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TRENDS, LEAD TIMES AND FORECASTING

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Trends, Lead Times and Forecasting

by

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Trends, Lead Times and Forecasting

Abstract

The local linear trend and global linear trend models embody extreme assumptions about trends. According to the local linear trend formulation the level and growth rate are allowed to rapidly adapt to changes in the data path. On the other hand, the global linear trend model makes no allowance for structural change. In this paper we introduce a new model that, as well as encompassing the global linear trend and local linear trend models, allows for a range of "in between" cases. The theoretical properties of the autocovariance and forecast functions for this model suggest that it should be useful when neither a local linear trend nor a global linear trend is appropriate. A comparison of forecasting performance using real time series provides further support for this hypothesis.

Key Words

Integrated autoregressive moving average models, Structural time series models, Trend forecasting, Forecast competition

Introduction

The concept of a trend plays an important role in the analysis of economic time series. The evolution of economies means that trends arise in many series which, when viewed over a period of time that is long in relation to the length of the series itself, display a general tendency to rise or fall. In recent times particular emphasis has been given to the question of stochastic or deterministic formulations for the trend in models for a wide range of macro-economic time series (Nelson and Plosser 1982, Perron 1989, de Jong and Whiteman 1994). These distinctly different formulations, it is claimed, have profoundly different implications for macro-economic policy formulation, economic theory and forecasting.

A possible candidate for the trend amongst the stochastic formulations is the ARIMA(0,2,2) model. It allows both the level and growth rate to evolve over time in response to structural change and, as a consequence, it is often termed a local linear trend model. Variations of this theme have been considered by Harvey and Todd (1983) and Gersch and Kitagawa (1983) .

In many cases of practical interest the ARIMA(0,2,2) formulation has provided relatively good within sample fit. Within sample fit, however, is of limited usefulness when interest is in long run forecasts of the trend. Compared to its within sample performance the long run predictive performance of the ARIMA(0,2,2) model has not been as good. For instance, Gardner and McKenzie (1976) found that a local linear trend model augmented with an autoregressive dampening term on the growth rate generated better long run forecasts than this model. In related work Meese and Geweke (1984) found that an approach incorporating a deterministic linear pre-filter also resulted in better long run forecasts than an approach based on differencing the series. Gersch and Kitagawa (1983) have also proposed a particular form of local linear trend model. Their concept of trend, however, is related to smoothness. For the data sets that they examined they found that the minimisation of a criterion based on multi-step prediction errors generated better long run forecasts than the minimisation of the same criterion based on 1-step prediction errors.

In this paper the notion of predictive accuracy is used to motivate a new model for trends. Whilst the new model encompasses both the global linear trend and ARIMA(0,2,2) specifications as special cases it also allows for a wide range of "in between" cases. An analysis of the predictive performance of the new model vis a vis the ARIMA(0,2,2) and global linear trend specifications demonstrates significant improvements in forecast performance for several macro-economic time series. Our analysis also provides further support for the hypotheses that within sample measures of fit are inadequate for choosing the best forecasting model.

2. Framework

Our approach is motivated by considering the extreme cases of the local linear trend and global linear trend models. A local linear trend model for a time series $\{y_t\}$ can be written in terms of a single source of disturbances $\{e_t\}$ (Snyder 1985, Aoki and Havenner 1991) as

$$y_t = \mu_{t-1} + \delta_{t-1} + e_t \quad (1)$$

$$\mu_t = \mu_{t-1} + \delta_{t-1} + \alpha_1 e_t \quad (2)$$

$$\delta_t = \delta_{t-1} + \alpha_2 e_t, \quad (3)$$

μ_t and δ_t being level and growth rates at time t, and the disturbances $e_t \sim NID(0,\sigma^2)$. The model can be viewed as the structural representation for an ARIMA(0,2,2) model so that the parameters α_1, α_2 satisfy the invertibility conditions $0 \le \alpha_1 \le 2$, and $0 \le \alpha_2 \le 4 - 2\alpha_1$. Heuristically the model can be thought of as providing a local linear approximation to the data path. A key feature is the ability of the level and growth rate to adapt to new information as it becomes available. At the other extreme a global linear trend model of the form

$$y_t = \mu + \delta t + e_t, \qquad (4)$$

in which the coefficients μ and δ represent an intercept and global growth rate, does not allow for any structural change.

In generating predictions with the local linear trend model the most recent values for μ_t and δ_t are used. Where the series of interest is highly autocorrelated or there are pronounced business cycle effects, however, estimates of these parameters can be overly influenced by short term factors, leading to quantities that are inappropriate for long run prediction. On the other hand predictions from the global linear trend model can be quite awry when there has been substantial structural change.

The local linear trend and global linear trend models embody rather extreme assumptions. Suppose, however, that interest is in forecasting the trend *h* periods into the future. Then it can be argued that the growth rate for a typical time *t-h* should be calculated in such a way that it provides the best prediction of the series for time *t*. This is the basic strategy behind the minimisation of multi-step prediction errors. Rather than changing the estimation criterion, however, we seek to embed this requirement in the model itself. Such a model is given by

$$y_t = \mu_{t-h} + h\delta_{t-h} + e_t \qquad (5)$$

$$\mu_t = \mu_{t-1} + \delta_{t-1} + \alpha_1 e_t \qquad (6)$$

$$\delta_t = \delta_{t-1} + \alpha_2 e_t \qquad (7)$$

The model, which will be termed an adaptive trend model of lag *h*, denoted AT(*h*), effectively imposes a requirement that μ_{t-h} and δ_{t-h} be chosen on the basis of their *h*-step predictive ability.

Differencing (5) twice and collecting like terms gives the reduced form restricted ARIMA(0,2,h+1) representation

$$\nabla^2 y_t = -\theta_{h+1} e_{t-h-1} - \theta_h e_{t-h} - \theta_2 e_{t-2} - \theta_1 e_{t-1} + e_t . \qquad (8)$$

with $\theta_{h+1} = \alpha_1 + (h-1)\alpha_2, \theta_h = -(\alpha_1 + h\alpha_2), \theta_2 = -1, \theta_1 = 2$.

The reduced form highlights the impact of shocks up to lag h+1 on the current value of the series. Formulae for the autocovariances for $\nabla^2 y_t$ for various values of h are shown in Table 1. The mean squared prediction error is given by

$$\mathrm{PMSE}(j) = \sigma^2 \left(1 + \sum_{k=1}^{j-h} (\alpha_1 + k\alpha_2)^2 \right), \qquad (9)$$

a quantity which remains constant for $j \leq h$ and increases thereafter.

The representation (8) is invertible if the roots of its characteristic equation expressed, in terms of the forward shift operator F (ie $Fe_t = e_{t+1}$), as

$$F^{h+1} + \theta_1 F^h + \theta_2 F^{h-1} + \theta_h F + \theta_{h+1} = 0,$$

all lie within the unit circle. θ_{h+1}, being the product of the roots, satisfies the condition $\theta_{h+1} < 1$, or equivalently,

$$\alpha_1 + (h-1)\alpha_2 < 1. \qquad (10)$$

If (8) is invertible then α_1 and α_2 satisfy the condition (10). Futhermore, as the number of terms in the product that formsθ_{h+1} increases with h, so the left-hand side of (10) converges to 0. Both α_1 and α_2 also converge to zero by necessity, implying that the AT(h) model approaches a global linear trend as h increases. This is supported by numerical solution of the stability region for a range of values for α_1, α_2 and h. The admissible regions for α_1 and α_2, shown in Figure 1, appear to reduce markedly as h is increased. Note that these computations indicate that α_1 and α_2 are always nonnegative in the admissable region, a result that we have been unable to verify analytically from the characteristic equation.

Interestingly, the AT(h) approach does not work in an unobserved component framework. An unobserved component representation of the AT(h) model is given by

$$y_t = \mu_{t-h} + h\delta_{t-h} + \varepsilon_t \tag{11}$$

$$\mu_t = \mu_{t-1} + \delta_{t-1} + \eta_t \tag{12}$$

$$\delta_t = \delta_{t-1} + \xi_t \tag{13}$$

where ε_t, η_t and ξ_t are zero mean serially and mutually uncorrelated random variables with variances σ^2_ε, σ^2_η and σ^2_ξ. The reduced form for this model, however, is simply the restricted ARIMA(0,2,2) model

$$\nabla^2 y_t = \left(h - (h-1)B\right)\xi_{t-h} + (1-B)\eta_{t-h} + (1-B)^2\varepsilon_t. \tag{14}$$

3. Experimental Design

Empirical analysis of the performance of the AT(h) model was undertaken using the extended version of the Nelson and Plosser data compiled in Schotman and van Dijk (1991). This is the same data set used in de Jong and Whiteman (1994). The data set contains 14 macro-economic time series for the United States economy. The longest series commences in 1860 and all of the series terminate in 1988. As has been the convention with this data set all of the series, except for Interest Rates, were logarithmically transformed prior to estimation.

Estimation and forecasting were performed using the state space representation of the various models. To illustrate, the state space representation for the AT(4) model is given by

$$y_t = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 4]\beta_{t-1} + e_t, \tag{15}$$

where $\beta'_t = [\mu_t \quad \delta_t \quad \mu_{t-1} \quad \delta_{t-1} \quad \cdots \quad \mu_{t-3} \quad \delta_{t-3}]$. The state vector is updated according to

$$\begin{bmatrix} \mu_t \\ \delta_t \\ \mu_{t-1} \\ \delta_{t-1} \\ \mu_{t-2} \\ \delta_{t-2} \\ \mu_{t-3} \\ \delta_{t-3} \end{bmatrix} = \left[\begin{array}{cc:cc:cc:cc} 1 & 1 & & & & & & \\ 0 & 1 & & & & & & \\ \hdashline 1 & 0 & & & & & & \\ 0 & 1 & & & & & & \\ \hdashline & & 1 & 0 & & & & \\ & & 0 & 1 & & & & \\ \hdashline & & & & 1 & 0 & & \\ & & & & 0 & 1 & & \end{array}\right] \begin{bmatrix} \mu_{t-1} \\ \delta_{t-1} \\ \mu_{t-2} \\ \delta_{t-2} \\ \mu_{t-3} \\ \delta_{t-3} \\ \mu_{t-4} \\ \delta_{t-4} \end{bmatrix} + \begin{bmatrix} \alpha_1 e_t \\ \alpha_2 e_t \\ 0 \\ \\ \\ \\ \\ 0 \end{bmatrix}. \tag{16}$$

Estimates of the state vector were computed using a square root covariance filter (Snyder and Saligari 1996). The entire state vector was initialised with a diffuse prior distribution. Estimates of α_1 and α_2 were obtained by maximising the marginal or diffuse likelihood (Ansley and Kohn 1985, de Jong 1991). The constraints on α_1 and α_2 required for a stable model were imposed by numerically solving the characteristic equation (A9). Predictions were generated by substituting the maximum likelihood estimates of α_1 and α_2 into the model.

Estimation results have been reported for the logarithmically transformed data, permitting comparison with earlier work. The forecast comparison, however, has been based on the original data. Denote the logarithmically transformed data by y_t and the forecast and forecast error variance of an observation at a particular time $T+j$ based on information up to time T by $y_{T+j|T}$ and $\sigma^2_{T+j|T}$. Except for the Interest Rate and Unemployment series, forecasts for the original data have been based on $exp\left(y_{T+j|T} + \sigma^2_{T+j|T}/2\right)$ (Granger and Newbold 1976). In the case of Interest Rates no transformation was necessary, whilst for Unemployment, the estimate of $\sigma^2_{T+j|T}$ from the local linear trend model was found to be unacceptably large and therefore the forecast for the original series was obtained from the naive transformation $exp\left(y_{T+j|T}\right)$.

Forecast accuracy was evaluated across time periods as described in Fildes (1992). Using this approach a number of forecasts were generated from different origins. The resulting forecast errors at each lead-time were treated as samples and the median and inter-quartile range of the absolute percentage errors (APE) evaluated for each lead-time. This can be contrasted with the strategy of obtaining forecast errors from a single forecast origin and then operating on the forecast errors across lead-times to obtain a summary measure. The former approach makes it possible to discern differences in forecast accuracy at various lead-times and also helps to overcome the possible dependence of the results on an arbitrarily chosen forecast origin. To ascertain the statistical significance of differences between the various series of forecast errors a Wilcoxon matched pairs signed ranks test was used (Flores 1989).

The approach used can be summarised as follows. Denote the sample size by T. The maximum forecast horizon used in the evaluation was 18. Commencing with a sample containing the first T-27 data points, the following steps were carried out.

1. Logarithmically transform the series.
2. Estimate the model.
3. Generate out of sample forecasts from 1 to 18 steps ahead.
4. Obtain the forecast for the original series and compute forecast errors.
5. Append the next observation to the sample.

Steps (1) to (5) were repeated 27 times so that, for each series, 27 1-step forecasts, 26 2-step forecasts, ... 10 18-step forecasts were obtained. The resulting forecast errors were used to evaluate predictive performance at each horizon.

4. Empirical Results

The results of the analysis can be found in the tables below. The results for the AT(h) models in which α_1 and α_2 were estimated to be zero have not been reported as these are effectively global linear trend models. For example, for the Unemployment series α_1 and α_2 were estimated to be zero for h>1. Therefore results have only been reported for the local linear trend and global linear trend models.

For all series the best within sample fit was achieved by the local linear trend model. In most cases the differences were substantial. For example, in the case of Interest Rates the local linear trend model produced a prediction error variance of 3462, the AT(3) model produced a prediction error variance of 13714 and the global linear trend model produced a prediction error variance of 37439.

As expected the results for 1-step forecasting largely replicated the within sample results. With the exception of Unemployment the lowest median absolute percentage error (APE) was achieved by the local linear trend model. Overall there appeared to be a tendency for the AT(h) and global linear trend models to perform better as the forecast horizon was increased. Thus the summary in table 3 shows a large number of series for which the local linear trend model produces the lowest median APE at short forecast horizons. At forecast horizon 18, however, the local linear trend model produces the lowest median APE for only five series.

In many cases the differences between the series of forecast errors are statistically significant. This can be ascertained from the results of the Wilcoxon test. The test is applied in a pairwise fashion. Therefore an entry in one of the tables shows the statistic arising from a test between the forecast errors generated by the models in the matching row and column. Statistics which are statistically significant at a 10% level have been emphasised. For example, in the case of GNP Deflator shown in Table 11, the 18-step forecast errors from the local linear trend model are significantly different from those from the AT(5) model and the median absolute percentage errors are 49.27 and 44.23 respectively. The discrepancy between forecast performance and within sample fit is highlighted by the 1-step prediction error variances which are 0.002 and 0.030. This situation is also evident for Real GNP and the Standard and Poor 500 Index. In the case of Real GNP the AT(2) model produces a lower median APE than the local linear trend model at forecast horizon 6. The prediction error variances, however, are 0.003 for the local linear trend model and 0.013 for AT(2). In the case of the Standard & Poor 500 Index the prediction error variances for the local linear trend and AT(2) models are 0.023 and 0.073. Whereas the respective 18-step median absolute percentage errors are 39.21 and 6.71.

The most startling result is for Money Velocity. The prediction error variances are 0.004 and 0.017 for the local linear trend and AT(4) models. For the local linear trend model the 6, 12 and 18-step median absolute percentage errors are 5.77, 14.27 and 18.84. This compares with 3.75, 4.79 and 2.95 for AT(4)!

We note that the AT(h) model does not lead to improvements in forecasting performance in all cases. Thus for the CPI and Money Stock series the local linear trend model produces the lowest median APE at all horizons. The local linear trend model also performs well for Nominal GNP. The result for CPI is surprising in lieu of the fact that the AT(h) model performs well for the GNP deflator series.

5. Conclusions

The AT(h) models produced improvements in forecasting performance at medium and long horizons for the majority of the series examined. In many cases the improvements were significant. As forecast performance is an important indicator of model adequacy it would appear that, for these series, the AT(h) model provides a better representation than either the local linear trend or global linear trend models. For most series, values of h ranging from 2 to 4 seemed to be appropriate. This appears to provide support for the argument that most of the improvement arises from smoothing out the effect of a short term cyclical component.

Turning from the question of performance for a particular data set to the more general problem of modelling trends in univariate time series, the AT(h) model would appear to provide an important bridge between the local linear trend and global linear trend formulations. Furthermore the AT(h) model lends weight to anecdotal evidence supporting the inclusion of additional moving average terms in ARIMA models.

On a final note, our analysis also provides further support for the hypotheses that within sample fit is a poor determinant of out of sample forecasting performance.

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Table 1: Autocovariance function for equation (10) for various values of h

h=2	$\gamma(0)=\left[(1+\alpha_1+2\alpha_2)^2+(\alpha_1+\alpha_2)^2+5\right]\sigma_e^2$ $\gamma(1)=-\left[(1+\alpha_1+2\alpha_2)(\alpha_1+\alpha_2+2)+2\right]\sigma_e^2$ $\gamma(2)=\left[(1+\alpha_1+2\alpha_2)+2(\alpha_1+\alpha_2)\right]\sigma_e^2$ $\gamma(3)=-\left[\alpha_1+\alpha_2\right]\sigma_e^2$ $\gamma(j)=0 \qquad j\geq 4$
h=3	$\gamma(0)=\left[(\alpha_1+2\alpha_2)^2+(\alpha_1+3\alpha_2)^2+6\right]\sigma_e^2$ $\gamma(1)=\left[(\alpha_1+3\alpha_2)(1-\alpha_1-2\alpha_2)-4\right]\sigma_e^2$ $\gamma(2)=\left[-3\alpha_1-8\alpha_2+1\right]\sigma_e^2$ $\gamma(3)=\left[3\alpha_1+7\alpha_2\right]\sigma_e^2$ $\gamma(4)=-\left[\alpha_1+2\alpha_2\right]\sigma_e^2$ $\gamma(j)=0 \qquad j\geq 5$
h>3	$\gamma(0)=\left[((h-1)\alpha_2+\alpha_1)^2+(\alpha_1+h\alpha_2)^2+6\right]\sigma_e^2$ $\gamma(1)=\left[-(\alpha_1+h\alpha_2)((h-1)\alpha_2+\alpha_1)-4\right]\sigma_e^2$ $\gamma(2)=\left[(\alpha_1+h\alpha_2)+1\right]\sigma_e^2$ $\gamma(h-1)=\left[(1-3h)\alpha_2-3\alpha_1\right]\sigma_e^2$ $\gamma(h)=\left[(3h-2)\alpha_2+3\alpha_1\right]\sigma_e^2$ $\gamma(h+1)=-\left[(h-1)\alpha_2+\alpha_1\right]\sigma_e^2$ $\gamma(j)=0 \qquad j=3,\ldots,h-2$ and $j\geq h+2$

Figure 1: Stability regions for AT(h) models

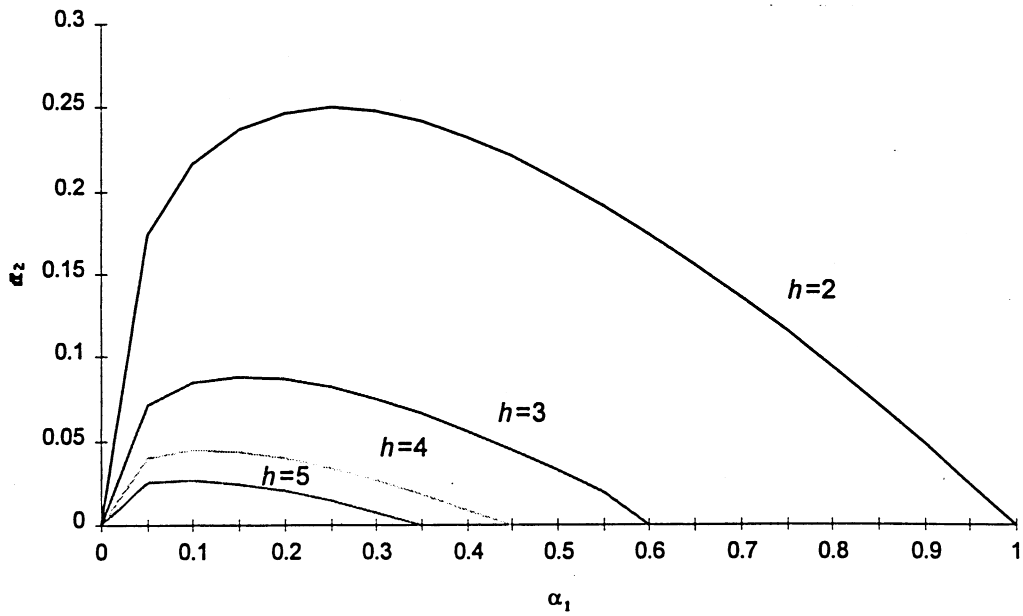

Table 2: Series used in empirical work. All series are for the United States of America.

Number	Description
1	Consumer Price Index 1860-1988
2	Employment 1890-1988
3	GNP Deflator 1889-1988
4	Interest Rates 1900-1988
5	Index of Industrial Production 1860-1988
6	Money Stock 1889-1988
7	Nominal GNP 1909-1988
8	per capita Real GNP 1909-1988
9	Real GNP 1909-1988
10	Real Wages 1900-1988
11	Standard & Poor 500 Index 1871-1988
12	Unemployment 1890-1988
13	Velocity of Money 1869-1988
14	Nominal Wages 1900-1988

Table 3: Model producing the lowest median APE for each series and forecast horizon

Forecast horizon	Model	Series number
1-step	Local	1, 2 ,3 ,4 ,5 ,6 ,7 ,8 ,9 ,10 ,11 ,13 ,14
	AT(h)	
	Global	12
2-step	Local	1, 2, 3, 6, 11, 13, 14
	AT(h)	4, 5, 7, 8, 9, 10
	Global	12
3-step	Local	1, 3, 6, 7, 10, 11, 13, 14
	AT(h)	2, 4, 8, 9
	Global	5, 12
6-step	Local	1, 6, 7, 8, 10, 14
	AT(h)	2, 3, 4, 9, 11, 13
	Global	5, 12
12-step	Local	1, 6, 7, 10
	AT(h)	2, 3, 4, 11, 13, 14
	Global	5, 8, 9, 12
18-step	Local	1, 2, 6, 7, 14
	AT(h)	3, 4, 11, 13
	Global	5, 8, 9, 10, 12

Figure 2: US Consumer Price Index 1860-1988

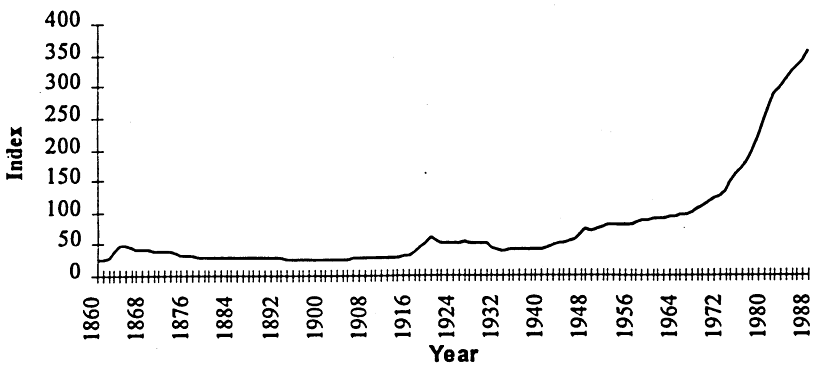

Table 4: Estimation results for Consumer Price Index using the sample 1860-1988

	σ^2	α_1	α_2
Local	**0.00188**	1.642	0.116
AT(2)	0.0129	0.638	0
AT(3)	0.0188	0.410	0
AT(4)	0.0298	0.295	0
AT(5)	0.0506	0.254	0
Global	0.140		

Table 5: Median APE and inter-quartile range for Consumer Price Index

	Forecast Horizon					
	1	2	3	6	12	18
Local	**1.60**	**4.36**	**7.64**	**20.45**	**36.72**	**53.60**
	1.80	4.11	4.27	10.92	16.48	5.11
AT(2)	4.61	4.83	7.77	22.76	42.29	56.98
	5.52	6.05	9.26	13.73	13.23	2.83
AT(3)	11.00	12.38	12.68	24.99	45.37	60.81
	10.47	9.27	9.06	15.22	12.56	3.50
AT(4)	11.47	11.96	12.50	23.16	46.48	61.79
	15.01	14.92	14.21	15.28	11.82	1.21
AT(5)	19.23	19.80	20.22	25.86	47.47	64.39
	13.39	12.31	11.86	11.21	9.84	2.71
Global	42.92	44.72	46.48	51.96	65.69	71.24
	27.44	26.17	24.89	24.41	13.42	1.69

Table 6: Wilcoxon signed ranks test statistics for Consumer Price Index. Values critical at a 10% level are shown in bold.

	12 step					18 step				
	AT(2)	AT(3)	AT(4)	AT(5)	GLB	AT(2)	AT(3)	AT(4)	AT(5)	GLB
LLT	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
AT(2)		**0**	**0**	**0**	**0**		**0**	**0**	**0**	**0**
AT(3)			**15**	**0**	**0**			**7**	**0**	**0**
AT(4)				**18**	**0**				**0**	**0**
AT(5)					**0**					**0**

Figure 3: US Employment 1890-1988

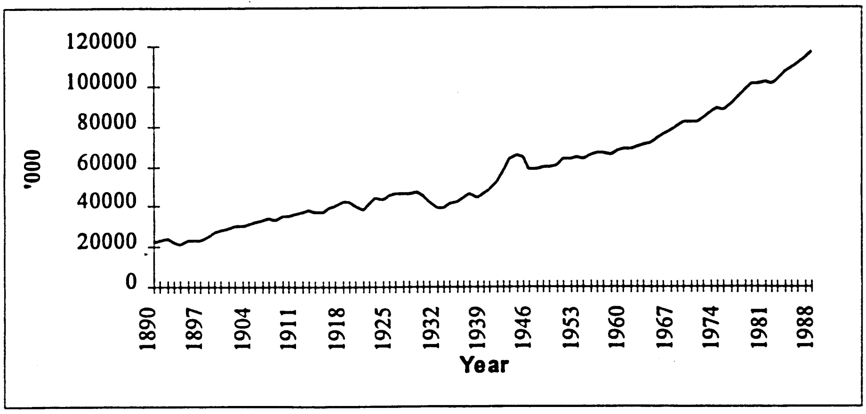

Table 7: Estimation results for Employment using the sample 1890-1988

	σ^2	α_1	α_2
Local	**0.00109**	1.397	0
AT(2)	0.00442	0.377	0
AT(3)	0.00642	0.212	0
Global	0.00584		

Table 8: Median APE and inter-quartile range for Employment

	Forecast Horizon					
	1	2	3	6	12	18
Local	**0.71**	**1.61**	2.08	2.73	2.89	**2.32**
	1.08	2.02	1.83	3.10	1.74	3.60
AT(2)	1.72	1.77	1.97	1.62	3.85	4.47
	1.10	1.15	2.05	2.36	1.48	2.77
AT(3)	1.71	1.71	**1.60**	**1.33**	**2.72**	4.20
	1.97	1.83	1.65	1.78	2.09	1.21
Global	3.04	3.02	3.13	2.73	3.93	3.59
	3.14	3.11	3.08	2.58	3.06	1.72

Table 9: Wilcoxon signed ranks test statistics for Employment. Values critical at a 10% level are shown in bold.

	12 step			18 step		
	AT(2)	AT(3)	GLB	AT(2)	AT(3)	GLB
LLT	**34**	56	66	**3**	13	26
AT(2)		**18**	50		12	12
AT(3)			55			18

Figure 4: US GNP Deflator 1889-1988

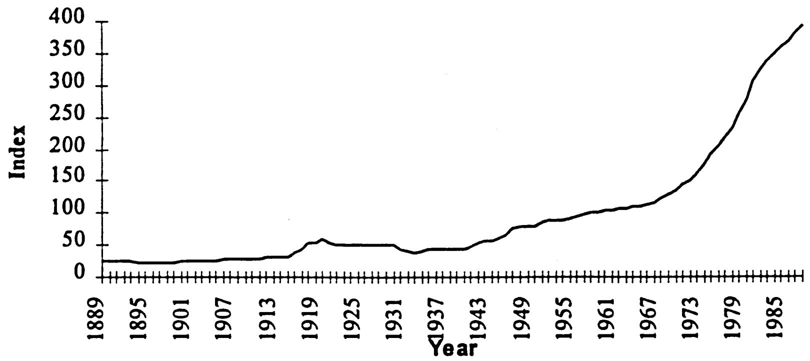

Table 10: Estimation results for GNP Deflator using the sample 1889-1988

	σ^2	α_1	α_2
Local	**0.00204**	1.283	0.219
AT(2)	0.00974	0.621	0
AT(3)	0.0182	0.368	0
AT(4)	0.0255	0.312	0
AT(5)	0.0299	0.347	0
Global	0.0531		

Table 11: Median APE and inter-quartile range for GNP Deflator

	Forecast Horizon					
	1	2	3	6	12	18
Local	**1.48**	**4.04**	**7.00**	15.28	33.57	49.27
	1.55	4.28	6.73	13.99	17.30	5.51
AT(2)	4.36	4.74	9.23	17.96	36.65	50.18
	4.30	4.48	8.20	14.62	11.60	3.38
AT(3)	8.55	8.79	9.17	18.31	37.09	50.46
	7.73	7.63	7.80	18.74	14.05	4.72
AT(4)	9.14	9.93	10.67	**14.49**	34.98	48.21
	13.02	13.02	13.28	19.66	14.68	3.07
AT(5)	9.42	11.05	12.33	15.63	**31.58**	**44.23**
	15.47	15.59	15.36	19.93	17.80	5.23
Global	27.47	29.35	31.21	36.61	48.72	54.32
	28.38	28.35	27.81	25.26	14.55	2.39

Table 12: Wilcoxon signed ranks test statistics for GNP Deflator. Values critical at a 10% level are shown in bold.

	12 step					18 step				
	AT(2)	AT(3)	AT(4)	AT(5)	GLB	AT(2)	AT(3)	AT(4)	AT(5)	GLB
LLT	9	**22**	41	63	**0**	9	21	12	**1**	**0**
AT(2)		**31**	41	**1**	**0**		25	**3**	**3**	**0**
AT(3)			**13**	**0**	**0**			**4**	**0**	**0**
AT(4)				**0**	**0**				**0**	**0**
AT(5)					**0**					**0**

Figure 5: US Interest Rates 1900-1988

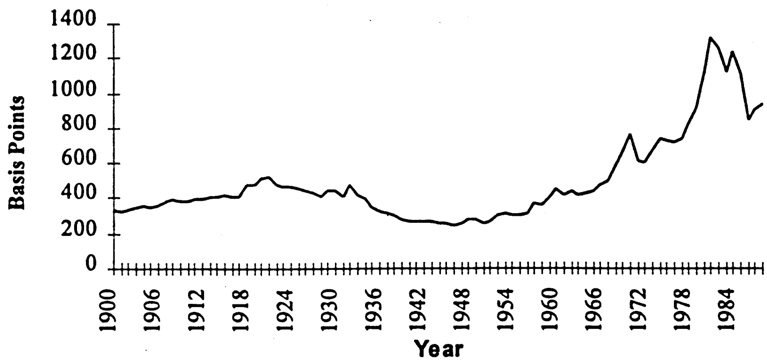

Table 13: Estimation results for Interest Rates using the sample 1900-1988

	σ_2	α_1	α_2
Local	**3462**	1.312	0
AT(2)	9965	0.594	0
AT(3)	13714	0.0040	0.0260
AT(4)	14433	0.0112	0.0238
AT(5)	14489	0.220	0
Global	37439		

Table 14: Median APE and inter-quartile range for Interest Rates

	Forecast Horizon					
	1	2	3	6	12	18
Local	**8.44**	12.47	21.04	22.08	24.59	44.85
	9.86	13.83	20.78	23.96	11.37	17.45
AT(2)	12.70	13.84	16.03	21.77	38.12	53.59
	14.36	18.47	21.82	15.35	19.52	31.69
AT(3)	10.06	12.08	13.39	14.08	**20.29**	38.24
	15.65	16.48	17.10	10.61	27.68	22.51
AT(4)	11.29	**12.00**	**12.54**	**12.51**	21.53	**36.61**
	12.70	13.42	13.61	11.76	21.15	22.57
AT(5)	12.07	13.66	14.53	20.03	31.24	52.03
	16.31	18.11	19.71	14.86	14.63	22.13
Global	36.06	38.12	40.72	47.90	58.78	70.33
	13.52	13.88	14.18	10.07	7.57	13.02

Table 15: Wilcoxon signed ranks test statistics for Interest Rates. Values critical at a 10% level are shown in bold.

	12 step					18 step				
	AT(2)	AT(3)	AT(4)	AT(5)	GLB	AT(2)	AT(3)	AT(4)	AT(5)	GLB
LLT	68	44	45	46	**0**	**0**	19	23	**0**	**0**
AT(2)		**12**	**30**	**32**	**0**		**0**	**0**	**0**	**0**
AT(3)			**34**	**0**	**0**			26	**0**	**0**
AT(4)				**0**	**0**				**0**	**0**
AT(5)					**0**					**0**

Figure 6: US Industrial Production 1860-1988

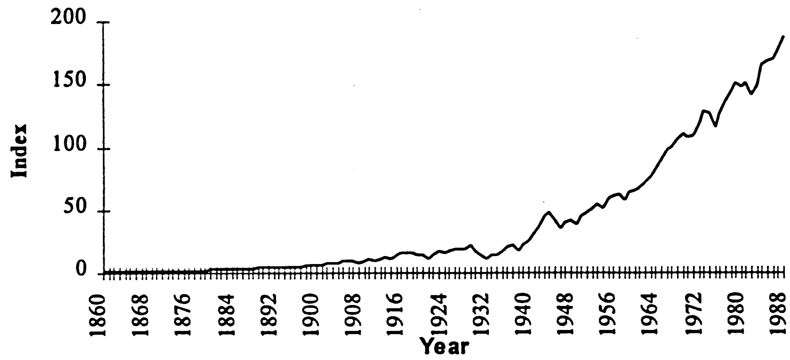

Table 16: Estimation results for Industrial Production using the sample 1860-1988

	σ^2	α_1	α_2
Local	**0.00919**	1.062	0
AT(2)	0.0224	0.289	0
AT(3)	0.0284	0.149	0
Global	0.0303		

Table 17: Median APE and inter-quartile range for Industrial Production

	Forecast Horizon 1	2	3	6	12	18
Local	**3.25**	6.20	6.65	12.57	23.15	46.27
	3.11	6.52	5.36	8.01	12.38	20.99
AT(2)	4.72	**4.83**	6.92	11.08	21.96	29.66
	6.12	6.05	6.42	7.81	17.44	21.19
AT(3)	7.60	7.73	8.06	13.21	19.42	26.73
	6.65	6.88	6.90	11.23	20.44	16.35
Global	6.39	6.37	**5.97**	**10.46**	**16.29**	**24.15**
	12.57	13.31	13.54	15.46	17.84	10.91

Table 18: Wilcoxon signed ranks test statistics for Industrial Production. Values critical at a 10% level are shown in bold.

	2 step			3 step			12 step			18 step		
	AT(2)	AT(3)	GLB	AT(2)	AT(3)	GLB	AT(2)	AT(3)	GLB	AT(2)	AT(3)	GLB
LLT	129	**74**	**66**	143	**91**	**81**	**24**	**25**	**18**	**0**	**0**	**0**
AT(2)		**62**	**62**		**69**	**70**		**45**	**27**		**7**	**9**
AT(3)			**82**			**72**			**25**			**14**

Figure 7: US Money Stock 1889-1988

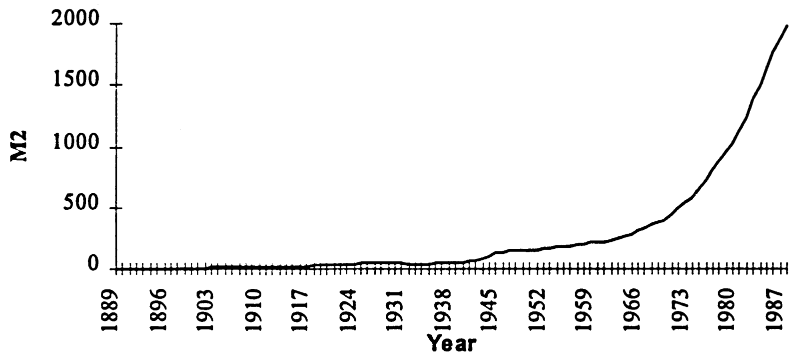

Table 19: Estimation results for Money Stock using the sample 1889-1988

	σ_2	α_1	α_2
Local	**0.00224**	1.373	0.414
AT(2)	0.0141	0.653	0
AT(3)	0.0276	0.420	0
Global	0.0426		

Table 20: Median APE and inter-quartile range for Money Stock

	Forecast Horizon					
	1	2	3	6	12	18
Local	**1.58**	**2.56**	**3.29**	**5.62**	**12.03**	**24.91**
	1.59	3.62	3.17	5.76	14.97	25.62
AT(2)	2.98	2.94	4.99	11.34	25.90	36.78
	3.29	3.62	5.57	5.49	3.31	1.88
AT(3)	4.88	4.79	4.78	12.98	27.73	38.28
	6.16	6.04	6.53	6.24	5.00	5.87
Global	18.54	18.95	19.38	21.85	26.81	34.11
	15.26	16.78	18.47	21.94	17.77	11.85

Table 21: Wilcoxon signed ranks test statistics for Money Stock. Values critical at a 10% level are shown in bold.

	12 step			18 step		
	AT(2)	AT(3)	GLB	AT(2)	AT(3)	GLB
LLT	**0**	**0**	**7**	**0**	**0**	**0**
AT(2)		40	63		12	**11**
AT(3)			60			**10**

Figure 8: US Nominal GNP 1909-1988

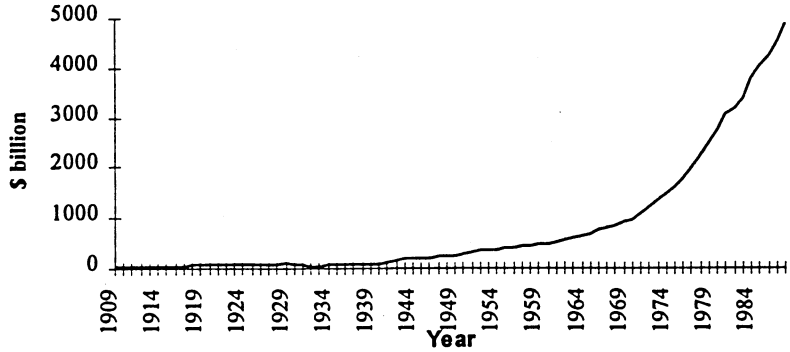

Table 22: Estimation results for Nominal GNP using the sample 1909-1988

	σ^2	α_1	α_2
Local	**0.00633**	1.433	0.0226
AT(2)	0.0326	0.424	0
AT(3)	0.0456	0.367	0
Global	0.0906		

Table 23: Median APE and inter-quartile range for Nominal GNP

	Forecast Horizon					
	1	2	3	6	12	18
Local	**1.98**	3.89	**4.71**	**9.88**	**23.74**	**34.01**
	1.77	4.53	6.11	9.72	8.34	2.42
AT(2)	3.41	**3.51**	5.58	14.81	30.85	41.36
	6.08	6.24	9.33	11.56	9.71	2.66
AT(3)	5.74	5.62	5.90	11.39	27.90	37.78
	4.66	4.79	5.00	7.77	7.84	4.65
Global	26.06	28.75	31.38	37.37	48.28	54.97
	17.05	17.10	17.55	13.41	6.76	0.96

Table 24: Wilcoxon signed ranks test statistics for Nominal GNP. Values critical at a 10% level are shown in bold.

	12 step			18 step		
	AT(2)	AT(3)	GLB	AT(2)	AT(3)	GLB
LLT	**0**	**21**	**0**	**0**	**4**	**0**
AT(2)		**0**	**0**		**0**	**0**

AT(3)	**0**	**0**

Figure 9: US per capita Real GNP 1909-1988

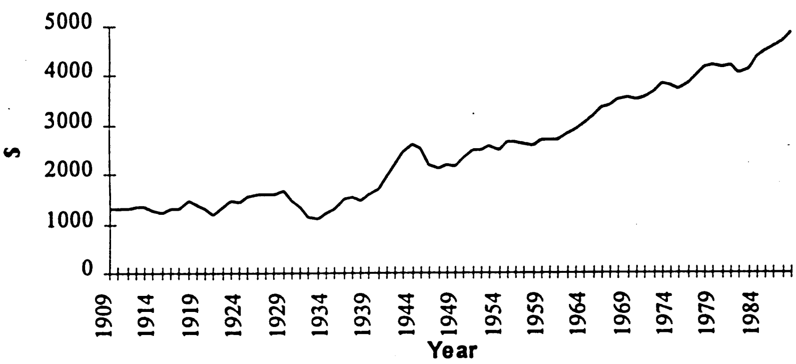

Table 25: Estimation results for per capita Real GNP using the sample 1909-1988

	σ^2	α_1	α_2
Local	**0.00307**	1.314	0
AT(2)	0.0128	0.374	0
Global	0.0139		

Table 26: Median APE and inter-quartile range for per capita Real GNP

	Forecast Horizon					
	1	2	3	6	12	18
Local	**1.94**	3.64	4.36	**3.45**	4.37	9.72
	1.99	3.15	3.67	6.99	7.06	7.27
AT(2)	3.34	**3.55**	**3.43**	3.60	5.01	6.50
	2.88	2.76	4.17	4.22	4.24	4.35
Global	4.35	4.70	5.03	4.71	**4.12**	**4.78**
	4.55	4.78	5.04	6.59	6.70	3.68

Table 27: Wilcoxon signed ranks test statistics for per capita Real GNP. Values critical at a 10% level are shown in bold.

	2 step		3 step		12 step		18 step	
	AT(2)	GLB	AT(2)	GLB	AT(2)	GLB	AT(2)	GLB
LLT	154	**63**	134	**50**	**14**	**1**	**1**	**0**
AT(2)		**61**		**57**		**4**		**5**

Figure 10: US Real GNP 1909-1988

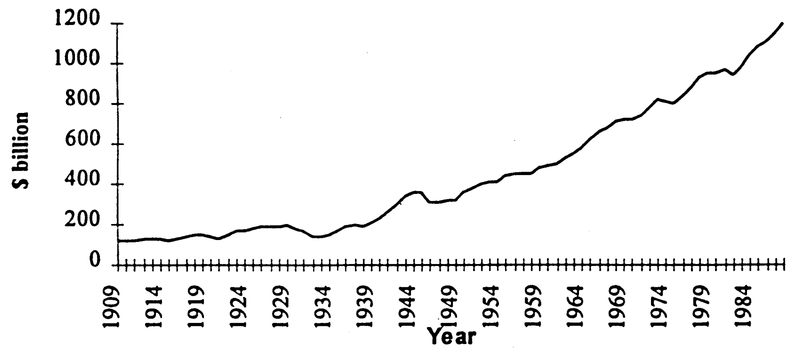

Table 28: Estimation results for Real GNP using the sample 1909-1988

	σ^2	α_1	α_2
Local	**0.00299**	1.321	0
AT(2)	0.0127	0.371	0
Global	0.0143		

Table 29: Median APE and inter-quartile range for Real GNP

	Forecast Horizon					
	1	2	3	6	12	18
Local	**1.72**	3.00	3.87	5.21	8.04	15.23
	1.78	3.35	4.07	4.95	7.54	11.40
AT(2)	2.63	**2.84**	**3.62**	**5.14**	8.16	9.51
	2.82	2.85	3.64	4.87	7.39	4.59
Global	5.96	5.77	5.61	5.17	**4.30**	**3.12**
	5.36	6.09	6.64	7.34	6.75	6.03

Table 30: Wilcoxon signed ranks test statistics for Real GNP. Values critical at a 10% level are shown in bold.

	2 step		3 step		12 step		18 step	
	AT(2)	GLB	AT(2)	GLB	AT(2)	GLB	AT(2)	GLB
LLT	166	**71**	145	**56**	**14**	**0**	**1**	**0**
AT(2)		**70**		**57**		**0**		**0**

Figure 11: US Real Wages 1900-1988

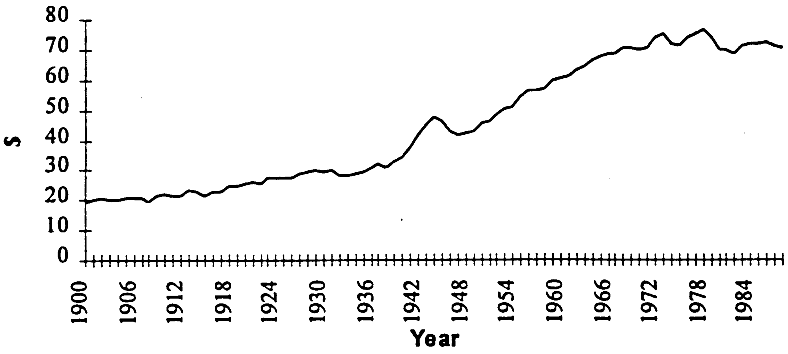

Table 31: Estimation results for Real Wages using the sample 1900-1988

	σ^2	α_1	α_2
Local	**0.00122**	1.250	0
AT(2)	0.00416	0.423	0
AT(3)	0.00611	0.0044	0.0273
Global	0.00929		

Table 32: Median APE and inter-quartile range for Real Wages

	Forecast Horizon					
	1	2	3	6	12	18
Local	**1.40**	2.74	**3.35**	**8.25**	**22.86**	37.18
	1.81	3.45	4.53	8.28	12.77	6.35
AT(2)	2.21	**2.31**	3.96	9.45	25.52	36.06
	4.12	4.20	4.71	9.90	14.32	6.30
AT(3)	7.39	9.38	10.15	18.43	35.29	71.62
	7.19	7.36	8.21	8.00	3.59	8.56
Global	8.18	8.54	8.87	8.27	23.11	**27.02**
	15.72	16.67	17.69	20.59	24.59	10.36

Table 33: Wilcoxon signed ranks test statistics for Real Wages. Values critical at a 10% level are shown in bold.

	12 step			18 step		
	AT(2)	AT(3)	GLB	AT(2)	AT(3)	GLB
LLT	47	**0**	44	**4**	**0**	**1**
AT(2)		**0**	47		**0**	**1**
AT(3)			**6**			**0**

Figure 12: US Standard & Poor's Index 1871-1988

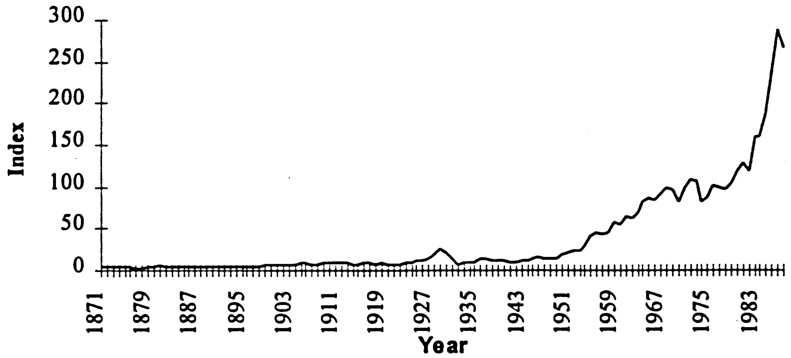

Table 34: Estimation results for Standard & Poor 500 Index using the sample 1871-1988

	σ^2	α_1	α_2
Local	**0.0234**	1.281	0
AT(2)	0.0734	0.320	0
AT(3)	0.0942	0.242	0
AT(4)	0.112	0.249	0
Global	0.189		

Table 35: Median APE and inter-quartile range for Standard & Poor 500 Index

	Forecast Horizon					
	1	2	3	6	12	18
Local	**9.56**	**11.60**	**14.00**	22.35	42.36	39.21
	8.28	14.06	14.31	23.89	33.53	28.18
AT(2)	11.28	11.78	16.23	**20.95**	**24.79**	**6.71**
	13.50	13.33	16.25	25.76	20.09	17.00
AT(3)	13.52	14.76	15.96	21.23	28.81	9.93
	18.16	18.91	19.43	27.76	20.36	16.51
AT(4)	22.69	23.89	24.68	31.55	39.99	22.49
	24.72	24.80	24.02	29.39	15.31	23.34
Global	44.74	45.75	46.30	47.16	49.73	61.39
	21.94	23.55	22.85	22.27	8.75	6.75

Table 36: Wilcoxon signed ranks test statistics for Standard and Poor 500 Index. Values critical at a 10% level are shown in bold.

	12 step				18 step			
	AT(2)	AT(3)	AT(4)	GLB	AT(2)	AT(3)	AT(4)	GLB
LLT	**5**	**9**	**14**	**0**	**0**	**0**	**1**	**0**
AT(2)		**0**	31	**0**		**0**	19	**0**
AT(3)			33	**0**			19	**0**
AT(4)				**0**				**0**

Figure 13: US Unemployment 1890-1988

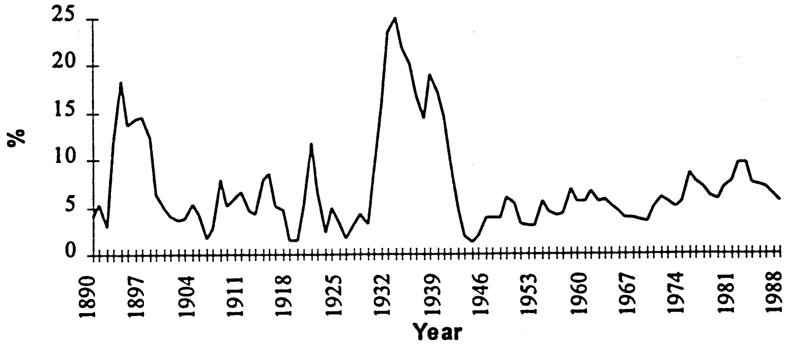

Table 37: Estimation results for Unemployment using the sample 1890-1988

	σ^2	α_1	α_2
Local	**0.195**	1.277	0
Global	0.411		

Table 38: Median APE and inter-quartile range for Unemployment[1]

	Forecast Horizon					
	1	2	3	6	12	18
Local	**12.35**	23.08	28.68	**23.82**	34.45	44.30
	11.82	15.14	22.42	19.71	18.41	22.36
Global	19.80	**21.97**	**25.44**	31.34	**34.32**	**34.99**
	22.39	21.71	21.01	23.53	19.21	7.18

Table 39: Wilcoxon signed ranks test statistics for Unemployment. Table entries are the statistics from a test between the local linear trend and global linear trend models. Values critical at a 10% level are shown in bold.

Horizon				
2	3	6	12	18
73	**64**	**53**	38	23

[1] The naive transformation $exp\left(y_{T+j|T}\right)$ has been used to obtain forecasts for the original series due to unacceptably large values for the forecast error variance obtained from the local linear trend model.

Figure 14: US Money Velocity 1871-1988

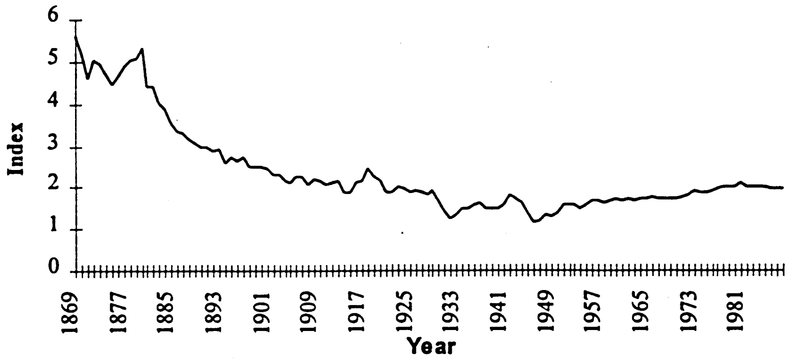

Table 40: Estimation results for Money Velocity using the sample 1871-1988

	σ^2	α_1	α_2
Local	**0.00405**	1.113	0.0303
AT(2)	0.0113	0.295	0.0103
AT(3)	0.0151	0.158	0.0089
AT(4)	0.0171	0.127	0.0086
AT(5)	0.0182	0.126	0
Global	0.0613		

Table 41: Median APE and inter-quartile range for Money Velocity

	Forecast Horizon					
	1	2	3	6	12	18
Local	**1.86**	**2.24**	**3.11**	5.77	14.27	18.84
	1.41	2.46	3.32	4.27	10.42	5.62
AT(2)	3.98	4.10	5.60	11.34	22.34	29.83
	2.19	2.78	2.98	6.35	4.21	3.34
AT(3)	3.76	3.80	3.79	5.54	9.82	11.07
	2.73	2.86	2.96	4.69	3.73	7.77
AT(4)	3.59	3.70	3.75	**3.75**	4.79	**2.95**
	2.61	2.45	3.19	5.24	2.55	8.15
AT(5)	4.62	4.75	4.74	5.37	**3.05**	9.48
	5.79	6.06	6.44	4.63	2.57	5.76
Global	33.62	34.85	35.98	39.69	46.59	52.47
	4.55	4.79	4.77	4.33	1.96	2.50

Table 42: Wilcoxon signed ranks test statistics for Money Velocity. Values critical at a 10% level are shown in bold.

	12 step					18 step				
	AT(2)	AT(3)	AT(4)	AT(5)	GLB	AT(2)	AT(3)	AT(4)	AT(5)	GLB
LLT	**0**	**4**	**15**	**0**	**0**	**0**	**0**	**3**	**0**	**0**
AT(2)		**0**	**2**	**0**	**0**		**0**	**3**	**0**	**0**
AT(3)			31	**4**	**0**			19	**0**	**0**
AT(4)				**11**	**0**				**0**	**0**
AT(5)					**0**					**0**

Figure 15: US Nominal Wages 1900-1988

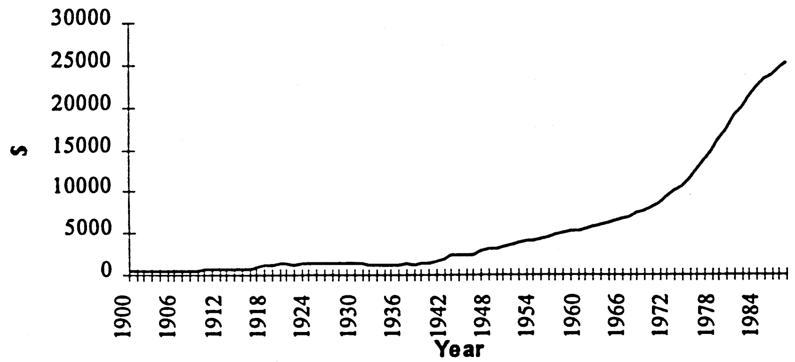

Table 43: Estimation results for Nominal Wages using the sample 1900-1988

	σ^2	α_1	α_2
Local	**0.00299**	1.477	0
AT(2)	0.0163	0.474	0
AT(3)	0.0252	0.376	0
AT(4)	0.0288	0.445	0
Global	0.0463		

Table 44: Median APE and inter-quartile range for Nominal Wages

	Forecast Horizon					
	1	2	3	6	12	18
Local	**1.33**	**3.43**	**5.25**	**9.28**	23.11	**29.26**
	1.73	2.94	5.05	10.83	12.99	5.38
AT(2)	3.11	3.64	5.67	11.36	25.38	33.22
	4.41	4.48	6.27	14.05	13.04	4.64
AT(3)	5.51	5.19	5.63	11.04	25.01	33.62
	5.88	6.16	6.43	11.62	12.49	5.21
AT(4)	8.72	8.85	8.07	9.87	**22.83**	40.22
	12.13	11.43	11.33	13.71	28.16	13.99
Global	18.00	19.83	21.96	27.09	35.98	42.01
	16.84	17.73	18.76	16.91	9.25	2.86

Table 45: Wilcoxon signed ranks test statistics for Nominal Wages. Values critical at a 10% level are shown in bold.

	12 step				18 step			
	AT(2)	AT(3)	AT(4)	GLB	AT(2)	AT(3)	AT(4)	GLB
LLT	**0**	**0**	60	**0**	**0**	**0**	**6**	**0**
AT(2)		**31**	51	**0**		**26**	**9**	**0**
AT(3)			49	**0**			**9**	**0**
AT(4)				**19**				**19**