MONASH

WP 3/96

ISSN 1032-3813
ISBN 0 7326 0784 1

MONASH UNIVERSITY

AUSTRALIA

TESTING FOR STRUCTURAL CHANGE IN COINTEGRATED REGRESSION MODELS: SOME COMPARISONS AND GENERALIZATIONS

Kang Hao

Working Paper 3/96
June 1996

DEPARTMENT OF ECONOMETRICS

TESTING FOR STRUCTURAL CHANGE IN COINTEGRATED REGRESSION MODELS: SOME COMPARISONS AND GENERALIZATIONS

Kang Hao

Department of Econometrics
Monash University
Clayton, Victoria 3168
Australia

Key words and phrases: cointegration; partial structural change; robust cointegration test; asymptotic distribution.

JEL classification: C22, C52

ABSTRACT

This paper compares and generalizes some testing procedures for structural change in the context of cointegrated regression models. The Lagrange Multiplier (LM) tests proposod by Hansen (1992) are generalized to testing for partial structural change. An exponential average LM test is also suggested following the idea of Andrews and Ploberger (1992). In particular, an optimal test for cointegration is developed. We also propose a new cointegration test which is robust to a possible one-time discrete jump in the intercept. We tabulate the asymptotic critical values for the above tests and conduct a small Monte Carlo simulation to investigate their finite sample performance.

1. INTRODUCTION

When an econometric model is used for forecasting or policy simulations, an implied assumption is the structural stability of such a model. The detection of structural change will not only provide evidence on whether a particular economic theory or policy is correct or not, it can also lead directly to improvements in forecasting performance. Therefore, it has become a routine practice for econometricians to test the structural stability assumption in econometric models.

Recently, Hansen (1992) developed the limiting theory for the Lagrange Multiplier (LM) test for structural change in the context of cointegrated regression models. Making use of the fully modified OLS (FM) estimation method of Phillips and Hansen (1990), Hansen (1992) derived the asymptotic distribution of various test statistics against different alternatives of interest and found that they are free of nuisance parameters but depend upon the stochastic process describing the regressors. At the same time, Quintos and Phillips (1993) proposed a LM test against the random walk alternative which corresponds with Hansen's approach. They argue that while Hansen's tests apply to the full vector of cointegrating coefficients, their LM test can be applied to subvectors of the cointegrating vector as well as the full cointegrating vector. Such a formulation is especially useful in empirical work, since it provides a means of isolating the variables that are responsible for the failure of the null hypothesis.

This paper further investigates the problem of testing for structural change in the context of cointegrated regression models. Such a problem is particularly important in that cointegrated regression models are often estimated over long sample periods, and the structural stability assumption is more likely to be violated.

A direct comparison of Hansen's approach with Quintos and Phillips' approach (section 3) shows that against the same alternative hypothesis, the various test statistics only differ in terms of the choice of the weighting matrix. Then following the idea of Andrews and Ploberger (1992), an average exponential form of the LM test which is asymptotically optimal in terms of weighted average power can be easily constructed (section 4). It is also found that Hansen's tests can be directly extended to testing for subvectors of the cointegrating vector, or, in other words, for partial structural change. As a special case, testing the null hypothesis of cointegration against the alternative of no cointegration is equivalent to testing the constancy of one coefficient, the intercept, against a random walk alternative. In this sense, a new test statistic can be formulated to test for this particular partial structural change. An interesting finding is that such a test is exactly the same as the test for a unit root first proposed by Kwiatkowski, Phillips, Schmidt and Shin (1992) and extended to the problem of testing the null hypothesis of cointegration by Harris and Inder (1994). Section 4 also includes a new optimal test of the null of cointegration, constructed in the average exponential form of the LM test against a particular partial structural change.

As observed by Hansen, a LM test against the random walk alternative also has good power against the discrete jump alternative. Therefore the cointegration test of Harris and Inder (1994) cannot discriminate between the random walk in the residuals and a discrete jump in the intercept. A robust cointegration test is thus suggested in section 4 to overcome this problem.

This paper is organized as follows. Section 2 sets up the structure of the cointegration model and briefly describes the method of FM estimation. Section 3 compares the various test statistics suggested by Hansen and Quintos and Phillips. Section 4 gives the average exponential form of the LM test and extend Hansen's tests to testing for partial structural change. It also derives the new optimal test of the null of cointegration against the alternative of no cointegration and the robust cointegration test. Section 5 conducts a Monte Carlo experiment to investigate and compare the behavior of the various tests. The conclusion is given in section 6.

To represent the asymptotics concisely, here and elsewhere in this paper, all limits apply as $T \to \infty$. Integrals (such as $\int_0^1 B$) are understood to be taken with respect to Lebesgue measure (that is $\int_0^1 B(r)dr$) when otherwise unspecified. Let [.] denote "integer part".

2. THE COINTEGRATED REGRESSION MODEL

Consider the cointegrated regression model

$$\begin{aligned} y_t &= \alpha + x_t'\beta + u_t, \\ x_t &= x_{t-1} + v_t, \qquad t = 1,\ldots,T \end{aligned} \tag{1}$$

where α is a scalar, β is a $k\times 1$ vector of unknown parameters, x_t is a $k\times 1$ vector of regressors, and u_t is a stationary error, hence y_t and x_t are cointegrated and $(-1, \beta')'$ is the cointegrating vector.

Let $\zeta_t = [u_t, v_t']$ be a k+1 dimensional process which satisfies the multivariate invariance principle as set out by Phillips and Durlauf (1986). Let

$$R_T(r) = R_{[Tr]} = \sum_1^{[Tr]} \zeta_t, \text{ then } T^{-1/2} R_{[Tr]} \Rightarrow W(r) = (W_0(r), W_1(r)),$$

where W(r) is a k+1 dimensional Brownian motion and partitioned in conformity with ζ_t. The covariance matrix of W(r) is

$$\psi = \begin{bmatrix} \omega_0^2 & \psi_{01} \\ \psi_{10} & \psi_1 \end{bmatrix} = \lim_{T\to\infty} T^{-1} E\left(\sum_1^T \zeta_t\right)\left(\sum_1^T \zeta_t\right)' = \Sigma + \Lambda + \Lambda'$$

where

$$\Sigma = \lim_{T\to\infty} T^{-1} E\left(\sum_1^T \zeta_t \zeta_t'\right) = \begin{bmatrix} \sigma_0^2 & \Sigma_{01} \\ \Sigma_{10} & \Sigma_1 \end{bmatrix}, \Lambda = \lim_{T\to\infty} T^{-1} \sum_{t=2}^T \sum_{j=1}^{t-1} E(\zeta_t \zeta_t') = \begin{bmatrix} \lambda_0^2 & \Lambda_{01} \\ \Lambda_{10} & \Lambda_1 \end{bmatrix},$$

and they are partitioned in conformity with ζ_t.

Define $\Delta = \Sigma + \Lambda = \begin{bmatrix} \Delta_0 & \Delta_{01} \\ \Delta_{10} & \Delta_1 \end{bmatrix}$ and denote consistent estimators of ψ and Δ as $\hat{\Psi}$ and $\hat{\Delta}$, respectively. Partition $\hat{\Psi}$ and $\hat{\Delta}$ as ψ and Δ. Set

$$\hat{\omega}_{0.1}^2 = \hat{\omega}_0^2 - \hat{\psi}_{01}\hat{\psi}_1^{-1}\hat{\psi}_{10}, \hat{\Delta}_{10}^+ = \hat{\Delta}_{10} - \hat{\Delta}_1 \hat{\psi}_1^{-1}\hat{\psi}_{10}.$$

Define the transformed dependent variable

$$y_t^+ = y_t - \hat{\psi}_{01}\hat{\psi}_1^{-1} v$$

with FM disturbances

$$u_t^+ = u_t - \hat{\psi}_{01}\hat{\psi}_1^{-1} v_t \ .$$

The cointegrated regression model (1) can be transformed to

$$y_t^+ = \alpha + x_t'\beta + u_t^+ = z_t'\gamma + u_t^+ \qquad (2)$$

where

$$z_t = (1, x_t')', \gamma = (\alpha, \beta')'.$$

The FM estimator of γ is then given by

$$\hat{\gamma}^+ = \begin{pmatrix} \hat{\alpha}^+ \\ \hat{\beta}^+ \end{pmatrix} = \left(\sum_1^T z_t z_t'\right)^{-1} \left(\sum_1^T z_t y_t^+ - T\begin{pmatrix} 0 \\ \hat{\Delta}_{10}^+ \end{pmatrix}\right) \qquad (3)$$

with FM residuals

$$\hat{u}_t^+ = y_t^+ - z_t'\hat{\gamma}^+ .$$

$$\text{Set } \hat{s}_t = (z_t'\hat{u}_t^+ - \begin{pmatrix} 0 \\ \hat{\Delta}_{10}^+ \end{pmatrix}), \tag{4}$$

then $\sum_{T=1}^{T} \hat{s}_t = 0$. Therefore $\hat{s}_t$ can be regarded as the first order conditions or score vectors of the cointegrated regression model. $\hat{s}_t$ play a very important role in the forming of the LM test statistics. For convenience of comparison, denote

$$\hat{s}_t^0 = x_t'\hat{u}_t^+ - \hat{\Delta}_{10}^+ .$$

Then $\hat{s}_t$ can be written as $\hat{s}_t = \left(\hat{u}_t^+ , \hat{s}_t^{0'} \right)'$.

3. A COMPARISON OF DIFFERENT APPROACHES

Modify (1) to incorporate possible structural change by allowing γ to depend on time,

$$\begin{aligned} y_t &= z_t'\gamma_t + u_t, \\ z_t &= (1, x_t')', x_t = x_{t-1} + v_t, \qquad t = 1, ..., T \end{aligned} \tag{5}$$

The null hypothesis can be formulated as

$$H_0 : \gamma_1 = \gamma_2 = \ ... \ = \gamma_T = \gamma .$$

Different test statistics can be constructed against several alternatives of interest. The first test models γ_t as obeying a single structural change at unknown time [Tr] for $0<r<1$. In this case, the alternative is

$$H_1 : \gamma_t = \begin{cases} \gamma_1 & t \le [Tr] \\ \gamma_2 & t > [Tr] \end{cases} \tag{6}$$

A LM test of H_0 against H_1 is given by

$$\sup LM = \sup_{r \in \Pi} LM_T(r), LM_T(r) = \hat{S}_T(r)' \left[\hat{\omega}_{0.1}^2 V_T(r) \right]^{-1} \hat{S}_T(r) \tag{7}$$

where $\hat{S}_T(r) = \sum_{t=1}^{[Tr]} \hat{s}_t$ is the partial sum of the first order conditions which are given by (4), Π is a subset of (0, 1) and

$$V_T(r) = M_T(r) - M_T'(r)M_T(1)^{-1}M_T(r), M_T(r) = \sum_{1}^{[Tr]} z_t z_t'.$$

The second and third tests model γ_t as a martingale process,

$$\gamma_t = \gamma_{t-1} + \varepsilon_t, \quad E(\varepsilon_t | \Xi_{t-1}) = 0, \quad E(\varepsilon_t \varepsilon_t') = \delta^2 G_t$$

where Ξ_t is some increasing sequence of σ-field to which γ_t is adapted and G_t is some known covariance matrix which measures the parameter stability in the t'th period. In this context, the hypothesis testing problem becomes

$$H_0: \delta^2 = 0 \text{ against } H_1^R: \delta^2 > 0.$$

Hansen (1992) shows that a LM test of H_0 against H_1^R is given by

$$L = \int_\Pi \hat{S}_T(r)' G_T(r)^{-1} \hat{S}_T(r). \tag{8}$$

By choosing different $G_T(r)$, we can get different test statistics. If we choose $G_T(r) = \hat{\omega}_{0.1}^2 M_T(1)$, then we get the L_c test of Nyblom (1989) which was originally proposed in the context of stationary regression models,

$$L_c = \int_\Pi \hat{S}_T(r)' \left[\hat{\omega}_{0.1}^2 M_T(1)\right]^{-1} \hat{S}_T(r). \tag{9}$$

If we choose $G_T(r) = \hat{\omega}_{0.1}^2 V_T(r)$, this corresponds to the meanLM test of Hansen (1990) under stationary regression models,

$$\text{meanLM} = \int_\Pi LM_T(r) = \int_\Pi \hat{S}_T(r)' \left[\hat{\omega}_{0.1}^2 V_T(r)\right]^{-1} \hat{S}_T(r). \tag{10}$$

On the other hand, Quintos and Phillips (1993) focus on the coefficients of the nonstationary regressors. Under their specification, the cointegrated regression model is

$$\begin{aligned} y_t &= \alpha + x_t'\beta_t + u_t, \\ x_t &= x_{t-1} + v_t \qquad t = 1, \ldots, T. \end{aligned} \tag{11}$$

β_t is supposed to follow a random walk,

$$\beta_t = \beta_{t-1} + \eta_t, \; \eta_t \sim N(0, \sigma_\eta^2 \Sigma_\eta). \tag{12}$$

The hypothesis under interest becomes $H_0: \sigma_\eta^2 = 0$ against $H_1^R: \sigma_\eta^2 > 0$.

Quintos and Phillips derived a LM test of parameter constancy against the random walk alternative under the cointegrated regression model with or without a constant term. For the model with a constant term, their test statistic is given by

$$Q-P = T^{-3}\hat{u}^{++\prime} \underset{\sim}{D}_X \underset{\sim}{L}(I_T \otimes \hat{\Psi}_1^{-1}) \underset{\sim}{L}' \underset{\sim}{D}'_X u^{++} / \hat{\omega}_{0.1}^2 \quad (13)$$

where $\underset{\sim}{D}_X = \mathrm{diag}(x_1', x_2', \ldots, x_T')$, $\underset{\sim}{L} = \begin{bmatrix} I_K & & 0 \\ \vdots & \ddots & \\ I_k & \cdots & I_k \end{bmatrix}$ and $\hat{u}^{++}$ is defined to make

$T^{-1}\sum_{1}^{[Tr]} x_t \hat{u}_t^{++} = T^{-1}\sum_{1}^{[Tr]} x_t \hat{u}_t^{+} - r\Delta_{10}$.

Making use of the notation of score vector in the context of the cointegrated regression model, the Quintos-Phillips test statistic can be expressed

$$Q-P = \frac{1}{T^3 \hat{\omega}_{0.1}^2} \sum_{j=1}^{T} \left\{ \left[\sum_{t=1}^{j} \left(\hat{u}_t^{+} x_t' - \Delta_{10}' \right) \right] \hat{\psi}_1^{-1} \left[\sum_{t=1}^{j} \left(x_t \hat{u}_t^{+} - \Delta_{10} \right) \right] \right\}$$

$$= \int_0^1 \hat{S}_T^{0\prime}(r) \left[T^2 \hat{\omega}_{0.1}^2 \hat{\psi}_1 \right]^{-1} \hat{S}_T^{0}(r). \quad (14)$$

We see that the L_c test, the meanLM test and the Q-P test are simply weighted averages of the squared partial sums of the first order conditions (4). They only differ in terms of how to choose the weighting matrix. For the L_c test, choosing $[M_T(1)]^{-1}$ as the weighting matrix results in a constant weighting matrix. As observed by Hansen (1990) for the case of stationary regression models, the effect of such a choice is to place unequal weights across $S_T(r)'G_T(r)^{-1}S_T(r)$ because its asymptotic expectation is r(1-r). It varies over r and attains the maximum at r = 0.5. Thus the L_c test places more weight on the middle observations and has difficulty in detecting early or late structural change. The same happens to the Q-P test which uses the long run covariance matrix as the weighting matrix. This problem is overcome by replacing $G_T(r)$ with $\hat{\omega}_{0.1}^2 V_T(r)$ and this leads to the meanLM test.

The meanLM test differs from the supLM test simply by the choice of norm. While the supLM test picks the largest from the T elements, the meanLM test calculates the average of these T elements. They are based on the same components but have particular power against the one-time discrete jump and random walk alternatives, respectively.

Although choosing $V_T(r)$ as the weighting matrix might be helpful in detecting structural change early or late in the sample, such a choice also has some adverse effect on the power of the supLM test and the meanLM test. Because both tests are based on the convergence of $[V_T(r)]^{-1}$, as $r \to 0$ or 1, $V_T(r)$ would not converge in distribution. In order to implement the tests, Hansen (1992) suggested selecting Π = [0.15, 0.85] to ensure the convergence of the test statistics. While this is an asymptotically useful approach, it also introduces an element of arbitrariness. When the structural change point is outside Π, the tests will lose power. On the other hand, the L_c test and the Q-P test are valid for $\Pi = (0.1)$, hence excluding any form of arbitrariness.

4. SOME GENERALIZATIONS

4.1 The expLM Test

The meanLM test is a particular form of the class of asymptotically optimal tests suggested by Andrew and Ploberger (1992) which is given by

$$\text{Exp}-\text{LM}_T(r) = (1+c)^{-k/2} \int_\Pi \exp\left[\frac{1}{2(1+c)}\text{LM}_T(r)\right] dJ(r), \tag{15}$$

where $\text{LM}_T(r)$ is just the standard LM test statistic for the null of no structural change versus the alternative of a particular type of structural change given the parameter r, J(.) is weighting function over values of r in Π, c is a scalar constant that depends on the chosen weighting function and determines power direction. Notice that

$$\lim_{c\to\infty} 2(\text{Exp}-\text{LM}_T - 1)/c = \int_\Pi \text{LM}_T(r)dr = \text{meanLM}.$$

Thus the limit as $c \to 0$ of the $\text{Exp-LM}_T(r)$ test is equal to the meanLM test.

The Monte Carlo simulation conducted by Andrews, Lee and Ploberger (1992) suggested that the power of such optimal tests is not very sensitive to changes in c. They also found that choosing $c = \infty$ results in a new test statistic which is slightly preferred to the meanLM test. Under such a choice,

$$\exp\text{LM} = \lim_{c\to\infty} \log\left[(1+c)^{k/2}\text{Exp}-\text{LM}_T\right] = \log \int_\Pi \exp\text{LM}_T(r) \tag{16}$$

Under the cointegration regression model (1), this corresponds to a new test statistic

$$\exp\text{LM} = \log \int_\Pi \exp\left\{\frac{1}{2}\hat{S}_T(r)'\left[\hat{\omega}^2_{0.1}V_T(r)\right]^{-1}\hat{S}_T(r)\right\}. \tag{17}$$

Denote $V(r) = M(r) - M(r)M(1)^{-1}M(r)$, $S^*(r) = S(r) - M(r)M(1)^{-1}S(1)$

where $S(r)=\begin{bmatrix} B_{0.1} \\ \int_0^r B_1 dB_{0.1} \end{bmatrix}$, $M(r)=\begin{bmatrix} r & \int_0^r B_1' \\ \int_0^r B_1 & \int_0^r B_1 B_1' \end{bmatrix}$, $B_{0.1}$ and B_1 are independent standard Brownian motion with dimension 1 and k, respectively. Then by Theorem 2 of Hansen (1992), we have $LM_T(r) \overset{d}{\Rightarrow} LM(r) = S^*(r)'V(r)^{-1}S^*(r)$. By the continuous mapping theorem of Billingsley (1968, p.30),

$$\exp LM \overset{d}{\Rightarrow} \log \int_{\Pi} \exp\left[\frac{1}{2} S^*(r)'V(r)^{-1}S^*(r)\right]. \tag{18}$$

The asymptotic distribution of the expLM test does not depend on any nuisance parameters. It is, however, a function of the number of regressors.

4.2 LM Tests for Partial Structural Change

Although Hansen did not derive the test statistic for partial structural change, the extension is straighforward. Suppose we are interested in some subvector γ_1 of the cointegrating vector γ. Without loss of generality, we can set γ_1 as the first subset of γ, then $\gamma = (\gamma_1', \gamma_2')'$. Partition $\hat{S}_T(r)$, $M_T(r)$ and $V_T(r)$ in conformity with γ. Define following test statistics for partial structural change:

$$\sup LM^1 = \sup_{r \in \Pi} \hat{S}_T^1(r)'\left[\hat{\omega}_{0.1}^2 V_T^{11}(r)\right]^{-1} \hat{S}_T^1(r),$$

$$\text{mean}LM^1 = \int_{\Pi} \hat{S}_T^1(r)'\left[\hat{\omega}_{0.1}^2 V_T^{11}(r)\right]^{-1} \hat{S}_T^1(r),$$

$$L_c^1 = \int_{\Pi} \hat{S}_T^1(r)'\left[\hat{\omega}_{0.1}^2 M_T^{11}(1)\right]^{-1} \hat{S}_T^1(r),$$

$$\exp LM^1 = \log \int_{\Pi} \exp\left\{\frac{1}{2} \hat{S}_T^1(r)'\left[\hat{\omega}_{0.1}^2 V_T^{11}(r)\right]^{-1} \hat{S}_T^1(r)\right\}.$$

In order to derive the asymptotic distributions of the above test statistics, define the weighting matrix:

$$\Gamma = \text{diag}(1, \frac{1}{\sqrt{T}} \hat{\psi}_1^{-1/2}). \tag{19}$$

Partition Γ, $S^*(r)$, $M(r)$ and $V(r)$ in conformity with γ. Applying theorem 1 of Hansen (1992), we have

$$\frac{1}{T}\Gamma^{1\prime} M_T^{11}(1)\Gamma^1 \overset{d}{\Rightarrow} M^{11}(1), \frac{1}{T}\Gamma^{1\prime} V_T^{11}(r)\Gamma^1 \overset{d}{\Rightarrow} V^{11}(r), \frac{1}{\sqrt{T}}\Gamma^{1\prime} S_T^1(r) \overset{d}{\Rightarrow} \omega_{0.1} S^{*1}(r).$$

Again, by the continuous mapping theorem of Billingsley, we have the following asymptotic distributions of the above partial structural change test statistics:

$$\sup LM^1 = \sup_{r\in\Pi} \hat{S}_T^1(r)'\left[\hat{\omega}_{0.1}^2 V_T^{11}(r)\right]^{-1} \hat{S}_T^1(r),$$

$$\overset{d}{\Rightarrow} \sup_{r\in\Pi} S^{*1}(r)'\left[V^{11}(r)\right]^{-1} S^{*1}(r), \tag{20}$$

$$\text{meanLM}^1 = \int_\Pi \hat{S}_T^1(r)'\left[\hat{\omega}_{0.1}^2 V_T^{11}(r)\right]^{-1} \hat{S}_T^1(r)$$

$$\overset{d}{\Rightarrow} \int_\Pi S^{*1}(r)'\left[V^{11}(r)\right]^{-1} S^{*1}(r), \tag{21}$$

$$L_c^1 = \int_\Pi \hat{S}_T^1(r)'\left[\hat{\omega}_{0.1}^2 M_T^{11}(1)\right]^{-1} \hat{S}_T^1(r)$$

$$\overset{d}{\Rightarrow} \int_\Pi S^{*1}(r)'\left[M^{11}(1)\right]^{-1} S^{*1}(r), \tag{22}$$

$$\exp LM^1 = \log \int_\Pi \exp\left\{\frac{1}{2} \hat{S}_T^1(r)'\left[\hat{\omega}_{0.1}^2 V_T^{11}(r)\right]^{-1} \hat{S}_T^1(r)\right\}$$

$$\overset{d}{\Rightarrow} \log \int_\Pi \exp\left\{\frac{1}{2} S^{*1}(r)'\left[V^{11}(r)\right]^{-1} S^{*1}(r)\right\}. \tag{23}$$

Asymptotic distributions of the above test statistics depend only on the total number of explanatory variables in the cointegrated regression model, and the number of explanatory variables under interest for the partial structural change. Hence we can obtain the critical values for both full and partial structural change for various numbers of regressors. They are found by simulation using a GAUSS program with a sample size of 1000 and 20000 replications for one to five explanatory variables. The results are given from Tables 1 to 4. For the supLM test, the meanLM test, the meanLM test and the expLM test, we set $\Pi = [0.15, 0.85]$ while for the L_c test we set $\Pi = (0, 1)$. In particular, we tabulate critical values for the intercept change and the slope change, respectively[1].

[1] Our simulation results show that critical values for the meanLM test and the expLM test are nonmonotonic. While this is unusual, it happens in the literature which deals with the similar problem. See, for example, Tables 4B (a) to (d) of Quintos and Phillips (1993).

TABLE 1

(a) Asymptotic Critical Values of the supLM Test

Number of regressors (Excluding Constant)	10%	5%	1%
1	10.50	12.28	16.16
2	12.92	14.70	18.61
3	15.14	17.07	21.23
4	16.93	18.94	23.23
5	18.82	20.96	24.25

(b) Asymptotic Critical Values of the intercept supLM test

Number of regressors (Excluding Constant)	10%	5%	1%
1	7.95	9.51	13.15
2	8.57	10.11	13.44
3	9.07	10.61	14.21
4	9.39	10.91	14.66
5	9.79	11.37	14.97

(c) Asymptotic Critical Values of the slope supLM test at 10% level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	7.93				
2	8.23	10.84			
3	8.53	11.09	13.27		
4	8.69	11.27	13.39	15.21	
5	8.94	11.53	13.51	15.41	17.19

(d) Asymptotic Critical Values of slope SupLM Test at 5% Level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	9.52				
2	9.74	12.53			
3	10.04	12.81	15.10		
4	10.26	13.03	15.19	17.11	
5	10.55	13.27	15.41	17.35	19.26

(e) Asymptotic Critical Values of slope meanLM Test at 1% Level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	12.79				
2	13.10	16.25			
3	13.55	16.48	18.92		
4	13.87	16.80	19.12	21.23	
5	14.07	17.05	19.27	21.60	23.43

TABLE 2

(a) Asymptotic Critical Values of the meanLM Test

Number of regressors (Excluding Constant)	10%	5%	1%
1	3.678	4.525	6.630
2	5.086	6.125	8.323
3	6.568	7.738	10.218
4	7.847	9.072	12.103
5	9.172	10.540	13.505

(b) Asymptotic Critical Values of the intercept meanLM test

Number of regressors (Excluding Constant)	10%	5%	1%
1	1.992	2.600	4.111
2	1.885	2.415	3.765
3	1.828	2.315	3.502
4	1.751	2.205	3.212
5	1.745	2.120	3.172

(c) Asymptotic Critical Values of the slope meanLM test at 10% level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	2.037				
2	1.997	3.591			
3	1.945	3.573	5.073		
4	1.915	3.530	5.014	6.388	
5	1.888	3.542	5.022	6.430	7.788

(d) Asymptotic Critical Values of the slope meanLM Test at 5% Level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	2.644				
2	2.539	4.364			
3	2.494	4.404	6.078		
4	2.443	4.319	5.976	7.533	
5	2.440	4.346	5.994	7.515	9.044

(e) Asymptotic Critical Values of the slope meanLM test at 1% level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	4.063				
2	3.985	6.226			
3	4.009	6.350	8.317		
4	3.845	6.274	8.387	10.335	
5	3.788	6.225	8.207	10.193	11.711

TABLE 3

(a) Asymptotic Critical Values of the L_c Test

Number of regressors (Excluding Constant)	10%	5%	1%
1	0.4454	0.5726	0.8791
2	0.5530	0.6787	0.9930
3	0.6844	0.8264	1.1926
4	0.7919	0.9576	1.3514
5	0.8948	1.0817	1.4696

(b) Asymptotic Critical Values of the intercept L_c test

Number of regressors (Excluding Constant)	10%	5%	1%
1	0.2300	0.3144	0.5295
2	0.1643	0.2213	0.3888
3	0.1202	0.1600	0.2851
4	0.0941	0.1220	0.2019
5	0.0763	0.0970	0.1639

(c) Asymptotic Critical Values of the slope L_c test at 10% level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	0.2155				
2	0.1770	0.3535			
3	0.1506	0.3082	0.4864		
4	0.1271	0.2725	0.4442	0.6153	
5	0.1096	0.2477	0.3973	0.5600	0.7248

(d) Asymptotic Critical Values of slope L_c Test at 5% Level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	0.2899				
2	0.2397	0.4475			
3	0.2054	0.3923	0.6104		
4	0.1682	0.3485	0.5437	0.7450	
5	0.1477	0.3147	0.4880	0.6695	0.8772

(e) Asymptotic Critical Values of the slope L_c test at 1% level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	0.5014				
2	0.4144	0.6899			
3	0.3631	0.6090	0.8877		
4	0.2975	0.5411	0.8139	1.0906	
5	0.2659	0.4888	0.7227	0.9824	1.2009

TABLE 4

(a) Asymptotic Critical Values of the expLM Test

Number of regressors (Excluding Constant)	10%	5%	1%
1	2.554	3.205	4.777
2	3.476	4.199	5.840
3	4.415	5.199	7.143
4	5.215	6.125	8.044
5	6.030	6.944	9.025

(b) Asymptotic Critical Values of the intercept expLM test

Number of regressors (Excluding Constant)	10%	5%	1%
1	1.520	2.042	3.336
2	1.550	2.054	3.232
3	1.583	2.080	3.404
4	1.567	2.055	3.322
5	1.617	2.121	3.409

(c) Asymptotic Critical Values of the slope expLM test at 10% level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	1.533				
2	1.526	2.578			
3	1.534	2.600	3.563		
4	1.527	2.606	3.515	4.375	
5	1.572	2.655	3.558	4.429	5.282

(d) Asymptotic Critical Values of slope expLM Test at 5% Level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	2.022				
2	2.031	3.186			
3	2.038	3.249	4.276		
4	2.045	3.263	4.299	5.217	
5	2.087	3.325	4.305	5.227	6.116

(e) Asymptotic Critical Values of the slope expLM test at 1% level

Total regressors	Number of subset regressors				
(Excluding Constant)	1	2	3	4	5
1	3.324				
2	3.274	4.728			
3	3.365	4.696	6.004		
4	3.339	4.792	6.002	7.101	
5	3.440	4.809	6.009	7.083	8.045

4.3 An Optimal Cointegration Test

In the case of estimating a cointegrating relationship, a natural hypothesis to test is the assumption of cointegration itself. Assume that y_t and x_t are not cointegrated. This is equivalent to the statement that the error term u_t is I(1). Decompose u_t as

$$u_t = w_t + \varepsilon_t$$

where w_t is a random walk and ε_t is stationary. Hansen (1992) notice that "no cointegration" in model (1) is equivalent to one coefficient, the intercept, following a random walk by writing (1) as

$$y_t = \alpha_t + x'\beta_t + \varepsilon_t \tag{24}$$

where

$$\alpha_t = \alpha + w_t .$$

Equation (24) thus becomes a special case of equation (2). Hansen therefore concludes that the L_c test can be used as a test of the null of cointegration against the alternative of no cointegration. In particular, (24) can be regarded as a cointegrated regression model with partial structural change or, in other words, with the intercept following the random walk. Although Hansen did not specially develop the partial structural change version of the L_c test, it can be easily derived from (22) and has the form

$$L_c^0 = T^{-1}\sum_{j=1}^{T}\left[\sum_{t=1}^{j}\hat{u}_t^+\left(T\hat{\omega}_{0.1}^2\right)^{-1}\sum_{t=1}^{j}\hat{u}_t^+\right] = \frac{\sum_{j=1}^{T}\left(\sum_{t=1}^{j}\hat{u}_t^+\right)^2}{T^2\hat{\omega}_{0.1}^2} . \tag{25}$$

This is exactly the statistic of testing for unit root first proposed by Kwiatkowski et al. (1992) and extended to testing the null hypothesis of cointegration by Harris and Inder (1994).

As we observed in section 4.1, among the class of optimal tests for structural change given by (16), the expLM test is particulary appealing in terms of its power performance under the stationary regression model. Under the cointegrated regression model (1), an optimal test for cointegration is simply the intercept change version of the expLM test which is given by

$$\exp LM_T^0 = \log\left\{\frac{1}{T^*}\sum_{j/T\in\Pi}\exp\left[\frac{1}{2}\left(\sum_{t=1}^{j}\hat{u}_t^+\right)\left(\hat{\omega}_{0.1}^2 V_j^{11}\right)^{-1}\left(\sum_{t=1}^{j}\hat{u}_t^+\right)\right]\right\} \tag{26}$$

where

$$V_j^{11} = j\left\{\left[T-\sum_1^T x_t'\left(\sum_1^T x_t x_t'\right)^{-1}\sum_1^T x_t\right]^{-1}\left[(T-j)-\sum_1^T x_t'\left(\sum_1^T x_t x_t'\right)^{-1}\sum_{j+1}^T x_t\right]\right.$$

$$\left.+\sum_1^j x_t'\left[\sum_1^T x_t x_t' - \frac{1}{T}\sum_1^T x_t \sum_1^T x_t'\right]^{-1}\left[\frac{j}{T}\sum_1^T x_t - \sum_1^j x_t\right]\right\}$$

and $T^* = \sum_{j/T\in\Pi} 1.$

4.4 A Robust Cointegration Test

As observed earlier, due to the similarities between different test statistics, the rejection of the null of parameter constancy does not definitely imply the particular alternative the test was designed to detect. In particular, as a test of the null of cointegration, the L_c^0 test would have good power against a one-time discrete jump in the intercept as well as the random walk in regression residuals. Considering that large samples are often used to estimate the cointegration model, it is not unlikely that a one-time discrete jump may occur in the intercept. This leads us to suggest a test for cointegration which is robust to a possible discrete jump in the intercept. Such a new test is formed by including a dummy variable in the regression equation to capture the possible jump in the intercept. Since the jump point is unknown, the test statistic is formed by taking the minimum value of the L_c^0 statistic at each possible jump point. Consider the following model:

$$y_t = \mu_1 d_r + \mu_2 + x_t'\beta + u_t \quad , \qquad (27)$$

where d_r is a dummy variable with

$$d_r = \begin{cases} 1 & \text{if } t \le [Tr] \\ 0 & \text{if } t > [Tr] \end{cases},$$

and $0 < r < 1$. If the regression model is not cointegrated, this is equivalent to the statement that the error u_t is I(1). Decompose u_t as

$$u_t = w_t + \varepsilon_t$$

where w_t is a random walk and ε_t is stationary. Equation (27) can be expressed as

$$y_t = \mu_1 d_r + \mu_{2t} + x_t'\beta + \varepsilon_t,$$
$$\mu_{2t} = \mu_2 + w_t.$$

This is a cointegrated regression model with a non-stationary coefficient in the intercept term. Therefore a test of null of cointegration which is robust to the discrete jump is

$$L_c^* = \inf_{r\in\Pi} L_c^0 = \inf_{r\in\Pi} \frac{\sum_{j=1}^{T}\left(\sum_{t=1}^{j}\hat{u}_t^+\right)^2}{T^2\hat{\omega}_{0.1}^2} \tag{28}$$

where $\hat{u}_t^+$ is the FM residuals for model (27).

Let $Z^* = (z_1^*, z_2^*, \ldots, z_T^*)'$, $z_t^* = \begin{cases}(1,\ 1,\ x_{t1}, \ldots, x_{tk})' & \text{if } t/T \le r \\ (0,\ 1,\ x_{t1}, \ldots, x_{tk})' & \text{if } t/T > r\end{cases}$,

model (27) is then equivalent to $y_t = z_t^{*\prime}\begin{pmatrix}\mu_1\\ \mu_2\\ \beta\end{pmatrix} + u_t$ or $Y = Z^*\delta + u$.

Except for the intercept term, model (27) is the same as the general cointegrated regression model (1). Under the same notation given in section 2, (27) can be written as

$$y_t^+ = \mu_1 d_r + \mu_2 + x_t'\beta + u_t^+ \quad \text{or} \quad Y^+ = Z^*\delta + u^+. \tag{29}$$

The FM estimator of δ is

$$\hat{\delta}^+ = \begin{pmatrix}\hat{\mu}_1^+\\ \hat{\mu}_2^+\\ \hat{\beta}^+\end{pmatrix} = (Z^{*\prime}Z^*)^{-1}\left\{\begin{bmatrix}\sum_1^{[Tr]} y_t^+ \\ \sum_1^{T}\begin{pmatrix}y_t^+\\ x_t'y_t\end{pmatrix}\end{bmatrix} - \begin{pmatrix}0\\ 0\\ T\hat{\Delta}_{10}^+\end{pmatrix}\right\} \tag{30}$$

$$\text{where } Z^{*\prime}Z^* = \begin{bmatrix} Tr & Tr & \sum_1^{Tr} x_t' \\ Tr & T & \sum_1^{T} x_t' \\ \sum_1^{[Tr]} x_t & \sum_1^{T} x_t & \sum_1^{T} x_t x_t' \end{bmatrix}.$$

The FM residuals of model (29) are then given by

$$\hat{u}_t^+ = y_t^+ - z_t^{*\prime}\hat{\delta}^+. \tag{31}$$

To derive the asymptotic distribution of the test statistic L_c^*, define the weighting matrix

$$\Gamma = \mathrm{diag}(1,\ 1,\ \frac{1}{\sqrt{T}}\hat{\psi}_1^{-1/2}),$$

then we have following results:

$$\frac{1}{T}\Gamma(Z^{*\prime}Z^{*})\Gamma' = \frac{1}{T}\Gamma\begin{bmatrix} Tr & Tr & \sum_1^{[Tr]} x_t' \\ Tr & T & \sum_1^{T} x_t' \\ \sum_1^{[Tr]} x_t & \sum_1^{T} x_t & \sum_1^{T} x_t x_t' \end{bmatrix}\Gamma' \overset{d}{\Rightarrow} \begin{bmatrix} r & r & \int_0^r B_1' \\ r & 1 & \int_0^r B_1' \\ \int_0^r B_1 & \int_0^r B_1 & \int_0^r B_1 B_1' \end{bmatrix} = Q(r),$$

$$\frac{1}{\sqrt{T}}\Gamma(Z^{*\prime}u^{+}) = \frac{1}{\sqrt{T}}\Gamma\sum_1^{T} z_t^{*} u_t^{+} = \begin{bmatrix} \frac{1}{\sqrt{T}}\sum_1^{[Tr]} u_t' \\ \frac{1}{\sqrt{T}}\sum_1^{T} u_t' \\ \frac{1}{T}\hat{\psi}_1^{-1/2}\sum_1^{T} x_t u_t^{+} \end{bmatrix} \overset{d}{\Rightarrow} \omega_{0.1}\begin{bmatrix} B_{0.1}(r) \\ B_{0.1}(1) \\ \int_0^1 B_1 dB_{0.1} \end{bmatrix} = \omega_{0.1} f(r),$$

$$\frac{1}{T}\Gamma\sum_1^{[T\tau]} z_t^{*} = \begin{bmatrix} T^{-1}\min([T\tau],[Tr]) \\ T^{-1}[T\tau] \\ T^{-3/2}\ \hat{\psi}_1^{-1/2}\sum_1^{[T\tau]} x_i \end{bmatrix} \overset{d}{\Rightarrow} \begin{bmatrix} \min(\tau, r) \\ \tau \\ \int_0^{\tau} B_1 \end{bmatrix}.$$

$$\sqrt{T}\Gamma^{-1}(\hat{\delta}^{+} - \delta) = \sqrt{T}\Gamma^{-1}(Z^{*\prime}Z^{*})^{-1} Z^{*\prime}u^{+}$$

$$= \left[\frac{1}{T}\Gamma(Z^{*\prime}Z^{*})\Gamma'\right]^{-1}\left[\frac{1}{\sqrt{T}}\Gamma(Z^{*\prime}u^{+})\right] \overset{d}{\Rightarrow} \omega_{0.1}\ Q(r)^{-1} f(r). \qquad (32)$$

$$\frac{1}{\sqrt{T}}\sum_1^{[T\tau]}\hat{u}_i^{+} = \frac{1}{\sqrt{T}}\sum_1^{[T\tau]} u_i^{+} - \frac{1}{\sqrt{T}}\sum_1^{[T\tau]} z_i^{*\prime}\ (\hat{\delta}^{+} - \delta)$$

$$= \frac{1}{\sqrt{T}}\sum_1^{[T\tau]} u_i^{+} - T^{-1}\left(\sum_1^{[T\tau]} z_i^{*\prime}\Gamma\right)\left[\sqrt{T}\Gamma^{-1}(\hat{\delta}^{+} - \delta)\right]$$

$$\overset{d}{\Rightarrow}\begin{cases} W_{0.1}(\tau) - [\tau,\ \tau,\ \int_0^{\tau} B_1']\omega_{0.1} Q(r)^{-1} f(r) & \text{if } \tau \le r \\ W_{0.1}(\tau) - [r,\ \tau,\ \int_0^{r} B_1']\omega_{0.1} Q(r)^{-1} f(r) & \text{if } \tau > r \end{cases} \qquad (33)$$

Thus $$L_c^* = \inf_{r\in\Pi} \frac{\sum_{j=1}^{T}\left(\sum_{t=1}^{j}\hat{u}_t^+\right)^2}{T^2\hat{\omega}_{0.1}^2}$$

$$\overset{d}{\Rightarrow} \inf_{r\in\Pi} \left\{ \int_0^r \left[B_{0.1}(\tau)-(\tau,\ \tau \int_0^r B_1')\,Q(r)^{-1} f(r)\right]^2 d\tau \right.$$

$$\left. + \int_r^1 \left[B_{0.1}(\tau)-(r,\ \tau,\ \int_0^r B_1')\,Q(r)^{-1} f(r)\right]^2 d\tau \right\}, \qquad (34)$$

which is free of any nuisance parameters.

Due to the requirements of recursive calculations, it is particularly time-consuming to calculate the critical values of the robust cointegration test L_c^* by simulation. Hence the asymptotic critical values are found by simulation using a GUASS program with a sample size of only 300 and with 3000 replications for one to five explanatory variables. Since the asymptotic distribution of the L_c^* test is dependent on the convergence of $Q(r)^{-1}$, therefore Π is set to be [0.15, 0.85]. The results are given in Table 5.

TABLE 5

Asymptotic Critical Values of the Robust Cointegration Test

Number of regressors (Excluding Constant)	10%	5%	1%
1	0.06228	0.07545	0.11590
2	0.05041	0.06012	0.08460
3	0.04128	0.04856	0.06837
4	0.03480	0.04084	0.05265
5	0.03064	0.03560	0.04644

5. A MONTE CARLO EXPERIMENT

5.1 Experimental Design

A Monte Carlo Study was conducted to investigate and compare the finite sample properties of the various tests discussed in this paper. In particular, we consider the following cointegrated regression model

$$y_t = (1, x_t)\gamma_t + u_t,$$
$$x_t = x_{t-1} + v_t, \qquad t = 1,\ldots,T, \qquad (35)$$

where $\gamma_t = (\alpha_t, \beta_t')$, u_t and v_t are generated by the processes

$$u_t = \rho_{11}u_{t-1} + \rho_{12}\eta_{1t},$$

$$v_t = \rho_{21}u_t + w_t,\ w_t = \rho_{22}w_{t-1} + \rho_{23}\eta_{2t}$$

with η_{1t} and η_{2t} being independently and identically distributed standard normal variables. Three sets of parameter values are used in our Monte Carlo study. They are, respectively

1: $(\rho_{11}, \rho_{12}) = (0, 1)$, $(\rho_{21}, \rho_{22}, \rho_{23}) = (0, 0, 1)$;

2: $(\rho_{11}, \rho_{12}) = (0.2, 0.98)$, $(\rho_{21}, \rho_{22}, \rho_{23}) = (0.25, 0.25, 0.935)$;

3: $(\rho_{11}, \rho_{12}) = (0.5, 0.866)$, $(\rho_{21}, \rho_{22}, \rho_{23}) = (0.25, 0.25, 0.935)$.

Since the Q-P test is derived under the assumption of no intercept change, we therefore confine the structural change in the slope for the fair comparison. The alternative hypothesis is a one-time discrete jump and the random walk, respectively. Against a one-time discrete jump, we consider

$$\gamma_t = \begin{cases} (1,\ 1)' & \text{when } t \le [Tr] \\ (1,\ 1.5)' & \text{when } t > [Tr] \end{cases} \tag{36}$$

where [Tr] is the break point and occurs at r = 0.1, 0.3, 0.5, 0.7, 0.9.

Against the random walk alternative, we consider

$$\alpha_t = \alpha_0 = 1,\ \beta = \beta_{t-1} + 0.03\xi_{2t},\ \beta_0 = 1,\ \xi_{2t} \sim N(0, 1). \tag{37}$$

As we observed in section 4.3, to test the null hypothesis of cointegration against the alternative of no cointegration is equivalent to testing the intercept following a random walk. In order to compare the performance of two "cointegration" tests, the L_c^0 test and the $\exp LM_T^{0}$ test, we consider the random walk alternative in the intercept,

$$\alpha_t = \alpha_{t-1} + 0.2\xi_{1t},\ \alpha_0 = 1,\ \xi_{1t} \sim N(0, \sigma_\xi^2),\ \beta_t = \beta_0 = 1, \tag{38}$$

where σ_ξ^2 equals 0 or 1. When σ_ξ^2 equals 0, it implies that there exists cointegration in model (35). When σ_ξ^2 equals 1, it implies that there is no cointegration in model (35).

To investigate the performance of the robust cointegration test, the null hypothesis is designed to be a one-time discrete jump in the intercept,

$$H_0: \gamma_t = \begin{cases} (1,\ 1)' & \text{when } t \le [Tr] \\ (1.5,\ 1)' & \text{when } t > [Tr] \end{cases}. \tag{39}$$

Under the alternative hypothesis, on the other hand, the intercept follows a random walk which is given by (38).

Against alternative hypotheses (36) and (37), we consider both the full structural change version and the slope change version of the expLM test, the supLM test, the meanLM test and the L_c test. The Q-P test corresponds to the slope change version of above tests in the sense that there is no full structural change version of the Q-P test. We use superscript "1" to denote the slope change version of each test. Against the alternative hypothesis (38), we consider the L_c^0 test and the $\exp LM_T^0$ test. We also consider the robust cointegration test L_c^* against the same alternative but the null hypothesis is (39). In experiment 1 the regressors are exogenous and Δx_t are serially uncorrelated. In experiments 2 and 3, x_t are endogenous as well as having serially correlated innovations. At the same time, we introduce autocorrelation in regression residuals u_t. For each experiment we do 1000 replications and record the rejection frequencies of the various tests using 5% asymptotic critical values.[2] The sample size is 100 and 250, respectively.

5.2 Results on the Estimated Sizes and Powers

Tables 6 and 7 report the estimated sizes and powers of various tests against alternative hypotheses (36) and (37). Table 8 reports the estimated sizes and powers of the L_c^0 test and the $\exp LM_T^0$ test against the alternative hypothesis (38). Table 9 reports the estimated sizes and powers of the L_c^* test. The Q-P test and three versions of the L_c test are evaluated across the full sample while the other tests are evaluated by restricting $\Pi = [0.15, 0.85]$.

We observe that all of the tests demonstrate reasonable size performance and the accuracy in approximating the nominal size typically improves as T increases. For the two versions of the supLM test, their estimated sizes are always significantly below their nominal size. A similar result was found by Gregory and Nason (1992) in their Monte Carlo study of various tests of structural change. In contrast, the estimated sizes of other tests are much closer to the nominal size. In particular, estimated sizes of the

[2] We also calculated size-corrected powers of above tests. Results are similar to those based on their asymptotic critical values and thus are not reported here.

TABLE 6
Rejection Frequencies with 5% Asymptotic Critical Values
(Slope change, T = 100)

/r	0.0	0.1	0.3	0.5	0.7	0.9	rw
				Experiment 1			
expLM	.045	.184	.879	.924	.892	.534	.372
L_c	.050	.164	.517	.764	.821	.490	.361
meanLM	.054	.165	.713	.892	.852	.423	.327
supLM	.026	.122	.838	.878	.814	.430	.272
$expLM^1$	.045	.183	.941	.967	.939	.509	.466
L_c^1	.049	.111	.594	.832	.856	.591	.443
$meanLM^1$	.046	.154	.851	.937	.888	.347	.411
$supLM^1$	.028	.148	.898	.942	.895	.488	.369
Q-P	.056	.080	.340	.553	.582	.377	.310
				Experiment 2			
expLM	.030	.132	.719	.708	.625	.277	.276
L_c	.047	.138	.363	.514	.545	.278	.299
meanLM	.041	.134	.495	.663	.608	.238	.273
supLM	.011	.066	.608	.527	.439	.169	.155
$expLM^1$	.036	.144	.862	.820	.710	.274	.398
L_c^1	.047	.097	.451	.656	.634	.332	.386
$meanLM^1$	.039	.129	.724	.764	.652	.213	.345
$supLM^1$	.018	.094	.801	.740	.623	.221	.272
Q-P	.049	.074	.192	.346	.358	.155	.285
				Experiment 3			
expLM	.018	.036	.226	.242	.225	.066	.061
L_c	.036	.074	.163	.231	.273	.122	.113
meanLM	.033	.056	.201	.297	.273	.088	.090
supLM	.004	.007	.080	.066	.056	.016	.016
$expLM^1$	.026	.059	.461	.436	.374	.081	.168
L_c^1	.043	.059	.262	.407	.386	.148	.210
$meanLM^1$	.035	.068	.391	.466	.379	.091	.140
$supLM^1$	.007	.023	.272	.198	.171	.029	.064
Q-P	.043	.060	.084	.129	.134	.030	.200

TABLE 7
Rejection Frequencies with 5% Asymptotic Critical Values
(Slope change, T = 250)

/r	0.0	0.1	0.3	0.5	0.7	0.9	rw
				Experiment 1			
expLM	.049	.626	1.000	.996	.978	.864	.991
L_c	.051	.527	.895	.978	.970	.845	.940
meanLM	.052	.519	.998	.991	.971	.773	.983
supLM	.038	.609	1.000	.995	.965	.848	.983
$expLM^1$	.055	.582	1.000	1.000	.984	.823	.997
L_c^1	.060	.308	.855	.971	.976	.850	.972
$meanLM^1$	.054	.443	1.000	.999	.981	.689	.983
$supLM^1$	.042	.580	1.000	.997	.981	.842	.994
Q-P	.053	.155	.707	.891	.872	.681	.781
				Experiment 2			
expLM	.044	.599	.991	.899	.831	.663	.935
L_c	.048	.477	.743	.832	.798	.631	.856
meanLM	.049	.470	.949	.882	.809	.577	.939
supLM	.025	.555	.983	.868	.773	.620	.881
$expLM^1$	.049	.530	.995	.953	.877	.580	.954
L_c^1	.051	.249	.743	.860	.850	.632	.911
$meanLM^1$	.045	.389	.984	.927	.835	.460	.932
$supLM^1$	.033	.520	.992	.922	.844	.593	.939
Q-P	.052	.108	.536	.743	.698	.374	.727
				Experiment 3			
expLM	.042	.333	.849	.653	.549	.362	.658
L_c	.044	.281	.460	.529	.551	.362	.588
meanLM	.046	.262	.675	.644	.572	.304	.677
supLM	.015	.265	.809	.549	.446	.274	.487
$expLM^1$	.041	.324	.936	.762	.638	.306	.756
L_c^1	.049	.163	.543	.641	.607	.358	.728
$meanLM^1$	.042	.233	.852	.724	.609	.247	.769
$supLM^1$	.022	.300	.910	.686	.540	.282	.634
Q-P	.050	.054	.205	.306	.257	.059	.449

Q-P test and three versions of the L_c test seem closer than those of the expLM test and the meanLM test. A comparison between three versions of the expLM test and two versions of the meanLM* test shows that they have essentially the same size performance. There is little difference in terms of accuracy of approximating the nominal size. We also observe that the robust cointegration test L_c^* has very good estimated sizes[3].

TABLE 8
Rejection Frequencies with 5% Asymptotic Critical Values
(Null Hypothesis: Cointegration)

	$\sigma_\xi = 0$			$\sigma_\xi = 1$		
	Exp.1	Exp.2	Exp.3	Exp.1	Exp.2	Exp.3
			T = 100			
$expLM^0$	.044	.040	.032	.731	.524	.160
L_c^0	.048	.044	.042	.465	.308	.151
			T = 250			
$expLM^0$	.048	.050	.045	.991	.980	.921
L_c^0	.050	.050	.050	.971	.960	.864

TABLE 9
Rejection Frequencies with 5% Asymptotic Critical Values
(Null Hypothesis: One-Time Discrete Jump)

Estimated Sizes (r = 0.5)				Estimated Powers		
Exp.1	Exp.2	Exp.3		Exp.1	Exp.2	Exp.3
			T = 100			
.029	.023	.017		.190	.127	.059
			T = 250			
.052	.034	.035		.936	.820	.374

In terms of power performance, we see that all of the tests are more powerful in experiment 1 than in experiments 2 and 3. The introduction of endogeneity as well as serially correlated innovations leads to a decline in power. At the same time, the sample

[3] Since the estimated sizes of the L_c^* test are very close. We therefore only reported its estimated sizes when r = 0.5 in table 9.

size plays a very significant role. When T is increased from 100 to 250, the estimated powers of these tests improve dramatically. In addition, the restriction on Π with [0.15, 0.85] has very little effect on estimated powers of the expLM test, the meanLM test and the supLM test. When r = 0.1 or 0.9, it is outside the range of $\Pi = [0.15, 0.85]$ or, in other words, the structural change occurs outside the range in which the above test statistics are evaluated. In this case, their estimated powers are still comparable to those of the L_c test. This suggests that the expLM test, the meanLM test and the supLM test are not very sensitive to the choice of Π.

We now briefly discuss the results on testing for the slope change. A comparison of the estimated powers of the full structural change version of various tests shows that against a one-time discrete jump, the expLM test is almost always more powerful than the other tests. A few exceptions occur in experiment 3 when T is small. When T is large, the power gain of applying the expLM test is quite obvious. Our results also show that with the exception of the expLM test, the supLM test outperforms the other tests when T is large. When T is small, the performance of the L_c test and the meanLM test is very similar, they are preferred to the supLM test. When the alternative is the random walk, the expLM test still dominates the other tests in experiment 1. In experiments 2 and 3, however, the L_c test seems more powerful when T is small. When T is large, on the other hand, the meanLM test demonstrates some power advantage over the other tests.

With respect to the estimated powers of the slope change version of above tests as well as the Q-P test, the $expLM^1$ test obviously outperforms the other tests against either alternative. There is often a significant power improvement from using the $expLM^1$ test, particularly when T is small. The performance of the L_c^1 test, the $meanLM^1$ test and $supLM^1$ test is reasonably similar. It is also quite clear that the Q-P test performs worst among all the tests under comparison. The only exception occurs in experiment 3 when T is small. In this case, Q-P test is slightly preferred to the other tests against random walk alternative.

Our results also suggest that the slope change version of a test typically outperforms its corresponding full structural change version. Exceptions mainly occur when r is far from the middle of the sample, and against the one-time discrete jump alternative. However, in cases where the full structural change version of a test is more powerful, the power difference is small and often negligible. On the other hand, in cases where the slope change version of such a test is more powerful, there is often a substantial power improvement. Our results suggest that when the partial structural change occurs and such information is available, the correct specified partial structural change version of a test might be significantly more powerful than the full structural change version of such a test.

In terms of the estimated powers of the L_c^0 test and the expLM^0 test, the expLM^0 test is consistently more powerful than the L_c^0 test. Sometimes the power discrepancy is large. This can be easily observed when T is small. We therefore conclude that as a cointegration test, the expLM^0 test possesses some desirable properties.

With respect to the estimated powers of the L_c^* test, our results suggest that the test has very good power when T is large. When T is small, it is harder to distinguish the random walk from a one-time discrete jump, yet the test has reasonable power in experiments 1 and 2. We also observe that when endogeneity and autocorrelation are introduced, the estimated powers of the L_c^* test drop very quickly.

6. CONCLUSION

This paper has investigated the problem of testing for structural change in the context of cointegrated regression models. A direct comparison of different approaches suggests that various test statistics only differ in terms of the choice of either different weighting matrix or different norm. Following the idea of Andrews and Ploberger (1992), we derived the asymptotic distribution of the expLM test and generalized it, together with the LM tests of Hansen (1992), to testing for partial structural change. Further, we suggested a new test for cointegration which is robust to the discrete jump in the intercept.

The finite sample properties of the various tests were investigated via a Monte Carlo experiment. It is found that both full and partial structural change versions of the expLM test typically outperform its competitors. Our results also suggested that the correctly specified partial structural change version of a test usually outperforms the full structural change version of a test. In particular, the expLM^0 test possesses some desirable finite sample properties. We also found that when robust cointegration test L_c^* performs well when the sample size is reasonably large.

ACKNOWLEDGEMENTS

I would like to thank Brett Inder, David Harris and two referees for many helpful comments on an early draft of this paper. I also thank Bruce Hansen for supplying his program to calculate the fully modified least squares estimator. Remaining errors are my own.

REFERENCES

Andrews, D.W.K. (1993) "Testing for parameter instability and structural change with unknown change point," *Econometrica*, 61, 821-856.

Andrews, D.W.K. and Ploberger, W. (1994) "Optimal tests when a nuisance parameter is present only under the alternative," *Econometrica*, 62, 1383-1414.

Andrews, D.W.K, Lee, I. and Ploberger, W. (1992) "Optimal changepoint tests for normal linear regression," Cowles foundation discussion paper 1016R.

Billingsley, P. (1968) *Convergence of Probability Measures*, Wiley, New York.

Gregory, A.W. and Nason, J.M. (1992) "Testing for structural breaks in cointegrated relationships," Queen's University.

Hansen, B.E. (1990) "Lagrange multiplier tests for parameter instability in non-linear models," University of Rochester.

Hansen, B.E. (1992) "Tests for parameter instability in regressions with I(1) processes," *Journal of Business and Economic Statistics*, 10, 321-355.

Harris, D. and Inder, B. (1994) "A test of the null hypothesis of cointegration," forthcoming in: *Nonstationary time series analysis and cointegration*, C. Hargreaves (Ed), in the series *Advanced Texts in Econometrics*, C.W.J. Granger and G.E. Mizon (Eds).

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Shin, Y. (1992) "Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?" *Journal of Econometrics*, 54, 159-178.

Nyblom, J. (1989) "Testing for constancy of parameters over time," *Journal of the American Statistical Association*, 84, 223-230.

Phillips, P.C.B. and Durlauf, S.N. (1986) "Multiple time series regression with integrated process", *Review of Economic Studies*, 53, 473-496.

Phillips, P.C.B. and Hansen, B. (1990) "Statistical inference in instrumental variables regression with I(1) process," *Review of Economic Studies,* 57, 99 - 125.

Quintos, C.E. and Phillips, P.C. B. (1993) "Parameter constancy in cointegrating regressions," *Empirical Economics*, 18, 675-706.