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TESTING FOR SERIAL CORRELATION IN THE PRESENCE OF DYNAMIC HETEROSCEDASTICITY

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ABSTRACT

A test for the presence of serial correlation is routinely carried out as a test for efficiency in financial markets. The problems inherent in such testing in the presence of dynamic heteroscedasticity are addressed in this paper. The accuracy of using standard critical values of serial correlation tests in the presence of autoregressive conditional heteroscedasticity (ARCH), generalized ARCH (GARCH), normal and non-normal disturbances is investigated. Tests examined include the conventional Durbin-Watson, Box-Pierce, Ljung-Box, Lagrange multiplier tests, proposed ARCH-corrected versions of these tests, and the robust tests of Diebold (1986) and Wooldridge (1992).

Standard serial correlation tests are derived assuming that the disturbances are homoscedastic, but this study shows that asymptotic critical values are not accurate when this assumption is violated. Asymptotic critical values for the ARCH(2)-corrected LM, BP and BL tests are valid only when the underlying ARCH process is strictly stationary, whereas Wooldridge's robust LM test has good size and power properties overall. These tests exhibit similar behaviour even when the underlying process is GARCH (1, 1). When the regressors include lagged dependent variables, the sizes and powers of the corrected tests depend on the coefficient of the lagged dependent variables, and the ratio of signal to noise. They appear to be robust across various disturbance distributions.

Key words: Serial correlation tests; ARCH-corrected tests; ARMA-ARCH models.

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1. Introduction

The problem of testing for serial correlation arises frequently in applied research involving economic and financial time series data. For example, omitted variables and inadequate modelling of functional form can give rise to correlated errors. A test of serial correlation, therefore, can be a test for misspecification of a model. Non-synchronicity due to infrequent trading of financial assets or inefficiency in financial markets results in serially correlated individual asset returns. A test for the absence of serial correlation in asset returns then can be a test for market efficiency [see Fama (1965) and Bollerslev and Hodrick (1992)] and synchronous trading [see Scholes and Williams (1977) and Lo and MacKinlay (1990)]. These are but two examples which illustrate both the importance of testing for serial correlation and how this issue can arise in different contexts.

Engle (1982) and Bollerslev (1986) demonstrated that autoregressive conditional heteroscedastic (ARCH) behaviour may be commonly present in a time series context. ARCH-type processes that emerge from evolving variance over time have the ability to capture the volatility clustering and leptokurtosis characteristic of financial time series of various frequencies; for example, see Bollerslev *et al.* (1992). A non-normal ARCH or generalized (GARCH) process is often required for a satisfactory representation of the distributional behaviour of asset returns, as shown by Baillie and DeGennaro (1990), Engle and Gonzalez-Rivera (1991) and others. See Bollerslev, Chou and Kroner (1992) and Bera and Higgins (1992) for extensive surveys of this ARCH literature.

Both phenomena, serial correlation and ARCH processes, have been found to occur simultaneously in models involving economic and financial variables, mainly due to time varying autoregressive parameters. Recent studies by Weiss (1986), Tsay (1987), Bera, Higgins and Lee (1992), Bollerslev and Hodrick (1992) and Bolleslev and Wooldridge (1992) consider the theory and applications of such ARMA-ARCH models, and demonstrate that the issue of testing for serial correlation in the presence of ARCH behaviour deserves attention.

The limiting distributions of many serial correlation tests are derived assuming independent identically distributed (i.i.d.) normal disturbances. In empirical studies involving time series this ideal assumption is often violated, and these tests can be biased. Since an indication of serially correlated errors has far-reaching implications for econometric modelling, it is important that tests for this behaviour have correct size and good power in finite samples, particularly when the underlying assumptions are violated.

The main objective of this study is to investigate the robustness of the popular Durbin-Watson (DW), Lagrange multiplier (LM), Box-Pierce (BP) and Ljung-Box (LB) tests and their corrected versions against autoregressive disturbances in the presence of dynamic heteroscedastic disturbances with normal or non-normal distributions.

Diebold (1986) addressed the question of robustness of the BP and LB tests in the presence of ARCH and recommended using ARCH-corrected standard errors. Although empirical evidence using an observed time series supports his claim, the performance of these tests with unobserved series needs to be evaluated. This is important as serial correlation is commonly present, for example, in the disturbance term of the regression model and our simulation study is designed to address this issue. Recently, Wooldridge (1991) proposed LM-type tests for serial correlation in the presence of ARCH and showed that they are robust when the dynamics are completely specified. The properties of these robust LM-type tests in finite samples remain unknown, though Small (1993) has undertaken some investigation in small samples.

We also suggest corrections similar to those of Wooldridge to the conventional DW, BP, LB and LM tests and examine their properties. Here we assess the finite-sample size properties of the standard tests, Diebold's and Wooldridge's robust tests and our ARCH-corrected tests and compare their performance also when the underlying disturbance process is normal or non-normal ARCH and GARCH.

The model and the tests are discussed in the next section and a Monte Carlo experiment and the results are reported in section 3. Section 4 gives an illustrative example and section 5 concludes the paper.

2. The Model and the Tests

Consider the model

$$y_t = x_t'\beta + u_t, \qquad t = 1, \dots T \tag{1}$$

where y_t is a dependent variable, $x_t = [x_{t1},\dots x_{tk}]$ is a kx1 vector of variables, which may include stochastic and non-stochastic variables, lagged regressors and lagged values of y_t, and β is a kx1 vector of unknown parameters, and u_t follows the stationary AR(m) process

$$u_t = \rho_1 u_{t-1} + \dots + \rho_m u_{t-m} + e_t, \qquad t = m+1, \dots, T \tag{2}$$

where $\rho_1, \dots, \rho_m$ are unknown autoregressive parameters. In order to ensure the stationarity of (2), we assume that roots of $1 - \rho_1 L - \dots - \rho_m L^m = 0$, where L is the lag operator, lie outside the unit circle. The term e_t is assumed to be of the form

$$e_t = \sigma_t z_t \tag{3}$$

where $\sigma_t > 0$, $\{z_t\}$ is i.i.d. with $E(z_t) = 0$ and $\mathrm{var}(z_t) = 1$, and for some function h,

$$\sigma_t^2 = E(u_t^2 \mid \Phi_{t-1}) = h(\Phi_{t-1}) \tag{4}$$

where Φ_{t-1} is the information set available at time t-1. This model is widely used in finance. Our interest lies in testing for serial correlation in model (1), but appropriate tests would depend on the functional form of h. Although several different functional forms have been suggested in the literature, we restrict attention to the well known GARCH(p,q) process

$$\sigma_t^2 = \sigma_0^2 + \sum_{i=1}^{q} \alpha_i e_{t-1}^2 + \sum_{j=1}^{p} \gamma_t \sigma_{t-j}^2 \tag{5}$$

where $\sigma_0^2 > 0$, $\alpha_i \geq 0$, $i = 1, \dots, q$ and $\gamma_j \geq 0$, $j = 1, \dots, p$ [see Bollerslev (1986)]. Nelson and Cao (1992) show that the non-negativity conditions can be relaxed somewhat when the process is GARCH.

Stationary and integrated GARCH(p,q) processes have been of interest in many empirical studies. Therefore, we assume that

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \gamma_j \leq 1.$$

If $\gamma_j = 0$, $j = 1, \dots, p$, then (5) reduces to the ARCH(q) disturbance process.

We wish to test the null hypothesis, given $E(u_t^2 \mid \Phi_{t-1}) = \sigma_t^2$,

$$H_0 : \rho_1 = ... = \rho_m = 0$$

against the alternative hypothesis

$$H_1\text{: Not all } \rho_j = 0, j = 1, ..., m$$

The Durbin-Watson test, although most often used to test against AR(1) disturbances, can be regarded as a test for disturbances with a first-order autocorrelated component [see King and Evans (1988)]. Against higher order AR(m) disturbances, the BP, LB and LM tests, denoted by BPm, LBm and LMm respectively, are used frequently. These test statistics are defined as

$$DW1 = \sum_{t=2}^{T}(\hat{u}_t - \hat{u}_{t-1})^2 / \sum_{t=1}^{T}\hat{u}_t^2,$$

$$BPm = T\sum_{i=1}^{m}\hat{\rho}_i^2,$$

$$LBm = T(T+2)\sum_{i=1}^{m}(T-i)^{-1}\hat{\rho}_i^2,$$

and $$LMm = (T-m)R^2,$$

where $\hat{u}_t$, $t = 1, ..., T$, are the OLS residuals of model (1), $\hat{\rho}_i = \Sigma\hat{u}_t\hat{u}_{t-i} / \Sigma\hat{u}_t^2$, $i = 1, ..., m$, and R^2 is the coefficient of determination of the regression of $\hat{u}_t$ on x_t and $(\hat{u}_{t-1}, ..., \hat{u}_{t-m})$. The test statistics other than DW1 have a chi-squared distribution with m degrees of freedom $(\chi^2_{(m)})$ asymptotically under the null hypothesis. All are derived under the assumption of homoscedastic and normal disturbance distributions.

Diebold's corrected BP and LB tests, denoted DBPm and DLBm respectively, are defined as

$$\text{DBPm} = T \sum_{i=1}^{m} \left[\frac{\hat{\sigma}^4}{\hat{\sigma}^4 + \hat{\tau}_i^2} \right] \hat{\rho}_i^2$$

$$\text{DLBm} = T(T+2) \sum_{i=1}^{m} \left[\frac{\hat{\sigma}^4}{\hat{\sigma}^4 + \hat{\tau}_i^2} \right] (T-i)^{-1} \hat{\rho}_i^2 ,$$

respectively, where $\hat{\tau}_i^2$ is an estimate of the ith autocovariance of $\hat{u}_t^2$ defined as

$$\hat{\tau}_i^2 = T^{-1} \Sigma(\hat{u}_t^2 - \hat{\sigma}^2)(\hat{u}_{t-i}^2 - \hat{\sigma}^2)$$

and $\hat{\sigma}^4$ is the square of an estimate of the unconditional second moment of $\hat{u}_t$ defined as

$$\hat{\sigma}^4 = [T^{-1} \Sigma \hat{u}_t^2]^2 .$$

Diebold (1986) has shown that these tests are asymptotically $\chi^2_{(m)}$ under the null hypothesis and the normality assumption. Although the exact expressions for τ_i^2 and σ^4 can be derived for an ARCH process, they need to be estimated in practice, which is done in our simulation study.

Wooldridge's ARCH-corrected LM test, denoted RLMm, is robust for testing H_0 in time-series models with completely specified dynamics. The construction of RLMm involves the following steps:

(i) Obtain the fitted values denoted here by $\hat{h}_t$, t=1, ..., T from the linear regression

$$\hat{u}_t^2 = \theta_0 + \theta_1 \hat{u}_{t-1}^2 + \ldots + \theta_q \hat{u}_{t-q}^2 + v_t , \qquad t = 1, \ldots, T.$$

(ii) Define $x_t^* = x_t / \sqrt{\hat{h}_t}$ and $\tilde{u}_t = \hat{u}_t / \sqrt{\hat{h}_t}$, $t = 1, \ldots, T$.

(iii) Save the 1xm vector of residuals, say $\tilde{r}_t$, from the regression of each of the $\tilde{\lambda}_t$ on x_t^*, where $\tilde{\lambda}_t = (\tilde{u}_{t-1}, \ldots, \tilde{u}_{t-m})$.

(iv) Compute (T - SSR), where SSR is the sum of squares of residuals from the regression of 1 on $\tilde{u}_t \tilde{r}_t$. (T - SSR) $\sim \chi^2(m)$ asymptotically under H_0.

Before introducing our modified versions of the serial correlation tests, recall that an important assumption underlying the tests is that the disturbance terms have a constant variance, which is not the case in the presence of ARCH. This suggests that the DW1, BPm and LBm tests might be improved by replacing $\hat{u}_t$ with its standardised version. We therefore replace $\hat{u}_t$ by $\tilde{u}_t$ obtained in step (ii) of Wooldridge's procedure and denote the corresponding corrected versions by CDW1, CBPm and CLBm, respectively. We also include such a correction for the conventional LMm test computed as $(T\text{-}m)R^2$, where R^2 is the coefficient of determination of the linear regression of $\tilde{u}_t$ on x_t^* and $(\tilde{u}_{-1},\ldots,\tilde{u}_{t\text{-}m})$, and denote it by CLMm. The asymptotic distributions of these corrected tests are valid under the particular ARCH(q) model - including homoscedasticity - estimated in the preliminary stage, but are not robust to variance misspecification. However, Woolridge's corrected LM tests are asymptotically valid under any heteroscedasticity. We assess the properties of all corrected tests when the true model is GARCH (p.q) but the correction is made assuming the ARCH(q) process.

Bolleslev and Wooldridge (1992) proposed easily computable LM tests for AR-GARCH, and in a simulation study showed that their sizes and powers compare favourably with the standard Wald and LM tests when the disturbances are non-normal. These tests are not considered in this study. Bera, Higgins and Lee (1992) also proposed a LM test for serial correlation in the presence of ARCH/GARCH process which arises as a result of time varying serial correlation and Small (1993) considered its applications. However this test could not be applied directly to model (1).

We use Monte Carlo simulations to assess the properties of the corrected versions of the various tests and compare them with those of their uncorrected counterparts and the robust tests of Diebold and Wooldridge.

3. Empirical Evaluation of the Tests

A Monte Carlo experiment was conducted to assess the accuracy of the sizes of the abovementioned tests in the presence of ARCH and GARCH disturbances, using standard critical values. Some power comparisons were also undertaken. Selected size and power results only are presented in Tables 1 to 9. The complete set of results is available on request.

3.1 Experimental Design

Critical values were based on the assumption of standard i.i.d. normal errors in model (1) at the 1, 5 and 10 per cent nominal levels. Exact values were calculated for the DW1 test and tabulated chi-square values were used for the other tests with an asymptotic justification.

Monte Carlo simulations were based on 2,000 replications. In order to limit the simulation study to a manageable scale, we considered only the cases m = 1, 2, 5, 10, 20 in the disturbance process (2), and (p,q) = (0,2), (1,1) in model (5) which correspond to ARCH(2) and GARCH(1,1) processes, respectively. An ARCH(2) process can be generated as

$$e_t = \eta_t(1 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2)^{1/2}$$

where $(\alpha_1, \alpha_2)\ \varepsilon\ \Omega_1 = \{(\alpha_1, \alpha_2) \mid \alpha_1, \alpha_2 \geq 0 \text{ and } \alpha_1 + \alpha_2 \leq 1\}$ with η_t a random disturbance. A GARCH(1,1) process can be generated as

$$e_t = \eta_t(1 + \alpha_1 e_{t-1}^2 + \gamma_1 \sigma_{t-1}^2)^{1/2}$$

where $(\alpha_1, \gamma_1)\ \varepsilon\ \Omega_2 = \{(\alpha_1, \gamma_1) \mid \alpha_1, \gamma_1 \geq 0 \text{ and } \alpha_1 + \gamma_1 \leq 1\}$.

For an underlying ARCH(2) disturbance process, sizes were estimated at the grid points

$$\{(\alpha_1, \alpha_2): \alpha_1 = 0.0, 0.2, 0.4 \text{ and } \alpha_2 = 0.0, 0.4, 0.6\} \subset \Omega_1,$$

and when the process is GARCH(1,1) they were estimated at

$$\{(\alpha_1, \gamma_1): \alpha_1 = 0.2, 0.4 \text{ and } \gamma_1 = 0.0, 0.4, 0.6\} \subset \Omega_2.$$

The following regressor or X matrices, with T = 50, 100, 500, were used

X1: A constant dummy and the daily 90-day Australian Treasury bill rate commencing 16 September 1985 (k=2).

X2: A constant dummy and the daily spread between 90 and 180 days Australian Treasury bill rates commencing 16 September 1985 (k=2).

X3: A constant dummy, the 90-day bill rate and this variable lagged by one, two and three days (k=5).

X4: X1 and the first-order lagged dependent variable (k=3), where the coefficient of X is $\beta' = (0,1,\delta)$, with the coefficient of the lagged dependent variable, δ, set at 0.2, 0.4, 0.6 and 0.8, and σ = 0.07, 2, 7.

X5: X2 and the first-order lagged dependent variable (k=3), where the coefficient of X is $\beta' = (0,1,\delta)$, with the coefficient of the lagged dependent variable, δ, set at 0.2, 0.4, 0.6 and 0.8, and $\sigma = 2, 4, 7$.

With dynamic regressors, test characteristics can depend on the signal to noise ratio, which for X4 and X5 corresponds to $\Sigma x_{t2}^2 / \sigma$. Generally the signal to noise ratio is given by $||X^*\beta||/\sigma$, where X* is the matrix of regressors excluding the lagged dependent variable. To keep the experiment manageable, we chose only one set of values for β but a range of values for σ, mostly those which result in reasonable R^2 values for the model (1).

A number of fitted values of $\hat{h}_t$ were found to be negative, which is undesirable because the variables used to construct the test statistics are normalized by dividing by $\sqrt{\hat{h}_t}$ (see step (i) in Wooldridge's procedure). Hence, to ensure that these fitted values $\hat{h}_t$ were positive, the parameter estimates of the model with q = 2 were obtained by the method of least squares subject to the constraints $\theta_0 > 0$, $\theta_1, \theta_2 \geq 0$. [When investigating the possibility of using absolute values of $\hat{h}_t$ and log $(\hat{h}_t)$, the ARCH-corrected tests were found to have unacceptably high sizes but Wooldridge's robust tests were unaffected.]

To generate the random disturbances $\{\eta_t\}$, a standard normal distribution, which is symmetric with a kurtosis of 3, and a weighted mixture (MIXNOR) of normal distributions {0.1N(0,1) + 0.9N(0,3)} were used. Disturbances following six other distributions, each with a zero mean, unit variance and characterised by their skewness and kurtosis, were also generated, based on a generalisation of Tukey's lambda distribution. Parameter values were chosen from the table in Ramberg, Tadikamalla, Dudewicz and Mykytka (1979). Leptokurtosis is implied by a kurtosis or tail measure greater than 3. These distributions have, respectively: a right skewness of 0.5 and medium kurtosis or tail of 4 (RSMT) and heavy kurtosis (RSHT); a heavy right skewness of 0.8 and medium kurtosis of 4 (HRSMT) and heavy kurtosis of 9 (HRSHT); and symmetry with kurtosis of 6 (KURT6) and 9 (KURT9). These distributions enable a systematic investigation of the effect of skewness and kurtosis and were chosen to represent a range of alternative behaviour characteristic of financial and economic situations.

The powers of the ARCH-corrected DW test were computed against an AR(1) alternative hypothesis with $\rho_1 = 0.1, 0.3, 0.5, 0.7, 0.9$ and those of the other ARCH-corrected tests against AR(2) were computed at the grid points $\{(\rho_1, \rho_2) : \rho_1 = 0.1, 0.3, 0.4 \text{ and } \rho_2 = 0.1, 0.3, 0.5\}$. Note that the ARCH/GARCH behaviour is present also under the alternative hypothesis.

3.2 Size Comparisons

Empirical sizes at a nominal significance level of 5 per cent for the DW1, LM2, LM5, BP5 and LB5 tests and our proposed ARCH(2)-corrected versions in the presence of ARCH(2) disturbances are reported in Table 1, and those in the presence of GARCH(1,1) are presented in Table 2 over selected grid points. Corresponding sizes of Wooldridge's robust RLM2 and RLM5 tests and Diebold's DBP5 and DLB5 tests are reported in Table 3. These are all based on the non-stochastic matrix X1 with T = 50, 100 and 500. Empirical sizes of these tests, based on asymptotic normal critical values, for various non-normal disturbance distributions are reported in Table 4. Size comparisons for a stochastic regressor matrix X4 are shown in Table 5 for the proposed ARCH(2)-corrected tests.

The results reported in Table 1A reveal that when the disturbances follow an ARCH(2) process the sizes of the standard serial correlation tests first gradually and then more sharply increase as $\alpha_1 + \alpha_2$ increases to 1. The maximum sizes always occur at $\alpha_1 + \alpha_2 = 1$, i.e., when the process is integrated. The maximum size is near 0.4 for DW1, and can be as high as 0.7 for the BP5, LB5 and LM5 tests in large samples. *Ceteris paribus*, the sizes of the standard tests tend to increase as the sample size increases when the ARCH process is integrated or nearly integrated, indicating that their asymptotic critical values are not accurate when the assumption of homoscedastic errors is relaxed.

When the disturbances follow an ARCH(2) process, the ARCH(2)-corrected tests have sizes which are generally closer to the nominal level than their uncorrected counterparts (see Table 1B), though still usually exceeding it particularly for (near) integrated process and in large samples. Even when the underlying disturbance process is ARCH(1), corrected tests based on an over-parameterized ARCH(2) model show a marked improvement over the uncorrected tests, particularly when the process is stationary. Our ARCH(2)-corrected tests appear to have reasonably accurate sizes using asymptotic critical values only when the ARCH/GARCH process is strictly stationary, possibly because the estimates of the ARCH parameters are not well-behaved otherwise.

Overall the sizes of the standard Durbin-Watson tests are smaller than those of the other tests; the Box-Pierce tests are closer to the nominal level than the Ljung-Box test (with 5 lags); and the Lagrange multiplier tests perform better with lags of two (LM2) than with five (LM5) in some range of ARCH parameter values and sample sizes, whereas the reverse is true in the other ranges.

When the tests are corrected assuming ARCH(2) disturbances, similar size behaviour is observed when the true disturbances are GARCH(1,1), demonstrating the robustness of such a correction when the heteroscedastic form is inappropriate (see Table 2). The sizes of our proposed ARCH(2)-corrected tests are often closer to the nominal size in the GARCH(1,1) parameter space at the selected grid points than those corresponding to ARCH(2).

The ARCH-corrected tests DBP5 and DLB5 do not seem in this study to have accurate sizes (see Table 3A), whereas in Diebold's (1986) study the ARCH-corrected BP and LB tests do. A possible reason for this inconsistency is that his study and ours differ in two respects. His experiment involved an observed time series $y_t = e_t$, but we use residuals from the regression model with an unobserved disturbance term. In addition, Diebold used a closed form expression for the standard errors, assuming normal disturbance terms following an ARCH process of known order, whereas we estimated the standard errors and the corrected tests statistics are derived without such assumptions. Because of the poor size performance of these tests in most cases, their powers are not computed.

The Wooldridge ARCH(2)-corrected RLM2 test has close to the nominal size in almost all cases considered in this study (see Table 3) and the size of the RLM5 test is much lower for small samples but is reasonable for $T \geq 50$. A desirable property of Wooldridge's test, not shared by the others, is that its size is usually below the nominal level in all samples and is stable over the range of ARCH/GARCH parameter values in large samples. The RLM2 and RLM5 tests are notably robust when the underlying disturbance process is GARCH(1, 1) rather than ARCH(2) and the use of asymptotic critical values results in accurate sizes.

The sizes of all the tests appear reasonably stable across various underlying disturbance distributions, as demonstrated in Table 4. This is consistent with Evans (1992), where DW1 and other tests of serial correlation were found to be robust even when the disturbance distribution had no finite moments. *Ceteris paribus*, the tests are not significantly affected by skewness and no systematic effect of kurtosis was apparent on their sizes. These characteristics were evident also at the 1 and 10 per cent significance levels.

The sizes of the ARCH(2)-corrected tests for the stochastic X4 matrix (shown in Table 5) depend on δ and the signal to noise ratio, generally increasing as σ and/or δ increase. The RLM2 test size is below 0.05 in all cases considered here, whereas for the CLM2, CBP2 and CBL2 test sizes can be as high as 0.3, 0.1 and 0.1 respectively, particularly when the ARCH processes is integrated or is nearly so. *Ceteris paribus*, the sizes of all tests increase as the sample sizes increases, but generally remain below the nominal level when the process is stationary with the exception of CLM2. The CDW test size can be as low as 0.00 when $T = 50$.

3.3 Power Comparisons

Selected power calculations given in Tables 6 to 9 are based on ARCH(2)-corrected serial correlation tests using standard critical values at the 5 percent nominal level. For the non stochastic matrix X1 with T = 50, empirical powers for the corrected LM and BP tests against AR(2) are shown for AR(2)-ARCH(2) disturbances in Table 6: for normal disturbances in Table 6A, for disturbances which are right skewed with heavy kurtosis in Table 6B; and for heavily right skewed disturbances with medium kurtosis in Table 6C. Power results of corrected tests against AR(2) disturbance process are given in Table 7A as well as for the corrected Durbin Watson test against AR(1) disturbances in Table 7B, when the underlying process is normal AR(2)-GARCH(1,1). For the stochastic matrix X4 with T = 50, power results for the ARCH(2)-corrected DW test when the disturbance distribution is normal AR(1)-ARCH(2) are given in Table 8 and with T = 100 in Table 9, for the ARCH(2)-corrected LM and BP tests when the disturbance distribution is normal AR(2)-ARCH(2).

The ARCH-corrected tests appear to have reasonable powers for non-stochastic regressors, as seen in Table 6, increasing with higher values of the autoregressive parameters ρ_1 and ρ_2. The power properties of the tests when the disturbance distribution is non-normal and the regressors are non-stochastic differ relatively little from the normal case: when the distribution is leptokurtic, the powers of the corrected tests are marginally lower than those for normal distribution in most cases; when the disturbance distribution is skewed, the powers slightly exceed those for normal distribution, particularly when ρ_1 and ρ_2 values exceed 0.3. The overall power was generally high for all, and the tests can be ranked as CBL2, CBP2, CLM2 and RLM2 in terms of power. Wooldridge's RLM2 test however actually performs the best, given that its sizes are the lowest and the closest to the nominal sizes, particularly for larger values of the ARCH(2)/GARCH(1, 1) parameter values. However, with a heavily skewed disturbance distribution (Table 6C), the RLM2 test is consistently superior for $\alpha_1 + \alpha_2 \geq 0.4$.

The RLM test is more powerful than the other ARCH(2)-corrected tests in the presence of normal GARCH (1, 1) disturbances (see Table 7). Patterns similar to these for ARCH(2) disturbances were observed across all X matrices. These power results and the corresponding signs demand the effectiveness and rorbustness of ARCH corrections, even if the true model is some other form of dynamic heteroscedasticity.

The power against AR(1) of the corrected DW test varies from 0.1 to 1.00 as ρ_1 varies from 0.1 to 0.9, when the regressions are non-stochastic as seen from Table 8 with $\delta = 0$. Powers are quite reasonable with a tendency to marginally decline as the ARCH(2) parameters α_1 and/or α_2 increase.

However when the regressor matrix is stochastic, with $\delta \neq 0$ such that it includes a lagged dependent variable, powers increase as σ decreases and/or δ increases and are significantly lower for high σ and low δ parameter. The CDW test is most powerful with powers ranging from 0.003 to 1.00 for T = 50 and 100: generally the nominal size exceeded the power for the other tests for T = 50, but these are not reported here. For a stochastic regressor matrix the power of each of the tests generally increases with higher δ values, as evident in Table 8 and 9. Generally when the dynamic term coefficient is large the power is quite reasonable for T = 100. The powers of the ARCH(2)-corrected LM tests are above the nominal level for all ρ_1 and ρ_2

values (see Table 9). The CBL2 test performs better than the CBP2 test as expected, but surprisingly its power can be much smaller than the nominal size for small values of δ and T = 100. The RLM2 tests have lower power than the other corrected LM tests in all cases as a consequence of its lower size for large ARCH parameter values; this difference is noticeable only when (ρ_1, ρ_2) values are small.

4. An Illustrative Example

The Australian Treasury bill rates used in our experiment have been found to be I(1) variables with GARCH(1,1) disturbances [see Inder and Silvapulle (1993)] when using monthly observations. Serial correlation in the first differences of these bill rates was tested for, using monthly data for the period January 1973 to October 1992. The estimated uncorrected test statistics and corresponding corrected versions are:

Series	DW1		LM5	BP5	BL5
3 month rate	1.614		13.148	15.911	15.083
6 month rate	1.789		12.402	19.000	18.904
Series	CDW1	CLM5	RLM5	CBP5	CBL5
3 month rate	1.890	7.036	8.414	14.112	13.012
6 month rate	1.808	5.890	10.001	12.927	12.000

At the 5 percent level, the uncorrected statistics all exceed the critical values, indicating that the null of no serial correlation is rejected. In contrast, the CDW, CLM5, and RLM5 statistics are insignificant at the 5 per cent level, indicating acceptance of the null hypothesis. However, the CBP5 and CLB5 statistics are still significant at the 5 per cent level.

This example demonstrates that, in the presence of ARCH disturbances, tests for serial correlation may result in misleading inferences if this ARCH behaviour is not taken into account. ARCH-corrected tests may improve such testing.

5. Conclusion

Using a Monte Carlo simulation study, we investigated the validity of the standard critical values of the Durbin-Watson, Lagrange multiplier, Box-Pierce and Llung-Box tests and their ARCH-corrected versions plus Diebold's and Wooldridge's robust tests in the presence of ARCH/GARCH disturbances.

Our results suggest that sizes of standard serial correlation tests are higher than the nominal size when ARCH/GARCH disturbance behaviour is present but unaccounted for, and they increase sharply as the parameter values of the process increase. For all sample sizes, our proposed ARCH-corrected tests have sizes that are close to the nominal level only when the underlying ARCH/GARCH disturbance process is stationary. Diebold's tests have relatively poor size properties. Wooldridge's ARCH-corrected LM tests sizes appear the closest to the nominal level and are stable over a range of ARCH/GARCH parameter values in large samples. The Durbin-Watson test appears to be the next best.

The sizes of the ARCH-corrected serial correlation tests are marginally smaller when the underlying disturbances follow a GARCH rather than an ARCH process. In the presence of stationary ARCH behaviour, when the correlation tests are corrected assuming a slightly over-parameterized process, the sizes are appear close to the nominal level.

Taking account of the size properties of the tests, it is evident from power comparisons that the corrected tests have good powers when the regressors are non-stochastic even in small samples, whereas they have poor powers for stochastic regressors, particularly when the sample size and the coefficient of lagged dependent variable are small and the signal to noise ratio is large. Again taking size properties into account, generally the ARCH-corrected Durbin Watson test is most powerful against first order autoregressive disturbances and Wooldridge's robust LM test against higher orders. Wooldridge's test is most powerful in the presence of inappropriate form of dynamic heteroscedasticity. ARCH corrected DW and LM tests resulted in correct inference when applied to Australian Treasury Bill rates.

Given their good size and power properties when the disturbance process is either some form of dynamic heteroscedasticity or is homoscedastic, the use of ARCH-corrected tests is highly recommended: one can test for serial correlation without taking a stand on the disturbance variance process.

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Table 1

Empirical sizes, with normal ARCH(2) disturbances, based on standard 5% critical values for matrix X1.

	T=	50			100			500		
Test Statistics	α_2	$\alpha_1 = 0.0$	0.2	0.4	0.0	0.2	0.4	0.0	0.2	0.4
A: Standard serial correlation tests										
DW1	0.0	0.038	0.048	0.057	0.048	0.065	0.078	0.050	0.067	0.123
LM2		0.061	0.100	0.133	0.062	0.089	0.138	0.056	0.085	0.134
LM5		0.064	0.091	0.110	0.071	0.070	0.109	0.058	0.074	0.136
BP5		0.045	0.061	0.079	0.053	0.063	0.106	0.051	0.071	0.134
LB5		0.065	0.090	0.108	0.064	0.076	0.120	0.052	0.074	0.136
DW1	0.4	0.081	0.093	0.099	0.068	0.090	0.112	0.098	0.124	0.142
LM2		0.121	0.153	0.206	0.121	0.187	0.254	0.133	0.272	0.517
LM5		0.105	0.127	0.196	0.114	0.159	0.260	0.136	0.287	0.514
BP5		0.074	0.102	0.165	0.096	0.153	0.253	0.123	0.272	0.517
LB5		0.103	0.134	0.200	0.110	0.168	0.271	0.125	0.279	0.513
DW1	0.6	0.069	0.098	0.125	0.102	0.189	0.213	0.123	0.292	0.381
LM2		0.141	0.175	0.242	0.168	0.237	0.348	0.258	0.431	0.597
LM5		0.134	0.153	0.224	0.161	0.256	0.389	0.293	0.498	0.693
BP5		0.098	0.138	0.201	0.139	0.237	0.349	0.264	0.509	0.711
LB5		0.137	0.170	0.244	0.156	0.261	0.378	0.269	0.513	0.714
B: ARCH(2)-corrected serial correlation tests										
CDW1	0.0	0.041	0.047	0.054	0.051	0.058	0.061	0.052	0.055	0.049
CLM2		0.030	0.052	0.057	0.035	0.050	0.060	0.036	0.048	0.049
CLM5		0.056	0.064	0.063	0.065	0.047	0.053	0.051	0.042	0.046
CBP5		0.047	0.047	0.050	0.056	0.046	0.060	0.043	0.049	0.043
CLB5		0.063	0.073	0.069	0.065	0.055	0.069	0.046	0.050	0.045
CDW1	0.4	0.068	0.059	0.061	0.059	0.067	0.078	0.062	0.077	0.089
CLM2		0.034	0.072	0.107	0.032	0.077	0.132	0.037	0.074	0.151
CLM5		0.061	0.072	0.111	0.057	0.068	0.106	0.049	0.077	0.143
CBP5		0.045	0.060	0.082	0.053	0.063	0.089	0.045	0.061	0.101
CLB5		0.074	0.084	0.107	0.066	0.073	0.103	0.046	0.063	0.103
CDW1	0.6	0.052	0.067	0.088	0.068	0.090	0.095	0.052	0.095	0.099
CLM2		0.041	0.081	0.117	0.040	0.108	0.163	0.054	0.156	0.271
CLM5		0.075	0.084	0.118	0.063	0.098	0.172	0.071	0.140	0.267
CBP5		0.061	0.066	0.099	0.053	0.081	0.140	0.061	0.115	0.201
CLB5		0.084	0.096	0.132	0.069	0.096	0.160	0.062	0.120	0.207

Table 2

Empirical sizes, with normal GARCH (1, 1) disturbances, based on standard 5% critical values for matrix X1.

	T=	50		100		500	
Test Statistics	γ_1	$\alpha_1 = 0.2$	0.4	0.2	0.4	0.2	0.4
A: Standard serial correlation tests							
DW1	0.0	0.045	0.053	0.061	0.077	0.063	0.120
LM2		0.091	0.140	0.087	0.139	0.085	0.178
LM5		0.079	0.113	0.076	0.118	0.075	0.159
BP5		0.058	0.074	0.068	0.102	0.070	0.141
LB5		0.082	0.101	0.079	0.116	0.074	0.145
DW1	0.4	0.090	0.095	0.087	0.109	0.121	0.130
LM2		0.096	0.159	0.116	0.189	0.115	0.290
LM5		0.097	0.145	0.095	0.179	0.102	0.342
BP5		0.068	0.117	0.088	0.173	0.104	0.348
LB5		0.099	0.157	0.104	0.202	0.108	0.354
DW1	0.6	0.091	0.111	0.099	0.138	0.109	0.298
LM2		0.093	0.146	0.129	0.231	0.142	0.475
LM5		0.101	0.156	0.117	0.276	0.154	0.600
BP5		0.096	0.158	0.113	0.276	0.152	0.626
LB5		0.124	0.192	0.127	0.306	0.156	0.632
B: ARCH (2)-corrected serial correlation tests							
CDW1	0.0	0.042	0.050	0.053	0.059	0.053	0.050
CLM2		0.045	0.069	0.046	0.060	0.041	0.058
CLM5		0.060	0.064	0.045	0.047	0.044	0.053
CBP5		0.042	0.044	0.047	0.043	0.043	0.050
CLB5		0.069	0.064	0.058	0.054	0.044	0.053
CDW1	0.4	0.063	0.068	0.062	0.070	0.077	0.081
CLM2		0.050	0.070	0.064	0.090	0.058	0.099
CLM5		0.060	0.093	0.068	0.090	0.057	0.120
CBP5		0.041	0.059	0.052	0.072	0.053	0.078
CLB5		0.067	0.082	0.059	0.086	0.055	0.080
CDW1	0.6	0.062	0.080	0.059	0.098	0.060	0.100
CLM2		0.050	0.084	0.057	0.127	0.062	0.223
CLM5		0.083	0.140	0.079	0.201	0.078	0.345
CBP5		0.057	0.070	0.062	0.107	0.061	0.204
CLB5		0.089	0.098	0.071	0.124	0.062	0.211

Table 3

Empirical sizes of the Wooldridge's ARCH(2)-corrected robust LM test and Diebold's corrected BP and LB tests based on standard 5% critical values for matrix X1.

	T=	50			100			500		
Test Statistics	α_2	$\alpha_1 = 0.0$	0.2	0.4	0.0	0.2	0.4	0.0	0.2	0.4
A: Normal ARCH(2) disturbances.										
RLM2	0.0	0.049	0.057	0.051	0.048	0.048	0.051	0.053	0.046	0.048
RLM5		0.026	0.026	0.023	0.046	0.040	0.042	0.049	0.052	0.050
DBP5		0.064	0.135	0.139	0.070	0.140	0.142	0.120	0.147	0.151
DLB5		0.060	0.120	0.121	0.065	0.128	0.128	0.100	0.120	0.134
RLM2	0.4	0.056	0.055	0.047	0.046	0.054	0.054	0.049	0.051	0.054
RLM5		0.027	0.032	0.025	0.041	0.049	0.048	0.048	0.049	0.052
DBP5		0.097	0.124	0.142	0.100	0.128	0.140	0.113	0.130	0.137
DLB5		0.087	0.113	0.124	0.090	0.112	0.106	0.096	0.118	0.127
RLM2	0.6	0.052	0.049	0.052	0.051	0.054	0.041	0.048	0.049	0.043
RLM5		0.028	0.022	0.025	0.041	0.042	0.034	0.037	0.043	0.040
DBP5		0.112	0.157	0.162	0.120	0.139	0.145	0.129	0.136	0.128
DLB5		0.096	0.142	0.157	0.115	0.131	0.139	0.120	0.120	0.127
	γ_1	$\alpha_1 =$	0.2	0.4	$\alpha_1 =$	0.2	0.4	$\alpha_1 =$	0.2	0.4
B: Normal GARCH (1,1) disturbances.										
RLM2	0.0		0.041	0.046		0.045	0.059		0.053	0.046
RLM5			0.018	0.029		0.039	0.048		0.052	0.047
DBP5			0.128	0.130		0.131	0.131		0.129	0.138
DLB5			0.121	0.123		0.120	0.122		0.118	0.121
RLM2	0.4		0.046	0.048		0.054	0.054		0.045	0.051
RLM5			0.034	0.025		0.047	0.042		0.047	0.049
DBP5			0.119	0.138		0.122	0.131		0.123	0.129
DLB5			0.102	0.130		0.111	0.120		0.120	0.121
RLM2	0.6		0.049	0.052		0.047	0.052		0.046	0.050
RLM5			0.028	0.028		0.041	0.045		0.047	0.053
DBP5			0.148	0.150		0.129	0.130		0.127	0.129
DLB5			0.131	0.139		0.113	0.114		0.118	0.120

Table 4

Empirical sizes with ARCH (2) disturbances of ARCH(2)-corrected serial correlation tests, based on standard 5% critical values for matrix X1 and different disturbance distributions.

			Disturbance Distribution							
T	(α_1, α_2)	Test Statistics	NORMAL	MIXNOR	RSMT	RSHT	HRSMT	HRSHT	KURT6	KURT9
50	(0, 0)	CDW1	0.041	0.040	0.045	0.049	0.050	0.050	0.049	0.050
		CLM2	0.030	0.038	0.036	0.028	0.032	0.039	0.028	0.030
		CLM5	0.056	0.055	0.057	0.054	0.058	0.056	0.057	0.055
		CBP5	0.047	0.049	0.040	0.032	0.039	0.032	0.035	0.034
		CLB5	0.063	0.061	0.055	0.049	0.050	0.046	0.052	0.049
		RLM2	0.049	0.051	0.050	0.053	0.052	0.055	0.057	0.056
		RLM5	0.026	0.028	0.026	0.027	0.028	0.031	0.030	0.032
	(0.4, 0.4)	CDW1	0.061	0.065	0.068	0.072	0.070	0.073	0.077	0.075
		CLM2	0.107	0.109	0.108	0.101	0.109	0.104	0.104	0.104
		CLM5	0.111	0.110	0.105	0.109	0.098	0.100	0.104	0.100
		CBP5	0.082	0.080	0.083	0.072	0.075	0.080	0.079	0.078
		CLB5	0.107	0.108	0.094	0.092	0.090	0.099	0.093	0.095
		RLM2	0.047	0.050	0.051	0.052	0.050	0.050	0.052	0.055
		RLM5	0.025	0.027	0.030	0.030	0.021	0.028	0.029	0.032
100	(0,0)	CDW1	0.051	0.050	0.048	0.055	0.052	0.055	0.056	0.056
		CLM2	0.035	0.048	0.037	0.029	0.032	0.029	0.032	0.028
		CLM5	0.065	0.062	0.060	0.062	0.058	0.053	0.056	0.055
		CBP5	0.056	0.058	0.054	0.059	0.054	0.055	0.057	0.058
		CLB5	0.065	0.068	0.067	0.069	0.070	0.072	0.073	0.073
		RLM2	0.048	0.047	0.049	0.049	0.050	0.050	0.052	0.052
		RLM5	0.046	0.047	0.050	0.051	0.051	0.052	0.050	0.054
	(0.4, 0.4)	CDW1	0.078	0.075	0.090	0.092	0.087	0.089	0.099	0.091
		CLM2	0.132	0.120	0.129	0.130	0.126	0.129	0.129	0.128
		CLM5	0.106	0.110	0.117	0.115	0.113	0.116	0.115	0.114
		CBP5	0.089	0.092	0.110	0.105	0.102	0.112	0.109	0.115
		CLB5	0.103	0.103	0.110	0.120	0.125	0.109	0.108	0.105
		RLM2	0.054	0.054	0.054	0.053	0.053	0.055	0.055	0.058
		RLM5	0.048	0.050	0.050	0.049	0.050	0.050	0.051	0.054

Table 5

Estimated sizes with normal ARCH(2) disturbances of the ARCH(2)-corrected serial correlation tests based on standard 5% critical values for matrix X4.

	α_1 =	0.0	0.0	0.0	0.2	0.2	0.2	0.4	0.4	0.4
(T,σ,δ)	α_2 =	0.0	0.4	0.6	0.0	0.4	0.6	0.0	0.4	0.6
(50,2,0.2)	CLM2	0.054	0.075	0.093	0.059	0.079	0.132	0.060	0.121	0.142
	CBP2	0.006	0.020	0.017	0.010	0.029	0.039	0.013	0.033	0.034
	CBL2	0.010	0.025	0.023	0.013	0.038	0.047	0.014	0.039	0.051
	RLM2	0.015	0.023	0.019	0.018	0.024	0.033	0.019	0.030	0.022
	CDW	0.003	0.002	0.002	0.001	0.005	0.009	0.021	0.020	0.021
(50,2,0.8)	CLM2	0.062	0.071	0.100	0.050	0.094	0.129	0.078	0.140	0.162
	CBD2	0.019	0.024	0.024	0.015	0.027	0.037	0.016	0.035	0.049
	CBL2	0.021	0.030	0.031	0.021	0.032	0.049	0.020	0.046	0.059
	RLM2	0.024	0.029	0.027	0.022	0.020	0.031	0.028	0.025	0.026
	CDW	0.020	0.012	0.027	0.021	0.030	0.042	0.027	0.039	0.059
(50,0.07,0.2)	CLM2	0.053	0.062	0.097	0.057	0.081	0.152	0.049	0.132	0.139
	CBP2	0.007	0.024	0.018	0.013	0.049	0.061	0.014	0.044	0.049
	CBL2	0.009	0.026	0.020	0.018	0.048	0.057	0.013	0.049	0.071
	RLM2	0.020	0.023	0.018	0.019	0.028	0.038	0.024	0.038	0.030
	CDW	0.030	0.013	0.015	0.020	0.009	0.005	0.008	0.012	0.013
(50,0.07,0.8)	CLM2	0.062	0.069	0.095	0.040	0.098	0.138	0.082	0.140	0.152
	CBP2	0.023	0.023	0.021	0.011	0.030	0.045	0.021	0.031	0.045
	CBL2	0.025	0.028	0.029	0.024	0.032	0.055	0.027	0.043	0.058
	RLM2	0.028	0.031	0.030	0.028	0.025	0.035	0.031	0.029	0.031
	CDW	0.029	0.014	0.018	0.021	0.028	0.043	0.023	0.047	0.060
(100,2,0.2)	CLM2	0.055	0.060	0.115	0.070	0.089	0.138	0.072	0.154	0.202
	CBP2	0.012	0.017	0.030	0.021	0.026	0.045	0.010	0.041	0.077
	CBL2	0.013	0.019	0.032	0.023	0.030	0.049	0.012	0.045	0.085
	RLM2	0.013	0.017	0.020	0.025	0.019	0.025	0.015	0.026	0.024
	CDW	0.007	0.010	0.009	0.019	0.016	0.019	0.021	0.024	0.033
(100,2,0.8)	CLM2	0.032	0.078	0.126	0.061	0.096	0.157	0.076	0.158	0.271
	CBP2	0.014	0.036	0.037	0.020	0.024	0.044	0.020	0.046	0.086
	CBL2	0.015	0.039	0.041	0.024	0.028	0.048	0.023	0.055	0.092
	RLM2	0.021	0.026	0.032	0.026	0.018	0.019	0.020	0.019	0.017
	CDW	0.015	0.025	0.033	0.024	0.029	0.053	0.040	0.055	0.087
(100,0.07,0.2)	CLM2	0.059	0.055	0.102	0.075	0.079	0.117	0.068	0.148	0.182
	CBP2	0.016	0.012	0.029	0.022	0.028	0.044	0.009	0.038	0.075
	CBL2	0.023	0.018	0.030	0.033	0.035	0.047	0.010	0.042	0.089
	RLM2	0.015	0.016	0.024	0.034	0.020	0.023	0.012	0.029	0.024
	CDW	0.010	0.012	0.018	0.019	0.021	0.021	0.025	0.030	0.037
(100,0.07,0.8)	CLM2	0.025	0.075	0.131	0.062	0.095	0.130	0.074	0.160	0.288
	CBP2	0.018	0.040	0.038	0.024	0.023	0.042	0.018	0.049	0.096
	CBL2	0.018	0.044	0.044	0.027	0.028	0.047	0.021	0.048	0.092
	RLM2	0.016	0.025	0.034	0.031	0.019	0.018	0.019	0.018	0.016
	CDW	0.017	0.028	0.038	0.021	0.037	0.056	0.025	0.056	0.084

Table 6

Empirical powers against normal AR(2) disturbances of ARCH(2)-corrected serial correlation tests based on asymptotic 5% critical values for matrix X1 with T = 50, with different underlying disturbance distributions.

		$\alpha_1 =$	0.0	0.0	0.0	0.2	0.2	0.2	0.4	0.4	0.4
ρ_1	ρ_2	$\alpha_2 =$	0.0	0.4	0.6	0.0	0.4	0.6	0.0	0.4	0.6
A: Normal AR(2)-ARCH(2) disturbances											
0.1	0.1	CLM2	0.071	0.072	0.074	0.075	0.096	0.099	0.110	0.126	0.123
		CBP2	0.083	0.063	0.690	0.088	0.076	0.111	0.115	0.105	0.110
		CBL2	0.092	0.086	0.090	0.103	0.093	0.120	0.128	0.120	0.118
		RLM2	0.060	0.061	0.062	0.062	0.066	0.063	0.064	0.068	0.067
0.3		CLM2	0.251	0.279	0.280	0.293	0.342	0.280	0.340	0.345	0.346
		CBD2	0.283	0.280	0.310	0.315	0.354	0.303	0.303	0.349	0.333
		CBL2	0.323	0.315	0.358	0.362	0.386	0.337	0.376	0.355	0.350
		RLM2	0.250	0.258	0.240	0.248	0.245	0.248	0.243	0.243	0.255
0.4		CLM2	0.480	0.472	0.480	0.485	0.487	0.482	0.512	0.514	0.516
		CBP2	0.519	0.461	0.512	0.520	0.482	0.473	0.490	0.499	0.489
		CBL2	0.551	0.495	0.532	0.557	0.515	0.515	0.522	0.530	0.538
		RLM2	0.445	0.426	0.429	0.430	0.440	0.436	0.442	0.435	0.440
0.1	0.3	CLM2	0.313	0.321	0.311	0.272	0.353	0.313	0.348	0.360	0.362
		CBP2	0.344	0.326	0.315	0.300	0.342	0.308	0.336	0.342	0.350
		CBL2	0.375	0.360	0.350	0.330	0.371	0.333	0.366	0.372	0.379
		RLM2	0.282	0.280	0.288	0.290	0.285	0.288	0.280	0.286	0.289
0.3		CLM2	0.564	0.592	0.585	0.571	0.603	0.589	0.565	0.580	0.589
		CBP2	0.612	0.624	0.621	0.616	0.629	0.605	0.593	0.590	0.603
		CBL2	0.632	0.654	0.648	0.649	0.655	0.625	0.615	0.618	0.609
		RLM2	0.562	0.500	0.497	0.490	0.498	0.525	0.530	0.510	0.520
0.4		CLM2	0.720	0.721	0.721	0.724	0.732	0.732	0.721	0.708	0.715
		CBP2	0.762	0.746	0.752	0.757	0.750	0.757	0.755	0.714	0.714
		CBL2	0.789	0.766	0.770	0.774	0.769	0.785	0.776	0.750	0.760
		RLM2	0.703	0.680	0.672	0.678	0.690	0.686	0.689	0.685	0.690
0.1	0.5	CLM2	0.736	0.777	0.728	0.713	0.746	0.717	0.719	0.740	0.754
		CBP2	0.748	0.781	0.755	0.746	0.759	0.721	0.717	0.734	0.738
		CBL2	0.770	0.800	0.778	0.759	0.777	0.739	0.736	0.743	0.752
		RLM2	0.700	0.703	0.700	0.698	0.705	0.707	0.699	0.692	0.692
0.3		CLM2	0.906	0.883	0.891	0.880	0.851	0.867	0.876	0.866	0.868
		CBP2	0.926	0.895	0.899	0.909	0.877	0.882	0.879	0.867	0.862
		CBL2	0.938	0.904	0.918	0.917	0.886	0.894	0.897	0.878	0.890
		RLM2	0.812	0.810	0.811	0.805	0.809	0.803	0.805	0.798	0.800
0.4		CLM2	0.939	0.938	0.938	0.932	0.940	0.931	0.926	0.920	0.919
		CBD2	0.950	0.941	0.930	0.929	0.942	0.936	0.929	0.928	0.920
		CBL2	0.958	0.948	0.935	0.939	0.949	0.946	0.935	0.924	0.930
		RLM2	0.925	0.907	0.903	0.901	0.896	0.883	0.872	0.909	0.872

Table 6 (continued)

		$\alpha_1 =$	0.0	0.0	0.0	0.2	0.2	0.2	0.4	0.4	0.4
ρ_1	ρ_2	$\alpha_2 =$	0.0	0.4	0.6	0.0	0.4	0.6	0.0	0.4	0.6
B: RSHT - AR(2)-ARCH(2) disturbances											
0.1	0.1	CLM2	0.057	0.063	0.074	0.074	0.091	0.089	0.085	0.124	0.114
		CBP2	0.069	0.070	0.081	0.075	0.092	0.086	0.093	0.103	0.106
		CBL2	0.080	0.082	0.092	0.094	0.109	0.102	0.114	0.125	0.118
		RLM2	0.057	0.070	0.067	0.064	0.069	0.050	0.063	0.077	0.052
0.3		CLM2	0.261	0.276	0.295	0.301	0.278	0.296	0.281	0.304	0.349
		CBP2	0.299	0.315	0.312	0.334	0.280	0.300	0.304	0.310	0.333
		CBL2	0.327	0.347	0.351	0.368	0.317	0.349	0.330	0.338	0.362
		RLM2	0.271	0.216	0.214	0.250	0.207	0.182	0.227	0.206	0.180
0.4		CLM2	0.517	0.467	0.471	0.514	0.499	0.468	0.501	0.509	0.512
		CBP2	0.541	0.492	0.476	0.553	0.508	0.486	0.520	0.519	0.496
		CBL2	0.580	0.539	0.504	0.591	0.546	0.515	0.558	0.562	0.540
		RLM2	0.471	0.381	0.354	0.429	0.372	0.368	0.457	0.349	0.321
0.1	0.3	CLM2	0.315	0.329	0.366	0.335	0.359	0.364	0.362	0.372	0.365
		CBP2	0.337	0.343	0.373	0.356	0.349	0.364	0.362	0.372	0.361
		CBL2	0.359	0.368	0.405	0.382	0.381	0.397	0.387	0.399	0.392
		RLM2	0.265	0.271	0.301	0.250	0.260	0.254	0.234	0.198	0.211
0.3		CLM2	0.552	0.546	0.573	0.561	0.558	0.545	0.562	0.570	0.564
		CBP2	0.618	0.604	0.620	0.615	0.599	0.582	0.590	0.618	0.595
		CBL2	0.643	0.632	0.639	0.642	0.636	0.607	0.622	0.641	0.615
		RLM2	0.532	0.499	0.515	0.507	0.494	0.452	0.495	0.450	0.391
0.4		CLM2	0.716	0.716	0.714	0.736	0.721	0.718	0.721	0.741	0.726
		CBP2	0.774	0.746	0.750	0.780	0.754	0.734	0.738	0.747	0.745
		CBL2	0.796	0.765	0.785	0.799	0.776	0.760	0.755	0.771	0.768
		RLM2	0.673	0.649	0.659	0.711	0.642	0.596	0.658	0.603	0.571
0.1	0.5	CLM2	0.772	0.787	0.787	0.745	0.771	0.761	0.744	0.734	0.751
		CBP2	0.803	0.806	0.790	0.768	0.786	0.770	0.749	0.741	0.752
		CBL2	0.822	0.818	0.815	0.796	0.811	0.790	0.775	0.757	0.779
		RLM2	0.729	0.725	0.727	0.674	0.685	0.657	0.644	0.609	0.542
0.3		CLM2	0.878	0.886	0.878	0.883	0.889	0.885	0.882	0.867	0.880
		CBP2	0.913	0.902	0.893	0.902	0.898	0.903	0.899	0.893	0.888
		CBL2	0.928	0.914	0.902	0.916	0.910	0.915	0.913	0.901	0.896
		RLM2	0.870	0.871	0.848	0.855	0.842	0.805	0.839	0.834	0.773
0.4		CLM2	0.935	0.933	0.932	0.936	0.927	0.927	0.922	0.920	0.920
		CBD2	0.952	0.947	0.938	0.947	0.941	0.939	0.937	0.932	0.928
		CBL2	0.957	0.953	0.941	0.948	0.949	0.946	0.942	0.941	0.936
		RLM2	0.923	0.904	0.894	0.917	0.899	0.868	0.914	0.875	0.851

Table 6 (continued

ρ_1	ρ_2	$\alpha_1 =$	0.0	0.0	0.0	0.2	0.2	0.2	0.4	0.4	0.4
		$\alpha_2 =$	0.0	0.4	0.6	0.0	0.4	0.6	0.0	0.4	0.6
C: HRSMT - AR(2)-ARCH(2) disturbances											
0.1	0.1	CLM2	0.041	0.095	0.149	0.076	0.142	0.165	0.107	0.203	0.165
		CBP2	0.047	0.076	0.115	0.067	0.111	0.112	0.092	0.147	0.112
		CBL2	0.059	0.092	0.135	0.089	0.126	0.133	0.103	0.171	0.133
		RLM2	0.061	0.161	0.194	0.074	0.219	0.293	0.139	0.283	0.353
0.3		CLM2	0.249	0.260	0.305	0.274	0.310	0.357	0.307	0.378	0.357
		CBP2	0.286	0.271	0.273	0.309	0.297	0.305	0.295	0.337	0.305
		CBL2	0.316	0.298	0.299	0.388	0.329	0.344	0.321	0.367	0.344
		RLM2	0.235	0.358	0.463	0.264	0.462	0.519	0.317	0.490	0.574
0.4		CLM2	0.448	0.428	0.445	0.479	0.490	0.526	0.500	0.543	0.526
		CBP2	0.484	0.411	0.403	0.486	0.452	0.449	0.504	0.482	0.449
		CBL2	0.525	0.442	0.436	0.526	0.495	0.491	0.530	0.510	0.491
		RLM2	0.419	0.565	0.613	0.454	0.597	0.643	0.524	0.640	0.691
0.1	0.3	CLM2	0.285	0.379	0.425	0.291	0.345	0.390	0.331	0.400	0.390
		CBP2	0.318	0.355	0.361	0.313	0.327	0.347	0.315	0.325	0.347
		CBL2	0.345	0.390	0.388	0.342	0.346	0.367	0.336	0.354	0.367
		RLM2	0.232	0.356	0.442	0.326	0.439	0.563	0.400	0.523	0.575
0.3		CLM2	0.533	0.582	0.616	0.550	0.590	0.585	0.536	0.610	0.589
		CBP2	0.591	0.602	0.585	0.586	0.584	0.580	0.556	0.570	0.580
		CBL2	0.615	0.631	0.613	0.617	0.605	0.601	0.578	0.591	0.601
		RLM2	0.520	0.630	0.656	0.530	0.681	0.714	0.603	0.714	0.785
0.4		CLM2	0.732	0.693	0.728	0.705	0.692	0.698	0.688	0.709	0.698
		CBP2	0.768	0.711	0.716	0.735	0.706	0.679	0.696	0.703	0.679
		CBL2	0.796	0.733	0.740	0.762	0.723	0.699	0.715	0.720	0.699
		RLM2	0.681	0.764	0.775	0.690	0.820	0.836	0.724	0.815	0.851
0.1	0.5	CLM2	0.744	0.769	0.787	0.709	0.739	0.734	0.708	0.740	0.734
		CBP2	0.762	0.755	0.754	0.721	0.722	0.700	0.702	0.729	0.700
		CBL2	0.790	0.787	0.777	0.747	0.750	0.730	0.731	0.740	0.730
		RLM2	0.698	0.772	0.791	0.730	0.777	0.813	0.731	0.784	0.810
0.3		CLM2	0.903	0.896	0.910	0.859	0.873	0.849	0.866	0.851	0.849
		CBP2	0.919	0.898	0.899	0.886	0.873	0.847	0.877	0.850	0.847
		CBL2	0.930	0.907	0.913	0.894	0.886	0.858	0.889	0.857	0.858
		RLM2	0.849	0.892	0.902	0.855	0.910	0.940	0.869	0.921	0.924
0.4		CLM2	0.953	0.928	0.921	0.926	0.919	0.914	0.918	0.910	0.914
		CBP2	0.955	0.935	0.917	0.941	0.926	0.921	0.918	0.910	0.921
		CBL2	0.962	0.940	0.925	0.946	0.934	0.926	0.929	0.915	0.926
		RLM2	0.908	0.940	0.935	0.907	0.940	0.952	0.914	0.960	0.969

Table 7

Empirical powers of ARCH(2)-corrected serial correlation tests based on standard 5% critical values for matrix X1 with T = 50, when the underlying disturbance process is normal AR(2) - GARCH(1, 1)

		$\alpha_1 =$	0.2	0.2	0.2	0.4	0.4	0.4
ρ_1	ρ_2	$\gamma_1 =$	0.0	0.4	0.6	0.0	0.4	0.6
A: Powers against normal AR (2).								
0.1	0.1	CLM2	0.055	0.045	0.068	0.063	0.045	0.043
		CBP2	0.070	0.061	0.093	0.092	0.104	0.104
		CBL2	0.083	0.070	0.085	0.090	0.100	0.082
		RLM2	0.102	0.089	0.093	0.109	0.113	0.098
	0.3	CLM2	0.231	0.245	0.210	0.234	0.237	0.211
		CBD2	0.276	0.295	0.277	0.294	0.316	0.332
		CBL2	0.288	0.318	0.297	0.312	0.337	0.331
		CLM2	0.320	0.355	0.337	0.345	0.375	0.356
	0.4	CLM2	0.467	0.422	0.424	0.418	0.396	0.361
		CBP2	0.505	0.488	0.490	0.518	0.554	0.518
		CBL2	0.519	0.518	0.512	0.507	0.531	0.515
		RLM2	0.553	0.548	0.555	0.544	0.570	0.550
0.3	0.1	CLM2	0.242	0.244	0.233	0.200	0.213	0.184
		CBP2	0.291	0.298	0.306	0.314	0.340	0.339
		CBL2	0.314	0.311	0.313	0.306	0.327	0.317
		RLM2	0.342	0.344	0.337	0.330	0.354	0.352
	0.3	CLM2	0.460	0.505	0.418	0.454	0.445	0.449
		CBP2	0.537	0.570	0.557	0.559	0.571	0.575
		CBL2	0.582	0.615	0.595	0.585	0.600	0.590
		RLM2	0.615	0.640	0.624	0.611	0.629	0.617
	0.4	CLML	0.666	0.638	0.638	0.625	0.641	0.627
		CBP2	0.724	0.728	0.707	0.716	0.741	0.714
		CBL2	0.747	0.729	0.749	0.731	0.746	0.734
		RLM2	0.767	0.762	0.761	0.766	0.770	0.756
0.5	0.1	CLM2	0.671	0.656	0.640	0.618	0.597	0.593
		CBP2	0.742	0.750	0.726	0.717	0.744	0.732
		CBL2	0.760	0.763	0.744	0.725	0.751	0.721
		RLM2	0.781	0.785	0.762	0.744	0.767	0.741
	0.3	CLM2	0.857	0.838	0.843	0.830	0.814	0.795
		CBP2	0.894	0.889	0.892	0.892	0.867	0.874
		CBL2	0.909	0.901	0.912	0.895	0.876	0.886
		RLM2	0.912	0.913	0.928	0.905	0.889	0.897
	0.4	CLM2	0.908	0.897	0.900	0.893	0.882	0.885
		CBP2	0.932	0.914	0.936	0.917	0.921	0.916
		CBL2	0.949	0.929	0.941	0.926	0.922	0.927
		RLM2	0.952	0.937	0.944	0.933	0.929	0.931

	$\alpha_1 = 0.2$	0.2	0.2	0.4	0.4	0.4
ρ_1	$\gamma_1 = 0.0$	0.4	0.6	0.0	0.4	0.6
B: Powers against normal AR(1) of the correct DW test, CDW1						
0.1	0.089	0.089	0.101	0.107	0.116	0.116
0.3	0.415	0.422	0.429	0.453	0.437	0.452
0.5	0.829	0.833	0.847	0.841	0.798	0.827
0.7	0.979	0.984	0.975	0.969	0.973	0.968
0.9	1.000	0.997	0.995	0.995	0.999	0.999

Table 8

Estimated powers against normal AR(1) disturbances of the ARCH(2)-corrected DW test, based on standard 5% critical values for matrix X4 with T = 50, when the underlying disturbance distribution is normal AR(1)-ARCH(2).

	$\alpha_1 = 0.0$	0.0	0.0	0.2	0.2	0.2	0.4	0.4	0.4
(ρ_1,σ,δ)	$\alpha_2 = 0.0$	0.4	0.6	0.0	0.4	0.6	0.0	0.4	0.6
(0.1,1,0)	0.124	0.126	0.122	0.120	0.130	0.110	0.130	0.140	0.136
(0.3,1,0)	0.480	0.502	0.463	0.461	0.460	0.430	0.440	0.420	0.380
(0.5,1,0)	0.850	0.850	0.810	0.820	0.770	0.730	0.760	0.730	0.700
(0.7,1,0)	0.970	0.950	0.930	0.960	0.950	0.920	0.940	0.900	0.860
(0.9,1,0)	1.000	0.990	0.981	0.992	0.987	0.976	0.979	0.968	0.952
(0.1,2,0.2)	0.003	0.006	0.016	0.005	0.017	0.026	0.014	0.036	0.040
(0.3,2,0.2)	0.028	0.044	0.060	0.040	0.047	0.077	0.059	0.073	0.099
(0.5,2,0.2)	0.142	0.153	0.192	0.189	0.185	0.199	0.149	0.175	0.240
(0.7,2,0.2)	0.245	0.287	0.302	0.287	0.285	0.307	0.246	0.271	0.348
(0.9,2,0.2)	0.381	0.392	0.403	0.380	0.378	0.399	0.432	0.392	0.430
(0.1,2,0.8)	0.061	0.080	0.140	0.088	0.120	0.150	0.110	0.162	0.189
(0.3,2,0.8)	0.207	0.212	0.183	0.211	0.239	0.204	0.240	0.241	0.236
(0.5,2,0.8)	0.481	0.430	0.453	0.434	0.467	0.486	0.430	0.471	0.439
(0.7,2,0.8)	0.680	0.728	0.761	0.741	0.667	0.667	0.633	0.677	0.640
(0.9,2,0.8)	0.978	0.978	0.983	0.957	0.951	0.951	0.914	0.931	0.933
(0.1,0.07,0.2)	0.089	0.080	0.106	0.108	0.116	0.110	0.113	0.111	0.125
(0.3,0.07,0.2)	0.232	0.245	0.235	0.226	0.239	0.284	0.241	0.252	0.261
(0.5,0.07,0.2)	0.396	0.410	0.385	0.362	0.371	0.408	0.342	0.408	0.419
(0.7,0.07,0.2)	0.588	0.562	0.611	0.521	0.556	0.590	0.584	0.584	0.560
(0.9,0.07,0.2)	0.940	0.955	0.953	0.891	0.892	0.913	0.873	0.881	0.893
(0.1,0.07,0.8)	0.132	0.142	0.136	0.158	0.192	0.163	0.211	0.252	0.265
(0.3,0.07,0.8)	0.328	0.352	0.359	0.320	0.327	0.370	0.362	0.363	0.372
(0.5,0.07,0.8)	0.633	0.692	0.719	0.546	0.602	0.630	0.542	0.555	0.633
(0.7,0.07,0.8)	0.978	0.978	0.983	0.957	0.951	0.951	0.914	0.931	0.933
(0.9,0.07,0.8)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999

Table 9

Estimated powers against normal AR(2) distributions of ARCH(2)-corrected serial correlation tests based on asymptotic critical values at the 5 per cent nominal level for matrix X4 with T = 100, when the underlying distribution is normal AR(2)-ARCH(2)

	$\alpha_1 =$	0.0	0.0	0.0	0.2	0.2	0.2	0.4	0.4	0.4
$(\sigma,\rho_1,\rho_2,\delta)$	$\alpha_2 =$	0.0	0.4	0.6	0.0	0.4	0.6	0.0	0.4	0.6
(2,0.3,0.1,0.2)	CLM2	0.084	0.106	0.142	0.086	0.140	0.160	0.117	0.172	0.220
	CBP2	0.010	0.027	0.011	0.028	0.053	0.012	0.043	0.052	0.071
	CBL2	0.011	0.018	0.030	0.013	0.030	0.057	0.013	0.047	0.079
	RLM2	0.028	0.025	0.025	0.027	0.028	0.020	0.023	0.026	0.027
(2,0.3,0.1,0.8)	CLM2	0.606	0.675	0.748	0.584	0.637	0.690	0.610	0.645	0.680
	CBP2	0.464	0.521	0.576	0.422	0.475	0.489	0.416	0.443	0.483
	CBL2	0.479	0.542	0.591	0.041	0.489	0.505	0.437	0.463	0.494
	RLM2	0.436	0.462	0.486	0.395	0.399	0.386	0.379	0.362	0.367
(2,0.3,0.4,0.2)	CLM2	0.535	0.510	0.559	0.504	0.529	0.584	0.532	0.568	0.613
	CBP2	0.512	0.449	0.483	0.447	0.454	0.471	0.443	0.435	0.429
	CBL2	0.526	0.463	0.493	0.463	0.466	0.479	0.466	0.451	0.445
	RLM2	0.483	0.451	0.410	0.434	0.401	0.390	0.389	0.385	0.339
(2,0.3,0.4,0.8)	CLM2	0.903	0.898	0.873	0.818	0.877	0.881	0.920	0.889	0.891
	CBP2	0.870	0.825	0.795	0.870	0.807	0.798	0.858	0.796	0.799
	CBL2	0.885	0.830	0.809	0.882	0.815	0.805	0.868	0.810	0.810
	RLM2	0.844	0.799	0.756	0.834	0.755	0.752	0.790	0.752	0.730
(2,0.5,0.1,0.2)	CLM2	0.176	0.193	0.222	0.171	0.210	0.269	0.181	0.242	0.322
	CBP2	0.020	0.036	0.052	0.033	0.057	0.062	0.024	0.055	0.099
	CBL2	0.023	0.039	0.055	0.041	0.059	0.068	0.026	0.062	0.106
	RLM2	0.062	0.063	0.059	0.060	0.063	0.066	0.061	0.067	0.069
(2,0.5,0.1,0.8)	CLM2	0.961	0.966	0.975	0.948	0.947	0.953	0.929	0.936	0.953
	CBP2	0.927	0.946	0.940	0.893	0.895	0.887	0.861	0.879	0.884
	CBL2	0.932	0.947	0.947	0.899	0.899	0.896	0.871	0.885	0.897
	RLM2	0.918	0.928	0.923	0.898	0.890	0.872	0.882	0.842	0.802
(2,0.5,0.4,0.2)	CLM2	0.298	0.305	0.374	0.328	0.364	0.442	0.336	0.438	0.503
	CBP2	0.294	0.249	0.288	0.294	0.279	0.317	0.261	0.303	0.334
	CBL2	0.302	0.266	0.303	0.314	0.292	0.328	0.278	0.323	0.344
	RLM2	0.270	0.252	0.257	0.280	0.250	0.261	0.265	0.250	0.232
(2,0.5,0.4,0.8)	CLM2	0.978	0.987	0.986	0.980	0.975	0.974	0.978	0.970	0.976
	CBP2	0.983	0.983	0.979	0.978	0.966	0.955	0.970	0.952	0.955
	CBL2	0.985	0.983	0.983	0.982	0.970	0.958	0.972	0.956	0.959
	RLM2	0.970	0.974	0.969	0.971	0.950	0.936	0.945	0.929	0.898

Table 9 continued

	$\alpha_1 =$	0.0	0.0	0.0	0.2	0.2	0.2	0.4	0.4	0.4
$(\sigma,\rho_1,\rho_2,\delta)$	$\alpha_2 =$	0.0	0.4	0.6	0.0	0.4	0.6	0.0	0.4	0.6
(0.07,0.3,0.1,0.2)	CLM2	0.307	0.350	0.360	0.344	0.367	0.423	0.391	0.434	0.483
	CBP2	0.184	0.185	0.184	0.199	0.200	0.219	0.218	0.233	0.272
	CBL2	0.191	0.197	0.197	0.215	0.213	0.231	0.229	0.245	0.278
	RLM2	0.183	0.169	0.156	0.189	0.158	0.170	0.170	0.149	0.150
(0.07,0.3,0.1,0.8)	CLM2	0.827	0.782	0.771	0.842	0.754	0.871	0.841	0.803	0.831
	CBD2	0.752	0.664	0.604	0.762	0.633	0.749	0.700	0.639	0.655
	CBL2	0.767	0.681	0.620	0.777	0.651	0.701	0.722	0.651	0.672
	RLM2	0.743	0.690	0.650	0.735	0.624	0.684	0.695	0.560	0.530
(0.07,0.3,0.4,0.2)	CLM2	0.760	0.713	0.734	0.772	0.693	0.696	0.770	0.750	0.763
	CBP2	0.714	0.639	0.624	0.707	0.616	0.681	0.692	0.639	0.615
	CBL2	0.732	0.654	0.638	0.729	0.617	0.699	0.705	0.652	0.630
	RLM2	0.685	0.589	0.572	0.690	0.569	0.666	0.666	0.616	0.539
(0.07,0.3,0.4,0.8)	CLM2	0.963	0.968	0.977	0.943	0.941	0.940	0.936	0.931	0.926
	CBP2	0.946	0.948	0.954	0.915	0.899	0.900	0.883	0.885	0.886
	CBL2	0.949	0.960	0.959	0.910	0.911	0.930	0.880	0.881	0.878
	RLM2	0.926	0.913	0.929	0.872	0.852	0.860	0.790	0.763	0.740
(0.07,0.5,0.1,0.2)	CLM2	0.441	0.426	0.490	0.486	0.497	0.480	0.469	0.513	0.572
	CBP2	0.336	0.309	0.351	0.358	0.349	0.350	0.344	0.364	0.417
	CBL2	0.354	0.319	0.364	0.382	0.370	0.372	0.368	0.378	0.438
	RLM2	0.323	0.301	0.306	0.344	0.300	0.299	0.298	0.264	0.250
(0.07,0.5,0.1,0.8)	CLM2	0.992	0.973	0.986	0.975	0.981	0.986	0.981	0.978	0.973
	CBP2	0.989	0.963	0.977	0.974	0.972	0.970	0.969	0.969	0.958
	CBL2	0.990	0.965	0.979	0.974	0.973	0.974	0.974	0.970	0.958
	RLM2	0.980	0.943	0.946	0.957	0.950	0.951	0.958	0.921	0.902
(0.07,0.5,0.4,0.2)	CLM2	0.905	0.880	0.886	0.909	0.874	0.882	0.900	0.879	0.888
	CBP2	0.866	0.823	0.807	0.847	0.779	0.821	0.841	0.770	0.798
	CBL2	0.873	0.831	0.817	0.857	0.794	0.842	0.856	0.780	0.813
	RLM2	0.839	0.787	0.740	0.822	0.762	0.742	0.782	0.654	0.610
(0.07,0.5,0.4,0.8)	CLM2	1.000	1.000	1.000	1.000	1.000	0.982	0.998	0.992	0.990
	CBP2	0.998	1.000	0.999	1.000	1.000	0.987	0.999	1.000	1.000
	CBL2	1.000	1.000	0.984	1.000	1.000	0.999	1.000	1.000	1.000
	RLM2	0.998	1.000	0.980	0.992	0.994	0.990	0.995	0.989	0.986