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A Threshold Error Correction Model for Intraday Futures and Index Returns

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Abstract

Index-futures arbitragers only enter into the market if the deviation from the arbitrage relation is large enough to compensate for transaction costs and associated interest rate and dividend risks. We estimate the band around the theoretical futures price within which arbitrage is not profitable for most arbitragers, using a threshold autoregression model. Combining these thresholds with an error correction model, we can make a distinction between the effects of arbitragers and infrequent trading on index and futures returns.

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1. Introduction

Index and index-futures prices are approximately related by the well-known cost-of-carry model. In practice we often see slight deviations from this no-arbitrage relation, that are not arbitraged away immediately. This is caused by transaction costs, interest rate risk, worst-case dividend yield policies and short-selling restrictions. The use of less than the full basket of stocks also induces risk. Based upon these considerations we expect a band around the arbitrage value, in which for an arbitrage position the expected (risk-adjusted) returns do not exceed the expected costs.

In the literature (e.g. Ghosh [1993] and Wahab and Lashgari [1993]), it is mentioned that, under certain conditions, the futures and spot price are cointegrated. This results in an error correction model for the returns in which the futures and index returns are explained by past futures and index returns, and the deviation from the arbitrage relation in the previous period, which we shall call the error correction term or basis.

It has been widely documented (e.g. Kawaller, Koch and Koch [1987], Stoll and Whaley [1990], Chan [1992] and Koch [1993]) that futures prices tend to lead index prices. Thus, most of the time deviations from the no-arbitrage relation will occur when the futures react to news first, only later followed by the index. This will be both reflected by the significant impact of past futures returns on the current index return, and by the error correction term.

One interpretation of the error correction term is that it reflects the effect of arbitragers. If the futures price is too high relative to the index value, they will buy the stocks underlying the index and sell the futures contract. If the futures price is too low, they will do the reverse, i.e. sell the stocks underlying the index and buy the futures contract. These trades drive prices back to the equilibrium, i.e. the error correction term back towards zero. A second interpretation is the infrequent trading effect on the index. For example Miller, Muthuswamy, and Whaley (1994) investigate the mean-reversion of S&P 500's index basis changes, and conclude that infrequent trading causes this mean-reversion in most cases. First the futures price adjusts to new information, only later followed by the index since not every stock trades every short time period. In terms of the basis, both arbitrage and infrequent trading therefore cause the same pattern when futures lead the index: first the basis becomes nonzero due to a change in the futures price, and then it returns stepwise towards zero provided nothing else happens. This paper disentangles these sources of mean reversion.

It is obvious that due to transaction costs arbitragers will only cause the mean-reversion when the deviation is large. For smaller deviations the infrequent trading in the index will be effective. Both effects make the system non-linear. The impacts of arbitrage and infrequent trading will change, dependent on the deviation from the no-arbitrage relation. We will, therefore, explore the existence of different arbitrage regimes. First, we investigate the location of possible thresholds, breakpoints indicating a change in the pattern of the basis, and possibly also in the relations between the index and futures returns, and the error correction term. Second, we will estimate the error correction model in each regime. By estimating transaction costs we indicate which thresholds could indicate the band around the theoretical futures price in which arbitrage is not profitable.

This paper is organised as follows. Section 2 describes the cost-of-carry model and infrequent trading, and their impact on the basis. Section 3 presents the methodology. The data are described in Section 4. Section 5 elaborates upon the results of the Threshold Autoregressive model and the Threshold Error Correction model. Finally, Section 6 will conclude.

2. The effect of arbitrage and infrequent trading on the basis

The cost-of-carry model is often assumed to describe the relation between futures and index prices:

$$F_{t,T} = S_t * \exp[(r_{t,T} - q_{t,T})(T-t)], \tag{1}$$

where $F_{t,T}$ is the futures price at time t of a futures contract maturing at T, S_t is the current value of the index, $r_{t,T}$ is the risk-free interest rate on an investment for the period (t,T), and $q_{t,T}$ is the dividend yield on the index. Following the cost-of-carry relation, we define the basis or error correction term as

$$z_t = \ln F_{t,T} - \ln S_t - (r_{t,T} - q_{t,T})(T-t). \tag{2}$$

Three main reasons are given (e.g. MacKinlay and Ramaswamy [1988]) to explain why deviations from the arbitrage relation (1) can prolong for some time. First, setting up an arbitrage position involves transaction costs. Second, arbitrage is not risk-free. The marking to market principle causes interest rate risk. Furthermore, the dividend yield in equation (1) is an estimation of future dividends until maturity. In practice worst-case dividend policies must be taken into account. Also, arbitrage is often done by using less than the full basket of stocks, obviously creating non-representativeness risk. These risks are all larger when time to maturity is longer. Third, some market participants have short-selling restrictions.

On the other hand, among others Sofianos (1993) reports that often arbitrage positions are closed before expiration following profitable mispricing reversals. This unwinding option adds value to the arbitrage position. Thus, market participants can set up an arbitrage position when the costs still exceed the expected return from the deviation from the arbitrage relation. Of course, traders must then take into account the additional transaction costs of the early unwinding.

Equation (1) gives the long-term equilibrium between futures and spot prices. The above considerations, however, make it likely that the movement toward the long-term equilibrium does not always occur immediately. Only when the deviation from the arbitrage relation exceeds a critical threshold, do expected (risk-adjusted) returns exceed the expected costs and arbitragers will enter into the market. Thus, there is a band around the arbitrage value within which arbitrage is not profitable.

Many stocks in the index do not trade every minute. Since the index value is based upon the last transaction of each individual stock, the index will lag actual developments in the financial markets. When lagging stocks eventually trade, the index will be updated. Miller, Muthuswamy and Whaley (1994) point out that as a result of this infrequent trading phenomenon, reported basis changes will appear to be negatively correlated. Here, we will focus on the basis itself, which is stationary due to the cointegrating relation between futures and spot prices. If futures prices lead spot prices, deviations will originally occur after a change in the futures price, only later followed by the index. Thus, we expect the same pattern caused by infrequent trading as well as by arbitrage. An example of this pattern is given in Figures 1a and 1b.

The negative correlation between basis *changes* found by Miller, Muthuswamy and Whaley indicates that the effect of a decrease (increase) after an increase (decrease) in the basis level is larger than the subsequent movements of the basis in the same direction due to the step-wise adjustment of the index. For the basis *level* we obviously will find positive correlation, as consecutive basis observations will be most of the time either positive or negative. Here, however, we are interested in changes in this positive autocorrelation pattern. If arbitragers enter into the market when the deviation from zero of the basis is large enough, the next observations of the basis will move rapidly towards zero. This will also happen to a lesser extent when deviations are large (but not large enough to allow for arbitrage) and infrequent trading effects start to play a role. On the other hand, the basis will not change very much when deviations from zero are small. Suppose the true model for the basis is an AR(1). Then we expect this AR(1) coefficient to be close to one when deviations are small, reflecting that consecutive basis observations do not change very much. The AR(1) coefficient will be much smaller when deviations are large as to reflect the mean-reversion towards zero.

The above considerations lead to the conclusion that a linear AR(p) model will not be the correct model for the basis. The coefficients will depend on the magnitude of the basis. For this reason we will estimate a Threshold Autoregressive (TAR) model. This non-linear model incorporates a number of thresholds and in every regime a different AR model will apply with increasingly smaller coefficients the further out we get. This could also have consequences for the error correction model, which we will therefore estimate per regime.

3. Methodology

In this section we will discuss the threshold autoregressive (TAR) model. Since arbitragers will only enter into the market when the deviation of the basis from zero is large, the AR coefficients will be different in different regimes. This is exactly the way the TAR model works. It detects from the data deviations in the pattern of the basis, and provides a method to estimate the critical thresholds from the data.

3.1. Threshold autoregressive (TAR) model

For the basis, given by (2), a TAR model will be estimated. More formally,

$$z_t = \phi_0^{(j)} + \sum_{i=1}^{p} \phi_i^{(j)} z_{t-i} + \varepsilon_t^{(j)} \qquad r_{j-1} \leq z_{t-d} < r_j , \tag{3}$$

where j=1, ..., k and the threshold lag d is a positive integer. The thresholds are $-\infty = r_0 <$

$r_1 < ... < r_k = \infty$. $\{\varepsilon_t^{(j)}\}$ is i.i.d. $(0,\sigma^{(j)2})$.

The characteristics of the basis, z_t, depend on the regime the error correction term is in. We suppose that the basis is the stationary error correction term from the cointegration between the non-stationary prices in (1). This will be tested formally in Section 5. The process can still be stationary if in the central regime the error correction term follows a random walk. Chan, Petruccelli, Tong and Woolford (1985) derive conditions for which the process is still stationary. This is certainly the case if in all other regimes around the central regime the process is stationary. Even in case of a very wide band this is still true.

Tsay (1989) gives four main steps to estimate a TAR model like (3). The procedure is described in detail in the Appendix. In the first step the AR order p is selected using the Partial Autocorrelation Function (PACF) given in equation (A3). Furthermore, the set S of possible threshold lags d has to be selected, preferably by economic theory. The threshold lag gives an indication of the speed within which the market reacts to deviations from the no-arbitrage relation. In the second step arranged autoregressions for a given p and every element d of S are estimated. The data under consideration are sorted out from low to high based upon z_{t-d}. Next, for the first b observations an AR(p) model is estimated. Then the AR(p) model is estimated for the first b+1 observations, b+2 observations et cetera. Each time, the last residual is stored. Finally, these residuals are regressed upon the same variables as in the AR(p) model (p lags of z_t) (equation (A5)). In case of linearity, the associated F-test (equation (A6)) from this last regression will be low indicating that the lagged z's have no impact anymore on the residuals. If, however, the F-test is (too) high, linearity is rejected. In that case a TAR model is superior to the linear model. By comparing the F-tests for several d, the optimal delay-parameter d^* is chosen by maximising the F-statistic. In the third step, the threshold values are located using scatterplots. The t-values from the arranged autoregressions in step two are plotted against the threshold variable z_{t-d^*}. In the case of linearity, these t-values will converge to their true value. In the case of non-linearity, however, at the threshold values the t-values will deviate from their path. Thus, by using scatterplots, we can see how many threshold values r_j there are likely to be and what value they approximately have. In step four the AR order and the threshold values are refined, if necessary, in each regime by using linear autoregression techniques.

3.2. Threshold error correction model (TECM)

Balke and Fomby (1993) explore a general approach, called threshold cointegration. They apply their method to the market determined Fed Funds rate and the Discount rate which is set by the Federal Reserve. The method divides the data into groups according to the deviation from the arbitrage value. The thresholds for the group including the exact arbitrage value then presumably reflect the band around the arbitrage value, in which arbitrage is not profitable. In our case this is obviously not necessarily true. In fact, it will probably only be the case for the outer regimes. The central regimes could also reflect differences due to infrequent trading.

For every group we will estimate an error correction model. In this model futures and index returns are explained by past futures and index returns, and the deviation from the arbitrage relation in the previous period. If the threshold values are established from the TAR model for the basis, the following error correction model for the returns will be estimated for each regime:

$$\Delta \ln F_{t,T} = c_F + \sum_{k=1}^{L_1} \phi_{F,k} \Delta \ln F_{t-k,T} + \sum_{k=1}^{L_2} \theta_{F,k} \Delta \ln S_{t-k} + \gamma_F z_{t-1} + \varepsilon_{F,t} \tag{4}$$

$$\Delta \ln S_{t,T} = c_S + \sum_{k=1}^{L_3} \phi_{S,k} \Delta \ln F_{t-k,T} + \sum_{k=1}^{L_4} \theta_{S,k} \Delta \ln S_{t-k} + \gamma_S z_{t-1} + \varepsilon_{S,t} \tag{5}$$

where $F_{t,T}$ is the futures price at time t of a futures contract with maturity date T, S_t is the value of the index at time t, and Δ is the difference operator, e.g.
$\Delta \ln F_{t,T} = \ln F_{t,T} - \ln F_{t-1,T}$.

We expect that the effect of the error correction term in the regime reflecting the band around the arbitrage value will be much smaller than in the other regimes. Furthermore, there could be differences in the impact of arbitragers in the lower and upper regimes, since the lower regimes involve short-selling of stocks. Finally, the lead-lag coefficients between the futures and spot returns might be different across the regimes.

4. Data

We give an empirical example for the S&P 500 index and index-futures contract maturing in June and December 1993. The data set is provided by the Futures Industry Institute Data Center in Washington. The S&P 500 index is calculated every 15 seconds during the opening hours of the New York Stock Exchange (NYSE), i.e. 8:30 through 15:00 Chicago time. For the index-futures, traded at the Chicago Mercantile Exchange (CME), transaction prices are available with a time stamp to the nearest second.

From these data we calculate one-minute returns, using every minute the latest available price. Thus, 390 prices a day are constructed unless, of course, trading started later than 8:30 or ended before 15:00. In cases of no trading in a minute, the last available price is used. From every day we disregard the first 10 observations, since a lot of stocks do not trade in the very beginning of the trading. The returns are calculated as the difference of the natural logs of the prices. We do this for every day, getting 379 (or less) returns per day. This way, when stacking several days, overnight returns are avoided.

To calculate the basis we use the daily U.S. discount rate, the rate applied between banks, which did not change much during this period (e.g. the 1-month rate was between 3.01 and 3.12 percent for the whole period). Since dividends are paid when the stock market is closed, the remaining average of the basis (without dividends-adjustments) per day should approximately reflect these dividends and we need only a daily adjustment. We use the dividends reported in the *S&P 500 Information Bulletin* published by Standard and Poors to get the basis in equation (2). This is of course an approximation since we are using realised dividends, but it is the best estimate we have.

We will estimate the model in (4) and (5) for a whole month rather than for a single day. Thus, we hope to detect the structural characteristics and avoid one-time events affecting the results.

5. Results

We will perform the TAR analysis for the months May (June 1993 contract) and November (December 1993 contract). For both months, the futures price and index value are cointegrated. The formal tests are given in Table 1.

The results indicate that for both months the futures and index prices have a unit root (prices non-stationary, returns stationary), which is the first condition for cointegration. For both months a linear combination exists between the futures and index prices, which is stationary. Thus, the futures and index prices are cointegrated. The cointegration vectors are not significantly different from (1,-1). After adjusting the cointegration relationship using (1,-1) with the interest rate and dividends, the basis as defined by equation (2) is still stationary as can be seen from the bottom line of Table 1.

5.1. Location of the thresholds

Step 1: AR order and set of threshold lags

We will now estimate the threshold values of the TAR model for the error correction term. First, the AR order p is selected using the PACF in (A2), resulting in p=7 and p=4 for May and November, respectively. Next, we have to select the set S of possible threshold lags d. In 1976 the Super Designated Order Turnaround (DOT) system was developed by the NYSE. This system is an automated order-processing system that electronically links member-firm order rooms to the market makers on the exchange. A NYSE member is guaranteed execution and reporting within 3 minutes. Since arbitrage opportunities will be observed almost immediately, we will use $S = \{1, 2, 3, 4\}$.

Step 2: Non-linearity test and the optimal threshold lag

For the four possible d's the arranged autoregressions are estimated. The F-statistics in (A6) associated with the regression of the residuals in (A5) of these arranged autoregressions are reported in Table 2.

The results indicate that linearity is rejected for all threshold lags d, while this is most clear for d=1. Thus, we set d^* equal to one for both months.

Step 3: Number and location of thresholds

In the third step following the method of Tsay (1989), we need to establish the number and the location of the threshold values. For this purpose we use the arranged autoregressions based upon z_{t-d*}. For the constant and for the lags of z_t, the t-values can then be plotted against the threshold variable z_{t-d*}. In our case of non-linearity we expect to see unexpected deviations in the t-values after trespassing a candidate threshold. Since we look for more than one threshold we look at the t-values, sorting from low to high and from high to low. The reason for this is that, when trespassing a threshold at the end of the arranged autoregressions, already most of the data will be included reducing the effect[1]. We also plotted the coefficients from the arranged autoregressions against the threshold variable. As we will see, especially the t-values of the AR(1) coefficient become so large, that deviations are not observable anymore. From Section two, however, we expect to clearly see changes in the AR(1) coefficient in particular.

For May the t-values and coefficients for the constant and the first four lags of the basis are given in Figures 2a through 2j sorting the data according to z_{t-1} from low to high. In Figures 3a through 3j the data are sorted according to z_{t-1} from high to low.

Threshold candidates should be indicated by changes in the pattern of the t-values and in the coefficients. An exception are changes in the pattern at the moment the t-values are smaller than 1.96 indicating non-significance at the 5% level. We also should be careful at the start when we have a small amount of observations (we start the arranged autoregressions with the first 15 observations).

For the negative values of the basis (Figures 2a through 2j) we observe as possible candidates (approximately) **-0.16** (t-values constant become significant, while there is a drop in the coefficient; the latter can also be observed from the AR(1) coefficient in Figure 2d), **-0.14** (starting a steady rising pattern in both the constant and AR(1) coefficient), **-0.10** and **-0.06** (being a local maximum and absolute minimum, respectively, in Figure 2a). The figures of the second and third lag do not contribute any candidates, since the t-values indicate they are not significant. From Figure 2i we see that the fourth lag is only significant around -0.15 and -0.13, the coefficient showing a large drop around **-0.13.**

On the positive side (Figures 3a through 3j) reading the figures from the right to the left, we observe as possible candidates **0.20** (the constant becomes significant and the coefficient drops clearly), **0.18** and **0.19** (peaks in both plots, from 0.18 starting to decrease towards zero). The same candidates apply from the AR(1) t-values and coefficients. From Figure 3e it is noticeable that from the significant level around **0.10** the t-values move rapidly towards zero until **0.07**. From the last figures (3g trough 3j) only the

[1]Suppose there are two thresholds on either side of the average. Suppose we arrange the observations from low to high and we find the threshold at observation 1000. This will give a clear deviation in the path of t-ratios. If, however, the second threshold is at, say, observation 6000, the deviation will be less clear. The reason for this is that the previous t-ratio was already based upon 6000 observations, and as a result for example 50 extra observations will only slightly deviate the path of the t-ratios. Therefore we also estimate the arranged autoregressions from high to low to have a closer look at the second threshold.

AR(4) coefficient is significant around **0.07**.

For November the t-values and coefficients of the constant, and the four lags of the basis are given in Figures 4a through 4j and Figures 5a through 5j for sorting according to z_{t-1} from low to high and high to low, respectively.

For the negative values of the basis (Figures 4a through 4j), we observe as possible threshold candidates **-0.19** (a clearly changing point for all the coefficients and the constant although for some of them in the non-significant area), **-0.15** (a clear turning point in most figures as well), and **-0.10** (peak in both constant plots, turning point for the AR(2) t-values and coefficients although more towards **-0.09**; the latter also accounts for the AR(3) and AR(4) plots).

On the positive side (Figures 5a through 5j) we observe as possible candidates **0.26** (maximum in the constant plot although in the non-significant area, minimum for the AR(1) coefficient, and a sharp rise in the plots for the fourth lag), **0.22** (from this point on the constant and the AR(1) coefficient start to converge, a sharp drop in the AR(2) plots, and a turning point in the fourth lag, although the latter is closer to **0.21**). Closer to zero it is difficult to point out any clear candidate taking into account the significance of the coefficients.

Step 4: Refine the threshold candidates

The third step should also involve the establishment of the number of thresholds. Clearly, this is difficult as so far there exists no formal test for this. Since our primary interest is to find the band around the arbitrage value in which arbitragers will not enter into the market, we apply the following procedure:

(i) *Find two thresholds from the candidates which are far enough from zero as to reflect candidates for the no-arbitrage band*

(ii) *Find two thresholds between zero and the outer thresholds for investigating changes in the pattern involving infrequent trading*

For both steps we will use a grid search using the criterion of the least sum of squared errors. Fixing the number of regimes at five, we can optimise their location by adding the sum of squared errors of all the regimes and minimise this. For the grid search of the outer regimes we take into account the observed candidates, while for the other two thresholds we search the whole area. The reason for this is that we do not expect that infrequent trading will change radically at a certain point, but we think that the pattern might be different at a certain distance from zero.

To start with the month May, we observed as candidate on the outer negative side -0.16. A grid search in this area gives the optimal threshold **-0.158**. On the outer positive side we observed as candidates 0.18, 0.19 and 0.20. A grid search in this area provides the optimal threshold **0.204**. Next, we use a grid search for the entire area between -0.158 and 0.204 for two other thresholds. The sum of squared errors (SSE) criterion results in the thresholds **-0.073** and **0.072**.

For November we observed as candidate on the most negative side -0.19. The grid search in this area provides the threshold **-0.186**. On the outer positive side we observed as candidates 0.21, 0.22 and 0.26. A grid search results in the threshold **0.212**. The SSE criterion for the area in between these two thresholds results in the thresholds **-0.090** and **0.062.**

The above results are summarised in Table 3. Table 3 also gives the number of observations in every regime. We see that there is only a small number of observations in the outer regimes. To give an indication whether arbitrage would be possible in these outer regimes, while it is not likely in the other regimes, we calculate the bandwidth in terms of index-points. The average value of the index in May 1993 was equal to 445.25, while in November 1993 it was 462.89. The bandwidth in May is 0.362 (0.204+0.158) percent, or 1.61 index-points. In November the bandwidth is 0.398 (0.212+0.186) percent, or 1.84 index-points. If the bandwidth is symmetrical around the arbitrage value, then this would mean a deviation of 0.805 index-points in May and 0.92 index-points in November that would trigger arbitrage[2].

The bid-ask spread for a typical stock in the S&P 500 index is 0.125. With, for example, an average share price of 40 dollars, the average percentage spread would be 0.3125 percent. If the cash index level is half way between the bid and ask levels, half the bid-ask spread would be incurred as a cost. With the average index level in May of

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[2]This does not necessarily mean that all observations in the outer regimes will trigger arbitrage. Some of them may well be the result of infrequent trading. In practice arbitragers will compute the S&P index level using the bid or ask prices to observe a 'true' arbitrage opportunity.

445.25, this would mean a transaction cost of 0.5*0.003125*445.25 or 0.70 index-point. For a futures contract half the spread is 0.05 index-point. Thus, costs due to the spread are in this example 0.75 index-point. For November this is 0.77 index-point. On top of these costs an arbitrager will have to pay commission fees. The latter costs will, however, be small for a member firm of the New York Stock Exchange. If a position is hold until expiration, then there are extra costs due to the spread. The above argument, however, did not consider the early unwinding option. Considering the number of observations in the outer regimes, we expect that for an arbitrage position started several weeks before maturity, such an early unwinding possibility will quite likely occur.

5.2. The TAR model

With the thresholds found in the previous section, we can now estimate the AR model per regime. The results for May and November are given in Table 4. For May we see that the AR(1) coefficient is the smallest in the outer regimes reflecting rapid return towards zero when arbitrage is presumably possible. The middle regime has the highest AR(1) coefficient. The two regimes around the middle regime have a somewhat lower coefficient than in the middle regime. Since arbitrage is not likely to be possible in these regimes, this effect can be ascribed to the infrequent trading effect which has obviously a larger impact when there is some deviation from the arbitrage relation. This deviation could be caused by new information coming into the market moving the futures prices only later followed by the index due to infrequent trading.

For November we see a similar pattern, with the only exception that the AR(1) coefficient is very large in regime 5. We already observed from Figure 5d that the pattern found in all other figures of the AR(1) coefficient (from a certain point a steady convergence) was not present here. A possible reason is that the infrequent trading effect was quite severe in this case.

5.3. The Threshold Error Correction model

We can now estimate for each regime the error correction model given by equations (4) and (5). The results for the regimes 2, 3, and 4 are reported in Tables 5 and 6 for May and November, respectively. For May we observe a clear lead of the futures market on the spot market in all three regimes. In regime 2, where futures prices are relatively low compared to index prices, the coefficients of the lagged futures returns in the index equation are approxi-mately twice the magnitude of the coefficients in the other two regimes. If futures lead the spot, then this regime is reached in cases of negative news entering the market, seemingly increasing the infrequent trading effect (if this is one of the main causes the futures market leads the spot market). The error correction term is mainly significant for the index returns, also indicating the lead of the futures prices. The impact of the deviation of the arbitrage relation on the current index returns is clearly larger in regime 2 and 4 than in the middle regime.

Estimating the ECM for the outer regimes using only 1 lag of the index and futures return to save on the degrees of freedom, we find no significant impact of the error correction term, but the sign is correct and the magnitude nearly twice as large as in regimes 2 and 4. The large coefficient (0.551 and 0.426 for regime 1 and 5, respectively) of the lagged index return in the index equation indicates that part of the observations in the outer regimes are due to infrequent trading (thus they do not reflect arbitrage opportunities).

For November we find similar results as for May. Again the futures market seems to lead the spot market, and the coefficients of the lagged futures returns in the index equation are approximately twice as large in regime 2. The shorter lead can be partly ascribed to the much lower number of observations in this regime. As opposed to May, in November we find some significant impact the other way around, i.e. a lead of the spot market on the futures market. For the error correction term we again find that the magnitude of the coefficients is larger in regimes 2 and 4 than in regime 3. For the outer regimes the clearly significant lagged spot return in the index equation (coefficient 0.488) and the much smaller coefficient of the error correction term confirm our conclusion that the different behaviour of the upper outer regime in November is due to a relative large number of infrequent trading cases outside the no-arbitrage band.

7. Conclusion

The threshold error correction model allows us to explicitly model the behaviour of arbitragers. Index-futures arbitrage only occurs when the deviation from the arbitrage relation is large enough to offset the difference between the costs (and risks) and the expected return including the early unwinding option. The threshold autoregression approach provides the band in which arbitrage is presumably not profitable, or at least not for a large group of arbitragers. One difficulty is that the infrequent trading effect of the index imposes the same pattern on the basis: first we observe a deviation from the no-arbitrage relation, and then the basis moves back towards zero. Arbitrage opportunities cause this pattern only at a deviation of the no-arbitrage relation at which the expected return is positive. For the infrequent trading effect the change in the pattern will occur more gradually the further away the basis is from zero. The results indicate that also in the case that arbitrage is not possible, the pattern is different in the regimes next to the central regime.

The error correction models for the regimes show that the impact of the futures market on the spot market is larger when the basis is negative, and that the deviation from the no-arbitrage relation becomes more important for the current returns the further the futures price is away from its theoretical value.

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Appendix: Tsay's method for TAR models

In this appendix we describe Tsay's (1989) method to establish the threshold lag and the threshold values. The model we want to estimate is the following:

$$z_t = \phi_0^{(j)} + \sum_{i=1}^{p} \phi_i^{(j)} z_{t-i} + \varepsilon_t^{(j)} \qquad r_{j-1} \leq z_{t-d} < r_j, \tag{A1}$$

where j=1,...,k, with k the number of regimes, and d the threshold lag. The thresholds are $-\infty=r_0<r_1<...<r_k=\infty$. The AR order of the model is p.

Tsay suggests the following procedure to estimate p, d and the thresholds:

Step 1

Select the AR order p and the set of possible threshold lags S. For this purpose the partial autocorrelation function (PACF) of z_t may be used. This function is calculated by estimating

$$z_t = \phi_0 + \sum_{i=1}^{q} \phi_i z_{t-i} + \varepsilon_t \tag{A2}$$

for increasing order q and test the significance of ϕ_q. This coefficient is called the qth partial autocorrelation coefficient, say ϕ_{qq}. As a model for the data-generating process we choose an AR(p) model such that

$$\phi_{qq} \begin{cases} \neq 0 \; for \; q=p \\ = 0 \; for \; q>p \end{cases} \tag{A3}$$

The ϕ_{qq} are approximately normally distributed with mean zero and variance 1/N for $q>p$, where N is the sample size. This can be used to check the significance of ϕ_{qq}.

The set S of possible threshold lags can be chosen by own practical feeling about what will be the lag length of the reaction of the market towards the deviations.

Step 2

For a given p and every element of S arranged autoregressions are fitted and the threshold nonlinearity test is performed.

Suppose we have the AR(p) model for z_t. We refer to $(z_t, 1, z_{t-1},...,z_{t-p})$ as a *case* of data for the AR(p) model. An arranged autoregression is an autoregression with cases rearranged, based on the values of a particular regressor. For the TAR model (A1), arranged autoregression becomes useful if it is arranged according to the threshold variable z_{t-d}. Consider for example the case k=3. For a given TAR model with N observations, the threshold variable z_{t-d} may assume values $\{z_h,...,z_{N-d}\}$ where h equals max{1, p+1-d}. Let π_n be the time index of the nth smallest observation of $\{z_h,...,z_{N-d}\}$. We then can rewrite (A1) as

$$z_{\pi_n+d} \begin{cases} = \phi_0^{(1)} + \sum_{i=1}^{p} \phi_i^{(1)} z_{\pi_n+d-i} + \varepsilon_{\pi_n+d}^{(1)} & if \; n \leq L \\ = \phi_0^{(2)} + \sum_{i=1}^{p} \phi_i^{(2)} z_{\pi_n+d-i} + \varepsilon_{\pi_n+d}^{(2)} & if \; L<n\leq U \\ = \phi_0^{(3)} + \sum_{i=1}^{p} \phi_i^{(3)} z_{\pi_n+d-i} + \varepsilon_{\pi_n+d}^{(3)} & if \; n>U \end{cases} \tag{A4}$$

where L satisfies $z_{\pi_L} < r_1 < z_{\pi_{L+1}}$ and U satisfies $z_{\pi_U} < r_2 < z_{\pi_{U+1}}$. This is an arranged autoregression with the first L cases in the first regime, the second U-L cases in the second regime and the rest in the third regime. This way the data points are grouped so that all of the observations in a group follow the same linear AR model. We need to find the threshold values r_1 and r_2. Since the threshold values are unknown, however, one must proceed sequentially. The least squares estimates $\hat{\phi}_i^{(1)}$ are consistent for $\phi_i^{(1)}$ if there are sufficiently large numbers of observations in the first regime. In this case, the predictive residuals are white noise asymptotically and orthogonal to the regressors. On the other hand, when n arrives at or exceeds L the predictive residual for the observations with time index $\pi_{n+1}+d$ is biased because of the model change at this time. Hence, the predictive residual is a function of the regressors. Consequently, the orthogonality between the predictive residuals and the regressors is destroyed once the recursive autoregression goes on to the observations whose threshold values exceeds r_1.

The procedure then is as follows. Equation (A2) for cases rearranged according to the threshold variable z_{t-d} is estimated for the first m cases. Then every time the next case enters into the estimation. Each time the last residual is stored. These residuals are the predictive residuals. For the standardised residuals we then estimate

$$\hat{e}_{\pi_n+d} = \theta_0 + \sum_{i=1}^{p} \theta_1 z_{\pi_n+d-i} + \varepsilon_{\pi_n+d} \tag{A5}$$

for n=m+1,...,N-d-h+1, and compute the associated F statistic

$$\hat{F}(p,d) = \frac{(\sum \hat{e}_t^2 - \sum \hat{\varepsilon}_t^2)/(p+1)}{\sum \hat{\varepsilon}_t^2/(N-d-m-p-h)} \tag{A6}$$

where the summations are over all of the observations in (A5) and $\hat{\varepsilon}_t$ is the least squares residual of (A5). If the F-statistic exceeds the critical value of the F distribution with $p+1$ and $N\text{-}d\text{-}m\text{-}p\text{-}h$ degrees of freedom, linearity is rejected. We then prefer a TAR model. The optimal threshold lag d^* is then selected as to maximise the F-statistic.

Step 3

For given p and d^*, locate the threshold values by using scatterplots. For the above described procedure to calculate the predictive residuals, the arranged autoregression is estimated every time. The t-values of every AR-coefficient and the constant are also stored together with the value of the threshold variable $z_{t\text{-}d^*}$ from the last observation included in the regression. We then use the scatterplots of the t-values of the constant and the AR-coefficients against the threshold variable. The idea behind this plot is the following. When we have a linear model, the t-values show the significance of the coefficient, and when the coefficient is significant, the t-ratios converge gradually and smoothly to a fixed value as the recursion continues. This is also the case in a TAR model until the recursion reaches a threshold r. Then the estimate of the parameter starts to change and the t-ratio begins to deviate. In effect, the t-ratio starts to turn and, perhaps, changes direction at the threshold value. Therefore the plots give insight on the possible number of thresholds and their location.

Step 4

Refine the AR order and threshold values, if necessary, in each regime by using linear autoregression techniques. To refine the threshold values a grid search can be applied in a range considered reasonable from the scatterplots. A possible criterion is the sum of squared errors of all the regimes together, for example the sum of the squared residuals in equation (A4) in case of three regimes.

Table 1
Augmented Dickey Fuller tests on spot and futures prices, and cointegration

	May		November		critical value
	Futures	Index	Futures	Index	
levels (i)	-0.96	-1.15	-1.83	-2.44	-2.86
(ii)	-1.21	-1.48	-1.78	-2.39	-3.41
differences (i)	-48.0	-24.5	-37.6	-15.1	-2.86
cointegration[1]	-16.9	-16.8	-17.7	-17.7	-3.30
vector	1	-1.03	1	1.00	
basis[2]	-16.81		-16.67		-2.86

The following equations are estimated using OLS:

$$P_t - P_{t-1} = \theta_0 + \theta_1 * P_{t-1} + \sum_{i=1}^{L_1} \phi_i * (P_{t-i} - P_{t-i-1}) + e_{1t} \quad \text{(i)}$$

$$P_t - P_{t-1} = \tilde{\theta}_0 + \tilde{\theta}_1 * P_{t-1} + \sum_{i=1}^{L_2} \tilde{\phi}_i * (P_{t-i} - P_{t-i-1}) + \gamma * t + e_{1t} \quad \text{(ii)}$$

For both the levels and the first differences, the t-values of θ_1 are reported. Critical values are given in the last column. The null hypothesis of non-stationarity is rejected if the t-value of θ_1 is below the critical value.

1 $P_{1t} = c + \pi P_{2t} + z_t$ is estimated, first with the futures price as dependent variable (column futures), second with the index value as dependent variable (column index). For the resulting error term $\hat{z}_t$, equation (i) is estimated. The t-value of θ_1 is reported here.

2 For the basis given in equation (2) equation (i) is estimated. The t-value of θ_1 is reported here.

Table 2
Tsay Test for Threshold Nonlinearity for the difference between the index-futures price and the index value

threshold lag d[1]	F-statistic[2]	
	May '93	November '93
1	10.48	6.34
2	9.42	3.08
3	4.70	5.27
4	2.13	3.88

1 The data set is sorted out from low to high based upon z_{t-d}

2 The degrees of freedom are p for the denumerator (7 for May and 4 for November) and the number of observations for the numerator (7053 in May and 7689 in November) The 5% (1%) critical value equals 3.84 (6.63) and 3.00 (4.61) for May and November, respectively.

Table 3
Boundaries and number of observations of each regime

	May 1993		November 1993	
	Thresholds	# obs	Thresholds	# obs
Regime 1	(-∞ ;-0.158]	45	(-∞ ;-0.186]	19
Regime 2	(-0.158;-0.073]	787	(-0.186;-0.090]	225
Regime 3	(-0.073; 0.072]	4794	(-0.090; 0.060]	4463
Regime 4	(0.072; 0.204]	1391	(0.060; 0.212]	2890
Regime 5	(0.204; ∞)	36	(0.212; ∞)	92

Table 4

Results of the TAR model

Panel A: May 1993					
Regime	1	2	3	4	5
constant	-0.597	-0.00641	0.000206	0.00733	0.104
	(-1.33)	(-1.07)	(0.52)	(2.17)	(1.31)
z_{t-1}	0.573	0.761	0.931	0.900	0.557
	(2.30)	(11.45)	(55.17)	(24.44)	(1.91)
z_{t-2}	-0.0852	-0.0296	-0.00476	-0.00146	-0.370
	(-0.46)	(-0.59)	(-0.25)	(-0.039)	(-1.93)
z_{t-3}	-0.279	0.0460	-0.0154	-0.0527	0.113
	(-1.21)	(0.94)	(-0.82)	(-1.41)	(0.52)
z_{t-4}	0.440	-0.0256	0.0119	-0.0793	0.104
	(1.80)	(-0.50)	(0.65)	(-2.04)	(0.36)
z_{t-5}	-0.592	0.0473	0.000344	0.0732	0.180
	(-3.12)	(0.94)	(0.019)	(1.95)	(0.53)
z_{t-6}	0.469	-0.0316	0.00157	-0.0164	-0.142
	(1.79)	(-0.64)	(0.086)	(-0.42)	(-0.54)
z_{t-7}	-0.0825	0.123	0.0164	0.0680	-0.160
	(-0.34)	(3.23)	(1.19)	(2.43)	(-0.81)
Panel B: November 1993					
constant	0.0572	0.0102	0.00255	0.00859	-0.0309
	(0.89)	(0.77)	(6.06)	(4.87)	(-0.81)
z_{t-1}	0.544	0.705	0.892	0.855	0.962
	(1.76)	(5.83)	(51.56)	(36.23)	(5.64)
z_{t-2}	0.732	0.313	0.0124	0.0251	-0.165
	(1.84)	(3.75)	(0.65)	(0.98)	(-1.20)
z_{t-3}	-0.0173	-0.203	-0.00612	-0.00972	-0.186
	(-0.034)	(-2.54)	(-0.32)	(-0.38)	(-0.12)
z_{t-4}	-0.0811	0.181	0.0220	0.0118	0.315
	(-0.23)	(2.56)	(1.53)	(0.62)	(2.54)

The model estimated is

$$z_t = \phi_0^{(j)} + \sum_{i=1}^{p} \phi_i^{(j)} z_{t-i} + \varepsilon_t^{(j)} \qquad r_{j-1} \leq z_{t-d} < r_j,$$

where j=1, ..., 5 and the threshold lag d equals 1. The thresholds are $-\infty = r_0 < r_1 < ... < r_5 = \infty$. For May we have r_1=-0.158, r_2=-0.073, r_3=0.072, and r_4=0.204. For November we have r_1=-0.186, r_2=-0.090, r_3=0.062, and r_4=0.212.
T-values are given in parentheses.

Table 5
Error correction model for the index and index-futures in May 1993.

	Regime 2 $-0.158<z_{t-1}\leq-0.073$ 772 observations		Regime 3 $-0.073< z_{t-1} \leq 0.072$ 4809 observations		Regime 4 $0.072< z_{t-1} \leq 0.204$ 1391 observations	
	$\Delta\ln F_{t,T}$	$\Delta\ln S_t$	$\Delta\ln F_{t,T}$	$\Delta\ln S_t$	$\Delta\ln F_{t,T}$	$\Delta\ln S_t$
c	.000002	.00106[a]	.000001	.000001	-.000049	-.000084[a]
ϕ_1	-.157[a]	.0856[a]	-.0431[a]	.0417[a]	-.0383	.0400[a]
ϕ_2	-.0282	.230[a]	.0229	.104[a]	.0141	.117[a]
ϕ_3		.164[a]	.0389[b]	.106[a]	-.00349	.0883[a]
ϕ_4		.144[a]	.0258[c]	.0787[a]	-.0571[c]	.0917[a]
ϕ_5		.141[a]		.0615[a]		.0474[a]
ϕ_6		.134[a]		.0445[a]		.0108
ϕ_7		.0714[a]		.0502[a]		.0328[a]
ϕ_8		.0597[a]		.0267[a]		.00950
ϕ_9		.0471[b]		.0287[a]		.0287[b]
ϕ_{10}	-0.0692	.0408[c]		.0196[a]		
ϕ_{11}		.0391[c]		.0125[b]		
ϕ_{12}		.0573[a]		.00958		
θ_1		-.0308	-.0233	-.0530[a]	.0159	.0738[a]
θ_2		-.0493		-.0185	-.137[b]	
θ_3		-.134[a]				
γ	.000305	.00106[a]	-.000129	.000130[a]	.000728[b]	.000999[a]
adj R^2	.023	.263	.003	.121	.007	.208

The following model is estimated for each regime, with L_1, L_2, L_3 and L_4 based on significant coefficients:

$$\Delta\ln F_{t,T} = c_F + \sum_{k=1}^{L_1} \phi_{F,k}\Delta\ln F_{t-k,T} + \sum_{k=1}^{L_2} \theta_{F,k}\Delta\ln S_{t-k} + \gamma_F z_{t-1} + \varepsilon_{F,t},$$

$$\Delta\ln S_{t,T} = c_S + \sum_{k=1}^{L_3} \phi_{S,k}\Delta\ln F_{t-k,T} + \sum_{k=1}^{L_4} \theta_{S,k}\Delta\ln S_{t-k} + \gamma_S z_{t-1} + \varepsilon_{S,t},$$

where $\Delta\ln F_{t,T}$ and $\Delta\ln S_t$ are the futures and index return at time t, respectively. z_t is the difference between the natural logs of the futures price and the index value.
[a], [b], and [c] correspond to significance levels of 1%, 5%, and 10%, respectively.

Table 6
Error correction model for the index and index-futures in November 1993.

para-meter	Regime 2 -0.186<z_{t-1}≤-0.090 225 observations		Regime 3 -0.090< z_{t-1} ≤0.062 4463 observations		Regime 4 0.062<z_{t-1} ≤0.212 2890 observations	
	$\Delta\ln F_{t,T}$	$\Delta\ln S_t$	$\Delta\ln F_{t,T}$	$\Delta\ln S_t$	$\Delta\ln F_{t,T}$	$\Delta\ln S_t$
c	.000068	.000079	.000007[c]	-.000120[a]	.000039[b]	-.000034[a]
ϕ_1	-.256[a]	.0811[b]	.00406	.0473[a]	-.0393[c]	.0307[a]
ϕ_2	.107	.121[a]		.0514[a]	.0169	.0699[a]
ϕ_3	-.142[b]	.123[a]		.0561[a]	.0154	.0671[a]
ϕ_4		-.00509		.0533[a]	.0519[a]	.0428[a]
ϕ_5				.0470[a]		.0337[a]
ϕ_6				.0204[a]		.0219[b]
ϕ_7				.0347[a]		
ϕ_8				.0307[a]		
ϕ_9				.0126[c]		
ϕ_{10}				.0232[a]		
ϕ_{11}				.0178[a]		
ϕ_{12}				.0152[b]		
ϕ_{13}				.0137[b]		
θ_1	.0382	-.00967	.118[a]	.0221	.0889[b]	.0814[a]
θ_2	.315[b]	.282[a]	.0104	.0303[b]		.0730[a]
θ_3		.181[b]	.0676[b]	.0453[a]		.0418[b]
θ_4		.155[c]	.0865[a]	.0278[c]		.0402[b]
θ_5						.0181
γ	.000712	.00115[b]	-.000209[c]	.000386[a]	-.000473[a]	.000441[a]
adj R^2	.119	.234	.008	.119	.008	.137

The following model is estimated for each regime, with L_1, L_2, L_3 and L_4 based on significant coefficients:

$$\Delta\ln F_{t,T} = c_F + \sum_{k=1}^{L_1} \phi_{F,k}\Delta\ln F_{t-k,T} + \sum_{k=1}^{L_2} \theta_{F,k}\Delta\ln S_{t-k} + \gamma_F z_{t-1} + \varepsilon_{F,t},$$

$$\Delta\ln S_{t,T} = c_S + \sum_{k=1}^{L_3} \phi_{S,k}\Delta\ln F_{t-k,T} + \sum_{k=1}^{L_4} \theta_{S,k}\Delta\ln S_{t-k} + \gamma_S z_{t-1} + \varepsilon_{S,t},$$

where $\Delta\ln F_{t,T}$ and $\Delta\ln S_t$ are the futures and index return at time t, respectively. z_t is the difference between the natural logs of the futures price and the index value.
[a], [b], and [c] correspond to significance levels of 1%, 5%, and 10%, respectively.

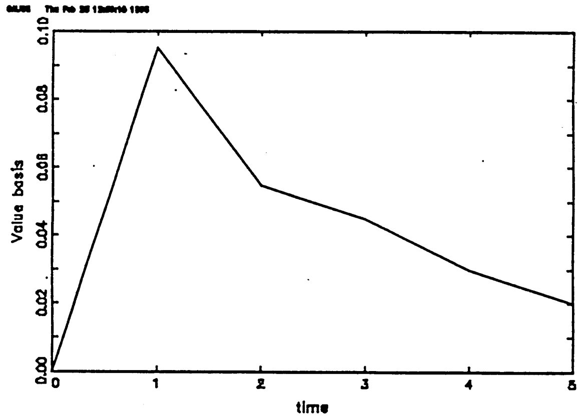

Fig 1a: Supposed pattern after positive news

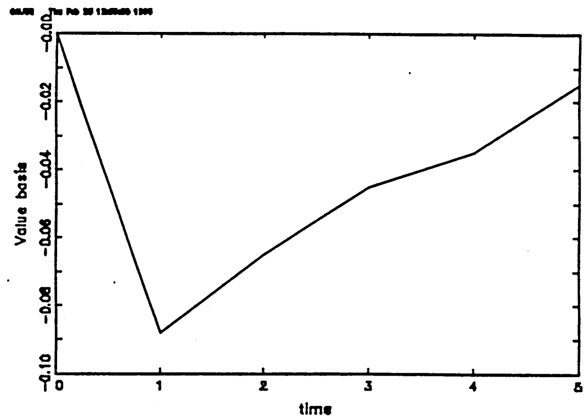

Fig 1b: Supposed pattern after negative news

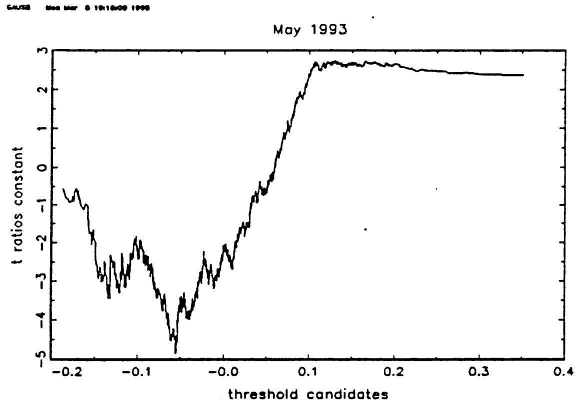

Fig 2a: Scatterplot t-values constant
Sorted from low to high

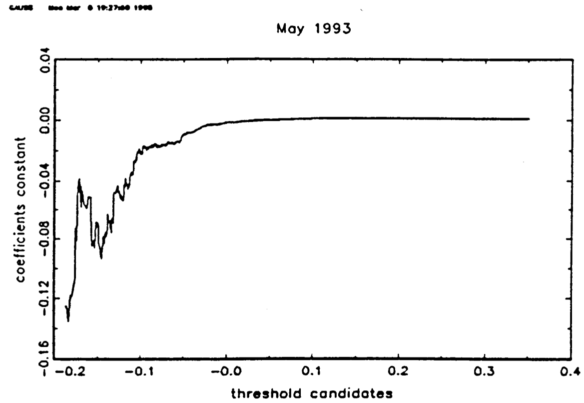

Fig 2b: Scatterplot coefficients constant
Sorted from low to high

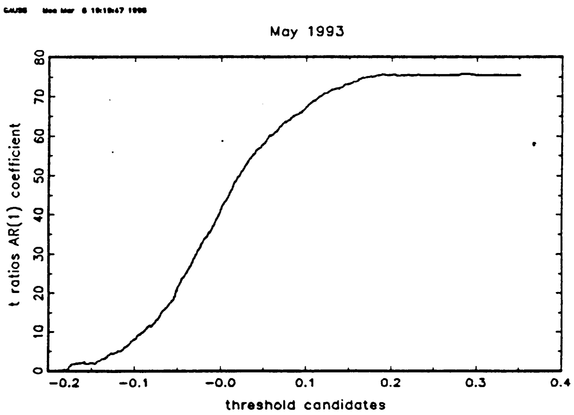

Fig 2c: Scatterplot t-values z_{t-1}
Sorted from low to high

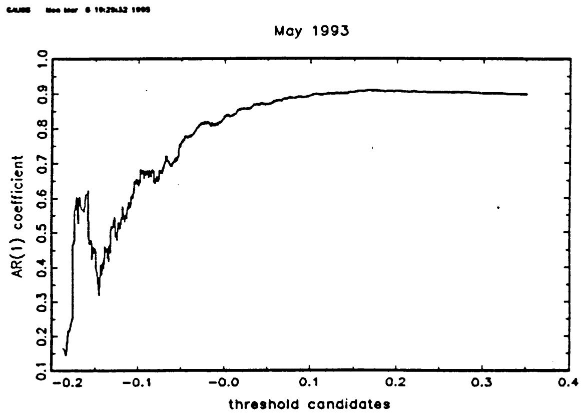

Fig 2d: Scatterplot coefficients z_{t-1}
Sorted from low to high

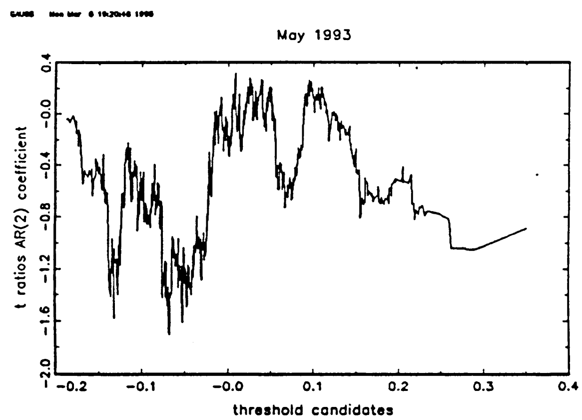

Fig 2e: Scatterplot t-values z_{t-2}
Sorted from low to high

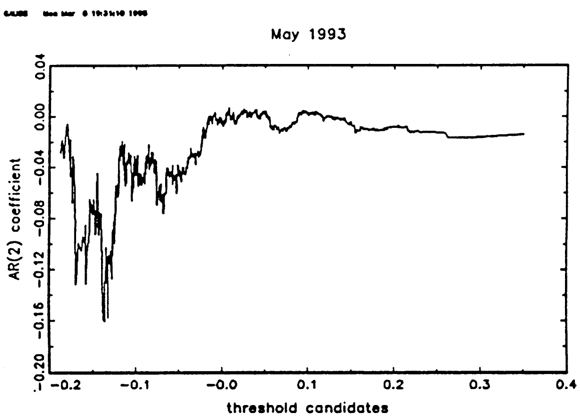

Fig 2f: Scatterplot coefficients z_{t-2}
Sorted from low to high

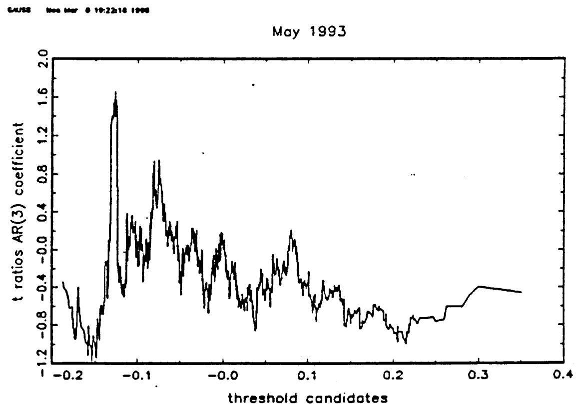

Fig 2g: Scatterplot t-values z_{t-3}
Sorted from low to high

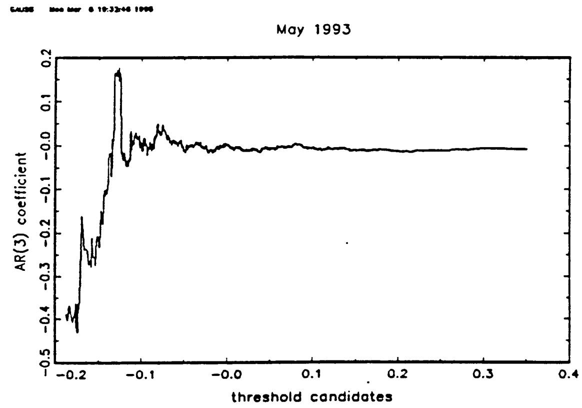

Fig 2h: Scatterplot coefficients z_{t-3}
Sorted from low to high

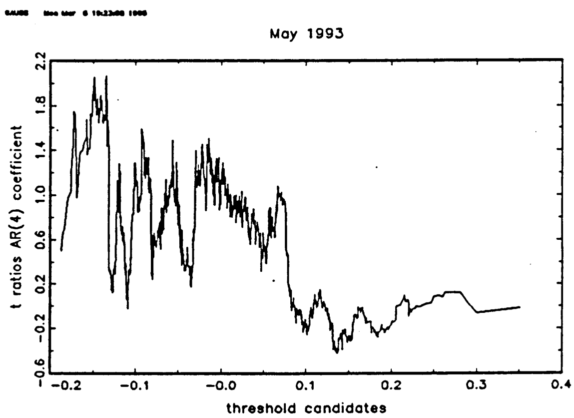

Fig 2i: Scatterplot t-values z_{t-4}
Sorted from low to high

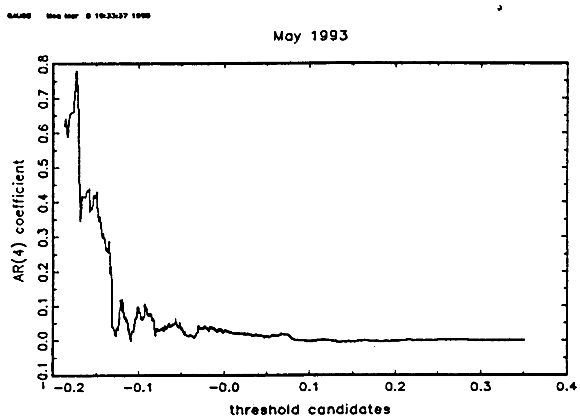

Fig 2j: Scatterplot coefficients z_{t-4}
Sorted from low to high

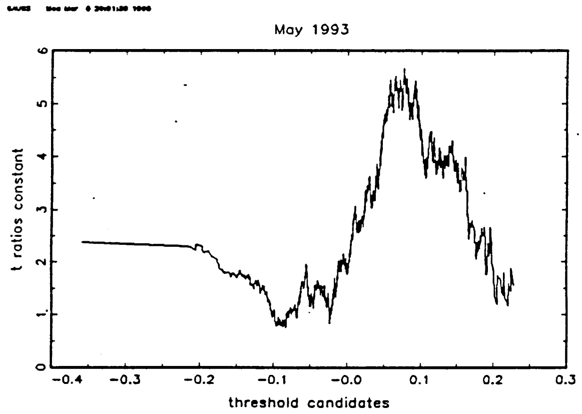

Fig 3a: Scatterplot t-values constant
Sorted from high to low

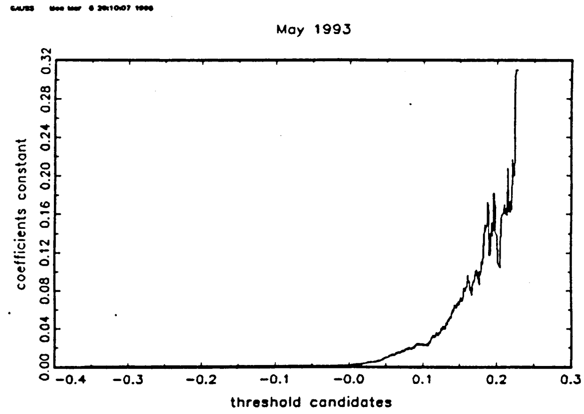

Fig 3b: Scatterplot coefficients constant
Sorted from high to low

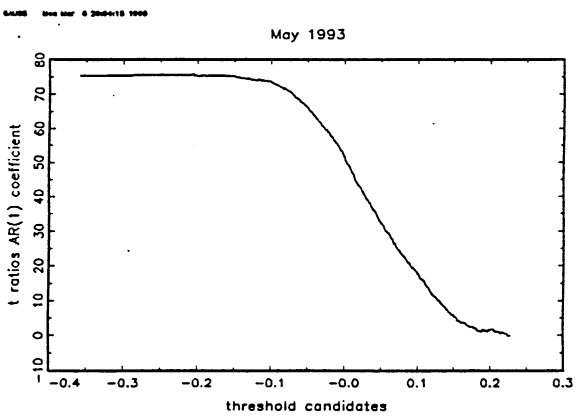

Fig 3c: Scatterplot t-values z_{t-1}
Sorted from high to low

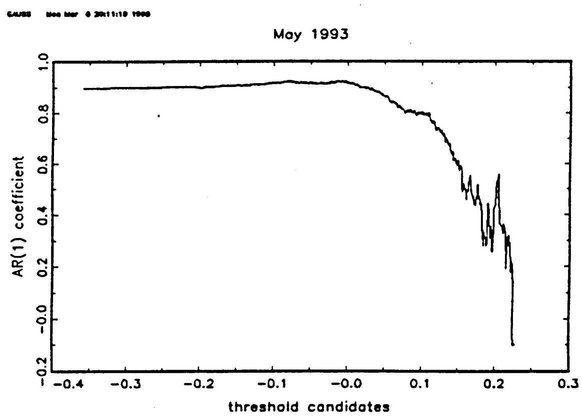

Fig 3d: Scatterplot coefficients z_{t-1}
Sorted from high to low

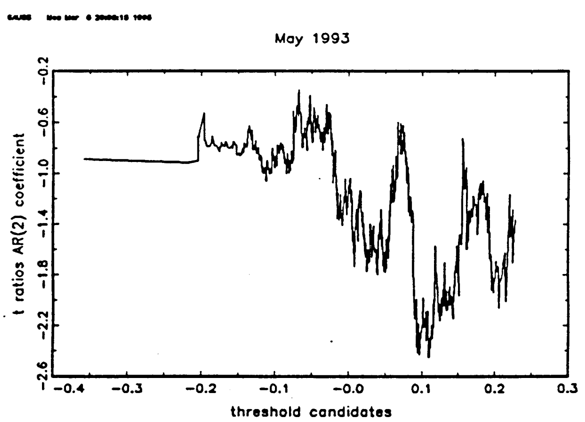

Fig 3e: Scatterplot t-values z_{t-2}
Sorted from high to low

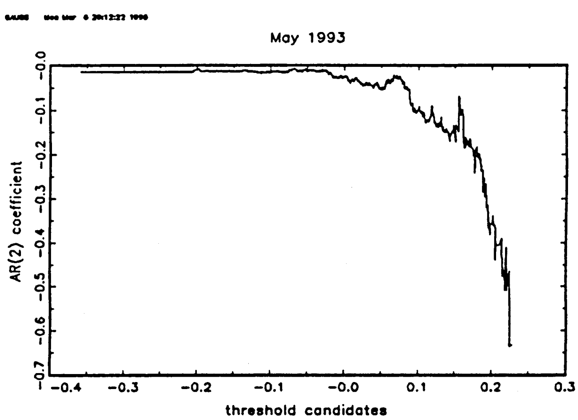

Fig 3f: Scatterplot coefficients z_{t-2}
Sorted from high to low

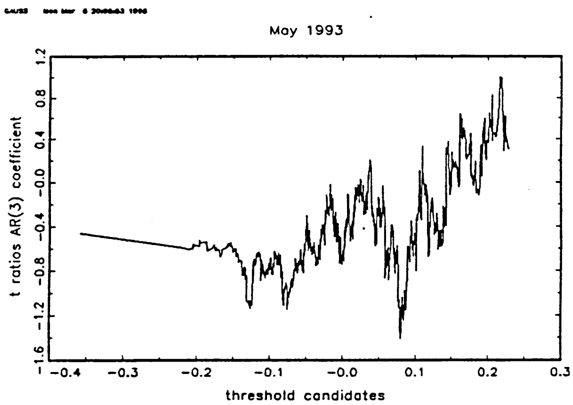

Fig 3g: Scatterplot t-values z_{t-3}
Sorted from high to low

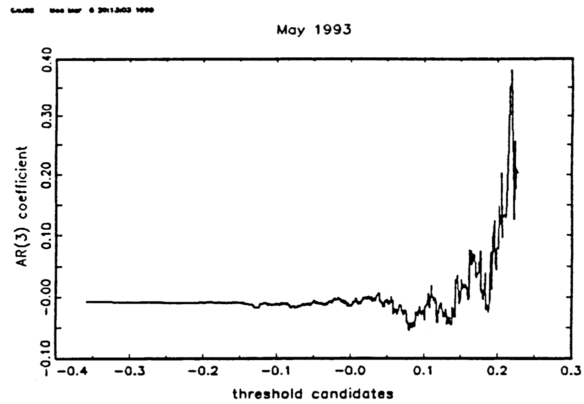

Fig 3h: Scatterplot coefficients z_{t-3}
Sorted from high to low

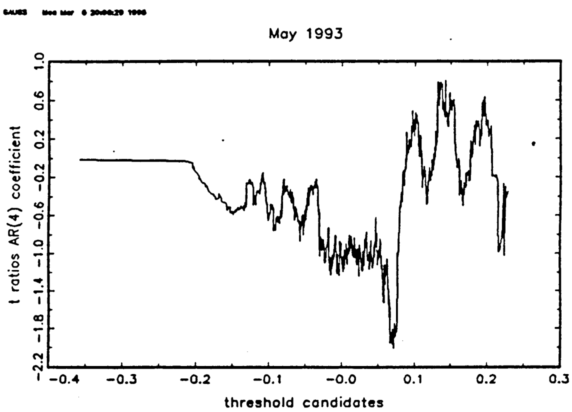

Fig 3i: Scatterplot t-values z_{t-4}
Sorted from high to low

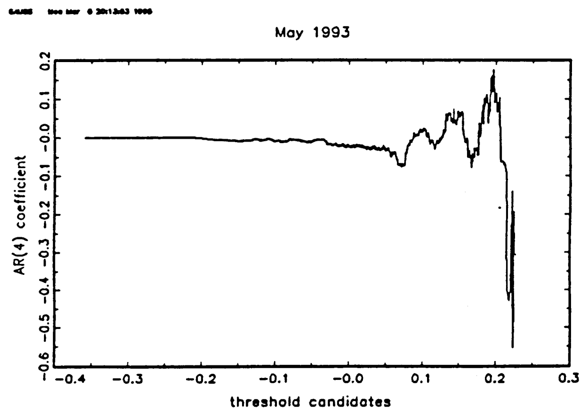

Fig 3j: Scatterplot coefficients z_{t-4}
Sorted from high to low

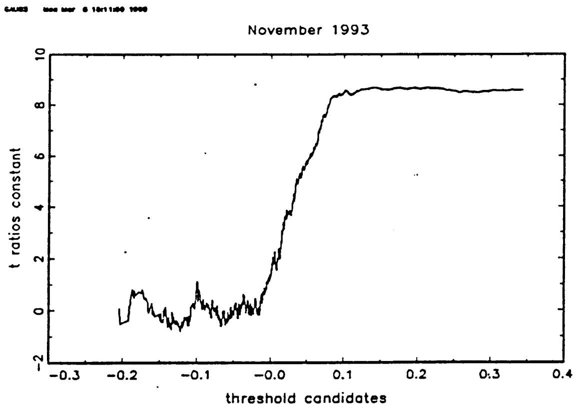

Fig 4a: Scatterplot t-values constant
Sorted from low to high

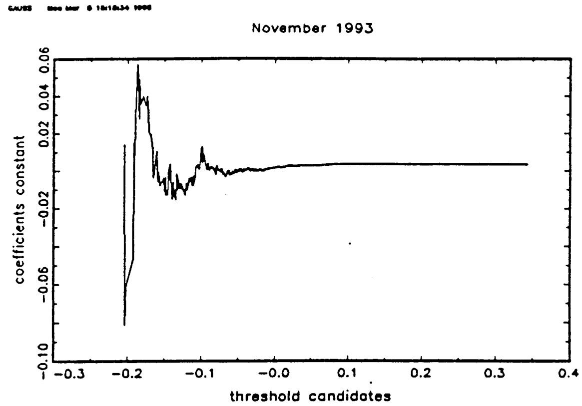

Fig 4b: Scatterplot coefficients constant
Sorted from low to high

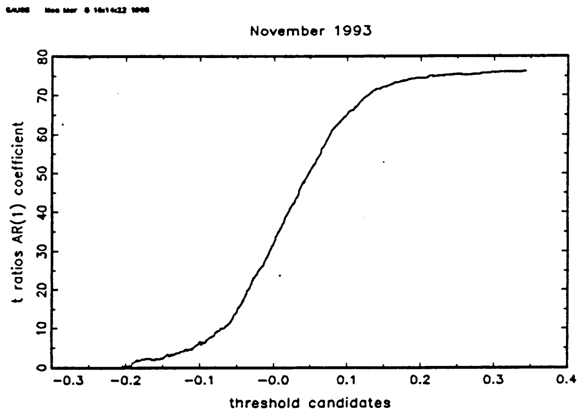

Fig 4c: Scatterplot t-values z_{t-1}
Sorted from low to high

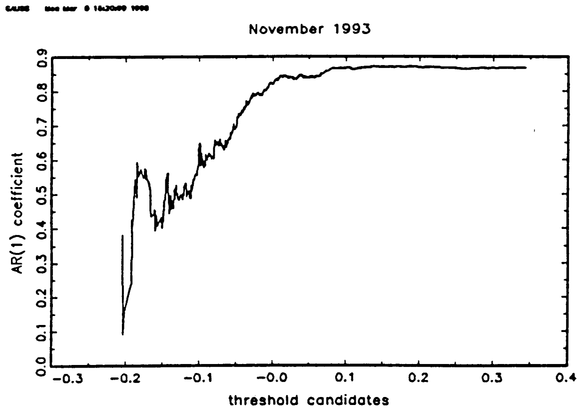

Fig 4d: Scatterplot coefficients z_{t-1}
Sorted from low to high

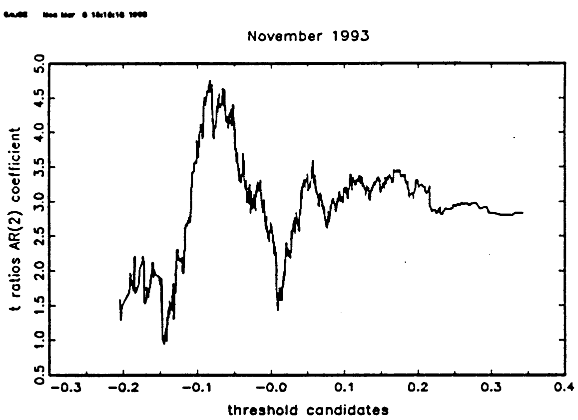

Fig 4e: Scatterplot t-values z_{t-2}
Sorted from low to high

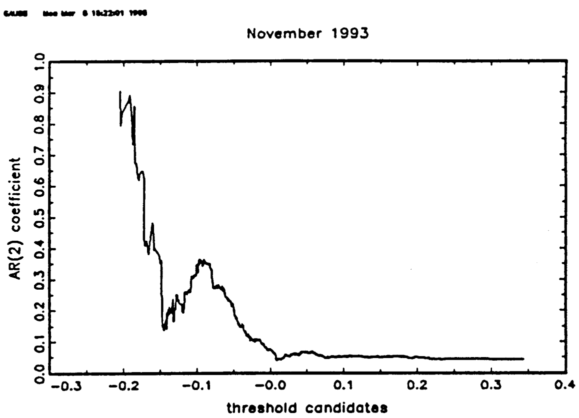

Fig 4f: Scatterplot coefficients z_{t-2}
Sorted from low to high

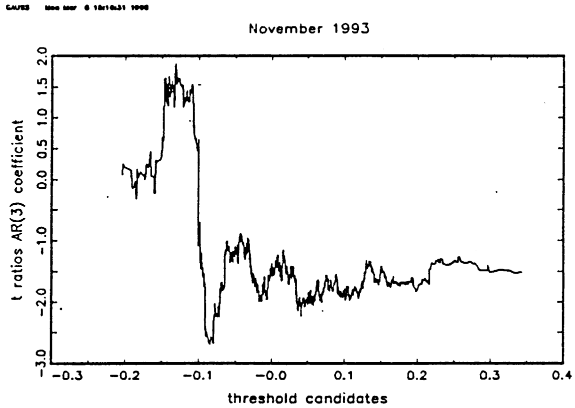

Fig 4g: Scatterplot t-values z_{t-3}
Sorted from low to high

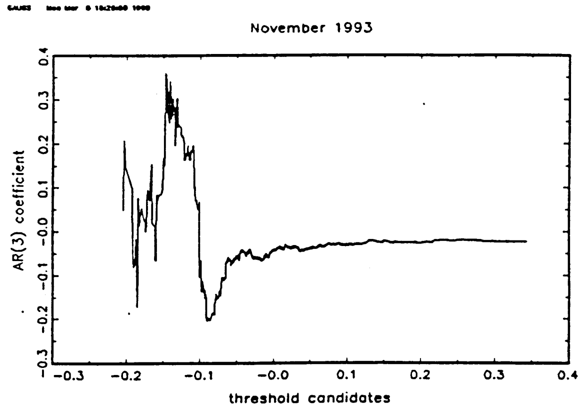

Fig 4h: Scatterplot coefficients z_{t-3}
Sorted from low to high

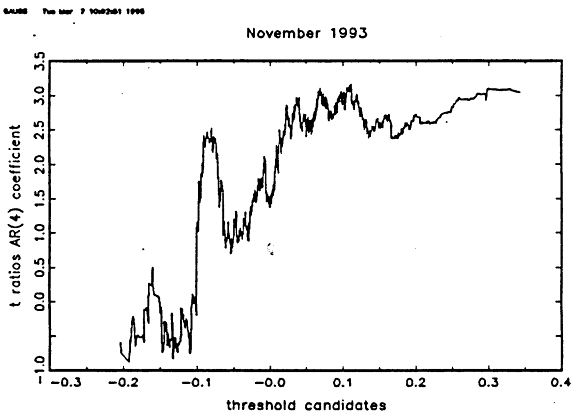

Fig 4i: Scatterplot t-values z_{t-4}
Sorted from low to high

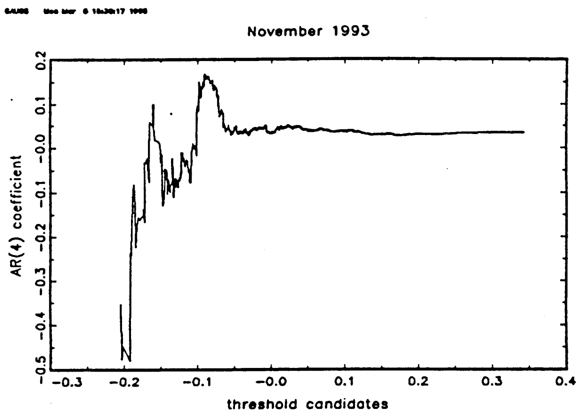

Fig 4j: Scatterplot coefficients z_{t-4}
Sorted from low to high

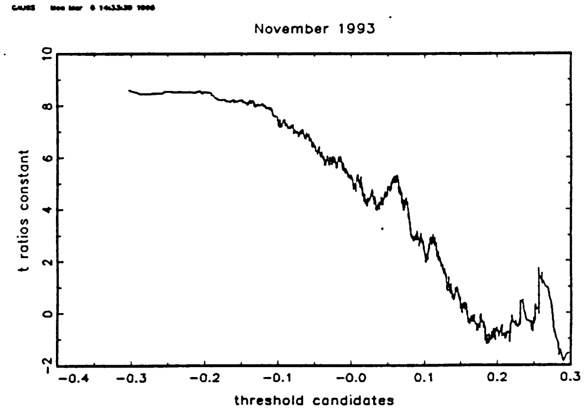

Fig 5a: Scatterplot t-values constant
Sorted from high to low

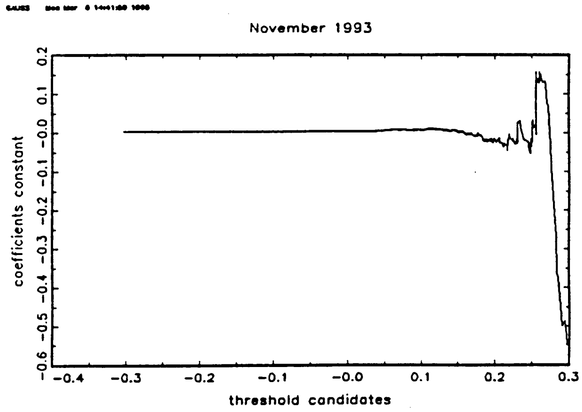

Fig 5b: Scatterplot coefficients constant
Sorted from high to low

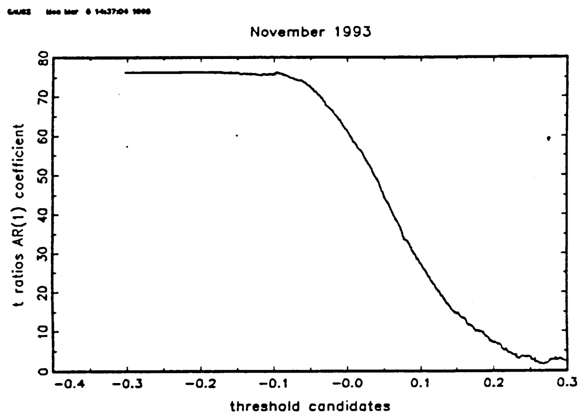

Fig 5c: Scatterplot t-values z_{t-1}
Sorted from high to low

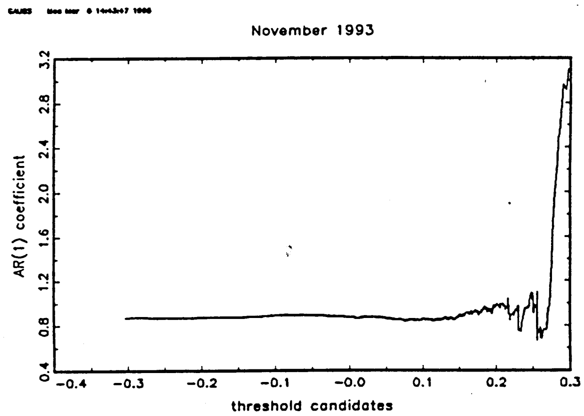

Fig 5d: Scatterplot coefficients z_{t-1}
Sorted from high to low

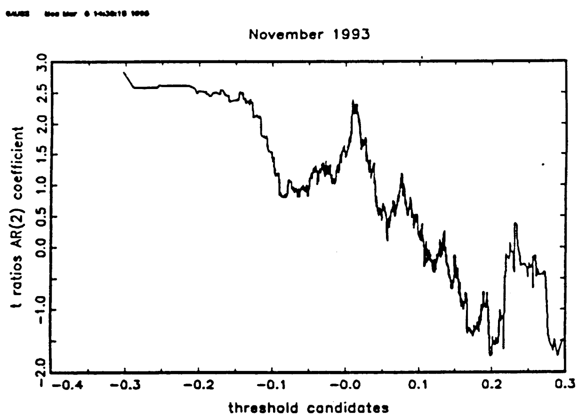

Fig 5e: Scatterplot t-values z_{t-2}
Sorted from high to low

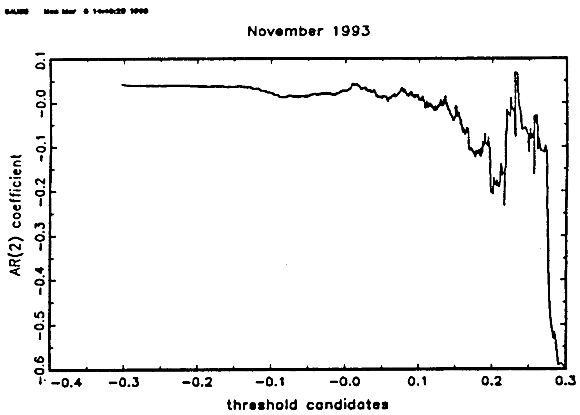

Fig 5f: Scatterplot coefficients z_{t-2}
Sorted from high to low

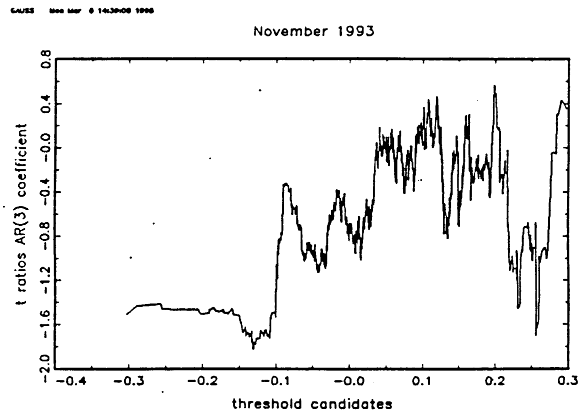

Fig 5g: Scatterplot t-values z_{t-3}
Sorted from high to low

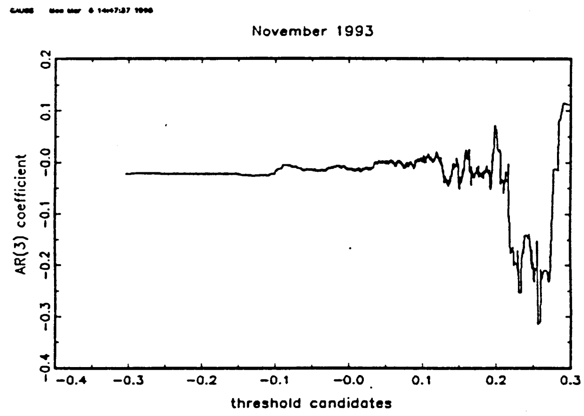

Fig 5h: Scatterplot coefficients z_{t-3}
Sorted from high to low

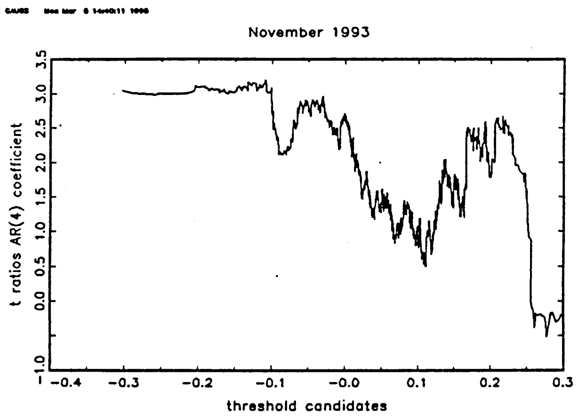

Fig 5i: Scatterplot t-values z_{t-4}
Sorted from high to low

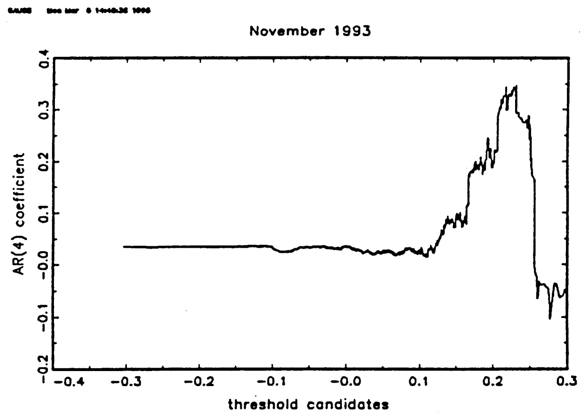

Fig 5j: Scatterplot coefficients z_{t-4}
Sorted from high to low