MONASH

WP 18/95

ISSN 1032-3813
ISBN 0 7326 0779 5

MONASH UNIVERSITY

AUSTRALIA

A MODIFIED FLUCTUATION TEST FOR STRUCTURAL CHANGE

Kang Hao and Brett A. Inder

Working Paper 18/95
October 1995

DEPARTMENT OF ECONOMETRICS

HKBI1.doc

A MODIFIED FLUCTUATION TEST
FOR STRUCTURAL CHANGE

Kang Hao and Brett Inder

Monash University

Address Correspondence to:
Brett Inder
Department of Econometrics
Monash University
Clayton Vic 3168
Australia

Ph: 61 3 990 52303 Fax: 61 3 990 55474
Email: Inder@Monash.edu.au

Abstract

This paper investigates the problem of testing for structural change for diagnostic purposes. We propose a modified form of the fluctuation test of Ploberger et al. (1989). The modified fluctuation test has the same asymptotic distribution as the fluctuation test but much better finite sample performance. A comparison of the supF test of Andrews (1993) shows that both tests are actually based on the same components.

Acknowledgements: Part of this research was funded by an Australian Research Council Large Grant.

1. Introduction

This paper considers the problem of testing for structural change for diagnostic purposes. The most famous test for this purpose is probably the CUSUM test of Brown, Durbin and Evans (1975). However, the results of Ploberger and Krämer (1989) and Hansen (1991) show that the CUSUM test only has trivial local power against certain types of structural change. As an alternative, Ploberger, Krämer and Kontrus (1989) (henceforth PKK) propose a formal test called the fluctuation test which has non-trivial local power irrespective of time and type of structural change. The finite sample behaviour of the fluctuation test has been investigated by Sonberger and Krämer (1986), Krämer, Ploberger and Kontrus (1989) among others and intensively applied in empirical research (see Sonberger and Krämer (1986)).

On the other hand, a number of test statistics developed to test against various specified alternatives have also demonstrated the same desirable local power property as the fluctuation test. See, for example, Hansen (1990), Andrews (1993), Andrews and Ploberger (1992) among others. In particular, the supF test of Andrews (1993) seems of special interest. Although designed to detect a one-time discrete jump, it was also recommended by Andrews for use as a diagnostic test.

This paper considers a modified form of the fluctuation test. Following the same idea of the fluctuation test but employing a different choice of weighting matrix and a more careful consideration of the partial sample estimation, we derive a new test called the modified fluctuation test. The asymptotic distribution of the modified fluctuation test is found to be free of nuisance parameters. We also investigate the relationship between the supF test and the modified fluctuation test. It is found that, although both tests stem from different classes of test procedures, they are actually based on the same ingredients. A Monte Carlo experiment is then conducted to compare the finite sample performance of the modified fluctuation test with the fluctuation test and supF test. The results show that the modified fluctuation test can be significantly more powerful than the fluctuation test in small samples. It is also preferred to the supF test in some cases, although neither dominates the other uniformly in finite samples.

The structure of this paper is as follows. In the next section we take a close look at the fluctuation test and propose our new test, the modified fluctuation test. The asymptotic distribution of the modified fluctuation test is also derived. Section 3 examines the relationship between the supF test and the modified fluctuation test. Section 4 discusses the experimental design of the Monte Carlo study and its results. Some conclusions are made in section 5.

2. A Modified Fluctuation Test

Consider the linear regression model

$$y_t = x_t'\beta_t + u_t \qquad t = 1,\ldots,T \tag{1}$$

where y_t is the dependent variable, x_t is a $(k\times1)$ vector of observations on the independent variables, β_t is a $(k\times1)$ vector of unknown regression coefficients, and u_t is an unobservable disturbance term. The null hypothesis is that $\beta_t = \beta_0$ for all time periods $t = 1,\ldots,T$.

We impose the following assumptions which are standard in this literature:

(A.1) The regressors x_t are non-stochastic.

(A.2) $\limsup\limits_{T\to\infty} \frac{1}{T}\sum_{1}^{T} \|x_t\|^{2+\delta} < \infty$ for some $\delta > 0$ ($\|\cdot\|$ is the Euclidean norm).

(A.3) $\lim\limits_{T\to\infty} \frac{1}{T}\sum_{t=1}^{n} x_t x_t' = \lim\limits_{\min(T,n)\to\infty} \frac{1}{T}\sum_{t=n+1}^{T+n} x_t x_t' = R$,

for some non-singular, non-stochastic $(k\times k)$ matrix R.

(A.4) The disturbances u_t are iid$(0,\sigma^2)$.

In fact, assumptions A.1 and A.4 can be weakened to allow, for example, for dynamic models; see PKK for details. Denote $X^t = (x_1, x_2,\ldots,x_t)'$, $Y^t = (y_1, y_2,\ldots,y_t)'$. The test proposed by PKK (1989) examines successive OLS parameter estimates

$\hat{\beta}^t = (X^{t\prime}X^t)^{-1}X^{t\prime}Y^t$ for t = k+1,...,T and rejects the null hypothesis whenever these estimates fluctuate too much. Their test statistic is

$$S^{(T)} = \max_{t=k,\dots,T} \frac{t}{\hat{\sigma}T}\|(X^{T\prime}X^T)^{1/2}(\hat{\beta}^t - \hat{\beta}^T)\|_\infty \ , \tag{2}$$

where $\|\cdot\|_\infty$ denote the maximum norm; specifically, for any $k\times 1$ vectors $\hat{\theta}^1$ and $\hat{\theta}^2$

$$\|\hat{\theta}^1 - \hat{\theta}^2\|_\infty \quad = \quad \max_{i=1,\dots,k} |\hat{\theta}_i^1 - \hat{\theta}_i^2|. \tag{3}$$

$\hat{\sigma}$ is a consistent estimate of the standard deviation of the disturbances. PKK (1989) suggest estimating σ by

$$\hat{\sigma} = \left[\frac{1}{T-k}\sum_t (y_t - x_t'\hat{\beta}^T)^2\right]^{1/2} . \tag{4}$$

The test statistic $S^{(T)}$ can be written as

$$S^{(T)} = \sup_{0\le r\le 1} \|B^{(T)}(r)\|_\infty , \tag{5}$$

where $B^{(T)}(r) = \dfrac{\tau(r)}{\hat{\sigma}T}(X^{T\prime}X^T)^{1/2}(\hat{\beta}^{\tau(r)} - \hat{\beta}^T)$, $\tau(r)$ is the largest integer less than or equal to k+r(T-k). $B^{(T)}(r)$ is a k-dimensional stochastic process whose trajectories are right continuous at each $r \in [0,1]$ and possess left-hand limits. Using the general result on the convergence in distribution of random elements, PKK (1989) showed that under H_0

$$B^T(r) \stackrel{d}{\Rightarrow} B(r) \ = \ W(r) - rW(1), \tag{6}$$

where $\stackrel{d}{\Rightarrow}$ denotes covergence in distribution as $T \to \infty$, W(r) is a k vector of independent Brownian Motion on (0,1). B(r) is thus a process known as "tied-down Brownian Motion". This process has well known boundary crossing probabilities. In particular, $P[\sup_{0\le r\le 1}\|B(r)\|_\infty \le x]$ is well known (Billingsley (1968, P.85)).

In the fluctuation test, the choice of $(X^{T'}X^{T})^{1/2}$ as the weighting matrix is somewhat arbitrary. The motive behind this is to standardize the differences $\hat{\beta}_i^t - \hat{\beta}_i^T$ as well as to facilitate the evaluation of the limiting distribution. Let [.] denote "integer part". Notice that

$$\hat{\beta}^{[Tr]} - \hat{\beta}^{T} = (X^{[Tr]'}X^{[Tr]})^{-1}\sum_{1}^{[Tr]} x_i u_i - (X^{T'}X^{T})^{-1}\sum_{1}^{T} x_i u_i,$$

so we can show that

$$\text{var}(\hat{\beta}^{[Tr]} - \hat{\beta}^{T}) = \sigma^2[(X^{[Tr]'}X^{[Tr]})^{-1} - (X^{T'}X^{T})^{-1}].$$

Compared with $(X^{T'}X^{T})^{1/2}$, another, and perhaps a better, choice of weighting matrix is $(X^{[Tr]'}X^{[Tr]})^{1/2}$, which varies consistently with the partial sample estimate $\hat{\beta}^{[Tr]}$. We thus define the first modified fluctuation test statistic B_1^T:

$$B_1^T = \sup_{0\le r\le 1} \|f_1^{(T)}(r)\|_\infty \tag{7}$$

where $f_1^{(T)}(r) = \dfrac{\sqrt{r}}{\hat{\sigma}} (X^{[Tr]'}X^{[Tr]})^{1/2}(\hat{\beta}^{[Tr]} - \hat{\beta}^{T})$.

From (7), it is clear that $f_1^{(T)}(r)$ is based on the difference between $\hat{\beta}^T$, the full sample estimate of β, and $\hat{\beta}^{[Tr]}$, the partial sample estimate of β which uses the first [Tr] observations. As r approaches 1 this test is likely to have poor power as $\hat{\beta}^{[Tr]}$ approaches $\hat{\beta}^T$. An alternative test which would not suffer from this problem would be one which uses the last T-[Tr] observations to get another partial sample estimate of β. We denote $\hat{\beta}^{T-[Tr]}$ as the partial sample estimator of β which uses the last T-[Tr] observations. We define the second modified fluctuation test statistic $B_2^{(T)}$:

$$B_2^{(T)} = \sup_{0\le r\le 1} \| f_2^{(T)}(r) \|_\infty \tag{8}$$

where $f_2^{(T)}(r) = \frac{\sqrt{1-r}}{\hat{\sigma}} (X^{T-[Tr]\prime} X^{T-[Tr]})^{1/2} (\hat{\beta}^{T-[Tr]} - \hat{\beta}^T)$.

$B_1^{(T)}$ and $B_2^{(T)}$ contain the different information concerning possible structural change in the regression model. $B_1^{(T)}$ is likely to have low power for structural change near the end of the sample, and $B_2^{(T)}$ will suffer power loss near the start of the sample. We can thus form a new test which combines the information provided individually by $B_1^{(T)}$ and $B_2^{(T)}$. Define the modified fluctuation test statistic $B^{(T)}$

$$B^{(T)} = \sup_{0\le r\le 1} \{ c\| f_1^{(T)}(r) \|_\infty + (1-c)\| f_2^{(T)}(r)\|_\infty \tag{9}$$

where c is any constant which satisfies $0 \le c \le 1$. If we choose $c = 0$, then $B^{(T)} = B_1^{(T)}$. If we choose $c = 1$, then $B^{(T)} = B_2^{(T)}$. Thus B_1^T and $B_2^{(T)}$ are included as special cases of $B^{(T)}$. The choice of c will be considered in section 4.

The asymptotic distribution of $B^{(T)}$ is an immediate consequence of the following results, which are of some interest in their own right.

Lemma 1. Under H_0 and the assumptions (A.1)-(A.4)

$$R^T(r) = \frac{1}{\hat{\sigma}} (X^{[Tr]\prime} X^{[Tr]})^{1/2} (\hat{\beta}^{[Tr]} - \beta_0) \overset{d}{\Rightarrow} \frac{1}{\sqrt{r}} W(r),$$

$$\overline{R}^T(r) = \frac{1}{\hat{\sigma}} (X^{[Tr]\prime} X^{[Tr]})^{1/2} (\hat{\beta}^T - \beta_0) \overset{d}{\Rightarrow} \sqrt{r}\, W(1),$$

where $W(r)$ is a k vector of independent Brownian Motion on (0,1).

Proof. Define a k-dimensional random vector

$$\varepsilon_t = x_t u_t .$$

Under our assumptions, ε_t obviously satisfies the conditions set out by Phillips and Durlauf (1986, p.475). Then following their Theorem 2.1, we have

$$\frac{1}{\sqrt{T}}(\sigma^2 R)^{-\frac{1}{2}}\left(\sum_{1}^{[Tr]} x_i u_i\right) \overset{d}{\Rightarrow} W(r) \text{ where } R = \operatorname{plim} \frac{1}{T}\sum_{1}^{T} x_i x_i'.$$

On the other hand,

$$\begin{aligned} R^T(r) &= \frac{1}{\hat{\sigma}}(X^{[Tr]'}X^{[Tr]})^{\frac{1}{2}}(X^{[Tr]'}X^{[Tr]})^{-1}\sum_{1}^{[Tr]} x_i u_i \\ &= \frac{1}{\sqrt{rT}}[\hat{\sigma}^2(X^{[Tr]'}X^{[Tr]})/Tr]^{-\frac{1}{2}}\sum_{1}^{[Tr]} x_i u_i. \end{aligned}$$

$$\begin{aligned} \bar{R}^T(r) &= \frac{1}{\hat{\sigma}}(X^{[Tr]'}X^{[Tr]})^{\frac{1}{2}}(X^{T'}X^{T})^{-1}\sum_{1}^{T} x_i u_i \\ &= \frac{\sqrt{r}}{\sqrt{T}}[\hat{\sigma}(X^{T'}X^{T})/T]^{-1}[(X^{[Tr]'}X^{[Tr]})/Tr]^{\frac{1}{2}}\sum_{1}^{T} x_i u_i. \end{aligned}$$

Since $(X^{T'}X^{T})/T \to R, (X^{[Tr]'}X^{[Tr]})/Tr \to R,$ and the Lemma follows.

Lemma 2. Under H_0 and the same assumptions as Lemma 1,

$$S^T(r) = \frac{1}{\hat{\sigma}}(X^{T-[Tr]'}X^{T-[Tr]})^{1/2}(\hat{\beta}^{T-[Tr]} - \beta_0) \overset{d}{\Rightarrow} \frac{1}{\sqrt{1-r}}(W(1) - W(r)),$$

$$\bar{S}^T(r) = \frac{1}{\hat{\sigma}}(X^{T-[Tr]'}X^{T-[Tr]})^{\frac{1}{2}}(\hat{\beta}^{T} - \beta_0) \overset{d}{\Rightarrow} \sqrt{1-r}\, W(1)$$

where W(r) is the same k-dimensional Brownian Motion as in Lemma 1.

Proof. Notice that

$$S^T(r) = \frac{1}{\hat{\sigma}}(X^{T-[Tr]'}X^{T-[Tr]})^{1/2}(X^{T-[Tr]'}X^{T-[Tr]})^{-1}\sum_{[Tr]+1}^{T} x_i u_i$$

$$= \frac{1}{\sqrt{1-r}\sqrt{T}}\ [\hat{\sigma}^2(X^{T-[Tr]'}X^{T-[Tr]})/T(1-r)]^{-1/2}(\sum_{1}^{T} x_i u_i - \sum_{1}^{[Tr]} x_i u_i).$$

$$\bar{S}^T(r) = \frac{1}{\hat{\sigma}}\ (X^{T-[Tr]'}X^{T-[Tr]})^{1/2}(X^{T'}X^T)^{-1}\sum_{1}^{T} x_i u_i$$

$$= \frac{\sqrt{1-r}}{\sqrt{T}}[(X^{T-[Tr]'}X^{T-[Tr]})/T(1-r)]^{1/2}[\hat{\sigma}(X^{T'}X^T)/T]^{-1}\sum_{1}^{T} x_i u_i.$$

Again by Theorem 2.1 of Phillips and Durlauf notice that the $R^T(r)$, $\bar{R}^T(r)$, $S^T(r)$ and $\bar{S}^T(r)$ are based on the same innovations $x_i u_i (i = 1, 2,\ldots,T)$, so Lemma 2 follows in a similar way to Lemma 1.

Theorem 1: Under H_0 and given assumptions (A.1)-(A.4), the statistic $B^{(T)}(r)$ has a well defined limiting distribution as $T \to \infty$ with distribution function

$$F(x) = 0 \qquad x < 0,$$

$$= [1 + 2\sum_{i=1}^{\infty}(-1)^i \exp(-2ix^2)]^k \qquad x \geq 0. \tag{10}$$

Proof. Since $f_1^{(T)}(r) = \sqrt{r}[R^T(r) - \bar{R}^T(r)]$, $f_2^{(T)}(r) = \sqrt{1-r}\ [S^T(r) - \bar{S}^T(r)]$.

Thus by Lemmas 1, 2 and the continuous mapping theorem of Billingsley (1968), we have

$$B^T(r) = \sup_{0\leq r\leq 1}\ \{\ c\|f_1^{(T)}(r)\|_\infty + (1-c)\|f_2^{(T)}(r)\|_\infty\}$$

$$= \sup_{0\leq r\leq 1}\ \{\ c\|\sqrt{r}\,[R^T(r) - \bar{R}^T(r)]\|_\infty + (1-c)\|\sqrt{1-r}\ [S^T(r) - \bar{S}^T(r)]\|_\infty\}$$

$$\overset{d}{\Rightarrow} \sup_{0\leq r\leq 1}\ \{\ c\|W(r)\ -\ rW(1)\|_\infty + (1-c)\ \|-(W(r) -\ rW(1))\|_\infty\}$$

$$= \sup_{0\leq r\leq 1}\ \|B(r)\|_\infty\ .$$

The distribution of $\sup_{0\le r\le 1} ||B(r)||_\infty$ is well known and given by (10). (See Billingsley (1968, P.85)). This completes the proof of the theorem.

Theorem 1 shows that the modified fluctuation test has exactly the same asymptotic null distribution as the fluctuation test statistic advocated by PKK. The critical values of the modified fluctuation test for various number of regressors can thus be found in PKK.

3. A COMPARISON WITH THE supF TEST

Although the supF test was originally developed as a test against a one-time discrete jump, it has been recommended by Andrews (1993) for use as a diagnostic test. Since both the supF test and the modified fluctuation test are based on the same norm, it would be worthwhile to further investigate their relationship.

The supF test statistic of Andrews (1993) is given by

$$\sup F = \sup_{r\in\Pi} F(r), \quad \text{with } F(r) = \frac{(\hat{u}'\hat{u} - \tilde{u}'\tilde{u} / k)}{\tilde{u}'\tilde{u} / (T-2k)}$$

where $\hat{u}$ is the vector of OLS residuals from fitting the model (1) under the null hypothesis of no structural change, $\tilde{u}$ is the vector of OLS residuals from fitting the model (1) under the alternative hypothesis of a one-time discrete jump with jump point [Tr]. In other words, $\tilde{u} = [\tilde{u}^{[Tr]'}, \tilde{u}^{T-[Tr]'}]'$ is the vector of OLS residuals from fitting the model

$$y = \begin{pmatrix} y^{[Tr]} \\ y^{T-[Tr]} \end{pmatrix} = \begin{pmatrix} X^{[Tr]} & 0 \\ 0 & X^{T-[Tr]} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} u^{[Tr]} \\ u^{T-[Tr]} \end{pmatrix}. \tag{12}$$

Denote $Z = \begin{pmatrix} X^{[Tr]} & 0 \\ 0 & X^{T-[Tr]} \end{pmatrix}$, $\gamma = ((\beta_1', \ \beta_2')'$. Then (12) can be written as $y = Z\gamma + u$. As shown by Johnston (1984, p.207), the F(r) test statistic (11) can be equivalently expressed as

$$F(r) = \frac{(\hat{\gamma}^* - \hat{\gamma})'(Z'Z)(\hat{\gamma}^* - \hat{\gamma})/k}{\tilde{u}'\tilde{u}/(T-2k)} \tag{13}$$

where $\hat{\gamma}^* = (\hat{\beta}^{T'}, \hat{\beta}^{T'})', \hat{\gamma} = (\hat{\beta}^{[Tr]'}, \ \hat{\beta}^{T-[Tr]'})'$.

Evaluating (13), we have

$$F(r) = \frac{(\hat{\gamma}^* - \hat{\gamma})'(Z'Z)(\hat{\gamma}^* - \hat{\gamma})/k}{\tilde{u}'\tilde{u}/(T-2k)}$$

$$= \frac{T-2k}{k\tilde{u}'\tilde{u}} \begin{pmatrix} \hat{\beta}^T - \hat{\beta}^{[Tr]} \\ \hat{\beta}^T - \hat{\beta}^{T-[Tr]} \end{pmatrix}' \begin{pmatrix} X^{[Tr]'}X^{[Tr]} & 0 \\ 0 & X^{T-[Tr]'}X^{T-[Tr]} \end{pmatrix} \begin{pmatrix} \hat{\beta}^T - \hat{\beta}^{[Tr]} \\ \hat{\beta}^T - \hat{\beta}^{T-[Tr]} \end{pmatrix}$$

$$= \frac{T-2k}{k\tilde{u}'\tilde{u}} \ [(\hat{\beta}^T - \hat{\beta}^{[Tr]})'(X^{[Tr]'}X^{[Tr]})(\hat{\beta}^T - \hat{\beta}^{[Tr]}) \ +$$

$$(\hat{\beta}^T - \hat{\beta}^{T-[Tr]})'(X^{T-[Tr]'}X^{T-[Tr]'})(\hat{\beta}^T - \hat{\beta}^{T-[Tr]})].$$

Observe that

$$f_1^{(T)}(r) = \frac{\sqrt{r}}{\hat{\sigma}} \ (X^{[Tr]'}X^{[Tr]})^{1/2}(\hat{\beta}^{[Tr]} - \hat{\beta}^T),$$

$$f_2^{(T)}(r) = \frac{\sqrt{1-r}}{\hat{\sigma}} \ (X^{T-[Tr]'}X^{T-[Tr]})^{1/2}(\hat{\beta}^{T-[Tr]} - \hat{\beta}^T).$$

Therefore

$$F(r) = \frac{T-2k}{k\tilde{u}'\tilde{u}} \ [\frac{\hat{\sigma}^2}{r} f_1^{(T)'}(r) f_1^{(T)}(r) + \frac{\hat{\sigma}^2}{1-r} f_2^{(T)'}(r) f_2^{(T)}(r)] \tag{14}$$

$$= \frac{\hat{\sigma}^2}{\tilde{u}'\tilde{u}/(T-2k)} \begin{pmatrix} f_1^{(T)}(r) \\ f_2^{(T)}(r) \end{pmatrix}' \begin{pmatrix} \frac{1}{r} I_k & 0 \\ 0 & \frac{1}{1-r} I_k \end{pmatrix} \begin{pmatrix} f_1^{(T)}(r) \\ f_2^{(T)}(r) \end{pmatrix}$$

$$= \frac{\hat{\sigma}^2}{k\tilde{\sigma}^2} [f_1^{(T)'}(r) f_1^{(T)}(r)/r + f_2^{(T)'}(r) f_2^{(T)}(r)/(1-r)]. \tag{15}$$

Comparing (15) with (9), we see that the supF test can essentially be expressed as a weighted average of the two squared components of the modified fluctuation test, except that the supF test uses a variance estimate obtained under the alternative instead of the null. In other words, the two tests differ primarily in how they use the information in the vectors $f_1^{(T)}(r)$ and $f_2^{(T)}(r)$. The supF test takes sums of squares of these elements, while the modified fluctuation test looks for the element which is largest in absolute terms.

We see then that despite the fact that the supF test and the modified fluctuation test are developed from different classes of test procedures, they are based on the same components. The different principles lead to the construction of similar test statistics. This confirms the claim that the supF test can be used quite effectively in a data analytic fashion.

Since the asymptotic distribution of the modified fluctuation test is valid for $r \in (0, 1)$, its asymptotic critical value is thus determined only by k, the number of regressors. On the other hand, the asymptotic distribution of the supF test is jointly decided by k and Π, a pre-specified subset of (0, 1) whose closure lies in (0, 1). Therefore, in addition to k, the number of regressors, the critical values of the supF test are also dependent on the choice of Π. This shortcoming can cause some inconvenience to the application of the supF test.

4. SOME MONTE CARLO EVIDENCE

A Monte Carlo study was conducted to investigate and compare the size and power properties of the fluctuation test, the modified fluctuation test and the supF test. The X matrix used in the comparison is

$$x_t = [1, \sin t]'$$

which is the same as that is used by Krämer, Ploberger and Schulter (1992).

Three sample sizes were used, small (T = 30), medium (T = 60) and large (T = 120). To compare the power properties, we consider both a one-time discrete jump and a random walk in β. Against a one-time discrete jump, a structural change in β is given by

$$\beta_t = \beta_0 + \Delta\beta, \qquad \text{when } t \geq T^*$$

$$\Delta\beta = b_1 / T^{1/2} (\cos\phi, \sin\phi)' .$$

On the other hand, against the random walk alternative, a structural change in β is given by

$$\beta_t = \beta_0 + \eta_t, \qquad \text{when } t \geq T^*$$

$$\eta_t = \eta_{t-1} + b_2 / T^{1/2} (v_{1t} \cos\phi, v_{2t} \sin\phi)'$$

where ϕ is the angle between the mean regressor which is given by

$$c = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} x_t = [1, \ 0]'$$

and the change vector b_1 (b_2). ϕ takes the value 0°, 30° 60° and 90°. v_{1t}, v_{2t} are independent N(0,1). We set $\beta_0 = (0, 0)'$ initially. $b_1 = 8.6$ and $b_2 = 2.0$. The b_1 and b_2 are selected so that the tests have reasonable power under the alternative hypothesis.

Under the alternative hypothesis, the structural change occurs at $T^* = [T\lambda]$. Against the one-time discrete jump alternative, $\lambda = 0.15, 0.3, 0.5, 0.7, 0.85$. Against the random walk alternative, we let $\lambda = 0, 0.2, 0.5$ and 0.8. Obviously $\lambda = 0$ implies a random walk at the beginning of the sample. For any combinations of $\phi, T, \lambda, N = 1000$ replications were performed to investigate the actual size performance of the tests.

For the fluctuation test and the modified fluctuation test, critical values only depend on the number of regressors. For the supF test, however, the critical values used in the experiment depend on the range of Π through the choice of r. We let $r = 0.1$, 0.05 and 0.025, respectively for T = 30, 60 and 120. Befoie the modified fluctuation test can be computed, a c value in (9) has to be chosen. A few c values are used in the experiments; the results for the modified fluctuation test reported in this chapter correspond to a c value of 0.5. Results seem largely insensitive to reasonable c values.

Observe that the fluctuation test and the modified fluctuation test estimate σ^2 under the null hypothesis while the supF test estimates σ^2 under the alternative of a one-time discrete jump. For a fair comparison, we also estimate σ^2 under such an alternative for both of the fluctuation test and the modified fluctuation test. Obviously this would not change their asymptotic distribution under the null while it may improve their power under the one-time discrete jump alternative. For convenience, we use M-Fluctuation to represent the modified fluctuation test.

Table 1 reports the rejection frequencies of the three tests using the asymptotic 1%, 5% and 10% significance levels. We observe that these tests tend to over-reject the true null hypothesis when sample size is small. Among them, the fluctuation test has the worst size distortion while the M-fluctuation test has much better performance. The supF test is somewhat between the fluctuation test and the M-fluctuation test. However, when sample size becomes larger, their size behavior improves quickly.

Tables 2 to 4 report size-corrected powers of the three tests. These tables eliminate the power distortions that arise due to under- or over-rejection under the null when asymptotic critical values are used. The estimated powers of all the tests clearly depend on the angle between mean regressor and the shift vector as well as the location of structural change, particularly when the alternative is random walk.

It is easy to observe that the M-fluctuation test consistently dominates the fluctuation test when sample size is small. Their power difference is reasonably large, exceeding 0.1 in most cases considered. When sample size becomes larger, the power advantage of the M-Fluctuation test gradually disappears. It is only slightly more powerful than the fluctuation test when T is 60. There is essentially no difference between the tests when T is increased to 120.

A comparison between the M-fluctuation test and the supF test shows that against a one-time discrete jump, the M-fluctuation test tends to have better power around the sample mid-point while the supF test is more powerful against an early or late structural change. Sometimes the power gain for the supF test over the M-fluctuation test is large. This is not surprising if we notice that unlike the supF test, the M-fluctuation test is unequally weighted across different values of r with asymptotic variance being equal to r(1-r). This variance attains its maximum at r = 0.5. Overall, it seems that the supF test is slightly preferred to the M-fluctuation test.

In terms of the random walk alternative, the M-fluctuation test typically outperforms the supF test by a small margin when the random walk occurs early or from the beginning of the sample. On the other hand, the supF test is slightly preferred against the random walk which occurs late in the sample. Against the random walk which occurs in the middle of the sample, the M-fluctuation test seems more powerful that the supF test with small and medium sample sizes, while the supF test looks better with large sample sizes.

In concluding, the M-fluctuation test seems to be a good alternative to the fluctuation test and a potential competitor to the supF test. It significantly improves the power performance of the fluctuation test and outperforms the supF test in certain cases. It also has better size behavior than both the fluctuation test and the supF test in small samples. Hao (1994) has provided further Monte Carlo evidence in support of the M-fluctuation test.

5. CONCLUSIONS

An important feature of the fluctuation test is that it has nontrivial local power irrespective of the particular type of structural change. This property makes it more attractive than other diagnostic tests such as the CUSUM test. In this paper we suggested a modified form of the fluctuation test through the employment of a different weighting matrix and a more careful consideration of partial sample estimation.

A comparison of the modified fluctuation test with the supF test of Andrews (1993) showed that although the two tests are proposed in different classes of test procedures, they are in fact based on the same ingredients. Both tests could thus be expected to have similar power performance whether as a particular test against discrete jump or a diagnostic test against a more general alternative hypothesis.

The size and power properties of the fluctuation test, the modified fluctuation test and the supF test were investigated through a Monte Carlo simulation. The modified fluctuation test has demonstrated the best size performance and is significantly more powerful than the fluctuation test in small samples. It is also preferred to the supF test in some cases, although none of them dominates the other uniformly in finite samples.

Table 1

Estimated null rejection frequency

T	Test	10%	5%	1%
T = 30	Fluctuation	0.178	0.111	0.042
	M-Fluctuation	0.105	0.062	0.020
	SupF	0.116	0.069	0.023
T = 60	Fluctuation	0.100	0.049	0.012
	M-Fluctuation	0.093	0.047	0.012
	SupF	0.092	0.050	0.014
T = 120	Fluctuation	0.082	0.038	0.008
	M-Fluctuation	0.082	0.040	0.008
	SupF	0.080	0.043	0.010

Table 2
Estimated size-adjusted powers at 5% nominal level (T = 30)

λ /	b	0^0	30^0	60^0	90^0
			One Time Discrete Jump		
			Fluctuation		
.15	8.6	.299	.222	.149	.173
.3	8.6	.691	.620	.397	.351
.5	8.6	.861	.719	.430	.488
.7	8.6	.658	.388	.182	.320
.85	8.6	.231	.197	.150	.151
			M-Fluctuation		
.15	8.6	.394	.314	.216	.244
.3	8.6	.805	.791	.605	.463
.5	8.6	.926	.856	.631	.620
.7	8.6	.781	.577	.343	.440
.85	8.6	.375	.346	.266	.228
			supF		
.15	8.6	.501	.476	.379	.302
.3	8.6	.737	.751	.604	.408
.5	8.6	.850	.799	.604	.476
.7	8.6	.727	.594	.430	.387
.85	8.6	.514	.472	.385	.276
			Random Walk		
			Fluctuation		
rw	2.0	.051	.047	.139	.281
.2	2.0	.316	.245	.449	.688
.5	2.0	.564	.489	.190	.055
.8	2.0	.033	.032	.033	.030
			M-Fluctuation		
rw	2.0	.100	.108	.254	.407
.2	2.0	.465	.332	.572	.819
.5	2.0	.705	.621	.304	.082
.8	2.0	.058	.042	.032	.035
			supF		
rw	2.0	.139	.102	.176	.290
.2	2.0	.389	.265	.481	.718
.5	2.0	.623	.583	.317	.097
.8	2.0	.103	.067	.034	.030

Table 3
Estimated size-adjusted powers at 5% nominal level (T = 60)

λ / b		0^0	30^0	60^0	90^0
			One Time Discrete Jump		
			Fluctuation		
.15	8.6	.384	.385	.273	.186
.3	8.6	.858	.762	.511	.490
.5	8.6	.953	.899	.678	.636
.7	8.6	.874	.769	.506	.452
.85	8.6	.332	.284	.189	.139
			M-Fluctuation		
.15	8.6	.384	.399	.299	.201
.3	8.6	.873	.776	.541	.523
.5	8.6	.957	.908	.711	.669
.7	8.6	.884	.784	.528	.492
.85	8.6	.355	.302	.198	.162
			supF		
.15	8.6	.509	.533	.431	.273
.3	8.6	.807	.768	.587	.449
.5	8.6	.892	.857	.673	.528
.7	8.6	.807	.749	.568	.418
.85	8.6	.507	.506	.351	.233
			Random Walk		
			Fluctuation		
rw	2.0	.971	.916	.427	.060
.2	2.0	.998	.984	.675	.066
.5	2.0	.852	.730	.299	.060
.8	2.0	.125	.108	.082	.070
			M-Fluctuation		
rw	2.0	.977	.918	.437	.067
.2	2.0	.998	.986	.693	.076
.5	2.0	.857	.746	.316	.072
.8	2.0	.136	.121	.101	.085
			supF		
rw	2.0	.950	.864	.405	.085
.2	2.0	.995	.972	.601	.073
.5	2.0	.772	.660	.243	.054
.8	2.0	.163	.155	.112	.076

Table 4
Estimated size-adjusted powers at 5% nominal level (T = 120)

λ / b		0^0	30^0	60^0	90^0
			One Time Discrete Jump		
			Fluctuation		
.15	8.6	.392	.299	.177	.193
.3	8.6	.862	.762	.523	.512
.5	8.6	.937	.866	.670	.671
.7	8.6	.853	.726	.483	.508
.85	8.6	.363	.247	.147	.156
			M-Fluctuation		
.15	8.6	.378	.303	.179	.191
.3	8.6	.859	.763	.534	.506
.5	8.6	.934	.871	.677	.667
.7	8.6	.846	.719	.484	.508
.85	8.6	.363	.247	.147	.156
			supF		
.15	8.6	.529	.453	.334	.259
.3	8.6	.804	.729	.556	.448
.5	8.6	.857	.809	.639	.527
.7	8.6	.788	.703	.531	.429
.85	8.6	.514	.405	.280	.227
			Random Walk		
			Fluctuation		
rw	2.0	.892	.768	.392	.371
.2	2.0	.960	.900	.914	.977
.5	2.0	.464	.660	.982	.996
.8	2.0	.662	.517	.190	.129
			M-Fluctuation		
rw	2.0	.888	.758	.381	.376
.2	2.0	.954	.897	.914	.979
.5	2.0	.454	.656	.981	.995
.8	2.0	.650	.508	.194	.133
			supF		
rw	2.0	.756	.596	.340	.319
.2	2.0	.964	.923	.857	.938
.5	2.0	.493	.820	.987	.996
.8	2.0	.697	.600	.274	.137

REFERENCES

Andrews, D.W.K. (1993) "Testing for parameter instability and structural change with unknown change point," *Econometrica*, 61, 821-856.

Andrews, D.W.K. and Ploberger, W. (1994) "Optimal Tests when a nuisance parameter is present only under the alternative," *Econometrica*, 62, 1383-1414.

Billingsley, P. (1968) *Convergence of Probability Measures,* Wiley, New York.

Brown, R.L., Durbin, J. and Evans, J.M. (1975) "Techniques for testing the constancy of regression relationships over time," *Journal of the Royal Statistical Society B,* 27, 149-163.

Hansen, B.E. (1990) "Lagrange multiplier tests for parameter instability in non-linear models," unpublished manuscript, Department of Economics, University of Rochester.

Hansen, B.E. (1991) "A comparison of tests for parameter instability: An examination of asymptotic local power," unpublished manuscript, Department of Economics, University of Rochester.

Hao, K. (1994) "Testing for structural change in linear regression models," unpublished Ph.D thesis, Monash University.

Johnston, J. (1984) *Econometric Methods*, McGraw-Hill.

Krämer, W. and Sonnberger, H. (1986) *The linear regression model under test,* Physica-Verlag, Heidelberg, West Germany.

Krämer, W., Ploberger, W. and Kontrus, K. (1989) "A comparison of two significance tests for structural stability in the linear regression model," in *Advances in Econometrics and Modeling*, edited by B. Raj, Kluwer Academic Publishers, Dondrecht, Netherlands.

Krämer, W. Ploberger, W. and Schlüter, I. (1991) "Recursive vs. OLS residuals in the CUSUM test," in *Economic Structural Change: Analysis and Forecasting*, edited by P. Hackl and Westlund, A., Physica-Verlag, Heidelberg, Germany.

Phillips, P.C.B. and Durlauf, S.N. (1986) "Multiple time series regression with integrated processes," *Review of Economic Studies,* 53, 473-496.

Ploberger, W. and Krämer, W. (1990) "The local power of the CUSUM and CUSUM of squares tests," *Econometric Theory,* 6, 335-347.

Ploberger, W., Krämer, W. and Kontrus, K. (1989) "A new test for structural stability in the linear regression model," *Journal of Econometrics,* 40, 307 - 318.