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ABSTRACT

Hypothesis tests designed to compare stationary time series usually require the series to be independent. In order to compare time series that may be influenced by one or more common factors, one has to assume that their underlying generating processes are related. In this paper we present a test statistic, which will be used to test for significant differences between generating processes of two time series that may be logically connected. The test statistic is based on the differences between estimated parameters of the autoregressive models which are fitted to the series.

1. INTRODUCTION

The comparison of time series has applications in various fields including economics, geology, engineering and climatology. Hypothesis tests designed to compare two stationary independent time series involving the use of fitted parameter estimates were considered by De Souza and Thomson (1982) and Maharaj (1996). Most other tests in the literature for the comparison of independent stationary series involve the use of the estimated spectra of the series. Some relevant studies are Swanepoel and Van Wyk (1986), Coates and Diggle (1986) and Diggle and Fisher (1991). In practice the application of these tests to real time series is limited since comparisons are often made between logically connected series. For example if we wish to compare gold production over a number of years between two countries, we need to take into account that global supply and demand influences production in the two countries. In this case the time series are not independent.

We will assume that if the series are not stationary, then the same order of differencing will be needed to make each one stationary. Just as in Maharaj (1996), it will also be assumed that ARMA models, converted to infinite order AR models truncated to order k, will be fitted to each series and the test statistic will be based on the difference between the AR(k) estimates of the two series under consideration. However in this paper it will be assumed that the disturbances of the two models are correlated. A test for significant differences between the generating processes of these logically connected series uses a statistic based on generalised least squares estimates of the AR parameters. This test is concerned only with testing for significant differences between the underlying stochastic nature of two series. In section 2 we present the test statistic which has an asymptotic chi-square distribution, and in section 3 we investigate the distributional properties, size and power of the test, for finite sample sizes by a Monte Carlo study. In section 4 we apply this test to economic time series and to climatological time series.

2. TEST OF HYPOTHESIS

Let Z_t be a zero mean univariate stochastic process and a_t be a univariate white noise process with mean 0 and variance, σ_a^2. Then Z_t is such that $Z_t \in L$, where L is the class of stationary and invertible ARMA models. Using the standard notation of Box and Jenkins (1976), such a model is defined as

$$\phi(B)Z_t = \theta(B)a_t$$

where

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$$

with the usual restrictions on the roots of $\phi(B)$ and $\theta(B)$.

Z_t can be expressed as

$$Z_t = \sum_{j=1}^{\infty} \pi_j Z_{t-j} + a_t$$

where

$$\Pi(B) = \phi(B)\,\theta^{-1}(B) = 1 - \pi_1 B - \pi_2 B^2 - \ldots$$

Let $\{x_t\}$ and $\{y_t\}$, $t = 1, 2, \ldots, T$, be two correlated stationary time series. Then using a definite criterion such as Schwartz's BIC for modelling AR structures, truncated AR(∞) models of order k_1 and k_2 can be fitted to $\{x_t\}$ and $\{y_t\}$, respectively. Define the vector of the AR(k_1) and AR(k_2) parameters of the generating processes X_t and Y_t, respectively as

$$\Pi'_x = \left[\pi_{1x}\ \pi_{2x}\ \ldots\ \pi_{k_1 x}\right]$$

and

$$\Pi'_y = \left[\pi_{1y}\ \pi_{2y}\ \ldots\ \pi_{k_2 y}\right].$$

Let

$$k = \max(k_1, k_2).$$

Then if $k = k_1 \neq k_2$

$$\pi_{jy} = 0 \qquad \text{for } j = k_2+1, k_2+2, \ldots, k$$

and if $k = k_2 \neq k_1$

$$\pi_{jx} = 0 \qquad \text{for } j = k_1+1, k_1+2, \ldots, k.$$

Then define

$$\Pi'_{kx} = \left[\pi_{1x}\ \pi_{2x}\ \ldots\ \pi_{kx}\right]$$

$$\Pi'_{ky} = \left[\pi_{1y}\ \pi_{2y}\ \ldots\ \pi_{ky}\ \right].$$

Given the series $\{x_t\}$ and $\{y_t\}$, $t = 1,2,\ldots T$, the hypotheses to be tested are

H_0: There is no significant difference between the generating processes of two stationary series i.e. $\Pi_{kx} = \Pi_{ky}$.

H_1: There is a significant difference between the generating processes of two stationary series. i.e. $\Pi_{kx} \neq \Pi_{ky}$.

Berk (1974) truncated the infinite order AR process to order k and obtained the AR estimates by the method of least squares. This gives valid asymptotic results providing k is chosen as a function of T, such that

$$\frac{k^3}{T} \rightarrow 0 \text{ and } \sqrt{T} \sum_{j=k+1}^{\infty} \left|\pi_{jx}\right| \rightarrow 0 \text{ as } T \rightarrow \infty,$$

where T is the length of the stationary series to which the AR(k) model is fitted. Bhansali (1978) derived the asymptotic normal distribution of these estimates.

The model to be considered is of the form of the "seemingly unrelated regressions" model, as proposed by Zellner (1962). The T-k equations of the models fitted to $\{x_t\}$ and $\{y_t\}$ can be expressed collectively as

$$\mathbf{x} = W_x \Pi_{kx} + \mathbf{a}_x$$

$$\mathbf{y} = W_y \Pi_{ky} + \mathbf{a}_y \qquad (2.1)$$

where

$$\mathbf{x}' = [x_{k+1}\ \ldots\ x_{T-1}\ x_T]$$

$$\mathbf{y}' = [y_{k+1}\ \ldots\ y_{T-1}\ y_T]$$

$$W_x = \begin{bmatrix} x_k & x_{k-1} & \cdot & \cdot & \cdot & x_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{T-2} & x_{T-3} & \cdot & \cdot & \cdot & x_{T-k-1} \\ x_{T-1} & x_{T-2} & \cdot & \cdot & \cdot & x_{T-k} \end{bmatrix}$$

$$W_y = \begin{bmatrix} y_k & y_{k-1} & \cdot & \cdot & \cdot & y_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{T-2} & y_{T-3} & \cdot & \cdot & \cdot & y_{T-k-1} \\ y_{T-1} & y_{T-2} & \cdot & \cdot & \cdot & y_{T-k} \end{bmatrix}$$

$$\Pi'_{kx} = \left[\pi_{1x} \quad \pi_{2x} \quad . \; . \; . \quad \pi_{kx}\right]$$

$$\Pi'_{ky} = \left[\pi_{1y} \quad \pi_{2y} \quad . \; . \; . \quad \pi_{ky}\right]$$

$$a'_x = \left[a_{k+1\,x} \quad . \; . \; . \quad a_{T-1\,x} \quad a_{T\,x}\right]$$

$$a'_y = \left[a_{k+1\,y} \quad . \; . \; . \quad a_{T-1\,y} \quad a_{Ty}\right]$$

and

$$E[a_x] = 0 \qquad E\left[a_x a'_x\right] = \sigma_x^2 I_{T-k}$$

$$E[a_y] = 0 \qquad E\left[a_y a'_y\right] = \sigma_y^2 I_{T-k},$$

where I_{T-k} is a (T-k) x (T-k) identity matrix. We will assume that the disturbances of the two models are correlated at the same points in time but uncorrelated across observations, i.e.

$$E(a_x a'_y) = \sigma_{xy} I_{T-k}\,.$$

The dimensions of x, y, a_x and a_y are (T-k) x 1, of Π_{kx} and Π_{ky} are k x1 and of W_x and W_y are (T-k) x k.

Then, assuming that a total of 2(T-k) observations are used in estimating the parameters of the two equations in (2.1), the combined model may be expressed as

$$\mathbf{Z} = \mathrm{W}\Pi + \mathbf{a}\,, \qquad (2.2)$$

where

$$\mathbf{Z} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}, \quad \mathrm{W} = \begin{bmatrix} \mathrm{W}_x & 0 \\ 0 & \mathrm{W}_y \end{bmatrix}$$

$$\Pi = \begin{bmatrix} \Pi_{kx} \\ \Pi_{ky} \end{bmatrix}, \quad \mathbf{a} = \begin{bmatrix} \mathbf{a}_x \\ \mathbf{a}_y \end{bmatrix}$$

and

$$\mathrm{E}(\mathbf{a}) = 0$$

$$\mathrm{E}(\mathbf{a}\mathbf{a}') = \mathrm{V} = \Sigma \otimes \mathrm{I}_{T\text{-}k}$$

where

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}.$$

Thus the generalised least squares estimator is

$$\hat{\Pi} = \left[\mathrm{W}'\mathrm{V}^{-1}\mathrm{W}\right]^{-1}\mathrm{W}'\mathrm{V}^{-1}\mathbf{Z}\,. \qquad (2.2)$$

Now assuming that $\mathbf{a}$ is normally distributed, then by results in Anderson (1971) and Amemiya (1985), $\hat{\Pi}$ is asymptotically normally distributed with mean Π and covariance matrix

$$\mathrm{Var}(\hat{\Pi}) = \left(\mathrm{W}'\mathrm{V}^{-1}\mathrm{W}\right)^{-1}. \qquad (2.3)$$

Now $\quad H_0 : \Pi_{kx} = \Pi_{ky}$

may be expressed as

$$H_0 : \mathrm{R}\Pi = 0\,,$$

where

$$R = [I_k \ \ -I_k]$$

and I_k is a k x k identity matrix. Hence $R\hat{\Pi}$ is asymptotically normally distributed with mean $R\Pi$ and covariance matrix

$$\mathrm{Var}(R\hat{\Pi}) = \left(RW'V^{-1}WR'\right)^{-1}. \tag{2.4}$$

Let

$$\mathbf{F} = \left(\mathrm{Var}(R\hat{\Pi})\right)^{-1/2}\left(R\hat{\Pi} - R\Pi\right). \tag{2.5}$$

Then substituting (2.2) into (2.5), **F** becomes

$$\mathbf{F} = \left[R\left(W'V^{-1}W\right)^{-1}R'\right]^{1/2} R\left(\left(W'V^{-1}W\right)^{-1}V\left(W\Pi + \mathbf{a}\right) - \Pi\right).$$

Under H_0,

$$\mathbf{F} = \left[R\left(W'V^{-1}W\right)^{-1}R'\right]^{1/2} R\left(W'V^{-1}W\right)^{-1}V\mathbf{a},$$

and under the assumption that

$$\mathbf{a} \sim N(\mathbf{0}, V),$$

$$E(\mathbf{F}) = \mathbf{0} \quad \text{and} \quad E\left(\mathbf{F}\mathbf{F}'\right) = I_k.$$

Hence

$$\mathbf{F} \overset{A}{\sim} N\left(\mathbf{0}, I_k\right).$$

Therefore

$$\mathbf{F}'\mathbf{F} = \left(R\hat{\Pi}\right)'\left[R\mathrm{Var}(\hat{\Pi})R'\right]^{-1}\left(R\hat{\Pi}\right) \overset{A}{\sim} \chi^2(k).$$

Since Σ is unknown, a feasible generalised least squares estimator of Π will have to be used. By Zellner (1962) least squares residuals may be used to estimate consistently the elements of Σ with $\hat{\sigma}_x^2 = \frac{\hat{\mathbf{a}}_x'\hat{\mathbf{a}}_x}{T-k}$, $\hat{\sigma}_y^2 = \frac{\hat{\mathbf{a}}_y'\hat{\mathbf{a}}_y}{T-k}$ and $\hat{\sigma}_{xy} = \frac{\hat{\mathbf{a}}_x'\hat{\mathbf{a}}_y}{T-k}$.

Hence the feasible generalised least squares estimator is

$$\hat{\Pi} = \left[W'\hat{V}^{-1}W\right]^{-1} W'\hat{V}^{-1}\mathbf{Z}, \tag{2.5}$$

with

$$\left(\hat{\mathrm{Var}}(\hat{\Pi})\right) = \left(W'\hat{V}^{-1}W\right)^{-1},$$

where

$$\hat{V} = \hat{\Sigma} \otimes I \qquad \text{and} \qquad \hat{\Sigma} = \begin{bmatrix} \hat{\sigma}_x^2 & \hat{\sigma}_{xy} \\ \hat{\sigma}_{xy} & \hat{\sigma}_y^2 \end{bmatrix}.$$

Since $\hat{\mathbf{V}}$ in nonsingular and

$$\operatorname{plim} \hat{V} = V \Rightarrow \operatorname{plim} \hat{\mathrm{Var}}\left(\hat{\Pi}\right) = \mathrm{Var}\left(\hat{\Pi}\right),$$

then under H_0

$$\begin{aligned} \mathbf{F} = {} & \left(\mathrm{Var}(R\hat{\Pi})\right)^{-1/2} R\hat{\Pi} \overset{A}{\sim} N(\mathbf{0}, I_k) \\ \Rightarrow \quad & \left(\hat{\mathrm{Var}}(R\hat{\Pi})\right)^{-1/2} R\hat{\Pi} \overset{A}{\sim} N(\mathbf{0}, I_k) \\ \Rightarrow \quad & D = \mathbf{F}'\mathbf{F} = \left(R\hat{\Pi}\right)' \left[R\hat{\mathrm{Var}}\left(\hat{\Pi}\right)R'\right]^{-1} \left(R\hat{\Pi}\right) \overset{A}{\sim} \chi^2(k). \end{aligned}$$

The statistic D presented thus has asymptotically a chi-squared distribution.

3. SIMULATION STUDY

To investigate the finite sample behaviour of the test statistic D, series of lengths 50 and 200 were simulated from a number of ARMA process. Distributional properties of the test based on D were checked by obtaining estimates of the mean, variance and skewness and size. This was done by applying the test to pairs of series simulated from AR(1) processes for $\phi = 0.1, 0.5, 0.9$, MA(1) processes for $\theta = 0.1, 0.5, 0.9$, AR(2) processes for $\phi_1 = 0.6$ $\phi_2 = 0.2$, MA(2) processes for $\theta_1 = 0.8$ $\theta_2 = -0.6$ and ARMA(1,1) processes for $\phi = 0.8$ $\theta = 0.2$. It was assumed that the correlation between the disturbances of the underlying generating processes of the series in each pair was in turn 0, 0.5 and -0.9. Estimates of size were obtained for the 5% and 1% significance levels. Estimates of power for the 5% and 1% significance levels were obtained by applying the test to pairs of AR(1) processes for $\phi =0.5$ versus 0.1, 0.2, 0.3, 0.4, 0.6, 0.7, 0.8 and 0.9. This was again done by assuming that the correlation between the series in each pair was in turn 0, 0.5 and -0.9. The order (up to 10) of the truncated AR model to be fitted to each series was determined by Schwartz's BIC. However in estimating the model in (2.1), the maximum order k was fitted to both the series in each pair. The test statistic D was then obtained. This was repeated 2000 times. As well as obtaining size and power estimates for the degrees of freedom corresponding to k each time, overall estimates of power and size were also obtained by aggregating over the various k values.

For series of length 50, size is considerably overestimated. The overall size estimates are shown in Table 1. No further analysis was done on series of length 50. For series of length 200, the estimates of the means, variances and skewness for the various degrees of freedom are very often fairly close to the theoretical means, variances and measures of skewness respectively. The measure of skewness was calculated from the ratio of (mean -median) and standard deviation. The results for which there were at least 100 test statistics corresponding to a particular degree of freedom are shown in Table 2. Size estimates for the series simulated from the AR models are fairly close to the predetermined significance levels when the correct order was fitted but size was often overestimated for other values of k. For the MA and ARMA models, for some values of k, the size estimates are fairly close to the predetermined significance levels but in other cases it is overestimated. Hence this often caused the overall estimates of size to be slightly overestimated. These results are shown in Tables 2 and 3. Overall power estimates are given in Table 4 and it is clear the test has reasonably good power.

Table 1 Overall Estimates of Size for T = 50

		Correlation		
Generating Process	Level of Significance	0	0.5	-0.9
AR(1)	5%	0.1520	0.1240	0.0935
$\phi=0.1$	1%	0.0570	0.0445	0.0280
$\phi=0.5$	5%	0.1590	0.1355	0.0885
	1%	0.0640	0.0445	0.0320
$\phi=0.9$	5%	0.1670	0.1320	0.0875
	1%	0.0595	0.0580	0.0330
MA(1)				
$\theta=0.1$	5%	0.1570	0.1235	0.0930
	1%	0.0590	0.0535	0.0345
$\theta=0.5$	5%	0.1350	0.1360	0.0925
	1%	0.0510	0.0515	0.0330
$\theta=0.9$	5%	0.1800	0.2065	0.1390
	1%	0.0740	0.0875	0.0585
AR(2)				
$\phi_1=0.6$ $\phi_2=0.2$	5%	0.1630	0.1420	0.0980
	1%	0.0610	0.0575	0.0310
MA(2)				
$\theta_1= 0.8$ $\theta_2=-0.6$	5%	0.1855	0.1660	0.1300
	1%	0.0715	0.0715	0.0545
ARMA(1,1)				
$\phi=0.8$ $\theta=0.2$	5%	0.1575	0.1405	0.1080
	1%	0.0645	0.0575	0.0290

Table 2 Estimates of Mean, Variance, Skewness and Size for T=200

Table 2a Correlation = 0

Generating Process	Degrees of freedom	Number of Test Statistics	Mean	Variance	Skewness	Size (5% sig. level)	Size (1% sig. level)
AR(1) ϕ=0.1	1	1662	1.0214	2.0306	0.3770	0.0511	0.0123
	2	240	3.4933	8.0018	0.2271	0.1750*	0.0417*
AR(1) ϕ=0.5	1	1623	1.0161	2.0533	0.3903	0.0462	0.0148
	2	252	3.5020	7.5468	0.1813	0.1706*	0.0357*
AR(1) ϕ=0.9	1	1648	1.0579	2.3850	0.3787	0.0564	0.0146
	2	249	3.3558	8.6883	0.2515	0.1888*	0.0522*
MA(1) θ=0.1	1	1618	1.0381	2.2504	0.3704	0.0544	0.0148
	2	269	3.7089	8.8782	0.3160	0.1970*	0.0595*
MA(1) θ=0.5	2	1037	2.2041	5.3012	0.3019	0.0665	0.0154
	3	627	3.4591	6.1579	0.2100	0.0686	0.0080
	4	184	5.6847	12.0599	0.1351	0.1630*	0.0272
MA(1) θ=0.9	4	111	4.7068	11.7943	0.0952	0.0541	0.0360
	5	327	5.4548	11.8254	0.2387	0.0581	0.0092
	6	462	6.5769	13.0084	0.1732	0.0800*	0.0108
	7	408	7.6273	18.0725	0.2292	0.0882*	0.0196
	8	344	8.9412	20.5206	0.1997	0.0930*	0.0262*
	9	192	10.7266	22.4665	0.1780	0.1094*	0.0643*
	10	140	12.7247	35.3618	0.2707	0.1714*	0.0643*
AR(2)	1	113	1.2847	2.6521	0.3474	0.0531	0.0265
ϕ_1=0.6	2	1572	2.0913	4.0517	0.2873	0.0541	0.0115
ϕ_2=0.2	3	208	3.9970	9.4500	0.2679	0.1106*	0.0337*
MA(2)	4	814	4.1453	8.1814	0.1929	0.0541	0.0074
θ_1= 0.8	5	469	5.7609	12.8913	0.1910	0.0918*	0.0171
θ_2=-0.6	6	258	7.5594	14.5066	0.0654	0.0930*	0.0310*
	7	297	8.3352	21.8085	0.2475	0.1111*	0.0337*
ARMA(1,1)	1	602	1.2775	3.2159	0.3935	0.0797*	0.0249*
ϕ=0.8	2	1145	2.2906	4.7083	0.3037	0.0655	0.0131
θ=0.2	3	176	4.5763	10.8567	0.2505	0.1705*	0.0450*

* significant at the 5% level

Table 2b Correlation = 0.5

Generating Process	Degrees of freedom	Number of Test Statistics	Mean	Variance	Skewness	Size (5% sig. level)	Size (1% sig. level)
AR(1) $\phi=0.1$	1	1641	1.0466	2.1346	0.3887	0.0609	0.0197
	2	259	3.7052	9.0065	0.2853	0.1313*	0.0579*
AR(1) $\phi=0.5$	1	1650	1.0713	2.3751	0.3832	0.0582	0.0133
	2	227	2.8520	6.7121	0.3348	0.1322*	0.0308*
AR(1) $\phi=0.9$	1	1640	1.0204	2.0791	0.3877	0.0573	0.0098
	2	280	2.6904	6.1286	0.2872	0.1120*	0.0200
MA(1) $\theta=0.1$	1	1621	0.9616	1.8854	0.3900	0.0432*	0.0093*
	2	262	2.9045	7.4221	0.3175	0.1260*	0.0496*
MA(1) $\theta=0.5$	1	112	1.0469	2.0740	0.4471	0.0538	0.0089
	2	1037	2.1237	4.8950	0.2780	0.0601	0.0155
	3	567	3.4209	7.4025	0.2173	0.0723	0.0176
	4	203	4.6443	12.0947	0.2297	0.0837*	0.0246
MA(1) $\theta=0.9$	4	165	4.5195	10.1064	0.3129	0.0909*	0.0242
	5	373	5.7407	14.9021	0.2414	0.0965*	0.0348*
	6	411	6.6279	13.6251	0.1027	0.0803*	0.0097
	7	381	7.9259	18.8673	0.2068	0.0866*	0.0262*
	8	307	9.4861	26.0783	0.2158	0.1433*	0.0325*
	9	196	10.6881	21.8468	0.1511	0.0765	0.0352*
	10	149	12.2770	25.1335	0.0682	0.1392*	0.0070
AR(2)	1	172	1.5739	5.0712	0.6384	0.0930*	0.0465*
$\phi_1=0.6$	2	1508	2.1354	4.5089	0.2910	0.0517	0.0146
$\phi_2=0.2$	3	212	3.8088	7.4842	0.3192	0.1038*	0.0235
MA(2)	4	815	4.0574	8.7903	0.2182	0.0541	0.0147
$\theta_1= 0.8$	5	457	5.6743	11.9856	0.1991	0.0656	0.0175
$\theta_2=-0.6$	6	242	7.1842	16.9086	0.1743	0.0785	0.0289*
	7	297	8.3022	18.3549	0.1975	0.1111*	0.0237*
ARMA(1,1)	1	680	1.3028	2.8338	0.3820	0.0838	0.0191
$\phi=0.8$	2	1051	2.1556	4.2158	0.2668	0.0533	0.0124
$\theta=0.2$	3	187	3.9038	8.2476	0.2849	0.0963*	0.0214

* significant at the 5% level

Table 2c Correlation = -0.9

Generating Process	Degrees of freedom	Number of Test Statistics	Mean	Variance	Skewness	Size (5% sig. level)	Size (1% sig. level)
AR(1) $\phi=0.1$	1	1703	1.0674	2.2951	0.3998	0.0575	0.0117
	2	206	2.2391	5.1043	0.3934	0.0922*	0.0100
AR(1) $\phi=0.5$	1	1713	1.0828	2.3795	0.3829	0.0613	0.0128
	2	191	2.4880	4.5684	0.3037	0.0681	0.0171
AR(1) $\phi=0.9$	1	1725	0.9845	1.8257	0.3845	0.0441	0.0064
	2	181	2.0377	3.2909	0.2483	0.0552	0.0000
MA(1) $\theta=0.1$	1	1717	1.0365	2.0798	0.3856	0.0559	0.0122
	2	203	2.3371	4.5538	0.2552	0.0690	0.0246
MA(1) $\theta=0.5$	1	250	1.0782	2.0458	0.4429	0.0640	0.0040
	2	1061	2.0778	4.1409	0.3134	0.0537	0.0123
	3	474	3.2280	6.3863	0.2113	0.0591	0.0105
	4	153	4.3478	8.6183	0.2543	0.0588	0.0065
MA(1) $\theta=0.9$	4	284	4.3926	9.6804	0.2835	0.0704	0.0211
	5	379	5.6586	11.5141	0.1974	0.0712	0.0211
	6	418	6.3640	14.3158	0.2326	0.0742	0.0167
	7	322	7.7944	19.5413	0.1912	0.1056*	0.0156
	8	261	8.4483	17.4808	0.2024	0.0536	0.0038
	9	149	9.9748	29.4605	0.1522	0.0805	0.0604
	10	109	10.2732	23.3131	0.1593	0.0550	0.0183
AR(2)	1	339	1.0426	2.5442	0.3234	0.0619	0.0206
$\phi_1=0.6$	2	1396	2.0935	4.4773	0.3104	0.0659	0.0150
$\phi_2=0.2$	3	165	3.4423	7.0527	0.2777	0.0606	0.0242
MA(2)	4	960	4.1243	9.7191	0.2587	0.0583	0.0208
$\theta_1= 0.8$	5	397	5.2624	11.6867	0.1340	0.0453	0.0126
$\theta_2=-0.6$	6	185	6.6477	16.0740	0.1027	0.1027*	0.0108
	7	224	7.2634	14.4319	0.2601	0.0536	0.0089
ARMA(1,1)	1	915	1.1620	2.4236	0.3923	0.0710*	0.0186
$\phi=0.8$	2	886	2.1222	4.2012	0.2978	0.0508	0.0113
$\theta=0.2$	3	134	3.3145	7.0200	0.2083	0.0597	0.0075

* significant at the 5% level

Table 3 Overall Estimates of Size for T = 200

			Correlation	
Generating Process	Level of Significance	0	0.5	-0.9
AR(1)				
ϕ=0.1	5%	0.0740*	0.0770*	0.0645*
	1%	0.0175	0.0195*	0.0120
ϕ=0.5	5%	0.0740*	0.0735*	0.0625
	1%	0.0215*	0.0200*	0.0130
ϕ=0.9	5%	0.0830*	0.0680*	0.0535
	1%	0.0225*	0.0130	0.0125
MA(1)				
θ=0.1	5%	0.0835*	0.0600	0.0580
	1%	0.0270*	0.0165	0.0135
θ=0.5	5%	0.0795*	0.0690*	0.0580
	1%	0.0145	0.0185*	0.0110
θ=0.9	5%	0.0880*	0.0985*	0.0740*
	1%	0.0220*	0.0245*	0.0215*
AR(2)				
ϕ_1=0.6 ϕ_2=0.2	5%	0.0700*	0.0680*	0.0660*
	1%	0.0185	0.0210*	0.0190
MA(2)				
θ_1= 0.8 θ_2=-0.6	5%	0.0880*	0.0770*	0.0625
	1%	0.0215*	0.0220*	0.0185
ARMA(1,1)				
ϕ=0.8 θ=0.2	5%	0.0850*	0.0715*	0.0595
	1%	0.0240*	0.0160	0.0135

* significant at the 5% level

Table 4 Overall Power Estimates for $T=200$ (AR(1) $\phi=0.5$ vs AR(1) $\phi\neq0.5$)

		Correlation		
Generating Process	Level of Significance	0	0.5	-0.9
AR(1) ϕ				
0.1	5%	0.9850	0.9995	1.0000
	1%	0.9400	0.9995	1.0000
0.2	5%	0.8875	0.9790	1.0000
	1%	0.7255	0.9290	1.0000
0.3	5%	0.5900	0.7955	1.0000
	1%	0.3625	0.5730	1.0000
0.4	5%	0.2155	0.3105	0.9220
	1%	0.0785	0.1335	0.7865
0.5	**5%**	**0.0740**	**0.0735**	**0.0625**
	1%	**0.0215**	**0.0200**	**0.0130**
0.6	5%	0.2485	0.3490	0.9510
	1%	0.1020	0.1615	0.8575
0.7	5%	0.7160	0.8835	1.0000
	1%	0.4710	0.7255	1.0000
0.8	5%	0.9720	0.9985	1.0000
	1%	0.9080	0.9900	1.0000
0.9	5%	1.0000	1.0000	1.0000
	1%	0.9995	1.0000	1.0000

4. APPLICATION

4.1 Loans Data

Total fixed loan commitments in thousands of dollars of all banks, finance companies and credit co-operative in Australia for the period Jan. 1985 to Nov. 1995 are examined. Of interest is whether, over the given period, there are significant differences in the lending patterns between the institutions The natural log transforms of these series are shown in Figure 1. It seems from this figure that, while lending is on different levels for the three institutions, the lending patterns over the given time period are similar for the banks and finance companies but differ for the banks and credit co-operatives and for the finance companies and credit co-operatives. Because the series are nonstationary, the first difference of the natural log transform of each series was obtained. All further analysis was carried out on these differenced series which were assumed to be stationary. Each series has 130 observations. The test was applied to each pair of series . The results are shown in Table 5. From the results it can be seen that

- there is some residual correlation between each pair of series. This would be expected since the same economic factors would affect lending commitments from the three types of institutions.
- there is not enough evidence to conclude that lending patterns between the banks and finance companies are significantly different, but there is strong evidence that lending patterns between the banks and credit cooperatives and between finance companies and credit cooperatives are significantly different. Since the means of the undifferenced bank and finance companies series are clearly not the same, the result of no significant difference between the underlying generating processes of

the corresponding differenced series clearly demonstrates that the test can distinguish between the underlying stochastic nature of the two series but not the underlying deterministic nature of the two series.

These results are in keeping with the observations made on examination of the series in Figure 1.

Table 5 Results of Loans Data Application

Pair	AR(k) fit	Residual Correlation	P-value
Banks vs Financial Co.	AR(9), AR(5)	0.3534	0.4107
Bank vs Credit Corp.	AR(9), AR(2)	0.4868	0.0034
Credit Corp. vs Financial Co.	AR(2), AR(5)	0.5416	0.0002

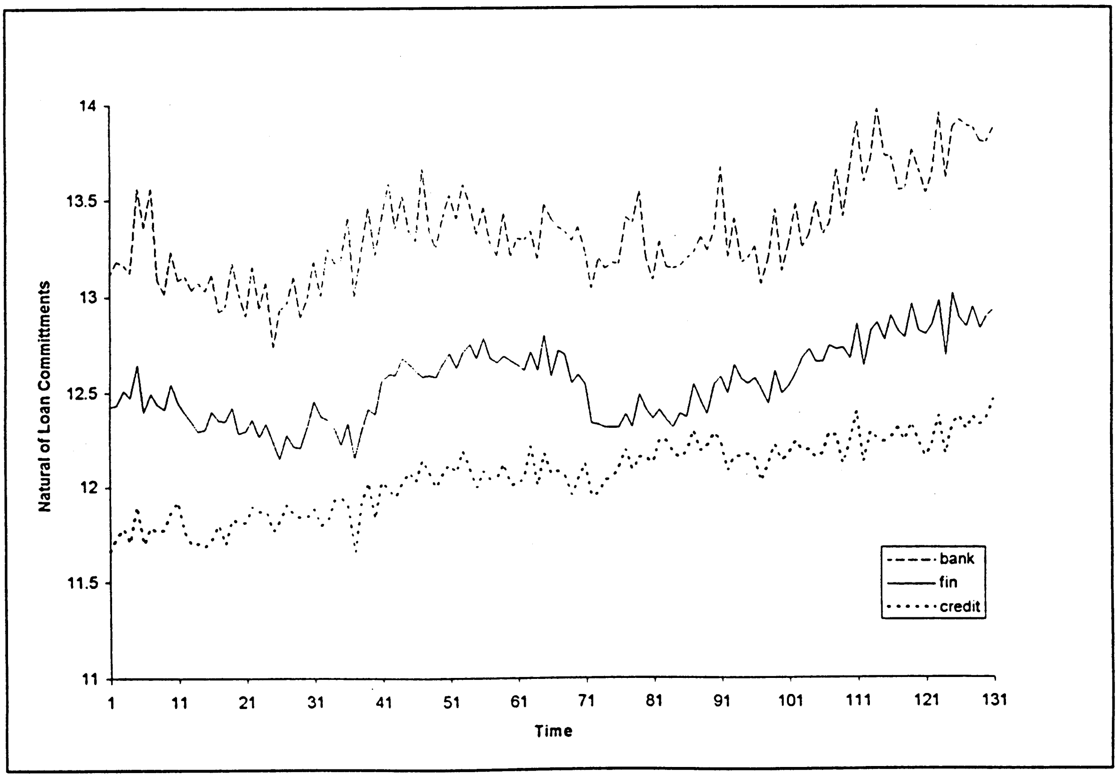

Figure 1: Total Loan Committments of the Banks, Finance Companies and Credit Co-operatives from January 1985 to November 1985

4.2 Tree Ring Data

In order to reconstruct historical climates from information from trees, one type of measurement that climatologists use are distances between the consecutive rings of trees. Figures 2, 3 and 4 show tree ring data series for three separate sites about 10 km. apart at about the same altitude on Mount Egmont on the North Island of New Zealand. Each data set consists of standardised distances between rings, averaged over a number of trees in a particular site. Standardisation allows samples with large differences in growth rates to be combined and can be used to remove any undesired growth trends present. Of interest is whether there are any significant differences between the growth patterns at the three sites, given that climatic conditions were assumed to be the same at the three sites. Each series consists of 352 observations.

The test was applied to each pair of series. The results are shown in Table 6. From the results it can be seen that

- there are almost no residual correlations between the series in each pair
- there is not enough evidence to conclude that there are significant differences between the underlying processes of the series in each pair.

Table 6 Results of Tree Ring Data Application

Pair	AR(k) fit	Residual Correlation	P-value
Site 1 vs Site 2	AR(8), AR(3)	-0.0033	0.0698
Site 1 vs Site 3	AR(8), AR(3)	0.0001	0.8814
Site 2 vs Site 3	AR(3), AR(3)	-0.0005	0.8783

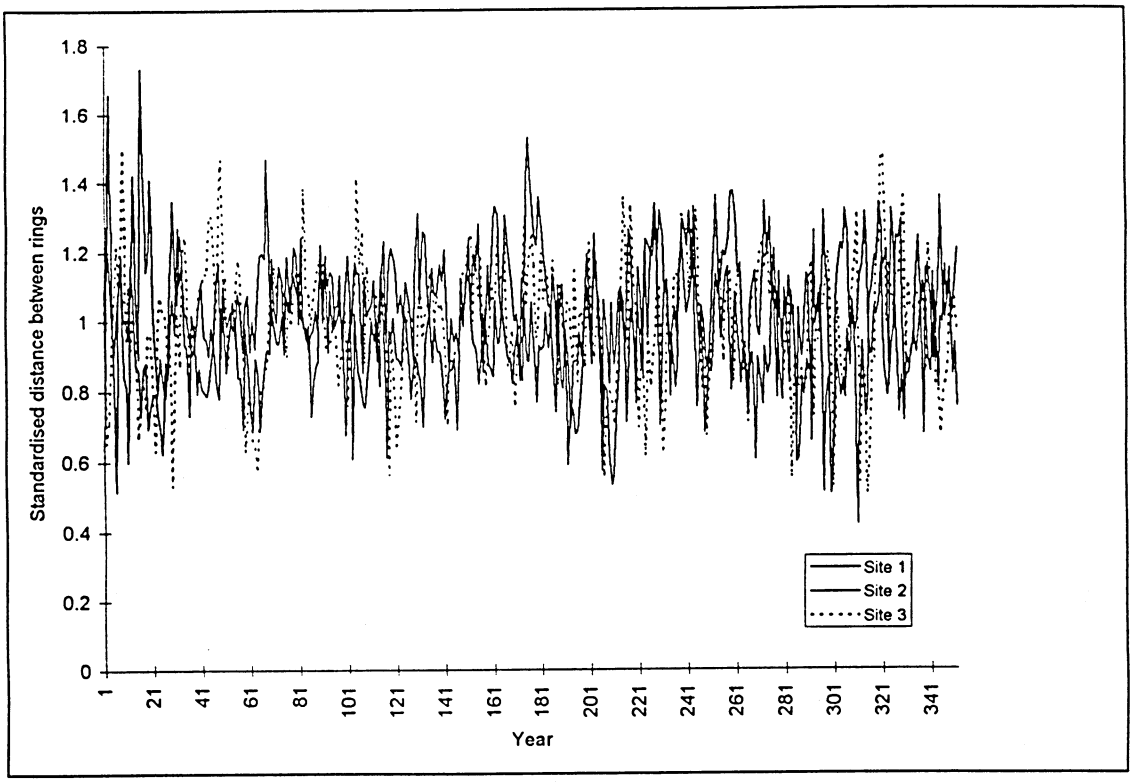

Figure 2 Tree Ring Series at Sites 1, 2 and 3 over 352 years

5. CONCLUDING REMARKS

From the simulation study is clear that for series of reasonable length, distributional approximations of the proposed test statistic to the chi-square distribution are reasonably adequate. The size of the test reasonably approximates the nominal size. Power estimates indicate fairly good power but further comparison with other tests would be needed to confirm this. From the results so far, it appears that the test can be quite successfully applied.

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