run effects matrix.

2.2 The test statistics

The tests under scrutiny are the Dickey-Fuller (DF) type tests, the Johansen and Juselius (1990) maximal eigenvalue statistic and two variants of Shin (1994), C and CI.

(a) Dickey-Fuller Test

Step 1: For two I(1) variables, x_t and y_t, use the OLS residuals from a regression of y_t on a constant and x_t, z_t.

Step 2: Form the regression $\Delta z_t = \rho z_{t-1} + u_t$ and estimate ρ by OLS and form the t-ratio.

Step 3: Compare the t-ratio with the tabulated critical values in Table 1 of MacKinnon (1991). The null hypothesis is that ρ=0, or that the OLS residuals contain a unit root. Therefore H_0 = Y and X are not cointegrated and H_a = Y and X are cointegrated.

(b) Augmented Dickey Fuller Test

Repeat Step 1 for the DF test, but include lagged first differences of the residuals of the first step in the autoregression.

Another variant of the DF type test is to include a time trend in the cointegrating regression in step 1. MacKinnon (1991) states that the distribution of the DF test statistic depends on the constant in the regression when there is no time trend. The test is invariant to the constant when the time trend is included, however it then depends on the coefficient of the time trend.

(c) Johansen and Juselius (1990) λ_{max} Test

This test is implemented using the Johansen procedure for estimating a vector autoregression and testing the rank of the long-run impact matrix. The null hypothesis is that the two I(1) variables are not cointegrated, the alternative is that the variables are cointegrated.

Step 1: Arrange the two I(1) variables x_t and y_t, into a vector $X_t = (x_t, y_t)'$.

Step 2: Estimate the differenced VAR by maximum likelihood under the null and alternative hypotheses, following Johansen and Juselius (1990). The process is as follows: Begin by transforming the model

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \ldots + \Pi_k X_{t-k} + \mu_0 + \mu_1 t + \varepsilon_t$$

into differences, noting that a deterministic trend is included since the DGP in the simulation study contains a trend.

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + \mu_0 + \mu_1 t + \varepsilon_t$$
$$\Gamma_i = -(I - \Pi_1 - \ldots - \Pi_i)$$
$$\Pi = -(I - \Pi_1 - \ldots - \Pi_k)$$

This is a first difference VAR model, except for the term ΠX_{t-k}. We are interested in the rank of the coefficient matrix Π, since this will contain information about any long run relationship between the variables in X_t. Since we know that the both variables in X_t are I(1), the rank will not be equal to two, hence we want to know whether the rank is zero, or greater than zero. If the rank equals zero, the two variables are I(1), but are not cointegrated. If the rank is one, then there is a unique cointegrating vector between the two variables. The aim of the Johansen and Juselius test is to find the statistical significance of the rank of Π. The rank of a matrix is the number of non-zero roots of the characteristic polynomial $|\Pi - \lambda \mathrm{I}| = 0$. The null hypothesis is

H_0: Rank(Π) = 0, or that all of the roots are zero;

and is tested against the alternative hypothesis

H_a: Rank(Π) > 0, or that at least one of the roots are greater than zero.

We do not know the true long run impact matrix Π, so it is estimated by maximum likelihood. The eigenvalues $\lambda_1 > \lambda_2$ are calculated and then the larger of the two, λ_1 is substituted into the test statistic:

$Q_r = -2\,ln(Q; r = 0 | r = 1) = -T\,ln(1 - \hat{\lambda}_1)$ and has a non standard distribution. In this case, p=2 variables and r=0 under H_0. Therefore one should look in the p-r=2 row of table 2 in Osterwald-Lenum (1992). Values in the upper tail of the distribution are evidence against the null.

(d) Shin (1994)

This test is different from the others because the null hypothesis is that of cointegration, whilst the alternative is that the variables are not cointegrated. Consider the two I(1) series x_t and y_t, and estimate the three following models:

$$y_t = \beta x_t + u_t \qquad (1)$$
$$y_t = \alpha_\mu + \beta_\mu x_t + u_t \qquad (2)$$
$$y_t = \alpha_\tau + \delta_\tau t + \beta_\tau x_t + u_t \qquad (3)$$

Now let $\hat{u}_t, \hat{u}_{\mu t}, \hat{u}_{\tau t}$ denote the OLS residuals from (1), (2) and (3) respectively. Now generate S_t, $S_{\mu t}$ and $S_{\tau t}$ as the partial sums of the OLS residuals from (1), (2) and (3). The test statistics for cointegration are:

$$CI = T^{-2}\sum_{t=1}^{T}\frac{S_t^2}{s^2(l)} \quad (4)$$

$$CI_\mu = T^{-2}\sum_{t=1}^{T}\frac{S_{\mu t}^2}{s_\mu^2(l)} \quad (5)$$

$$CI_\tau = T^{-2}\sum_{t=1}^{T}\frac{S_t^2}{s_\tau^2(l)} \quad (6)$$

where $s^2(l), s_\mu^2(l), s_\tau^2(l)$ are consistent semi-parametric estimators of the long-run variance of the regression error. The estimator chosen for this study was derived from Andrews (1991) using the Quadratic Spectral (QS) kernel with Automatic (data driven) Bandwidth Selection (See Appendix 1). This estimator was chosen because it has some optimality properties concerning efficiency and its rate of convergence.

The OLS estimator of the cointegrating vector (in this case scalar) β is super-consistent, but inefficient. An efficient estimator includes lags of the differenced x_t series as follows.

$$y_t = \tilde{\beta}x_t + \sum_{j=-K}^{K}\tilde{\pi}_j\Delta x_{t-j} + \tilde{u}_t \qquad (7)$$

$$y_t = \tilde{\alpha}_\mu + \tilde{\beta}_\mu x_t + \sum_{j=-K}^{K}\tilde{\pi}_{\mu j}\Delta x_{t-j} + \tilde{u}_{\mu t} \qquad (8)$$

$$y_t = \tilde{\alpha}_\tau + \tilde{\delta}_\tau t + \tilde{\beta}_\tau x_t \sum_{j=-K}^{K}\tilde{\pi}_{\tau j}\Delta x_{t-j} + +\tilde{u}_{\tau t} \qquad (9)$$

The test statistics are constructed in the same way as CI,

$$C = T^{-2}\sum_{t=1}^{T}\frac{\tilde{S}_t^2}{\tilde{s}^2(l)} \quad (10)$$

$$C_\mu = T^{-2}\sum_{t=1}^{T}\frac{\tilde{S}_{\mu t}^2}{\tilde{s}_\mu^2(l)} \quad (11)$$

$$C_\tau = T^{-2}\sum_{t=1}^{T}\frac{\tilde{S}_{\tau t}^2}{\tilde{s}_\tau^2(l)} \quad (12)$$

with the partial sum terms and the estimates of the long-run error variance calculated from the OLS estimates of (7), (8) and (9).

Shin (1994) shows that the limiting distributions of C and CI are equivalent and he tabulates the lower tail for several values. The hypothesis of cointegration should be rejected for 'large' values, that is, the test rejects cointegration at the α significance level if the test statistic $C > C_{1-\alpha}$.

It is instructive to consider the qualitative differences between the three types of tests, DF, Johansen, and Shin. The DF tests have a null of no cointegration and accept the null if the OLS residuals from the cointegrating regression follow a random walk. The Johansen type tests examine the long run impact matrix of the error correction model. In essence, this test is looking for a long run equilibrium relationship between the two (or more) variables. The Shin tests are different in that the null is cointegration and the alternative is no cointegration. The residual from the cointegrating regression is assumed to have two stochastic components. One is a random walk, and the other is an independent, contemporaneous innovation. The combined disturbance is stationary if the variance of the random walk innovation is zero. Hence the null of cointegration is equivalently expressed by having zero variance in the random walk component. If variance is constant across time, then the variance estimates using any subset of the sample should be approximately equal. If variance is increasing over time, as would be the case if the residual were I(1), then the variance of a small subset beginning at the first observation would be smaller than the variance of a longer subset which also begins at the first observation.

3. The Simulation Study

The behaviour of the cointegration test statistics is analysed under eight different scenarios for two pairs of aggregate time series which are designed to be cointegrated at the individual level. The design of the experiment is as follows. Firstly, six time series of length T are generated for N independent individuals.

$$
\begin{array}{lll}
y_{1i}^{(1)} \,\&\, y_{1i}^{(2)} \sim IMA(0,0) & \rightarrow y_{1i}^{(\cdot)}(t) = \mu + \varepsilon_{1i}^{(\cdot)}(t) & \\
y_{2i}^{(1)} \,\&\, y_{2i}^{(2)} \sim IMA(0,1) & \rightarrow y_{2i}^{(\cdot)}(t) = \mu + \varepsilon_{2i}^{(\cdot)}(t) + \theta\varepsilon_{2i}^{(\cdot)}(t-1) & i = 1,\ldots\ldots,N \\
y_{3i} \sim IMA(1,0) & \rightarrow \Delta y_{3i}(t) = \mu + \varepsilon_{3i}(t) & t = 1,\ldots\ldots,T \\
y_{4i} \sim IMA(1,1) & \rightarrow \Delta y_{4i}(t) = \mu + \varepsilon_{4i}(t) + \theta\varepsilon_{4i}(t-1) &
\end{array}
$$

Note that $\varepsilon_{ij}^{(k)} \sim iidN(0,1) \quad \forall i,j,k$

The drift parameter μ is set equal to one for all i and all series, the MA(1) parameter θ is set equal to 0.8 for all i and all series. These series are then aggregated into four series as follows.

$$
\begin{array}{ll}
x_1^{(1)} = \sum_{i=1}^{N} y_{3i} + \sum_{i=1}^{N} y_{1i}^{(1)}, & x_1^{(2)} = \sum_{i=1}^{N} \beta_i y_{3i} + \sum_{i=1}^{N} y_{1i}^{(2)} \\
x_2^{(1)} = \sum_{i=1}^{N} y_{4i} + \sum_{i=1}^{N} y_{2i}^{(1)}, & x_2^{(2)} = \sum_{i=1}^{N} \beta_i y_{4i} + \sum_{i=1}^{N} y_{2i}^{(2)}
\end{array}
$$

β_i is the heterogeneity parameter and nine possible scenarios for the heterogeneity are considered.

Case A: $\beta_i = \beta_0 + s\gamma_i$; where $\gamma_i \sim iidN(0,1)$.

a. s=0.1 β_0=0 d. s=1 β_0=0

b. s=0.1 β_0=1 e. s=1 β_0=1

c. s=0.1 β_0=10 f. s=1 β_0=10

Case B: $\beta_1 = 0$; $\beta_i = 1$, $i = 2, \dots, N$.

Case C: $\beta_1 = 10$; $\beta_i = 1$, $i = 2, \dots, N$.

Case D: $\beta_i = 1$, $i = 1, \dots, N$.

Case A is where all individuals are cointegrated with different cointegrating vectors. The vectors are random with a mean of β_0 and a standard deviation of s.

Cases B and C represent cases where all individuals except for one are cointegrated with the same vector (1, -1). In case B, the outlier is not cointegrated while in case C, the outlier is cointegrated with a much larger value in the cointegrating vector (1, -1/10).

Note that Case D is the comparison case, where the two series are cointegrated.

The cointegration analysis is then conducted for the pairs $x_1^{(1)} \& x_1^{(2)}$ and $x_2^{(1)} \& x_2^{(2)}$. The cointegrating relationships between the individual components of the aggregate time series under the different cases are as follows. Two I(1) time series, x_t and y_t, are cointegrated if a linear combination of them, $y_t - bx_t = u_t$, is a stationary I(0) process for some b. For the i'th individual, the contribution to the aggregate series is as follows.

$$x_1^{(1)}(i) = y_{3i} + y_{1i}^{(1)}, \quad x_1^{(2)}(i) = \beta_i y_{3i} + y_{1i}^{(2)}$$

Theoretically, the only aggregate series which is cointegrated is Case D.

Experiments were conducted for sample sizes 25, 50, 100, 250 and 500. The number of individuals (N) was 20 in all experiments. We had 1000 replications for all experiments.

4. Discussion of Results

The results are tabulated in Appendix 4. There are several important observations to be made, firstly, consider the behaviour of the test statistics one by one, and then compare the performance of the tests across datasets. The Dickey-Fuller type tests, which appear to have good power (for Case D, where the alternative is true), when the sample size increases to one hundred or more. It is also clear that including a time trend in the cointegrating regression reduces power. The null is **not** true for cases Aa-Af, B and C. The asymptotic behaviour of the OLS slope estimator in the cointegrating regression is important to the properties of the tests based on the residuals. Appendix 2 demonstrates that the OLS slope estimate converges asymptotically to $1/\bar{\beta}$, where $\bar{\beta}$ is the mean of the individual β_i's. The β_i's in cases Aa and Ad have a mean close to zero (for finite n), so appears not to converge. Appendix 3 shows that the divergence rate of the OLS residual from the cointegrating regression depends on the degree of relative heterogeneity across individuals. The tests are expected to find no cointegration more often when the divergence rate is highest. The order of relative heterogeneity of the coefficients is as follows.

Most Heterogeneous	Aa
	Ad
⇓	Ae
	Ab and Af
Least Heterogeneous	Ac

The outlier in case C is larger than in case B, hence we expect to find no cointegration more often in case C than case B. The DF test results reflect the ordering as cases Aa and Ad found cointegration least frequently, however as the sample size increases, they often find in favour of cointegration. Compare the results in all of the sample sizes with Case D (no heterogeneity). The test cannot tell the difference at all between D and Ab, not surprising as the mean coefficient values are equal and the standard deviation in Ab is only 0.1. Ac is also similar. Note also that the augmented versions found in favour of no cointegration in small samples for all of the cases, including Case D.

For the Johansen type tests results tell a similar story to those of the DF-type tests when the degree of relative heterogeneity is small (Cases Aa-Ac). Johansen does perform much better when there is more heterogeneity, Ad and Ae. When T=500, the tests are still oversized but at least they choose the right outcome most of the time. In smaller samples, Johansen finds in favour of no cointegration, but has the same finding for Case D. In B and C, where there is just one outlying individual with a different coefficient, Johansen performs favourably when the outlier is quite a long way from the other individuals (Case C).

The Shin test is difficult to compare since the null and alternative are reversed. The null is that the two aggregate time series are cointegrated. It is of little surprise that the null is not rejected for small sample sizes for any of the cases except the de-meaned versions (C_μ and CI_μ) on cases Aa and Ad (expecting Aa and Ad to reject the most). As the sample size increases, something curious happens, as cases Aa and Ad do not reject as often as the others. On the basis of the results for Case D, where the null is true, it appears the de-trended version based on an efficient estimator of the cointegrating regression is most appropriate (C_tau in the tables). At the 5% level, the Shin test appears to have reasonable power (except for case Ac) only when the sample size increases to 500.

To sum up, it can be said that the issue at hand seems to be whether or not the test can detect heterogeneity amongst the noise in the data. In the cases where the degree of heterogeneity is low, the noise obscures the differences between individuals. Recall that the heterogeneity can be detected from the regression of aggregate time series and not from observations of each individuals' cointegrating vectors. It is not entirely surprising that the tests do not perform very well in these sample sizes, although the divergence rate (of the OLS residual) suggests that all of the tests will be consistent.

As the outcomes for Case D indicate, for small samples the power of both the Engle-Granger type tests and the Johansen type tests is rather small, but for the usual medium sized samples they seem to be surprisingly powerful. Overparametrization of the VAR model exerts an obvious toll on the power of the Johansen type tests.

In most practical situations the errors of the cointegrating equation are serially correlated. For correctly specified VAR models the Johansen test is not sensitive to autocorrelated errors. Similarly, the Shin test shows little sensitivity to serial correlation. The Engle-Granger type tests and Johansen type test based on incorrectly specified VAR models are much more influenced by error autocorrelation.

Sample size is certainly a major factor influencing the outcome of the tests. The Shin test has a very low power against almost all cases in small or medium sized samples. Clearly, the choice of the test statistic matters, but (in our cases)the correct *CI* form only becomes powerful for 250 or more observations in most cases. The other tests, where the null is no cointegration, have incorrect sizes for almost all cases, and at most situations larger samples will actually make rejection more likely, *i.e.,* in more cases the conclusion will be that the aggregate time series are cointegrated.

As already seen, different assumptions about heterogeneity lead to clearly differing outcomes. First, comparing Cases B (which assumes that the time series are cointegrated with the same vector for all but one agents, for whom they are not cointegrated) and C (which assumes that the time series are cointegrated with the same vector for all but one agents, for whom they are cointegrated with a significantly different vector) the situation seems to be very different, especially for larger samples. There is a curious dichotomy in large samples: the Shin tests are much more likely to reject the null of cointegration for Case B and the other tests the null of no cointegration for the same Case B than for Case C. This contradiction is difficult to interpret. (Similar tendencies are also apparent in some other cases, *e.g.,* Ae and Af). The Engle-Granger and Johansen type tests conclude much more frequently that Case B is cointegrated which is curious, because they clearly are not, as one component is not cointegrated. It is less difficult to accept that the Shin-type tests will not reject cointegration in most instances for Case C; after all there at least all components are cointegrated.

All other cases represent stochastic homogeneity, the difference among the behaviour of the agents is random. However, Aa and Ad are very peculiar cases, because here the behaviour varies around the lack of cointegration. The time series of all agents will be cointegrated with probability 1, but the expected value of their cointegrating vectors indicates no cointegration. Case Aa represents a much smaller variation around the no cointegration situation. This difference in the variability of the cointegrating vectors (*i.e.,* behaviours) makes a clear difference between the two cases: the null is be rejected much more frequently for Case Aa than for Case Ad. The distributions of the estimated cointegration coefficients in those cases where tests indicated cointegration is necessarily bimodal: they are symmetrical around zero, however, zero is a clear case of no cointegration. However, the no cointegration area around zero is almost negligible for case Ad. For medium or larger samples, if one chooses categories for histogram in the way that zero is in the middle of a class it will look like a usual unimodal distribution. Thus, if we have a situation where cointegration emerges by chance only, but the behaviour of the individual agents is rather heterogeneous we are unlikely to find any indication of the theoretical no cointegration. The no cointegration region around zero is much larger for Case Aa and the distribution of the estimated cointegration coefficients is much wider. The frequency distribution is much more visibly bimodal in this more homogeneous case. However, Figures 1 to 3 demonstrate that the no cointegration region around zero quickly diminishes with increasing sample size.

In those cases, where cointegration is the conclusion in more than 10% of the experiments, the average of the estimated βs for these cases is reasonably close to the mean of the theoretical distribution for medium and larger samples, however, its variance is usually much larger than the theoretical one. Distributions are non-normal, but symmetrical in most cases, Case Ae being the strange exception for which many distributions are skewed. Figures 4 and 5 compare the frequency distributions of Cases Ab and Ae for sample size 250: The only difference between the two experiments is that the behaviour of the individuals is much more heterogeneous in Case Ae than in Case Ab. (The variance of the random component of the coefficient is ten times larger.) While the distribution of the estimated cointegration coefficient for those instances where DF test rejected the no cointegration null has an almost perfect normal distribution, the larger heterogeneity of the behaviours of Case Ae results in severely skewed distribution.

5. Conclusion

The results from the Monte Carlo study contained in this paper raise important questions regarding the use of aggregated time series in applied econometrics. Many econometric models are based on the idea of a homogeneous population, for example, the representative economic agent. The theory behind cointegration itself precludes two aggregate time series from being cointegrated if they are composed of individuals whose individual series are cointegrated with different cointegrating vectors. The residuals from a linear combination of the two aggregate series will always be I(1), however cointegrating relationships are often found between economic time series for which homogeneity is unlikely. Aggregate consumption and aggregate income is one such candidate as individuals would be expected to have different preferences for current and future consumption.

The Monte Carlo results presented in this paper suggest that tests for cointegrating relationships are unable to reject cointegration unless the degree of heterogeneity across individuals is large relative to the noise inherent in the data generating processes. As a practical issue, most economic data is not available at the disaggregated level and so an assumption of homogeneous individuals must be maintained. The divergence rate of the residual from the cointegrating relationship depends on the degree of relative heterogeneity across individuals, where less heterogeneity implies a slow divergence rate. The researcher is probably interested in long-run equilibrium behaviour in a model and so a slow divergence rate may appear to be no divergence at all. The point is that one should be cautious when testing for cointegration wherever there is a possibility of heterogeneity. Testing for the assumption of homogeneity is difficult given the lack of disaggregated data in the first place, hence the applied researcher ought to consider the likely degree of heterogeneity based on theoretical foundations.

The consequences of non-homogeneity of the behaviour of economic agents is far from being uniform. They partly depend on the true patterns of behaviour, and partly on the statistical information, on the length of the available time series. But in almost all situations the lack of homogeneity will seriously hamper statistical inference; the tests regularly used for detecting cointegration will not be reliable tools. Depending on the actual situation, they will fail to lead to correct inference in many cases. One positive result though is that if the individual cointegration vectors only differ in a random error component the estimated cointegrating vector will not be far away from the mean behaviour.

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Appendix 1 - Consistent Semi-parametric estimation of the long-run error variance in the Shin (1994) test.

If the I(1) variables x_t and y_t are cointegrated then permanent changes to x will have a permanent impact on y. The estimation of the variance of the long run error variance is problematic, especially in the presence of heteroscedasticity and/or autocorrelation, which is quite possible if there is a delay in the transmission of shocks from one variable to the other. We used the Heteroscedastic and/or Autocorrelation Consistent (HAC) covariance estimation method of Andrews (1991). As long as the residuals are not *too* heteroscedastic or autocorrelated, the resulting estimator will be consistent. Intuitively, the estimator takes a weighted average of the autocovariances, with the weights diminishing as the autocovariances become more distant in time. The weighting scheme is critical to the convergence rate of the estimator, and we chose the Quadratic Spectral kernel for its optimality properties.

The formula for the variance in (1) is as follows:

$$s^2(l) = T^{-1}\sum_{t=1}^{T}\hat{u}_t^2 + 2\sum_{j=1}^{T-1}\left\{k\left(\frac{j}{\hat{S}_T}\right)\cdot T^{-1}\sum_{t=j+1}^{T}\hat{u}_t\hat{u}_{t-j}\right\}$$

so that $k(.)$ is a function of how far apart the observations in each term are (j), and a Bandwidth parameter. The bandwidth parameter S_T was chosen automatically for the QS kernel and has the following formula.

$$\hat{S}_T = 1.3221(\hat{\alpha}(2)T)^{1/5};$$

$$\hat{\alpha}(2) = \frac{4\hat{\rho}^2}{(1-\hat{\rho})^4}$$

where $\hat{\rho}$ is the OLS estimator of the disturbance regressed on its own lag.

The Quadratic Spectral kernel which forms the weights is given as follows:

$$k(x) = \frac{25}{12\pi^2x^2}\left(\frac{sin(6\pi x/5)}{6\pi x/5} - cos(6\pi x/5)\right)$$

The resulting estimator of the long-run error variance will be optimal in terms of asymptotic mean-squared-error (Andrews (1991)) of the class of kernels which generate positive variances.

Appendix 2 - Asymptotic Properties of the OLS estimator in the presence of heterogeneity.

It is known that the OLS estimator of the slope estimate in the model $\mathbf{y_t=x_t'\beta} + \varepsilon_t$ is superconsistent. The behaviour of the test statistics above depend on the asymptotic properties of the cointegrating regressions. The theory of aggregating cointegrated variables suggests that there is **no** unique θ such that $\mathbf{y_t} - \theta_0 - \theta\mathbf{x_t}$ is I(0). It is instructive to examine whether the OLS estimator in the cointegrating regressions for the DGP's in the Monte Carlo study are converging to anything meaningful. If so, what is the relationship between the variance of the residual over time, the variance of the DGP and the degree of heterogeneity (as measured by the variance of what is essentially a random coefficient).

Theorem: Let

$$\begin{aligned} y_t &= \sum_{i=1}^{n} y_{3i(t)} + \sum_{i=1}^{n} \varepsilon_{1i(t)} \\ x_t &= \sum_{i=1}^{n} \beta_i y_{3i(t)} + \sum_{i=1}^{n} \varepsilon_{2i(t)} \end{aligned} \qquad \text{(A2.1)}$$

where

$$\begin{aligned} &\Delta y_{3it} = \mu + u_{it}; \\ &u_{it} \sim iid\left(0,\sigma_u^2\right)\forall i,t \\ &\varepsilon_{1it} = \alpha_0 + \upsilon_{1it} \\ &\varepsilon_{2it} = \alpha_0 + \upsilon_{2it}; \\ &\upsilon_{.it} \sim iid\left(0,\sigma_\upsilon^2\right) \end{aligned} \qquad \text{(A2.2)}$$

Then the OLS slope estimate in the regression of y_t on x_t and a constant, converges in probability to the inverse of the sample average of heterogeneous individual coefficients. ie, $\hat{\theta} \xrightarrow{p} \frac{1}{\overline{\beta}}; \;\; \overline{\beta} = n^{-1}\sum_{i=1}^{n}\beta_i$

Proof of the Theorem:

Rewrite x_t and y_t as follows. Firstly define

$$\upsilon_{1t} = \sum_{i=1}^{n} \upsilon_{1i(t)} \quad \upsilon_{2t} = \sum_{i=1}^{n} \upsilon_{2i(t)} \qquad \text{(A2.3)}$$

which can be viewed as iid innovations. Therefore,

$$
\begin{aligned}
y_t &= \alpha_0 n + \mu n t + \sum_{i=1}^{n}\sum_{j=1}^{t} u_{ij} + \upsilon_{1t} \\
x_t &= \alpha_0 n + \mu n t \bar{\beta} + \bar{\beta}\sum_{i=1}^{n}\sum_{j=1}^{t} u_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{t}\left(\beta_i - \bar{\beta}\right) u_{ij} + \upsilon_{2t} \qquad \text{(A2.4)} \\
\bar{\beta} &= n^{-1}\sum_{i=1}^{n}\beta_i
\end{aligned}
$$

Now draw attention to the terms involving u_{ij}. Let

$$
\begin{aligned}
u_t &= \sum_{i=1}^{n} u_{it} \\
\therefore \sum_{i=1}^{n}\sum_{j=1}^{t} u_{ij} &= \sum_{j=1}^{t} u_j = S_t
\end{aligned} \qquad \text{(A2.5)}
$$

where S_t has the interpretation of the partial sum of some iid innovations with zero mean. Let $r = t/T$ so that t = [Tr] is the integer part of Tr.

$$
T^{-1/2} S_{[Tr]} \xrightarrow{d} B_u(r) \qquad \text{(A2.6)}
$$

where $B_u(r)$ is a Brownian motion with zero mean and variance equal to $rn\sigma_u^2$.

Also define

$$
\begin{aligned}
\tilde{u}_t &= \sum_{i=1}^{n}\left(\beta_i - \bar{\beta}\right) u_{it} \\
\therefore \sum_{i=1}^{n}\sum_{j=1}^{t}\left(\beta_i - \bar{\beta}\right) u_{ij} &= \sum_{j=1}^{t} \tilde{u}_j = \tilde{S}_t
\end{aligned} \qquad \text{(A2.7)}
$$

which implies

$$
T^{-1/2} \tilde{S}_{[Tr]} \xrightarrow{d} \tilde{B}_u(r) \qquad \text{(A2.8)}
$$

This Brownian Motion has zero mean and variance equal to $rn\sigma_u^2\hat{\sigma}_\beta^2$ and $\hat{\sigma}_\beta^2 = n^{-1}\sum_{i=1}^{n}\left(\beta_i - \bar{\beta}\right)^2$ captures the degree of heterogeneity. Note that if all of the β_i are equal, then the distribution is degenerate.

Given (3), (5) and (7), we can rewrite x_t and y_t as follows:

$$
\begin{aligned}
y_t &= \alpha_0 n + \mu n t + S_t + \upsilon_{1t} \\
x_t &= \alpha_0 n + \mu n t \bar{\beta} + \bar{\beta} S_t + \tilde{S}_t + \upsilon_{2t}
\end{aligned} \qquad \text{(A2.9)}
$$

The OLS estimator in the cointegrating regression of y_t on a constant and x_t is

$$\hat{\theta} = \frac{\sum_{t=1}^{T} x_t y_t - T^{-1}\left(\sum_{t=1}^{T} x_t\right)\left(\sum_{t=1}^{T} y_t\right)}{\sum_{t=1}^{T} x_t^2 - T^{-1}\left(\sum_{t=1}^{T} x_t\right)^2} \qquad \text{(A2.10)}$$

Therefore we need to examine the asymptotic behaviour of each term on the right hand side of (10). Firstly, consider the sums.

$$\sum_{t=1}^{T} y_t = T\alpha_0 n + \mu n \sum_{t=1}^{T} t + T^{1/2}\sum_{t=1}^{T}\left(T^{-1/2}S_t\right) + \sum_{t=1}^{T} \upsilon_{1t}$$

$$= T\alpha_0 n + \mu n\left(\frac{T^2+T}{2}\right) + T^{3/2}\left(T^{-1}\sum_{t=1}^{T}\left(T^{-1/2}S_t\right)\right) + T\left(T^{-1}\sum_{t=1}^{T}\upsilon_{1t}\right)$$

$$\therefore T^{-2}\sum_{t=1}^{T} y_t = \underbrace{T^{-1}\alpha_0 n}_{\to 0} + \underbrace{\mu n\left(\frac{1+T^{-1}}{2}\right)}_{\to \mu n/2} + T^{-1/2}\underbrace{\left(T^{-1}\sum_{t=1}^{T}\left(T^{-1/2}S_t\right)\right)}_{\xrightarrow{d} \int_0^1 B_u(r)dr} + T^{-1}\underbrace{\left(T^{-1}\sum_{t=1}^{T}\upsilon_{1t}\right)}_{\xrightarrow{p} 0} \qquad \text{(A2.11)}$$

$$\Rightarrow T^{-2}\sum_{t=1}^{T} y_t \xrightarrow{p} \mu n/2$$

By similar reasoning,

$$T^{-2}\sum_{t=1}^{T} x_t \xrightarrow{p} \mu n\overline{\beta}/2 \qquad \text{(A2.12)}$$

Now, looking at the squares and cross products:

$$\sum_{t=1}^{T} x_t^2 = O(T^3) \qquad ; \qquad \sum_{t=1}^{T} x_t y_t = O(T^3) \qquad \text{(A2.13), (A2.14)}$$

$$\therefore T^{-3}\sum_{t=1}^{T} x_t^2 = \frac{1}{3}(\mu n\overline{\beta})^2 \qquad \therefore T^{-3}\sum_{t=1}^{T} x_t y_t = \frac{1}{3}(\mu n)^2\overline{\beta}$$

Therefore we return to the OLS estimator:

$$\hat{\theta} = \frac{T^{-3}\sum_{t=1}^{T} x_t y_t - \left(T^{-2}\sum_{t=1}^{T} x_t\right)\left(T^{-2}\sum_{t=1}^{T} y_t\right)}{T^{-3}\sum_{t=1}^{T} x_t^2 - \left(T^{-2}\sum_{t=1}^{T} x_t\right)^2} \qquad \text{(A2.15)}$$

Now substitute the limiting quantities in (11), (12), (13) and (14) into (15), and

$$\hat{\theta} \xrightarrow{p} \frac{1}{\overline{\beta}} \qquad \text{(A2.16)}$$

Q.E.D.

Appendix 3. Divergence rate of the Variance of the Residual.

Consider residual in the linear combination of the following:

$$e_t = y_t - \frac{1}{\overline{\beta}} x_t \tag{A3.1}$$

then

$$e_t = \alpha_0 n \left(1 - \frac{1}{\overline{\beta}}\right) + \upsilon_{1t} - \frac{1}{\overline{\beta}} \upsilon_{2t} - \frac{1}{\overline{\beta}} \tilde{S}_t \tag{A3.2}$$

and so

$$T^{-\frac{1}{2}} e_t \xrightarrow{d} \frac{1}{\overline{\beta}} \tilde{B}_u(r) \tag{A3.3}$$

where

$$\frac{1}{\overline{\beta}} \tilde{B}_u(r) \sim N\left(0, rn\sigma_u^2 \left(\frac{\hat{\sigma}_\beta^2}{\overline{\beta}^2}\right)\right) \tag{A3.4}$$

Thus the variance of the residual term is exploding at a rate which is proportional to the product of the variance of the u_{it}'s (the data) and the sample coefficient of variation of the β_i's (the degree of heterogeneity). On inspection, the variance of the normalised residual will collapse if and only if all of the β_i's are equal, that is Case D in our data generating processes, which is the only one which is formally cointegrated. Note also that the variance is proportional to the number of individuals which comprise the series'. In the context of testing for cointegration in the presence of heterogeneous individuals, the normalised residuals have a zero mean across all time, but exploding variance. The rate at which the variance explodes is crucial because the tests must be able to detect the increasing variance in a finite sample. We expect tests for cointegration using data generated under Case Ac to accept cointegration most often because the coefficient of variation will be close to 0.0001 since β_0 = 10 and s = 0.1. By similar argument, we expect cases Aa and Ad to reject cointegration most often since $\beta_0 = 0$ (so that $\overline{\beta}$ will be close to zero).

Appendix 4. Simulation Results

T = **25**: Number of replications which reject the null hypothesis at **1%**

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	30	813	827	99	276	805	754	126	812
ADF - no trend	0	79	65	7	10	65	67	5	83
DF - with trend	76	453	674	107	206	647	416	191	465
ADF - with trend	3	10	38	6	9	30	8	8	11
Johansen (k=2)	183	181	182	85	90	181	171	72	186
Johansen (k=3)	106	110	112	74	77	122	97	84	119
Johansen (k=5)	254	301	295	276	306	296	279	287	303
CI	5	2	0	12	1	0	1	0	0
CI_mu	431	0	0	76	0	0	0	2	0
CI_tau	40	21	6	33	31	5	16	29	16
C	23	0	0	3	0	0	0	1	0
C_mu	93	0	0	25	0	0	0	1	0
C_tau	1	0	0	0	0	0	0	0	0

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	0	54	48	1	7	42	45	5	56
ADF - no trend	0	13	13	1	5	16	14	1	13
DF - with trend	0	12	20	0	4	16	7	3	11
ADF - with trend	0	0	3	1	2	4	4	2	0
Johansen (k=2)	298	287	294	177	170	295	262	171	288
Johansen (k=3)	65	65	83	79	73	79	71	78	69
Johansen (k=5)	307	308	336	334	344	346	308	372	321
CI	6	0	0	29	4	0	0	21	0
CI_mu	390	0	0	195	25	0	0	91	0
CI_tau	168	7	2	124	58	2	9	93	5
C	3	0	0	4	0	0	0	7	0
C_mu	114	0	0	44	1	0	0	10	0
C_tau	34	1	1	14	9	0	0	11	1

Appendix 4. Simulation Results

T = **25**: Number of replications which reject the null hypothesis at **5%**

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	112	963	970	278	578	963	936	324	964
ADF - no trend	0	266	268	30	78	238	217	46	278
DF - with trend	236	780	901	306	475	890	726	435	791
ADF - with trend	14	59	136	23	30	111	53	40	57
Johansen (k=2)	447	419	431	221	246	415	401	208	435
Johansen (k=3)	283	305	328	205	243	319	287	233	317
Johansen (k=5)	481	492	518	507	508	506	483	504	493
CI	11	39	67	78	79	79	75	51	26
CI_mu	972	0	0	248	0	0	0	5	0
CI_tau	349	178	61	303	276	67	222	254	177
C	81	0	0	13	0	0	0	1	0
C_mu	184	0	0	70	0	0	0	2	0
C_tau	15	2	4	6	0	3	0	1	1

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	8	254	270	24	54	256	229	29	278
ADF - no trend	0	91	81	17	22	63	84	18	89
DF - with trend	16	79	123	16	31	117	74	19	88
ADF - with trend	2	17	35	6	6	35	15	6	16
Johansen (k=2)	579	550	611	393	428	608	525	394	554
Johansen (k=3)	212	214	234	202	204	226	216	218	212
Johansen (k=5)	535	531	549	538	576	555	527	598	526
CI	27	16	21	82	33	26	16	65	12
CI_mu	891	0	1	419	47	1	1	153	0
CI_tau	521	148	102	407	316	108	171	365	146
C	5	0	0	8	0	0	0	12	0
C_mu	271	0	0	86	4	0	0	25	0
C_tau	83	7	4	38	26	6	4	29	7

Appendix 4. Simulation Results

T = **50**: Number of replications which reject the null hypothesis at **1%**

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	483	1000	999	258	526	999	998	209	1000
ADF - no trend	19	529	568	21	38	482	392	18	579
DF - with trend	144	989	999	254	523	998	968	388	992
ADF - with trend	3	124	365	7	22	305	73	13	135
Johansen (k=2)	681	673	712	77	127	651	572	77	707
Johansen (k=3)	253	313	317	50	76	285	234	58	345
Johansen (k=5)	112	111	129	60	73	130	83	59	122
CI	0	11	3	10	12	5	42	1	2
CI_mu	733	0	0	55	0	0	0	2	0
CI_tau	32	71	5	31	81	16	104	55	56
C	61	0	0	5	0	0	0	1	0
C_mu	93	0	0	69	1	0	0	1	0
C_tau	2	0	0	3	4	0	3	4	0

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	55	603	656	9	26	565	438	3	653
ADF - no trend	3	135	164	4	7	132	85	3	164
DF - with trend	1	234	373	2	9	308	155	6	267
ADF - with trend	0	32	80	1	2	54	17	0	31
Johansen (k=2)	670	698	750	119	149	691	568	163	728
Johansen (k=3)	78	80	95	28	22	85	73	32	92
Johansen (k=5)	61	67	91	50	43	74	68	51	76
CI	0	0	0	21	2	0	0	11	0
CI_mu	313	0	0	163	27	0	0	88	0
CI_tau	148	8	5	128	28	9	16	85	10
C	0	0	0	9	2	0	0	4	0
C_mu	119	0	0	62	10	0	0	13	0
C_tau	78	0	0	37	4	0	0	17	1

Appendix 4. Simulation Results

T = **50**: Number of replications which reject the null hypothesis at **5%**

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	612	1000	1000	453	763	999	1000	423	1000
ADF - no trend	72	829	859	60	132	797	715	78	877
DF - with trend	329	1000	999	475	734	999	999	626	1000
ADF - with trend	20	372	695	36	86	629	279	70	395
Johansen (k=2)	925	918	913	251	345	882	847	231	929
Johansen (k=3)	597	621	661	162	203	613	541	174	652
Johansen (k=5)	319	334	367	194	216	343	302	189	352
CI	9	89	576	121	158	572	201	62	43
CI_mu	986	1	0	181	0	0	6	5	0
CI_tau	387	284	49	329	396	99	379	352	265
C	137	0	0	18	0	0	0	2	0
C_mu	186	0	0	145	1	0	0	3	0
C_tau	61	20	9	41	66	18	40	57	11

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	185	936	947	47	120	905	826	25	952
ADF - no trend	20	443	481	24	48	415	304	18	494
DF - with trend	11	674	805	21	72	747	525	35	704
ADF - with trend	6	161	284	20	24	239	112	14	177
Johansen (k=2)	887	900	928	308	385	898	819	377	929
Johansen (k=3)	269	283	294	90	106	270	227	105	302
Johansen (k=5)	215	258	261	154	148	253	211	168	280
CI	3	49	186	61	28	201	73	55	27
CI_mu	937	0	0	306	43	0	0	159	0
CI_tau	491	134	69	413	259	107	190	342	114
C	5	0	0	18	3	0	0	14	0
C_mu	460	0	0	127	15	0	0	40	0
C_tau	151	23	11	85	41	17	32	56	26

Appendix 4. Simulation Results

T = **100**: Number of replications which reject the null hypothesis at **1**%

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	820	1000	1000	361	684	1000	1000	267	1000
ADF - no trend	420	990	1000	18	73	967	816	23	997
DF - with trend	247	1000	1000	392	717	1000	1000	547	1000
ADF - with trend	4	849	985	11	50	920	561	16	915
Johansen (k=2)	992	1000	1000	91	204	999	970	102	1000
Johansen (k=3)	887	926	977	42	74	878	714	43	972
Johansen (k=5)	365	465	566	23	34	388	246	38	573
CI	18	63	140	12	15	191	253	1	9
CI_mu	809	0	0	34	0	1	0	2	0
CI_tau	55	178	13	54	161	94	359	52	118
C	72	0	0	17	0	0	0	1	0
C_mu	66	0	0	148	0	0	0	3	0
C_tau	9	17	0	7	22	30	83	6	3

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	561	999	999	6	51	995	942	2	1000
ADF - no trend	163	766	907	5	8	660	390	4	901
DF - with trend	4	981	999	8	60	981	840	16	996
ADF - with trend	1	464	695	5	5	485	194	3	582
Johansen (k=2)	986	995	1000	157	247	995	933	159	1000
Johansen (k=3)	418	498	615	13	18	422	245	18	610
Johansen (k=5)	171	230	350	20	18	200	101	31	301
CI	3	19	4	23	3	18	7	13	3
CI_mu	503	0	0	143	21	0	0	101	0
CI_tau	121	44	7	88	23	51	75	62	14
C	9	0	0	5	3	0	0	15	0
C_mu	254	0	0	62	5	0	0	34	0
C_tau	65	5	0	35	6	8	11	20	3

Appendix 4. Simulation Results

T = **100**: Number of replications which reject the null hypothesis at **5%**

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	853	1000	1000	565	832	1000	1000	501	1000
ADF - no trend	569	999	1000	96	188	994	952	91	1000
DF - with trend	457	1000	1000	604	878	1000	1000	746	1000
ADF - with trend	32	981	1000	66	172	991	825	89	994
Johansen (k=2)	1000	1000	1000	262	421	1000	992	260	1000
Johansen (k=3)	978	997	1000	137	214	982	919	148	999
Johansen (k=5)	698	767	869	97	114	683	518	132	859
CI	214	205	1000	142	176	997	457	44	45
CI_mu	973	13	2	122	2	39	116	5	1
CI_tau	393	472	52	331	503	245	634	417	384
C	262	0	0	40	0	0	0	1	0
C_mu	156	2	0	257	2	9	8	3	1
C_tau	132	71	25	107	200	91	247	145	28

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	685	1000	1000	48	172	1000	994	28	1000
ADF - no trend	342	964	991	35	59	907	712	21	992
DF - with trend	24	1000	1000	41	166	999	966	68	1000
ADF - with trend	9	811	962	23	40	802	493	27	908
Johansen (k=2)	998	1000	1000	363	470	998	988	374	1000
Johansen (k=3)	741	820	891	58	81	746	548	72	901
Johansen (k=5)	449	512	656	81	94	470	299	95	630
CI	100	156	939	64	30	785	229	62	39
CI_mu	925	0	0	275	43	0	0	154	0
CI_tau	428	210	63	360	241	206	313	315	123
C	134	0	0	17	5	0	0	20	0
C_mu	742	0	0	126	20	0	0	77	0
C_tau	196	105	30	124	68	87	158	104	54

Appendix 4. Simulation Results

T = **250**: Number of replications which reject the null hypothesis at **1%**

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	963	1000	1000	428	725	1000	1000	321	1000
ADF - no trend	868	1000	1000	27	87	999	973	17	1000
DF - with trend	409	1000	1000	465	818	1000	1000	632	1000
ADF - with trend	6	1000	1000	19	91	999	956	27	1000
Johansen (k=2)	1000	1000	1000	136	286	1000	1000	108	1000
Johansen (k=3)	999	1000	1000	45	105	998	967	37	1000
Johansen (k=5)	895	986	1000	23	28	917	585	25	1000
CI	563	407	1000	11	13	984	529	0	10
CI_mu	792	88	0	20	1	163	181	1	0
CI_tau	95	513	17	52	198	524	752	69	160
C	78	0	0	17	1	0	0	0	0
C_mu	46	18	0	190	3	86	10	3	0
C_tau	7	243	9	9	78	391	492	30	8

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	876	1000	1000	14	91	1000	995	4	1000
ADF - no trend	743	997	1000	3	9	974	772	4	1000
DF - with trend	28	1000	1000	9	107	1000	989	24	1000
ADF - with trend	2	995	1000	5	8	962	707	7	1000
Johansen (k=2)	1000	1000	1000	167	286	1000	991	167	1000
Johansen (k=3)	952	989	1000	11	18	938	697	10	1000
Johansen (k=5)	698	869	999	18	21	700	309	15	999
CI	299	209	992	23	3	652	134	8	5
CI_mu	490	0	0	136	26	0	0	84	0
CI_tau	92	274	19	103	25	314	327	56	28
C	262	0	0	17	5	0	0	25	0
C_mu	424	0	0	76	9	0	0	42	0
C_tau	61	191	6	48	12	213	216	30	22

Appendix 4. Simulation Results

T = **250**: Number of replications which reject the null hypothesis at **5%**

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	969	1000	1000	609	862	1000	1000	534	1000
ADF - no trend	893	1000	1000	107	194	1000	993	81	1000
DF - with trend	615	1000	1000	656	911	1000	1000	777	1000
ADF - with trend	36	1000	1000	85	216	1000	986	116	1000
Johansen (k=2)	1000	1000	1000	310	494	1000	1000	285	1000
Johansen (k=3)	1000	1000	1000	165	257	1000	991	142	1000
Johansen (k=5)	964	997	1000	116	145	978	803	109	1000
CI	775	565	1000	142	200	1000	690	59	41
CI_mu	954	267	2	66	11	432	592	7	1
CI_tau	442	749	63	333	552	726	913	414	451
C	447	0	0	35	1	0	0	3	0
C_mu	112	179	0	318	9	324	290	6	1
C_tau	187	443	42	184	338	586	720	262	48

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	895	1000	1000	75	240	1000	1000	32	1000
ADF - no trend	808	1000	1000	25	63	996	927	27	1000
DF - with trend	85	1000	1000	43	233	1000	998	99	1000
ADF - with trend	15	999	1000	23	51	995	869	31	1000
Johansen (k=2)	1000	1000	1000	377	516	1000	997	360	1000
Johansen (k=3)	981	998	1000	58	90	987	870	66	1000
Johansen (k=5)	879	961	1000	76	85	864	561	69	1000
CI	642	419	1000	63	35	978	458	50	44
CI_mu	824	108	1	233	39	130	49	153	0
CI_tau	408	539	80	389	241	566	646	289	149
C	595	0	0	34	7	0	0	36	0
C_mu	744	36	0	146	18	41	9	74	0
C_tau	250	422	48	205	120	457	525	147	113

Appendix 4. Simulation Results

T = **500**: Number of replications which reject the null hypothesis at **1%**

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	982	1000	1000	468	760	1000	1000	313	1000
ADF - no trend	943	1000	1000	38	88	998	983	35	1000
DF - with trend	565	1000	1000	485	839	1000	1000	674	1000
ADF - with trend	24	1000	1000	25	111	998	978	40	1000
Johansen (k=2)	1000	1000	1000	134	297	1000	1000	118	1000
Johansen (k=3)	999	1000	1000	50	110	999	973	38	1000
Johansen (k=5)	960	997	1000	22	40	959	706	19	1000
CI	867	632	1000	7	30	999	702	1	9
CI_mu	848	431	0	19	0	558	540	1	0
CI_tau	211	839	31	63	257	853	926	68	206
C	168	0	0	35	1	0	0	0	0
C_mu	61	315	0	230	2	498	188	3	0
C_tau	24	649	29	31	147	798	800	36	11

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	954	1000	1000	20	109	1000	999	11	1000
ADF - no trend	881	999	1000	5	15	991	854	5	1000
DF - with trend	157	1000	1000	11	141	1000	996	27	1000
ADF - with trend	9	1000	1000	2	13	990	828	6	1000
Johansen (k=2)	1000	1000	1000	184	288	1000	996	160	1000
Johansen (k=3)	991	1000	1000	12	20	991	806	9	1000
Johansen (k=5)	891	984	1000	11	13	877	434	17	1000
CI	640	491	1000	16	4	858	254	13	11
CI_mu	484	92	0	110	21	49	1	118	0
CI_tau	112	584	32	98	30	622	539	74	30
C	549	0	0	17	3	0	0	24	0
C_mu	445	44	0	69	14	22	0	59	0
C_tau	83	531	21	46	16	564	473	43	22

Appendix 4. Simulation Results

T = **500**: Number of replications which reject the null hypothesis at **5%**

Dataset 1

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	984	1000	1000	670	894	1000	1000	530	1000
ADF - no trend	955	1000	1000	116	225	1000	997	107	1000
DF - with trend	743	1000	1000	685	925	1000	1000	827	1000
ADF - with trend	74	1000	1000	87	234	1000	993	133	1000
Johansen (k=2)	1000	1000	1000	327	503	1000	1000	315	1000
Johansen (k=3)	1000	1000	1000	156	241	999	994	136	1000
Johansen (k=5)	983	999	1000	93	113	988	854	85	1000
CI	913	766	1000	100	216	1000	823	50	49
CI_mu	972	662	8	53	7	779	864	2	0
CI_tau	555	946	109	394	619	948	986	417	516
C	609	2	0	78	1	9	0	0	0
C_mu	127	582	2	340	6	741	661	5	0
C_tau	256	819	83	256	491	917	938	325	61

Dataset 2

	Aa	*Ab*	*Ac*	*Ad*	*Ae*	*Af*	*B*	*C*	*D*
DF - no trend	963	1000	1000	81	254	1000	1000	33	1000
ADF - no trend	909	1000	1000	38	64	997	942	27	1000
DF - with trend	278	1000	1000	51	277	1000	999	107	1000
ADF - with trend	46	1000	1000	15	51	997	931	32	1000
Johansen (k=2)	1000	1000	1000	390	523	1000	999	372	1000
Johansen (k=3)	998	1000	1000	60	93	997	924	64	1000
Johansen (k=5)	960	996	1000	72	77	956	658	72	1000
CI	819	670	1000	67	43	967	573	78	56
CI_mu	814	461	6	203	38	467	216	182	0
CI_tau	410	798	99	363	280	829	824	308	136
C	766	0	0	34	9	0	0	41	0
C_mu	782	394	2	143	25	397	152	111	1
C_tau	312	753	80	250	194	784	795	201	116

Appendix 5. Distributions of the OLS slope estimator in selected cases

Figure 1: Frequency distribution of the $\hat{\beta}_{OLS}$ values when the DF test rejects H_0.

Case Aa, dataset 1, 5%, sample size: 500

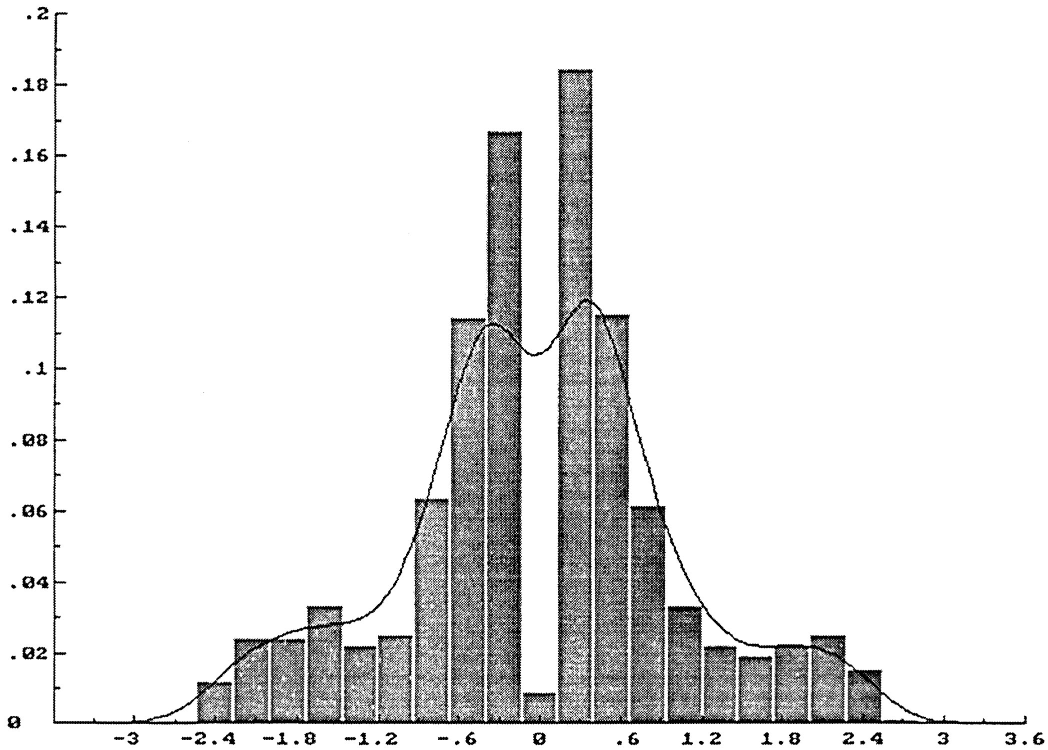

Figure 2: Frequency distribution of the $\hat{\beta}_{OLS}$ values when the DF test rejects H_0.

Case Aa, dataset 1, 5%, sample size: 250

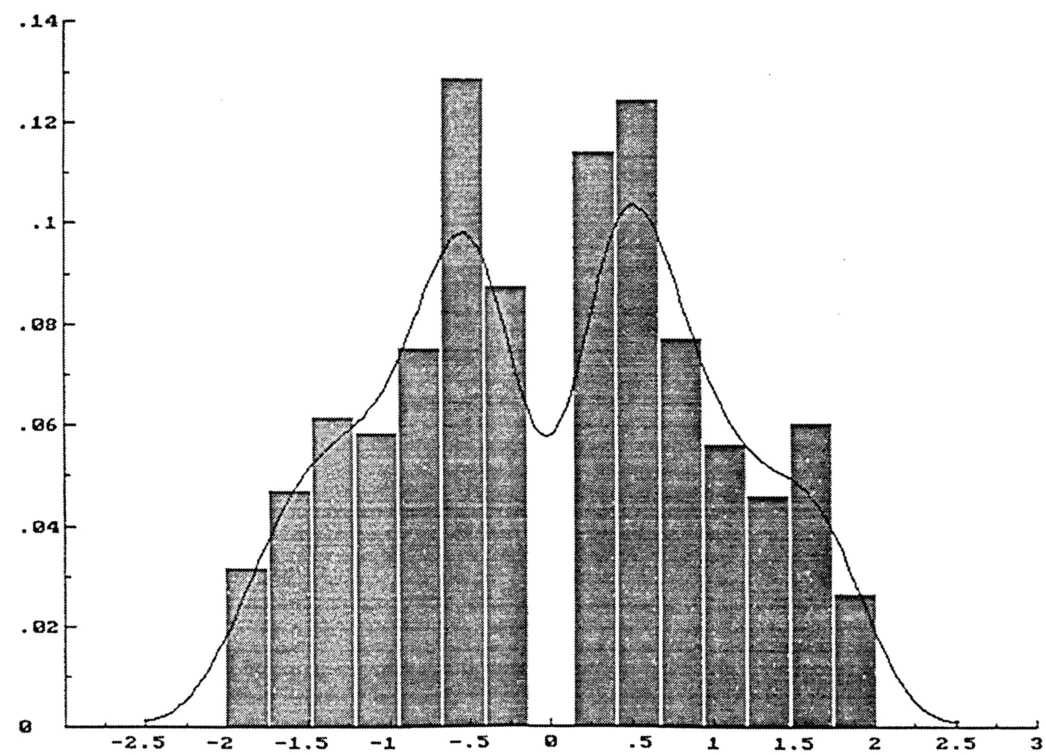

Figure 3: Frequency distribution of the $\hat{\beta}_{OLS}$ values when the DF test rejects H_0.

Case Aa, dataset 1, 5%, sample size: 100

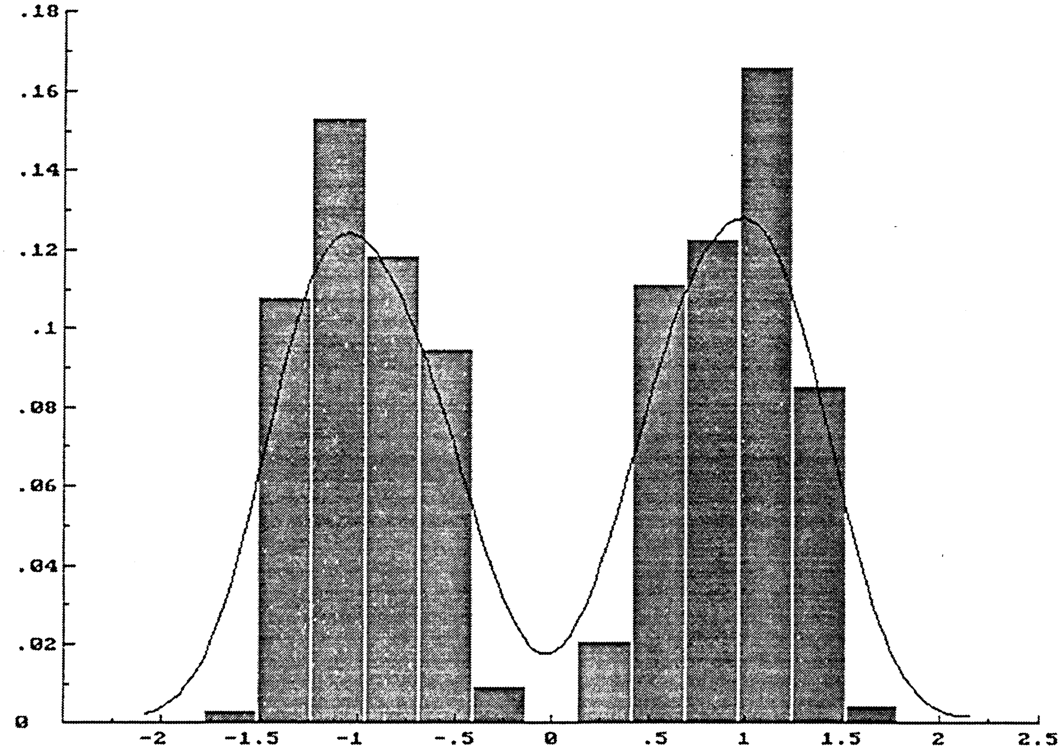

Figure 4: Frequency distribution of the $\hat{\beta}_{OLS}$ values when the DF test rejects H_0.

Case Ab, dataset 1, 5%, sample size: 250

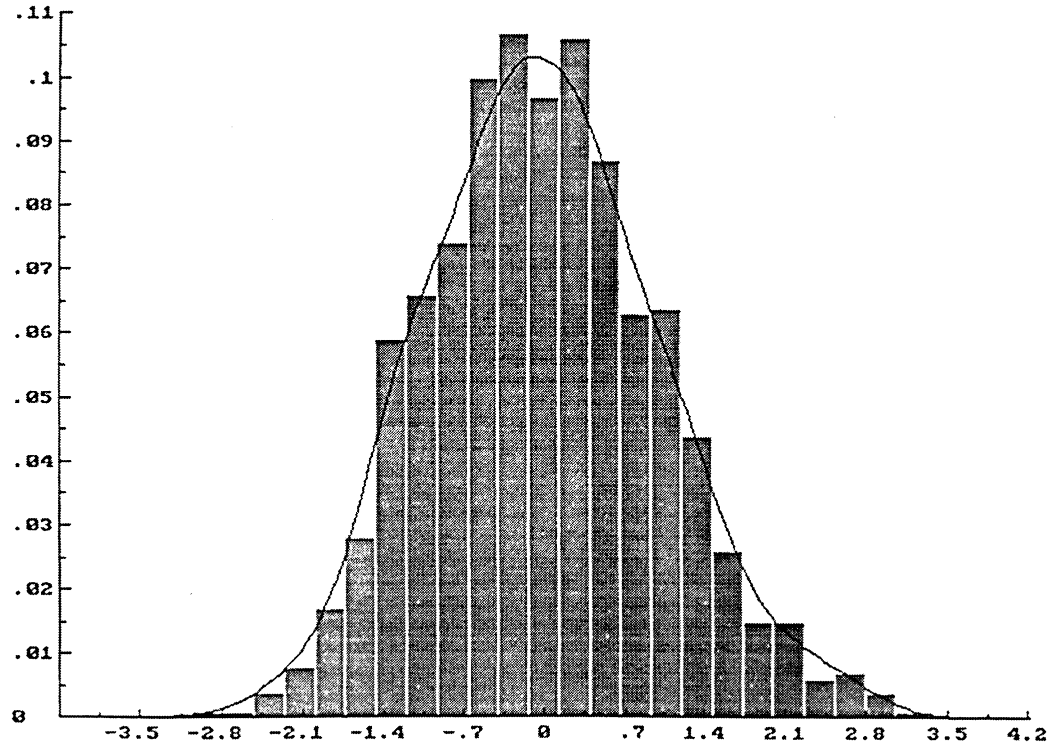

Figure 5: Frequency distribution of the $\hat{\beta}_{OLS}$ values when the DF test rejects H_0.

Case Ae, dataset 1, 5%, sample size: 250