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THE STOCHASTIC SPECIFICATION OF ATTRACTION MODELS

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The stochastic specification of attraction models.

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Abstract

Estimation of attraction models in marketing typically involves the use of the log-centering transformation. The resultant estimating equations are then linear in the parameters. The log-centering transformation also appears in the statistical analysis of compositional data (CODA). CODA techniques are applied to data on "shares" in a wide variety of disciplines. This paper uses CODA techniques to rationalize the stochastic specification of attraction models. It further shows that another transformation from CODA, the log-ratio transform, can yield simpler estimating equations. The results are illustrated using an empirical example.

J.E.L. Classification: M31, C39, C13

Keywords: compositional data analysis, attraction models.

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1. Introduction.

Attraction models for brand or market share analysis in marketing have the advantage that they automatically bound shares to lie between zero and one and to sum to one over all M brands in the market. That is they are logically consistent. In estimating attraction models the traditional approach is to apply a log-centering transform to the share data and then to apply least squares estimation techniques. Unfortunately, the properties of the implied error (stochastic) term in this approach are complicated. In this paper we consider the use of a modeling technique from the statistical literature, compositional data analysis, to specify the stochastic structure of attraction models.

The plan of the rest of this paper is as follows: Section 2 describes the compositional data approach to modeling share data, applies this approach to the specification of attraction models, contrasts this with the traditional approach and also discusses the identification issues that arise in modeling share data. Section 3 contains a brief case study to illustrate the application of the proposed modeling approach and, finally, section 4 contains some concluding remarks.

2. Model Specification.

2.1. Compositional Data Analysis.

The restriction of shares to the unit simplex has been recognized by researchers in many fields (see *inter alia* Aitchison (1986), Fry *et al* (1996a) and McLaren *et al* (1995)). In particular, this restriction causes problems for traditional multivariate statistical methods which are based upon the Normal distribution. It is, however, possible to develop a framework for the statistical analysis of data on shares. Such techniques are termed compositional data analysis, hereafter CODA, (Aitchison (1986)). The advantage of CODA techniques is that they provide a unifying set of distributional assumptions which allow for the use of traditional multivariate statistical methods.

In the statistical literature a *composition* consists of M *parts.* The parts are labels which identify the components into which a total has been sub-divided (*e.g.* the parts are brands and the total is total market volume sales). The *components* are the numerical proportions in which the parts appear (*i.e.* the shares). A composition is defined by taking the elements of a *basis* (*e.g.* individual brand volume sales) and dividing them by the *size* of the basis (*e.g.* total market volume sales). This operation takes elements defined as non-negative and constrains them to lie between zero and one and to sum to one (*i.e.* to lie on the unit simplex, S^{M-1}). It should be noted that this unit sum constraint reduces the dimension of the space on which the vector of components (shares) is defined to $M-1$. The major obstacle to the statistical analysis of compositional data is that the restriction to the unit simplex necessarily leads to the lack of an interpretable (covariance) structure and, as a result, the multivariate Normal distribution is inappropriate.

In order to apply statistical analysis techniques based upon the Normal distribution a one-to-one transformation is required to map the data on shares to data suitable for analysis using multivariate Normal based techniques. That is we need to map from the unit simplex, S^{M-1}, to R^{M-1} and produce an interpretable (covariance) structure. One such transformation is the additive log-ratio (ALR) transform:

$$y_i = \ln\left(\frac{s_i}{s_M}\right), \; i = 1, \ldots, M-1$$

with an associated Jacobian given by $jac(\mathbf{y} \mid \mathbf{s}) = (s_1 \ldots s_M)^{-1}$.

The inverse transformation, S^{M-1} to R^{M-1}, is the additive logistic transform and reconstructs the components as:

$$\begin{aligned} s_i &= \frac{\exp(y_i)}{1+\exp(y_1)+\ldots+\exp(y_{M-1})}, \quad i = 1, \ldots, M-1, \\ s_M &= \frac{1}{1+\exp(y_1)+\ldots+\exp(y_{M-1})} \\ &= 1 - s_1 - \ldots - s_{M-1}. \end{aligned}$$

These transformations form the heart of CODA techniques. To model compositional data we apply the ALR transform to produce log-ratio data and then apply traditional multivariate statistical techniques (*e.g.* multivariate regression) to the transformed data. To return to the composition we simply apply the inverse transform, the additive logistic.

A major benefit of this approach is that it is straightforward to derive the associated distribution theory (Aitchison (1986) pp. 115-119) for the random variables. In particular, if the log-ratio vector $\mathbf{y}$ has an $M-1$ dimension Normal distribution, $N(\boldsymbol{\mu}, \Sigma)$, then the composition, s, (the vector of shares) will follow an additive logistic normal distribution, $L(\boldsymbol{\mu}, \Sigma)$, defined on the unit simplex. The additive logistic normal distribution is particularly attractive in that, like the Normal distribution, it is capable of capturing the wide range of covariance structures encountered in observed data. Additionally within the CODA framework it can be shown that the basis $\mathbf{q}$ (*e.g.* the vector of brand volume sales) will follow a multivariate log-Normal distribution.

Before discussing the application of CODA techniques to attraction models for brand shares three further points need to be made. Firstly, the use of s_M as the denominator in the ALR transform is, at first, unusual in that the parts of the composition are treated asymmetrically. It is important, however, to note that reordering the parts and changing the component used as the denominator in the transform makes no difference to any statistical procedures. Thus all statistical procedures are invariant to the choice of the component used as the denominator. Secondly, the ALR is not the only transform that could be used. In particular, a centered log-ratio transform could be used and, as is discussed below, this centered version is related to the approach currently undertaken in the stochastic specification of attraction models. Finally, CODA techniques can also be used to specify models for the joint modeling of data on shares and the size of the basis (see Aitchison (1986) p. 220) and can easily be modified to deal with the case of observed shares which are zero (see Fry *et al* (1996b)).

2.2. Attraction Models.

A direct application of the CODA approach would involve modeling the log-ratio transformed data, $\mathbf{y}$, in terms of $\boldsymbol{\mu}$, and Σ. In particular, we may parameterize the mean, $\boldsymbol{\mu}$, to depend upon a set of (marketing) variables, $\mathbf{Z}$ and a set of parameters, $\boldsymbol{\theta}$, according to a multivariate regression model:

$$y_i = \ln\left(\frac{s_i}{s_M}\right) = \mu_i(\mathbf{Z}, \boldsymbol{\theta}) + u_i,$$

where $\mathbf{u} = [u_i]$ is a stochastic term which is distributed as multivariate Normal $(\mathbf{0}, \Sigma)$. The advantage of this model is that, within this framework, the shares are distributed as additive logistic normal and the basis as multivariate log-Normal. The remaining issue is the specification of the functional form for the $\mu_i(\mathbf{Z},\boldsymbol{\theta})$. By analogy with the arguments in Fry *et al* (1996a), the parameterization chosen should retain any parameter interpretations from the underlying (marketing) theory and, further, it should retain the logical consistency argument that shares from the model are restricted to the unit simplex. Such a parameterization is given by:

$$y_i = \ln\left(\frac{S_i(\mathbf{Z},\boldsymbol{\theta})}{S_M(\mathbf{Z},\boldsymbol{\theta})}\right) + u_i$$

where $S_i(\mathbf{Z},\boldsymbol{\theta})$ is a specification for the share of brand i which retains the logical consistency requirement. An appropriate choice is given by Kotler's (1984) *market share as share of marketing effort* representation - also rationalized in the attraction modeling literature (see Cooper (1993)) - in which:

$$S_i(\mathbf{Z},\boldsymbol{\theta}) = \frac{A_i}{\sum_{j=1}^{M} A_j} = \frac{A_i(\mathbf{Z},\boldsymbol{\theta})}{\sum_{j=1}^{M} A_j(\mathbf{Z},\boldsymbol{\theta})},$$

with $A_i(\mathbf{Z},\boldsymbol{\theta}) \geq 0 \; \forall \; i, \; \sum_i A_i(\mathbf{Z},\boldsymbol{\theta}) > 0$.

The estimating equations from this model specification are given by:

$$y_i = \ln(A_i(\mathbf{Z},\boldsymbol{\theta})) - \ln(A_M(\mathbf{Z},\boldsymbol{\theta})) + u_i.$$

The exact form of the equations will depend upon whether the marketing variables in $\mathbf{Z}$ enter into the model in a multiplicative competitive interaction (MCI) or a multinomial logit (MNL) form or a combination of the two. For example, if all the variables enter in the MCI form, $A_i(\mathbf{Z},\boldsymbol{\theta}) = \exp(\alpha_i) \times \prod_k Z_k^{\beta_{ik}}$,and, if they all enter in the MNL form,

$A_i(\mathbf{Z}, \boldsymbol{\theta}) = \exp(\alpha_i) \times \exp(\sum_k \beta_{ik} Z_k)$.

To illustrate the application of the CODA methodology we will consider two simple cases. First where p variables enter an attraction model (see Cooper (1993) pp. 285-290) in the MCI form. In this case the estimating equations are given by:

$$y_i = (\alpha_i - \alpha_M) + \sum_{k=1}^{p} (\beta_{ik} - \beta_{Mk}) \ln(Z_k) + u_i.$$

The second example is where the p variables enter a fully extended attraction model in an MNL form. In this case the estimating equations are:

$$y_i = (\alpha_i - \alpha_M) + \sum_{k=1}^{p} (\beta_{ik} - \beta_{Mk}) Z_k + u_i.$$

The extension to models which include a mixture of MCI and MNL terms is straightforward. It is also simple to show that if the attraction of a brand depends upon its attraction in the previous period (Cooper and Nakanishi (1988) Ch. 3) the CODA estimating equations are given by:

$$y_i = \ln(A_i(\mathbf{Z}, \boldsymbol{\theta})) - \ln(A_M(\mathbf{Z}, \boldsymbol{\theta})) + \phi \ln(y_i(-1)) + u_i.$$

It is convenient to compare the CODA approach with that traditionally taken in the estimation of attraction models as detailed in, *inter alia*, Cooper and Nakanishi (1988, Ch. 3 and Ch. 5) and Cooper (1993). To facilitate estimation the data is transformed using a centered log-ratio transform, $s_i^* = \ln(s_i/\tilde{s})$, where $\tilde{s}$ is the geometric mean of the M shares. This also yields estimating equations that are linear in the parameters and amenable to estimation by least squares methods. However, three problems are associated with this approach. Firstly, a relatively small problem is that the estimating equations involve deviations of variables from their (geometric) means. Secondly, there exists an identification problem in the estimation of the parameters. We deal with this identification problem in the next sub-section. Thirdly, and perhaps most importantly, the stochastic properties of the error term in the traditional estimating equations are complex (see Bultez and Naert (1975), Cooper and Nakanishi (1988), Ghosh *et al* (1984), McGuire *et al* (1977) and Nakanishi and Cooper (1974), (1982)).

The problem with the stochastic specification in the traditional approach stems from the fact that a stochastic component is typically incorporated in a multiplicative manner in the functional form for $A_i(\mathbf{Z}, \boldsymbol{\theta})$. In particular, a stochastic term is entered multiplicatively as $\exp(\varepsilon_i)$ into the specification yielding an expression for the "attraction of brand i, $A_i^\dagger(\mathbf{Z}, \boldsymbol{\theta})$ [$= \exp(\varepsilon_i) \times A_i(\mathbf{Z}, \boldsymbol{\theta})$]. Applying the centered log-ratio transformation in an MCI formulation yields:

$$s_i^* = (\alpha_i - \bar{\alpha}) + \sum_{k=1}^{p} \beta_{ik} \left(\ln(Z_k) - \ln\left(\tilde{Z}\right) \right) + \left(\ln(\varepsilon_i) - \ln(\tilde{\varepsilon}) \right),$$

and in a MNL formulation the estimating form is:

$$s_i^* = (\alpha_i - \bar{\alpha}) + \sum_{k=1}^{p} \beta_{ik} \left(Z_k - \bar{Z} \right) + (\varepsilon_i - \bar{\varepsilon}).$$

The properties of the error term in these estimating equations are that the errors are correlated across equations and, since there are M equations, the appropriate covariance matrix is not of full rank. This seriously complicates least squares or maximum likelihood estimation. This problem is the same as that which confronts researchers in economics when using Logit or Addilog forms for allocation models (see inter alia Bewley (1982a), (1982b), (1986) and Chavas and Segerson (1986) and thus appropriate solutions do exist to estimate these equations by either generalized least squares or maximum likelihood.

This traditional approach seriously complicates the distribution theory. A simpler approach is to incorporate the stochastic assumptions within the CODA approach. In particular, we argue, as before, that shares are distributed as additive logistic normal and then apply an appropriate transformation. This idea can be "grafted onto" the traditional approach since the centered log-ratio transform can also be used in CODA modeling. This would allow traditional market modelers to justify a simpler form for the error term in their estimating equations. In fact, since there is a one-to-one mapping between the centered log-ratio form and our preferred log-ratio approach, the two CODA approaches are identical (see Aitchison (1986) and McLaren et al (1995)). Thus the choice of one over another is purely a matter of convenience. Since, even within the CODA approach, the rank problem remains when using the centered log-ratio transformation it is our opinion that the log-ratio transform is preferable. Before discussing an empirical example of the application of the CODA approach we next discuss the topic of uniquely identifying the parameters in the underlying attraction model.

2.3. Identification Issues.

The fact that there are identification issues involved in the use of share equations of the form:

$$S_i(\mathbf{Z}, \boldsymbol{\theta}) = \frac{A_i(\mathbf{Z}, \boldsymbol{\theta})}{\sum_{j=1}^{M} A_j(\mathbf{Z}, \boldsymbol{\theta})},$$

has long been recognized (Theil (1969)). In particular, if we re-scale by multiplying by an arbitrary, non-zero, constant, say, $\exp(q)$, we find:

$$S_i(\mathbf{Z}, \boldsymbol{\theta}) = \frac{\exp(q) A_i(\mathbf{Z}, \boldsymbol{\theta})}{\sum_{j=1}^{M} \exp(q) A_j(\mathbf{Z}, \boldsymbol{\theta})} = \frac{A_i(\mathbf{Z}, \boldsymbol{\theta})}{\sum_{j=1}^{M} A_j(\mathbf{Z}, \boldsymbol{\theta})}.$$

As a result, only differences of parameters, or some other normalized parameters can be uniquely identified. This is recognized in the attraction modeling literature (see Cooper and Nakanishi (1988, pp108-109) and the following identifying restriction is imposed:

$$\sum_{j=1}^{M} \alpha_j = 0.$$

That is the sum of all brand specific constants is zero. Further, since elasticities for the $\mathbf{Z}$ variables (*e.g.* price elasticities) only depend upon parameter differences, no additional identifying restrictions are needed. That is, traditionally only parameter differences are estimated.

The identification problem obviously exists for our CODA based approach. To identify the model we also impose the "adding up" constraint to the constants. However, in contrast to the traditional approach, to identify the other parameters in the model (*e.g.* the coefficients on price, or ln(price), terms) we argue the following. That is if, say, price of one brand falls, then its share should increase. Further, the share of the other brands should either remain unchanged or fall. Overall, since the shares are constrained to the unit simplex, the net effect of the price change should be zero. Thus it must be the case that the (price) coefficients also sum to zero across the M brands.

The parameter restrictions discussed above will formally identify the attraction model.

There are, however, two other sets of parameter restrictions which should also be be considered when estimating attraction models. These restrictions concern homogeneity and symmetry of, say, price effects. Homogeneity implies that if all prices double then brand shares will remain unchanged. This can be imposed in two ways. Firstly, we can impose the parameter restriction that the M price coefficients sum to zero within each equation. However, more common in estimating attraction models is to utilize not "raw" prices, but prices *relative* to the market average price in the specification. In this second approach homogeneity is obviously imposed since if all prices double relative prices remain unchanged. Symmetry implies that the impact of, say, the price of brand i on the share of brand j is equal to the impact of the price of brand j on the share of brand i. That is $\beta_{ij} = \beta_{ji}$. This restriction is one that might either be imposed, not imposed or tested using an appropriate statistical test (*e.g.* a likelihood-ratio test).

3. Empirical Example.

In this section we describe the estimation of an attraction model using CODA techniques. The example uses the dataset from Case A in Broadbent and Fry (1995). The data comprises 119 observations on category volume sales share, price relative to the category average for five brands, hereafter brands A to E, two consumer promotions for brand A, TV advertising for brand A and All Commodity Volume sales. Selected descriptive statistics can be found in Table 1. The specification chosen for the attraction model was to specify the effect of relative price in A_i as MCI and the impact of all other variables as MNL. Other combinations of MCI and MNL effects were tried but were not as effective in modeling the data.

Table 1: Descriptive Statistics.

	Mean	Std. Dev.	Minimum	Maximum
Share A	0.4610	0.0358	0.3708	0.5263
Share B	0.0751	0.0165	0.0342	0.1363
Share C	0.0596	0.0109	0.0426	0.0911
Share D	0.3232	0.0402	0.2324	0.3972
Share E	0.0810	0.0510	0.0239	0.2521
Price A	103.7010	3.4767	99.6630	115.2188
Price B	103.9228	4.4625	93.7551	123.3333
Price C	102.9650	2.7779	95.9350	113.3333
Price D	88.8994	9.4017	66.6666	105.8693
Price E	97.6942	2.3042	88.4298	105.4852

The model was estimated by taking the log-ratio transform and then estimating a multivariate regression model. Estimation was carried out by maximum likelihood using the LSQ command TSP386, Version 4.3A (Hall *et al* (1995)), which is well suited to the estimation of multivariate regression models with cross equation constraints. The parameter estimates and "heteroscedastic consistent" standard errors (see White (1980)) obtained are shown in Table 2. It should be noted that the parameter estimates for brand E were obtained exploiting the "adding up" constraints discussed above.

Table 2: **CODA Estimation Results for Attraction Model.**

Variable	Equation: Brand A	Brand B	Brand C	Brand D	Brand E*
Constant	5.8280	-10.3059	9.0424	1.4976	-6.0621
	(1.7329)	*(-1.5693)*	*(1.3160)*	*(0.4234)*	*(-0.7602)*
ln(Price A)	-1.2974	0.9142	0.0941	0.1567	0.1324
	(-3.1665)	*(1.1499)*	*(0.1136)*	*(0.3638)*	*(0.1367)*
ln(Price B)	0.1111	-1.8679	-0.3635	-0.1810	2.3012
	(0.5692)	*(-4.6121)*	*(-0.9029)*	*(-0.8668)*	*(5.2189)*
ln(Price C)	0.0408	0.8295	-0.6647	0.4079	-0.6134
	(0.1817)	*(1.8625)*	*(-1.4379)*	*(1.7067)*	*(-1.1818)*
ln(Price D)	0.2593	1.2004	-0.1003	0.6627	-2.0221
	(1.7691)	*(4.0830)*	*(-0.3314)*	*(4.1337)*	*(-4.3879)*
ln(Price E)	-0.4482	0.9066	-0.8677	-1.2802	1.6896
	(-1.6351)	*(1.6894)*	*(-1.5462)*	*(-4.4325)*	*(2.9491)*
Brand A Promotion 1	0.0022	-0.0024	0.0033	-0.0005	-0.0026
	(2.5707)	*(-1.4280)*	*(1.9115)*	*(-0.6001)*	*(-0.9705)*
Brand A Promotion 2	0.0681	-0.0904	0.0659	-0.0546	0.0111
	(1.8619)	*(-1.2580)*	*(0.8798)*	*(-1.4105)*	*(0.2046)*
All Commodity Volume	0.0090	0.0089	-0.0064	0.0008	-0.0123
	(8.6895)	*(4.9420)*	*(-3.3720)*	*(0.8039)*	*(-4.8375)*
R^2	0.9401	0.9404	0.9018	0.9523	
d.w.	1.8723	1.8778	1.7697	1.8785	

* indicates derived estimates.
Heteroscedastic t-ratios in parentheses.
Maximized log-likelihood 440.675.

Autoregressive Coefficient, $\hat{\phi}$: 0.4625 *(13.7873)*

In the original Broadbent and Fry (1995) paper the interest was centered on a single equation model for brand A. The attraction model differs from that specification in that TV advertising and a trend term are found to be insignificant and in that a lagged share term (an inertia, loyalty or "partial adjustment" effect) has been found important. It is probable that this lagged term is capturing the "longer and broader" effects discussed by Broadbent and Fry (1995). As a result the terms in their single equation model concerning long term effects are not significant in the attraction model.

Overall, the results from the attraction specification agree with *a priori* beliefs. Symmetry was not imposed as it was felt not to be appropriate for this market. Most of the coefficients either have the correct sign or are insignificantly different from zero, but with the wrong sign. The exceptions, however, are the coefficients concerning certain price terms and the impact of brand A's consumer promotion on brand C's share. The price terms which do not correspond with our, *a priori*, beliefs concern brand E and, in particular, its own price term and cross price with brand D. Additionally, brand D's own price term has the wrong sign and is significant. To assist in the discussion of these results we present the estimated elasticities in Table 3. These elasticities are calculated using the formulae in Cooper (1993, p.286) and evaluating the formulae at the respective means.

Table 3: Estimated Elasticities Evaluated at Means.

Variable	Brand A	Brand B	Brand C	Brand D	Brand E
Price A	-0.8349	0.4336	0.0311	0.0745	0.0714
Price B	0.0939	-1.7125	-0.3195	-0.2704	1.2403
Price C	-0.1621	0.5711	0.3642	0.2659	-0.3306
Price D	0.0054	1.2688	0.0151	0.5091	-1.0899
Price E	0.0190	0.0840	-0.4723	-0.8169	0.9107
Brand A Promotion 1	0.0020	-0.0009	0.0025	-0.00001	-0.0018
Brand A Promotion 2	0.0018	-0.0017	0.0012	-0.0009	0.0002
All Commodity Volume	0.6118	1.0623	-0.3204	0.1564	-0.7651

In order to understand these anomalous results we need to consider the market in more detail than in the original study. Brand A is a "premium" brand from a leading manufacturer and is the only advertised brand, brands B and C are brands from major manufacturers, brand D is a composite brand consisting of generic/no-label brands and brand E is a composite of a number of individual brands from small manufacturers. In the light of this, it would appear that the results are similar to Case 3 in Cooper (1993, p.288). In other words, some of the brands in the market are "niche" brands and some of the brands appear to be aggressive in their price activity.

4. Conclusions.

This paper has taken a methodology for analyzing data on shares of a total, compositional data analysis, from the statistical literature and applied it to the stochastic specification of attraction models in marketing. We find that the suggested specification greatly simplifies the distributional theory underlying the estimation of attraction models. In particular, the CODA approach, extended such that the mean is parameterized on the basis of attraction theory yields a model which bounds shares to the unit simplex, has the same parameter interpretations as the "traditional" modeling approach and is straightforward to estimate as a multivariate regression model in the usual statistical packages. We illustrate the application of the CODA specification using a case study, the results of which are sensible. Moreover, since the CODA approach has added advantages in that it can easily be extended to incorporate both the joint modeling of shares and total market sales (size) and shares observed to be zero, the CODA approach would seem to have potential for future work with attraction models.

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