AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

MONASH

WP 12/95

ISSN 1032-3813
ISBN 0 7326 0773 6

MONASH UNIVERSITY

AUSTRALIA

LIBRARY
NOV 1 8 1995
WITHDRAWN
GIANNINI FOUNDATION OF
AGRICULTURAL ECONO

MARGINAL LIKELIHOOD BASED TESTS OF A SUBVECTOR OF THE PARAMETER VECTOR OF LINEAR REGRESSION DISTURBANCES

Ismat Ara and Maxwell L. King

Working Paper 12/95
September 1995

DEPARTMENT OF ECONOMETRICS

MARGINAL LIKELIHOOD BASED TESTS OF A SUBVECTOR OF THE PARAMETER VECTOR OF LINEAR REGRESSION DISTURBANCES

by

Ismat Ara and Maxwell L. King*

Department of Econometrics
Monash University
Clayton, Victoria 3168
Australia

Abstract

This paper is concerned with the problem of testing a subset of the parameters which characterize the error variance-covariance matrix in the general linear regression model. Formulae for likelihood ratio, Wald, Lagrange multiplier and asymptotically locally most mean powerful test statistics based on the likelihood of a maximal invariant statistic or an equivalent marginal likelihood are given. Specific applications discussed are the problems of testing against AR(4) disturbances in the presence of AR(1) disturbances and testing for a Hildreth-Houck (1968) random coefficient against the alternative of a Rosenberg (1973) random coefficient. Monte Carlo size and power calculations for these two testing problems are reported. These results provide further evidence that supports the proposed approach to test construction. It also suggests that better handling of nuisance parameters is likely to improve the small-sample properties of asymptotically based inference procedures.

---

* This research was supported in part by an ARC grant. We are grateful to Alan Morgan for research assistance. This paper was presented at the Econometrics Conference at Monash University, July 1995 and the Recent Advances in Econometric Theory Conference at Hitotsubashi University, August 1995. We are grateful to participants at these two conferences for their comments.

1. Introduction

In regression analysis involving non-experimental economic data, the specification of the covariance matrix of the disturbances is always a matter for concern. This is widely recognised in the extensive literature on testing linear regression disturbances; see for example Godfrey (1988), Judge et al. (1985), King (1987a, 1987b), Pagan and Hall (1983) and Pagan (1984). There is an emphasis in this literature on the three classical testing procedures based on the likelihood function, namely the likelihood ratio (LR), Wald (W) and Lagrange multiplier (LM) tests. On the other hand, some have questioned the accuracy in small-samples of these tests, see for example King (1987a), Honda (1988), Moulton and Randolph (1989) and Ara and King (1993).

Ara and King (1993) conjectured that the relatively poor performance of these tests in small samples is due to the presence of nuisance parameters that can cause biases in the estimates of key parameters in the test statistics. For tests of regression disturbances, the regression coefficients and any other parameters not under test are nuisance parameters. Ara and King suggested the use of invariance arguments to overcome this problem. This involves treating a maximal invariant statistic as the observed data and its density as the likelihood function. They proved that this is equivalent to constructing tests based on the marginal likelihood function. Estimates based on the marginal likelihood function are known to be less biased than those based on the profile or concentrated likelihood; see Tunnicliffe Wilson (1989) and Ara and King (1993). The latter's study suggests that the maximal invariant/marginal likelihood (MIML) approach produces more accurate asymptotic critical values for the

LM and LR tests when the null hypothesis is normal spherical disturbances. They were unable to report a similar improvement for the W test.

Rahman and King (1993) extended this work to the multivariant one-sided testing problem involving a subset of the parameter vector of this disturbance covariance matrix. In this setting, not all nuisance parameters can be eliminated by invariance arguments. The usual procedure in such a situation is to replace the nuisance parameters in the test statistics by maximum likelihood estimates. The MIML approach suggests the use of maximum MIML estimates for the nuisance parameters. There are two reasons for expecting this method to be superior to the classical approach. The first is the use of invariance arguments to reduce the number of nuisance parameters and the second involves the use of typically less biased estimates of those nuisance parameters which remain.

Rahman and King's (1993) principal concern was with the problem of testing against Hildreth-Houck (1968) random coefficients in the presence of first-order autoregressive (AR(1)) errors. They considered only the LM and King and Wu's (1990) asymptotic locally most mean powerful (ALMMP) tests and found improvements in both small-sample size accuracy and power when MIML based tests are used in place of their classical counterparts. In a subsequent study (Rahman and King (1994)), they extended their Monte Carlo study to include King's (1987b) approximate point optimal invariant (APOI) tests and concluded that the extra work required to apply APOI tests hardly seems worthwhile.

In this paper, we extend this work to include both one-sided and two-sided testing and a greater range of tests for the general case of testing a subset of the parameter vector of the disturbance covariance matrix. The plan of the paper is as follows. Section 2 outlines the theory behind the MIML approach to hypothesis testing in the context of the general linear regression model. Formulae for MIML-based LR, W, LM and ALMMP tests are given for the general testing problem. Section 3 discusses the application of these formulae to the problem of testing for AR(4) disturbances in the presence of AR(1) disturbances. The problem of testing for a Hildreth-Houck (1968) random coefficient against the alternative of a Rosenberg (1973) random coefficient in the linear regression model is the subject of section 4. Section 5 reports the results of a Monte Carlo size and power comparison for these two specific testing problems. Section 6 contains some concluding remarks.

2. Theory

Consider the normal linear model with non-spherical disturbances

$$y = X\beta + u, \qquad u \sim N(0, \sigma^2\Omega(\theta)), \tag{1}$$

where y is $n \times 1$, X is $n \times k$ nonstochastic and of rank $k < n$, and $\Omega(\theta)$ is a symmetric, positive definite matrix function of the unknown $p \times 1$ parameter vector θ.

Suppose θ is partitioned as $\theta' = (\theta_1', \theta_2')$ where θ_1 and θ_2 are $p_1 \times 1$ and $(p - p_1) \times 1$ subvectors, respectively. We are interested in testing $H_0{:}\theta_2 = 0$ against either $H_a{:}\theta_2 \neq 0$ or $H_a^+{:}\theta_2 > 0$, where > denotes each component is less than or equal to its corresponding component but there is at least one strict inequality. This testing problem is invariant with respect to transformations of the form

$$y \to \eta_0 y + X\eta \tag{2}$$

where η_0 is a positive scalar and η is a $k \times 1$ vector. As noted in Ara and King (1993) and Rahman and King (1993), the $m \times 1$ vector

$$v = Pz / (z'P'Pz)^{1/2}$$

is a maximal invariant under the group of transformations given by (2) where $m = n - k$, P is an $m \times n$ matrix such that $PP' = I_m$ and $P'P = I_n - X(X'X)^{-1}X' = M$ and $z = P'Py$ is the ordinary least squares (OLS) residual vector from (1).

The probability density function of v (see King, 1980) is

$$f(v;\theta)dv = \tfrac{1}{2}\Gamma(m/2)\pi^{-m/2}|P\Omega(\theta)P'|^{-1/2} a(\theta)^{-m/2} dv \tag{3}$$

where

$$\begin{aligned} a(\theta) &= v'(P\Omega(\theta)P')^{-1}v \\ &= \hat{u}'\Omega(\theta)^{-1}\hat{u}/z'z \quad , \end{aligned}$$

$\hat{u}$ is the generalized least squares (GLS) residual vector assuming covariance matrix $\sigma^2\Omega(\theta)$ and dv denotes the uniform measure on the surface of the unit m-sphere. Also note that from Tunnicliffe Wilson (1989), the marginal likelihood for θ can be written as

$$f_m(\theta|y) = |\Omega(\theta)|^{-1/2} |X'\Omega(\theta)^{-1}X|^{-1/2} (\hat{u}'\Omega(\theta)^{-1}\hat{u})^{-m/2} \tag{4}$$

and Ara and King (1993) have shown that as likelihoods of θ, (3) and (4) are equivalent.

By invariance arguments, our testing problem can be reduced to one of testing H_0 against H_a based on v with density (3) as the observed data. Equivalently in this case we could choose to base our inferences about θ on the marginal likelihood (4). We call this the MIML approach because (3) and (4) are equivalent likelihoods. In our case, θ_1 is a nuisance parameter vector which we have been unable to eliminate through either invariance or marginal likelihood arguments.

Let $\hat{\theta}$ denote the unrestricted maximum MIML estimator of θ; i.e., that value of θ that maximizes (3) (or equivalently (4)). Let $\tilde{\theta}$ denote the maximum MIML estimate of θ under the restriction that $\theta_2 = 0$. Thus $\tilde{\theta} = (\tilde{\theta}_1{}', 0')$. The MIML-based LR test of H_o against $H_a{:}\theta_2 \neq 0$ rejects H_0 for large values of

$$\log\left\{\frac{|\Omega(\tilde{\theta})||X'\Omega(\tilde{\theta})X|}{|\Omega(\hat{\theta})||X'\Omega(\hat{\theta})X|}\right\} + m\log\left\{\frac{\tilde{u}'\Omega(\tilde{\theta})^{-1}\tilde{u}}{\hat{u}'\Omega(\hat{\theta})^{-1}\hat{u}}\right\} \tag{5}$$

where $\tilde{u}$ is the GLS residual vector assuming $\theta = \tilde{\theta}$, and $\hat{u}$ is now the GLS residual vector for $\theta = \hat{\theta}$. (5) can also be written as

$$m\log\frac{\sum_{i=1}^{n}\left(\tilde{e}_i |H(\tilde{\theta})|^{-1/m} |\tilde{X}^{*\prime}\tilde{X}^{*}|^{1/2m}\right)^2}{\sum_{i=1}^{n}\left(\hat{e}_i |H(\hat{\theta})|^{-1/m} |\hat{X}^{*\prime}\hat{X}^{*}|^{1/2m}\right)^2} \tag{6}$$

where $H(\theta)$ is the Cholesky decomposition matrix of $\Omega(\theta)$, i.e. $H(\theta)'H(\theta) = \Omega(\theta)^{-1}$, $\tilde{X}^* = H(\tilde{\theta})X$, $\hat{X}^* = H(\hat{\theta})X$, and $\hat{e}$ and $\tilde{e}$ are the OLS residual vectors from the transformed regression

$$H(\theta)y = H(\theta)X\beta + H(\theta)u$$

with $\theta = \hat{\theta}$ and $\theta = \tilde{\theta}$, respectively.

In order to construct the W test we need to partition the information matrix $I(\theta)$, whose $(i,j)^{th}$ element is simplified from Ara and King (1993) as

$$I(\theta)_{ij} = \frac{1}{2(m+2)}\left[m\,\mathrm{tr}\left\{M(\theta)*D(\theta)_i\, M(\theta)*D(\theta)_j\right\} - \mathrm{tr}\left\{M(\theta)*D_i(\theta)\right\}\mathrm{tr}\left\{M(\theta)*D_j(\theta)\right\}\right] \tag{7}$$

where $\quad M(\theta)^* = I - \Omega(\theta)^{-1}X(X'\Omega(\theta)^{-1}X)^{-1}X'$

and $$D(\theta)_i = \frac{\partial\Omega(\theta)^{-1}}{\partial\theta_i}\Omega(\theta) = -\Omega(\theta)^{-1}\frac{\partial\Omega(\theta)}{\partial\theta_i} .$$

The submatrices are defined as

$$I(\theta) = \begin{bmatrix} I(\theta)_{11} & I(\theta)_{12} \\ I(\theta)_{21} & I(\theta)_{22} \end{bmatrix}, \tag{8}$$

where $I(\theta)_{11}$, $I(\theta)_{12}$ and $I(\theta)_{22}$ are $p_1 \times p_1$, $p_1 \times (p-p_1)$ and $(p-p_1)\times(p-p_1)$, respectively.

The MIML-based W test of H_0 against $H_a{:}\theta_2 \neq 0$ rejects H_0 for large values of

$$\hat{\theta}_2'\left[I(\hat{\theta})_{22} - I(\hat{\theta})_{21} I(\hat{\theta})_{11}^{-1} I(\hat{\theta})_{12}\right]\hat{\theta}_2, \tag{9}$$

where $\hat{\theta}_2$ is the lower $(p-p_1)\times 1$ subvector of $\hat{\theta}$. One further simplification is possible when $I(\hat{\theta})_{12} = I(\hat{\theta})_{21} = 0$ and the information matrix is block diagonal as then the W test statistic becomes

$$\hat{\theta}_2' I(\hat{\theta})_{22}\hat{\theta}_2 .$$

The construction of the LM and King and Wu's (1990) ALMMP tests needs the calculation of the score subvector and the partitions of the information submatrices under the null. Let $s(\tilde{\theta})$ denote the $(p-p_1)\times 1$ vector of scores with respect to the elements of θ_2 evaluated at $\theta_1 = \tilde{\theta}_1$ and $\theta_2 = 0$. Thus the i^{th} element of the score is

$$s(\tilde{\theta})_i = -\frac{m}{2}\left[\tilde{u}'\frac{\partial\Omega(\tilde{\theta})^{-1}}{\partial\theta_i}\tilde{u} \Big/ \tilde{u}'\Omega(\tilde{\theta})^{-1}\tilde{u}\right] - \frac{1}{2}\operatorname{tr}\left[\Delta(\tilde{\theta})\frac{\partial\Omega(\tilde{\theta})}{\partial\theta_i}\right], \tag{10}$$

where $i = p_1+1,\ldots,p$ and $\Delta(\tilde{\theta}) = \Omega(\tilde{\theta})^{-1} - \Omega(\tilde{\theta})^{-1}X(X'\Omega(\tilde{\theta})^{-1}X)^{-1}X'\Omega(\tilde{\theta})^{-1}$.

$s(\tilde{\theta})_i$ can again be simplified as

$$s(\tilde{\theta})_i = \frac{1}{2}\operatorname{tr}\left[M(\tilde{\theta}) * D(\tilde{\theta})_i\right] - \frac{m}{2}\left[\tilde{u}'\frac{\partial\Omega(\tilde{\theta})^{-1}}{\partial\theta_i}\tilde{u} \Big/ \tilde{u}'\Omega(\tilde{\theta})^{-1}\tilde{u}\right],$$

where $M(\tilde{\theta})^*$ and $D(\tilde{\theta})_i$ are defined as in (7) with θ replaced by $\theta = \tilde{\theta}$.

Let $I(\tilde{\theta})_{ij}$ denote the ij^{th} element of the $p \times p$ information matrix defined by (7) evaluated at $\theta_2 = 0$ and $\theta_1 = \tilde{\theta}_1$. We partition the information matrix $I(\tilde{\theta})_{ij}$ as in (8) and evaluate it at $\theta = \tilde{\theta}$. The LM test of H_0 against $H_a\!:\!\theta_2 \neq 0$ rejects H_0 for large values of

$$s(\tilde{\theta})'\left[I(\tilde{\theta})_{22} - I(\tilde{\theta})_{21} I(\tilde{\theta})_{11}^{-1} I(\tilde{\theta})_{12}\right]^{-1} s(\tilde{\theta}). \tag{11}$$

If the information matrix is block diagonal, (11) simplifies to

$$s(\tilde{\theta})' I(\tilde{\theta})_{22}^{-1} s(\tilde{\theta}) \ .$$

In the case of testing H_0 against H_a^+, we can construct an ALMMP test. This test rejects H_0 for large values of

$$\sum_{i=p_1+1}^{p} s(\tilde{\theta})_i \Big/ \left\{\ell' I(\tilde{\theta})_{22}\ell - \ell' I(\tilde{\theta})_{21} I(\tilde{\theta})_{11}^{-1} I(\tilde{\theta})_{12}\ell\right\}^{1/2}, \tag{12}$$

where ℓ is the $(p-p_1)\times 1$ vector of ones. If the information matrix is block diagonal, the denominator becomes

$$\frac{1}{2(m+2)}\sum_{i=p_1+1}^{p}\sum_{j=p_1+1}^{p}\left[m\,\mathrm{tr}\left\{M(\tilde{\theta})^* D_i(\tilde{\theta}) M(\tilde{\theta})^* D_j(\tilde{\theta})\right\} - \mathrm{tr}\left\{M(\tilde{\theta})^* D_i(\tilde{\theta})\right\}\mathrm{tr}\left\{M(\tilde{\theta})^* D_j(\tilde{\theta})\right\}\right].$$

3. Testing for AR(4) Disturbances in the Presence of AR(1) Disturbances

As noted by King (1989), the presence of first order autocorrelation in a quarterly regression model is a good reason to suspect additional high order seasonal autocorrelation. Indeed, the omission of relevant variables with seasonal components might lead to higher order effects in addition to first order autoregression in the disturbances. Thus it makes sense to test for a higher order autoregressive process in the disturbances when first order autocorrelation is present. Godfrey (1978a, 1978b) recommended the use of the LM test to test for higher order AR models. He observed that error processes are often modelled by low order autoregressive schemes which may in some cases be inappropriate. It is therefore important to be able to check the consistency of the error structure with the sample data and to check that there is no significant additional autocorrelation in the residuals.

Consider the linear regression model with the disturbances generated by the stationary AR(4) process

$$u_t = \theta_1 u_{t-1} + \theta_2 u_{t-2} + \theta_3 u_{t-3} + \theta_4 u_{t-4} + \varepsilon_t$$

where $\varepsilon_t \sim IN(0,\sigma^2)$. The $\Omega(\theta)$ matrix is defined by

$$\Omega(\theta) = \left[L_4' L_4 - NN'\right]^{-1}$$

(see Ljung and Box, 1979, or van der Leeuw, 1994) in which L_4 is the $n \times n$ matrix

$$L_4 = \begin{bmatrix} 1 & 0 & & & 0 & 0 \\ -\theta_1 & 1 & & & & 0 \\ \vdots & & & & & \\ -\theta_4 & -\theta_3 & & & & \\ 0 & -\theta_4 & & & & \\ \vdots & & \ddots & & 1 & 0 \\ 0 & 0 & -\theta_4 & \cdots & -\theta_1 & 1 \end{bmatrix}$$

and N is the $n \times 4$ matrix of zeros but with the top 4×4 block being

$$\begin{bmatrix} -\theta_4 & -\theta_3 & -\theta_2 & -\theta_1 \\ 0 & -\theta_4 & -\theta_3 & -\theta_2 \\ 0 & 0 & -\theta_4 & -\theta_3 \\ 0 & 0 & 0 & -\theta_4 \end{bmatrix}.$$

The Cholesky decomposition matrix $H(\theta)$ is an $n \times n$ lower triangular matrix

$$H(\theta)=\begin{bmatrix} h_{11} & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ h_{21} & h_{22} & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ h_{31} & h_{32} & h_{33} & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ h_{41} & h_{42} & h_{43} & h_{44} & 0 & \cdot & \cdot & \cdot & 0 \\ -\theta_4 & -\theta_3 & -\theta_2 & -\theta_1 & 1 & 0 & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & & \cdot & & & \\ \cdot & & \cdot & \cdot & & & \cdot & & \\ \cdot & & & \cdot & & & & 1 & 0 \\ 0 & \cdot & \cdot & 0 & -\theta_4 & -\theta_3 & -\theta_2 & -\theta_1 & 1 \end{bmatrix} \tag{13}$$

where $h_{44}=(1-\theta_4^2)^{1/2}$,

$$h_{4,4-i}=\frac{(-\theta_i-\theta_{4-i}\theta_4)}{h_{44}} \qquad i=1,2,3,$$

$$h_{33}=(1+\theta_1^2-\theta_3^2-\theta_4^2-h_{43}^2)^{1/2},$$

$$h_{22}=(1+\theta_1^2-\theta_3^2-\theta_4^2-h_{32}^2-h_{42}^2)^{1/2},$$

$$h_{32}=\frac{\left(-\theta_1-\theta_1\theta_2-\sum_{i=2}^{3}\theta_i\theta_{i+1}-h_{43}h_{42}\right)}{h_{33}},$$

$$h_{31}=(-\theta_2-\theta_2\theta_4-h_{43}h_{41}),$$

$$h_{21}=\frac{(-\theta_1-\theta_3\theta_4-h_{32}h_{31}-h_{42}h_{41})}{h_{22}},$$

and $$h_{11}=\left(1-\theta_4^2-\sum_{i=1}^{3}h_{i+1,1}^2\right)^{1/2}.$$

The matrix of first derivatives is

$$\frac{\partial\Omega(\theta)^{-1}}{\partial\theta_i}=\big(B_i(\theta)+B_i{}'(\theta)\big)-\big(C_i(\theta)+C_i{}'(\theta)\big) \tag{14}$$

where $B_i(\theta)$ is the $n\times n$ matrix whose left $n\times(n-i)$ matrix is

$$\begin{bmatrix} \theta_i & \theta_{i+1} & \cdots & \theta_4 & 0 & \cdots & 0 \\ \theta_{i-1} & \theta_i & & & & & \cdot \\ \cdot & & \cdot & & & & 0 \\ \theta_1 & & & \cdot & & & \theta_4 \\ -1 & \cdot & & & \cdot & & \cdot \\ 0 & & & & & \cdot & \cdot \\ \cdot & & & & & & \theta_i \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & -1 & \theta_1 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & -1 \end{bmatrix} .$$

and the remaining i columns are zeros and $C_i(\theta)$ is the $n \times n$ matrix of zeros whose top left $i \times 4$ block is identical to the top left $i \times 4$ block of $B_i(\theta)$.

We are interested in testing $H_0{:}\theta_2 = \theta_3 = \theta_4 = 0$ against the alternative that at least one θ_i is non-zero. Under the null hypothesis, u_t follows a stationary AR(1) process, i.e.,

$$u_t = \theta_1 u_{t-1} + \varepsilon_t, \qquad |\theta_1| < 1, \qquad \varepsilon_t \sim IN(0, \sigma^2).$$

In this case

$$\Omega(\theta) = \frac{1}{1-\theta_1^2} \begin{bmatrix} 1 & \theta_1 & \theta_1^2 & \cdot & \cdot & \cdot & \theta_1^{n-1} \\ \theta_1 & 1 & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & \cdot & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot & \theta_1 \\ \theta_1^{n-1} & \cdot & \cdot & \cdot & \cdot & \theta_1 & 1 \end{bmatrix} \tag{15}$$

$$\text{and } H(\theta) = \begin{bmatrix} \sqrt{1-\theta_1^2} & 0 & \cdot & \cdot & \cdot & 0 \\ -\theta_1 & 1 & \cdot & & & \cdot \\ 0 & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & -\theta_1 & 1 \end{bmatrix} . \tag{16}$$

The LR test statistic will be defined as (5) or (6) with $\Omega(\tilde{\theta})$ and $H(\tilde{\theta})$ given by (15) and (16) in which $\theta = \tilde{\theta}$. The W test statistic is (9) with the partitioned matrices defined as in (8) and the elements of the information matrix $I(\hat{\theta})$ given by (7) evaluated at $\theta = \hat{\theta}$. The LM and the ALMMP test statistics will be defined as (11) and (12), respectively, when $\Omega(\tilde{\theta})$ and $H(\tilde{\theta})$ are determined by (15) and (16), respectively, with $\theta = \tilde{\theta}$. The $\frac{\partial \Omega(\tilde{\theta})^{-1}}{\partial \theta_i}$ matrices are defined as in (14) evaluated at $\theta_1 = \tilde{\theta}_1$ and $\theta_2 = \theta_3 = \theta_4 = 0$.

4. Testing against a Rosenberg Coefficient in the Presence of a Hildreth-Houck Random Coefficient

In this section we consider the problem of testing a single time varying coefficient in the linear regression model. The null hypothesis is that the coefficient follows the Hildreth-Houck (1968) random coefficient (HRC) model and the alternative is that the coefficient follows Rosenberg's (1973) return to normalcy (RRN) model. The latter assumes the coefficient follows an AR(1) process while the former assumes it is independently distributed about a mean value. Thus the HRC model can be viewed as a special case of the RRN model.

Bos and Newbold (1984) considered this testing problem in their empirical investigation of systematic risk in the market model. They applied classical likelihood based LR and W tests and conjectured that these tests lacked power. For this reason, Brooks and King (1994) suggested the use of the APOI test and compared its small sample properties with those of the classical LR and W tests. These studies indicate that classical likelihood based tests cannot be relied upon to have good small sample properties. Below we discuss the construction of MIML based LR, W, LM and ALMMP tests for this testing problem. In the following section we report the results of a Monte Carlo study conducted to investigate the small sample properties of these tests.

Consider the linear regression model with a single varying coefficient, α_t,

$$y_t = \alpha_t x_t + z_t'\beta + \varepsilon_t, \quad \varepsilon_t \sim IN(0,\sigma^2), \qquad t = 1,\ldots,n, \tag{17}$$

where x_t is a scalar regressor, z_t is a $k \times 1$ vector of k non-stochastic regressors, β is a $k \times 1$ vector of unknown constant coefficients and ε_t is the disturbance term. If α_t is a HRC then

$$\alpha_t = \bar{\alpha} + a_t \tag{18}$$

with $$a_t \sim IN(0,\theta_1\sigma^2) \tag{19}$$

and a_t is independent of ε_t. The economic interpretation of α_t is that it has an instantaneous mean reversion property so that the effect of any shock on the coefficient does not carry over to future periods.

Alternatively, if α_t follows the RRN model then

$$\alpha_t - \bar{\alpha} = \theta_2(\alpha_{t-1} - \bar{\alpha}) + a_t, \tag{20}$$

where a_t is generated as (19) and is independent of ε_t. In this case we have an additional parameter θ_2 and α_t follows an AR(1) process. For the process to be stationary, θ_2 must lie between -1 and +1. From an economic point of view, a more meaningful restriction is $0 \le \theta_2 \le 1$ which implies smooth evolution of the coefficient over time. Also a negative value for θ_2 causes considerable difficulties in interpretation particularly when the time interval is reduced. The economic interpretation of the RRN model is that α_t still possesses a mean reversion property but it is not instantaneous. The speed of mean reversion depends on the value of the AR(1) parameter θ_2. The greater the speed of mean reversion, the smaller is the value of θ_2.

Under either (18) or (20), the model (17) can be written as

$$y_t = x_t\bar{\alpha} + z_t'\beta + w_t$$

where w_t is normally distributed with mean zero and the second order moments are determined by the α_t process. If α_t is generated by (18) and (19) then

$$\mathrm{var}(w_t) = \sigma^2(1+\theta_1 x_t^2)$$

$$\mathrm{cov}(w_t w_s) = 0 \qquad \text{for } t \neq s$$

and if α_t is generated by (19) and (20) then

$$\mathrm{var}(w_t) = \sigma^2\left(1+\frac{\theta_1 x_t^2}{1-\theta_2^2}\right)$$

$$\mathrm{cov}(w_t w_s) = \frac{\sigma^2\theta_1 x_t x_s \theta_2^{|t-s|}}{(1-\theta_2^2)} \qquad \text{for } t \neq s.$$

Our problem is one of testing $H_0{:}\theta_2 = 0$ against $H_a^+{:}\theta_2 > 0$.

The construction of the MIML based LR test requires the Cholesky decomposition matrix $H(\theta)$ to be constructed for both $\theta = \tilde{\theta}$ and $\theta = \hat{\theta}$. Following Brooks (1993), we define $H(\theta)$ as

$$H(\theta) = \bar{L}^{-1}T_1T_2$$

with $$\bar{L}_{11} = \left[\frac{1}{x_1^2} + \frac{\theta_1}{(1-\theta_2^2)}\right]^{1/2},$$

$$\bar{L}_{tt} = \left(\frac{\theta_2^2}{x_{t-1}^2} + \frac{1}{x_t^2} + \theta_1 - \bar{L}_{t,t-1}^2 \right)^{1/2},$$

$$\bar{L}_{t,(t-1)} = -\frac{\theta_2}{\left(x_{t-1}^2 \bar{L}_{t-1,t-1}\right)},$$

and remaining $\bar{L}_{ij}$ values being zero,

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & \cdot & \cdot & 0 \\ -\theta_2 & 1 & 0 & \cdot & \cdot & 0 \\ 0 & -\theta_2 & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & -\theta_2 & 1 \end{bmatrix},$$

$$T_2 = \begin{bmatrix} x_1^{-1} & 0 & \cdot & \cdot & 0 \\ 0 & x_2^{-1} & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & x_n^{-1} \end{bmatrix}.$$

By multiplying the above three matrices, we find that the $H(\theta)$ matrix has the form of a lower triangular matrix with elements

$$H(\theta)_{tt} = \frac{\ddot{A}_{tt}}{x_t}$$

$$H(\theta)_{ts} = \left(\ddot{A}_{ts} - \theta_2 \ddot{A}_{t,s+1}\right), \qquad s = 1,2,\ldots,(t-1) \qquad (21)$$

with $$\ddot{A}_{tt} = \frac{1}{\bar{L}_{tt}}$$

$$\ddot{A}_{ts} = \frac{\left[\bar{L}_{t,t-1} \ddot{A}_{t-1,s}\right]}{\bar{L}_{tt}}; \qquad s = 1,2,\ldots,(t-1).$$

The matrix $H(\hat{\theta})$ can then be constructed from $H(\theta)$ by evaluating at $\theta = \hat{\theta}$. Similarly, $H(\widetilde{\theta})$ can be constructed using $\theta_1 = \widetilde{\theta}_1$ and $\theta_2 = 0$. The MIML based LR test statistic will then follow from (5) or (6).

The calculation of the MIML based W test needs the construction of the first derivative matrix $\frac{\partial \Omega(\hat{\theta})}{\partial \theta_i}$, as well as $H(\hat{\theta})$. The elements of the $\frac{\partial \Omega(\theta)}{\partial \theta_i}$ matrix are defined as

$$\frac{\partial \Omega(\theta)_{tt}}{\partial \theta_1} = \frac{x_t^2}{(1-\theta_2^2)}$$

$$\frac{\partial \Omega(\theta)_{tt}}{\partial \theta_2} = \frac{2\theta_1 x_t^2 \theta_2}{(1-\theta_2^2)^2}$$

$$\frac{\partial \Omega(\theta)_{ts}}{\partial \theta_1} = \frac{x_t x_s \theta_2^{|t-s|}}{(1-\theta_2^2)}$$

$$\frac{\partial \Omega(\theta)_{ts}}{\partial \theta_2} = \frac{\theta_1 x_t x_s}{(1-\theta_2^2)^2}\left[(t-s)\theta_2^{(t-s)-1} - (|t-s|-2)\theta_2^{(t-s)+1}\right] \qquad \text{for } t > s. \qquad (22)$$

These elements of the $\frac{\partial \Omega(\theta)}{\partial \theta_i}$ matrix are then evaluated at $\theta_1 = \hat{\theta}_1$ and $\theta_2 = \hat{\theta}_2$, allowing the MIML based W test statistic to be calculated from (9).

For the LM and ALMMP tests we need to construct the $H(\widetilde{\theta})$ and $\frac{\partial \Omega(\widetilde{\theta})^{-1}}{\partial \theta_i}$ matrices. The elements of the $\frac{\partial \Omega(\widetilde{\theta})}{\partial \theta_i}$ matrices are obtained from (22) as follows

$$\frac{\partial \Omega(\widetilde{\theta})_{tt}}{\partial \theta_1} = x_t^2,$$

$$\frac{\partial \Omega(\widetilde{\theta})_{tt}}{\partial \theta_2} = 0,$$

$$\frac{\partial\Omega(\tilde{\theta})_{ts}}{\partial\theta_1} = 0 \qquad \text{for } t \neq s$$

$$\frac{\partial\Omega(\tilde{\theta})_{ts}}{\partial\theta_2} = \tilde{\theta}_1 x_t x_s, \qquad \text{for } |t-s| = 1,$$

$$= 0, \qquad \text{for } |t-s| > 1.$$

The elements of the $\frac{\partial\Omega(\tilde{\theta})^{-1}}{\partial\theta_i}$ matrices are then obtained using the relationship

$$\frac{\partial\Omega(\tilde{\theta})^{-1}}{\partial\theta_i} = -\Omega(\tilde{\theta})^{-1}\frac{\partial\Omega(\tilde{\theta})}{\partial\theta_i}\Omega(\tilde{\theta})^{-1}$$

and $$\Omega(\tilde{\theta})^{-1}_{tt} = \frac{1}{(1+\tilde{\theta}_1 x_t^2)}$$

$$\Omega(\tilde{\theta})^{-1}_{ts} = 0, \quad \text{for } t \neq s.$$

The MIML based LM test statistic is therefore obtained using equation (11). Note here that the ALMMP test statistic is the square root of the LM test statistic in this case as θ_2 is a scalar parameter. The asymptotic null distribution of the LM statistic is χ^2 with one degree of freedom and the asymptotic null distribution of the ALMMP test statistic is $N(0,1)$.

The LR and W tests are based on MIML estimates of θ_2 under the constraint $\theta_2 > 0$. These tests are now one-sided versions of the original LR and W tests and their asymptotic null distributions are probability mixtures of chi-square distributions, i.e.,

$$\frac{1}{2}\chi^2_{(0)} + \frac{1}{2}\chi^2_{(1)}.$$

See Gourieroux, Holly and Monfort (1980) and also Wu and King (1994) for more details about one sided LR and W tests. The critical region of the LR test at level α

will therefore be of the form $\mathrm{LR} > c$, when c is defined by $\Pr[\mathrm{LR} > c | H_0] = \alpha$. To obtain the required asymptotic size, we therefore use the $\chi^2_{(1)}$ critical value at the 2α level of significance.

5. Monte Carlo Size and Power Comparisons

In order to explore the small-sample size and power properties of the MIML based tests, we conducted two Monte Carlo experiments. The first experiment concentrated on the problem of testing for general AR(4) disturbances in the presence of AR(1) disturbances as outlined in section 3. The size and powers of the MIML based tests, denoted by MLR, MW and MLM, were compared with those of their classical likelihood counterparts, namely the LR, W and LM tests. The second experiment concentrated on the problem of testing for a Rosenberg coefficient in the presence of a Hildreth-Houck random coefficient as outlined in section 4. As it is a one sided testing problem, we have also included the MIML based ALMMP test (MALMMP) and the classical likelihood based ALMMP tests for comparison along with those tests of the first experiment.

5.1 Experimental Design

The following $n \times k$ X matrices were chosen for the data generation process:

X1: $(n \times 2)$ A constant and a linear time trend. The time trend is the regressor with the varying coefficient.

X2: $(n \times 4)$ A constant and three quarterly seasonal dummy variables. This data set was used only for the first experiment.

X3: $(n \times 3)$ A constant, the quarterly seasonally adjusted Australian household disposable income and private consumption expenditure series, commencing 1959(4). The consumption series which is lagged one quarter, is the regressor with the varying coefficient.

X4: $(n \times 3)$ A constant, quarterly Australian private capital movements and Government capital movements commencing 1968(1). The latter is the regressor with the varying coefficient. For the first experiment, the two additional regressors are the two variables lagged one quarter.

X5: $(n \times 6)$ A constant, quarterly Australian private capital movements and Government capital movements commencing 1968(1) and a full complement of quarterly seasonal dummies.

For testing the presence of general AR(4) disturbances, size and powers were estimated for X1, X2, X3 and X4 with k = 5. For testing against the Rosenberg coefficient, the data matrices used were X1, X3, X4 and X5.

It is important to note here that the Hildreth-Houck parameter θ_1 is the ratio of the random coefficient disturbance variance to the regression disturbance variance. Its contribution to the variance of the composite disturbance w_t depends on the scale of all the regressors. We transform x_t $(t = 1,2,\ldots,n)$ to $\ddot{x}_t$ using the equation

$$\ddot{x}_t = \frac{x_t - \min(x_t)}{\max(x_t) - \min(x_t)} + 1$$

where $\min(x_t)$ and $\max(x_t)$ are the minimum and maximum of the x_t series respectively. The testing problem is unchanged by this kind of transformation in the x_t's. The variance of w_t under the null hypothesis becomes

$$\sigma_t^2 = \sigma^2(1+\theta_1 \ddot{x}_t^2)$$

so that $$\Omega(\theta)_{tt} = (1+\theta_1 \ddot{x}_t^2),$$

and under the alternative hypothesis,

$$\Omega(\theta)_{tt} = 1 + \frac{\theta_1 \ddot{x}_t^2}{(1-\theta_2^2)}$$

and $$\Omega(\theta)_{ts} = \frac{\theta_1 \ddot{x}_t \ddot{x}_s \theta_2^{|t-s|}}{(1-\theta_2^2)} \qquad \text{for } t \neq s.$$

It is now possible to examine the coefficient of variation in σ_t^2 under the null hypothesis for different values of θ_1. This then allows us to choose reasonable bounds on the θ_1 values according to the coefficient of variation values. For more details about the choice of θ_1 bounds, see Evans and King (1985).

For each testing problem, the experiment was conducted in two parts. The first part of the study involved a comparison of estimated sizes using asymptotic critical values. As sizes vary for different values of θ_1, we estimated sizes for a range of values of θ_1. For the first problem, the values used were $\theta_1 = 0, 0.1, 0.2, \ldots, 0.9$. For the second problem, the θ_1 values were chosen according to the coefficient of variation in σ_t^2 and so that the chosen values reflect a range of coefficient of variation in σ_t^2. These are $\theta_1 = .001, .05, .2, .5, 1, 3, 7, 30, 100, 200$.

The second part involved the use of Monte Carlo methods to estimate five percent critical values for each of the tests at each of the θ_1 values. Note here that the size of each test is a function of θ_1 and may not be constant over the null parameter space. Thus the maximum size may be greater than the nominal significance level. We therefore controlled the maximum probability of a type 1 error by choosing the highest critical value. For each test the largest critical value was used to calculate exact powers. For the first testing problem, powers were calculated at the following $(\theta_1, \theta_2, \theta_3, \theta_4)$ parameter combinations:

(0.0, 0.0, 0.0, 0.0), (0.0, 0.5, 0.0, 0.0), (0.2, 0.6, 0.0, 0.0),

(0.4, 0.0, 0.0, 0.4), (0.3, 0.2, 0.0, 0.0), (0.3, 0.2, 0.2, 0.0),

(0.3, 0.2, 0.2, 0.2), (0.2, 0.5, 0.1, 0.1), (0.1, -0.3, 0.0, 0.3),

(0.1, 0.4, -0.2, 0.0).

For the second testing problem, the following (θ_1, θ_2) parameter combinations were used:

(0.5, 0.3), (0.5, 0.5) (0.5, 0.8), (3, 0.3), (3, 0.5),

(3, 0.8), (30, 0.3), (30, 0.5) and (30, 0.8).

A nominal significance level of five percent and 1000 replications were used throughout. The two different sample sizes used were 30 and 60. All tests are invariant to β and σ^2 and therefore these parameter values were set to one in the simulations. The IMSL subordinates DBCLSF, DUMPOL and DBCPOL were used to maximise the likelihood functions.

5.2 Size Results

Tables 1 and 2 report the estimated sizes of the six tests against AR(4) disturbances in the presence of AR(1) disturbances when asymptotic critical values at the five percent nominal level were used. The corresponding estimated sizes of the eight tests for a Rosenberg coefficient are presented in tables 3 and 4. In both cases, sizes are estimated for each value of θ_1.

Tables 1 and 2 reveal that all estimated sizes of the classical likelihood based LR and W tests for AR(4) disturbances in presence of AR(1) disturbances are significantly above 0.05. This is true for all X matrices and both sample sizes. In particular, the sizes of the W test are very high and clearly unreliable. However, there is a clear sign of improvement in size as n increases from 30 to 60. The LM test has acceptable sizes except for higher values of θ_1. The sizes tend to become significantly above 0.05 at the upper boundary of θ_1 values. For example, in the case of design matrix X2, the estimated sizes of the LM test are significantly above 0.05 when θ_1 = 0.7, 0.8 and 0.9. The sizes decreased with the increase of n from 30 to 60 for data matrices X1 and X2. For X4, sizes increase with the increase in sample size and for X3, sizes increase except when $\theta_1 = 0.3$, 0.7, 0.8 and 0.9.

The improvement in estimated sizes when the marginal likelihood method is applied is remarkable both in the case of the MLR and MW tests. The most reliable test seems to be the MLR test. Its estimated sizes are much closer to 0.05 compared to those of the LR test and significantly higher than 0.05 only for the data matrix X4 (n = 30) with θ_1 = 0.4, 0.5, and 0.6. However, this is not the case when n is increased from 30 to 60. The sizes are closer to 0.05 for the X3 and X4 matrices with the increase of n from 30 to 60, the only exception being X3 with $\theta_1 = 0.9$. For X1 and X2, sizes are slightly reduced with the increase in n, but show a better approximation to the desired size near the boundary of θ_1.

There is a large improvement in the sizes of the MW test compared to those of the W test, although they are still significantly above 0.05 in more than half of the cases. For the data matrices X1, X2 and X3 with θ_1 = 0.0, 0.1, 0.2 and 0.3 the differences between the estimated sizes and nominal size of the MW test are not significant. The behaviour of the MW test mentioned in Ara and King (1993) seems slightly improved, possibly because of the use of MIML estimates of θ_1.

The LM test seems to have better estimated sizes compared to the MLM test for most θ_1 values. However, the sizes of MLM test are less variable compared to the LM test and particularly improved at the upper bound of θ_1 values, i.e., at $\theta_1 = 0.7$, 0.8 and 0.9. The estimated sizes of the MLM test are not significantly different from 0.05 with the only exceptions occurring at $n = 30$ and $\theta_1 = 0.9$ for the X1 and X2 matrices and at $n = 60$ and $\theta_1 = 0.2$, 0.3, 0.4, 0.5, 0.6 for the X2 matrix. Based on actual size being the maximum size, the MLM test is clearly better than the LM test. Our results regarding the LM and MLM tests are consistent with those reported by Rahman and King (1993).

Tables 3 and 4 report the estimated sizes of eight tests against a Rosenberg coefficient in the presence of a Hildreth-Houck random coefficient in the linear regression model. Asymptotic critical values at the 5% nominal level were used for the LM, MLM, ALMMP and MALMMP tests. Asymptotic critical values of the $\chi^2_{(1)}$ distribution at the 10% nominal level are used for the LR, MLR, W and MW tests as the asymptotic null distributions of their test statistics are mixtures of $\chi^2_{(0)}$ and $\chi^2_{(1)}$ distributions as discussed at the end of section 4.

Most of the estimated sizes of the LR test are significantly below 0.05, the few exceptions are for X5 and X4 with $n = 30$ and for X1 and X5 with $n = 60$ for the upper half of θ_1 values. In contrast, in the case of the MLR test, typically there is no significant difference between the estimated sizes and the nominal size. The sizes are overestimated only for X4 ($n = 30$) and for X1 ($n = 60$) when $\theta_1 = 3, 7$.

The estimated sizes of the W test are mostly significantly below 0.05 at the 1% level of significance, the only exceptions are X5 ($n = 30$) and X4 ($n = 30$). Our results regarding low sizes of the LR and W tests are consistent with the findings of Bos and Newbold (1984) and Brooks and King (1994). The sizes improve everywhere except for X5 ($n = 30$) when the MW test is used. The sizes for X5 improve when the sample size increased to 60, because then the sizes of the W test are significantly different from 0.05. The improvement in sizes of the MW test are significant when the sample size is 60 for all X matrices and also for X4 when $n = 30$. For X4 ($n = 60$), the MW test seems to overestimate the sizes for lower half of the θ_1 range.

The sizes of the LM test increase with the increase in sample size. There appears to be a tendency for the sizes to be slightly above 0.05 and they are significantly above 0.05 for most of the θ_1 values in the cases of X3 and X5 with $n = 60$. In contrast, the sizes of the MLM test are not significantly different from 0.05. For the X1 data matrix, both the LM and MLM tests show similar size behaviour. For the X3 and X4 matrices, LM test sizes are closer to 0.05 when $n = 30$, but when $n = 60$ LM test sizes became significantly above 0.05 for most of the θ_1 values and the MLM test sizes are closer to 0.05. For the X5 matrix, the MLM test has better sizes except for the first two θ_1 values.

The ALMMP test sizes are mostly significantly below 0.05, the only exception occurs for X4 ($n = 30$) and for X1 ($n = 60$) when $\theta_1 = 3, 7, 30, 100, 200$. The sizes of the ALMMP test for X4 are significantly below 0.05 when $n = 60$. On the other hand, there is no significant difference between the estimated sizes of the MALMMP test and the nominal size except for X4 with $n = 30$.

For all data matrices except X4, the MALMMP test gives the most accurate sizes among all the tests, at least for the lower half of the θ_1 range. The LM test tends to have good size properties towards the lower bound of θ_1, whereas the MLM test tends to perform better towards the upper bound of θ_1, in which case the LM test gives significantly high sizes, the MLR test is the next best test in terms of size properties in general, but sometimes has more accurate sizes compared to the LM and MLM tests. The sizes of the MW test are also reasonably improved due to the use of marginal likelihood estimates of the nuisance parameter θ_1.

Hence among all the tests, the MLR test seems to have the best estimated sizes. At the upper bound of the θ_1 range, the LM test sizes are mostly significantly different from the nominal size. Based on actual sizes being the maximum size, the MLM test is clearly better than the LM test. Hence, the MLM test is the next best candidate. Only in the case of the W test, are the sizes significantly higher than the nominal size. However, the improvement in size after using the MIML approach is clear. In the one sided testing situation, the MALMMP test gives most accurate sizes among all the tests. This is also evident in the results reported by Ara and King (1993) and is not surprising because this test is particularly designed for one-sided testing problems. Overall it seems clear that the use of MIML improves the accuracy of the asymptotic critical values for all the tests although this improvement is not as clear cut as for testing against AR(4) disturbances.

5.3 Power Results

Estimated powers of the six tests for AR(4) disturbances in presence of AR(1) disturbances are presented in tables 5 and 6. Exact critical values have been computed at the 5% level using Monte Carlo simulation for each of the θ_1 values. The largest simulated critical value was then used to calculate power of each test thus ensuring that the maximum size is approximately 0.05, as mentioned in section 5.1. The tables show that the tests' powers increase with the increase in sample size. This is true for each data set.

Among the classical likelihood based tests, the LR test dominates the LM and W tests in general and the W test dominates the LM test. Exceptions occur on few occasions. The LM test dominates at the parameter combinations (0.1, -0.3, 0, 0.3) and (0.4, 0, 0, 0.4) for most X matrices. The W test dominates the LM test at few points on the alternative parameter space mostly when $n = 60$.

When the MIML approach is applied, powers of the MLR and MLM tests are increased everywhere in the alternative parameter space. This is true for all the X matrices and both sample sizes. In some cases it increases substantially, i.e., around 20% for the MLR test and 30% for the MLM test. The increase in average power is from 8% to 15% in both cases. The power of the MW test is also improved when the MIML approach is used with very few exceptions. The average increase is 2% to 8% in this case. It therefore appears that the power curves of the MIML based tests are higher than those of the classical likelihood based tests. Among the three tests, the MLR test has the highest power and the MLM test has the second highest power in general. The MLM test is the most powerful in very few cases and occasionally its power is slightly below that of the MW test.

We now discuss the estimated powers of the tests for a Rosenberg coefficient in the presence of a Hildreth-Houck random coefficient. The power results are presented in tables 7 and 8. The θ_1 values were chosen according to the degree of coefficient of variation in the composite disturbance variance. For each θ_1 value, three different θ_2 values have been chosen to represent different degrees of autocorrelation in the alternative parameter space. The tables reveal that the powers of all the tests increase with increases in θ_1 and θ_2 values.

The LM test shows the lowest power among all the classical likelihood based tests. Power differences among the LR, W and ALMMP tests are, in most cases, less than 4%. The ALMMP test shows a slightly better power for half the regressors, i.e., for X1 (n = 60), X3 (n = 30) and X4 (n = 30, 60), otherwise no test dominates the others. The power differences of the LM test with the other three tests are sometimes more than 30%. Its power gets closer to those of the other tests only at parameter combinations (3, 0.8) and (30, 0.8) when $n = 60$.

When the MIML approach is used, the MLR test has better power than that of the LR test when n = 60 except at 3 points for X4 and at most of the points in the alternative parameter space when n = 30. The MW test performed relatively poorly compared to the W test. The difference in power is sometimes more than 15%. The findings in Brooks and King (1994) and Bos and Newbold (1984) also show that the LR test lacks in power, but the W test does not lack in power although its sizes are typically different from the nominal size. The average power difference between the

MALMMP test and ALMMP test is less than 2%. However, the MALMMP test performed slightly better except for X4 and X3 when n = 30. The improvement in power of the MLM test is rather noticeable. The power curves of the MLM test are clearly higher than those of the LM test. Power improvements are sometimes more than 20%. We note that Rahman and King (1993) reported a similar finding in their study.

Among all the tests, the MLR and MALMMP tests performed best in terms of power. The power advantage of the MLR test over MALMMP test is less than 4% and no one dominates the other on average.

6. Conclusion

This paper is concerned with testing the covariance matrix of the disturbances that involve nuisance parameters which cannot be eliminated by invariance arguments. We outlined the construction of MIML based LR, LM, W and ALMMP tests extending the work of Ara and King (1993) and Rahman and King (1993). The Monte Carlo experiment we report shows that the use of MIML based tests rather than their traditional counterparts does improve both size and power properties of the tests in finite samples. The level of improvement is higher than that reported by Ara and King when no nuisance parameters are present in the marginal likelihood. This is particularly evident from the ALMMP test. In the case of the ALMMP test, the sizes clearly improve and the power also improves slightly. This is in contrast to the results of Ara and King who report identical powers when simulated critical values are used. The additional improvement in our case seems to come from the use of maximum MIML estimates of the nuisance parameters which are more nearly unbiased than the classical maximum likelihood estimates. This is also true for the LR and LM tests. Both the size and power of the W test have improved when the MIML approach is used for testing the form of autocorrelation. Only in the case of the W test and testing for a Rosenberg coefficient, did we not see an improvement in power after using the MIML approach, although the sizes did improve. A possible reason could be that one sided testing is more likely to have asymptotic problems than two-sided testing. Power curves can be wrongly centered and recentering them can give rise to lower power in part of the parameter space and higher power elsewhere. It is a gamble.

All the evidence reported above strongly supports the MIML approach to test construction. It also suggests an important implication for econometricians who are forced to apply asymptotically based inference procedures to short data sets. It does seem that the small-sample properties of these procedures can be improved by better handling of any nuisance parameters. The full potential of this improvement for highly parameterised systems of equations that are common in econometrics has yet to be investigated.

References

Ara, I. and King, M.L. (1993) Marginal likelihood based tests of regression disturbances. Paper presented at the 1993 Australasian Meeting of the Econometric Society, Sydney.

Bos, T. and Newbold, P. (1984) An empirical investigation of the possiblity of stochastic systematic risk in the market model. *Journal of Business*, 57, 35-41.

Brooks, R.D. (1993) Testing varying coefficient models: Theory and applications, unpublished Ph.D. thesis, Department of Econometrics, Monash University.

Brooks, R.D. and King M.L. (1994) Testing Hildreth-Houck against return to normalcy random regression coefficients. *Journal of Quantitative Economics*, 10, 33-52.

Evans, M.A. and King, M.L. (1985) A point optimal test for heteroscedastic disturbances. *Journal of Econometrics*, 27, 163-178.

Godfrey, L.G. (1978a) Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables. *Econometrica*, 46, 1293-1302.

Godfrey, L.G. (1978b) Testing for higher order serial correlation in regression equations when the regressors include lagged dependent variables. *Econometrica*, 46, 1303-1310.

Godfrey, L.G. (1988) *Misspecification tests in econometrics : The Lagrange multiplier principle and other approaches*, (Cambridge University Press, Cambridge).

Gourieroux, C., Holly, A. and Monfort, A. (1980) Kuhn-Tucker, likelihood ratio and Wald tests for nonlinear models with inequality constraints on the parameters, Discussion Paper No. 770, *Harvard Institute of Economic Research*, Harvard University.

Hildreth, C. and Houck, J.P. (1968) Some estimators for a linear model with random coefficients. *Journal of the American Statistical Association*, 63, 584-595.

Honda, Y. (1988) A size correction to the Lagrange multiplier test for heteroscedasticity. *Journal of Econometrics*, 38, 375-386.

Judge, G.G., Griffiths, W.E., Hill, R.C., Lutkepohl, H. and Lee, T.C. (1985) *The theory and practice of econometrics, 2nd ed.*, John Wiley and Sons, New York.

King, M.L. (1980) Robust tests for spherical symmetry and their application to least squares regression. *Annals of Statistics* 8, 1265-1271.

King, M.L. (1987a) Testing for autocorrelation in linear regression models : A survey. In *Specification analysis in the linear model.* (eds. King, M.L and Giles, D.E.A.). Routledge and Kegan Paul, London, 19-73.

King, M.L. (1987b) Towards a theory of point optimal testing. *Econometric Reviews*, 6, 169-218.

King, M.L. (1989) Testing for fourth-order autocorrelation in regression disturbances when first-order autocorrelation is present. *Journal of Econometrics*, 41, 285-301.

King, M.L. and Wu, P.X. (1990) Locally optimal one-sided tests for multiparameter hypotheses. Presented at the Sixth World Congress of the Econometric Society, Barcelona, Working paper 2/90, Department of Econometrics, Monash University, Melbourne.

Ljung, G.M. and Box, G.E.P. (1979) The likelihood function of stationary autoregressive-moving average models. *Biometrika*, 66, 265-270.

Moulton, B.R. and Randolph, W.C. (1989) Alternative tests of the error components model. *Econometrica*, 57, 685-693.

Pagan, A.R. (1984) Model evaluation by variable addition. In *Econometrics and quantitative economics* (eds. Hendry, D.F. and Wallis, K.F.). Basil Blackwell, Oxford, 103-133.

Pagan, A.R. and Hall, A.D. (1983) Diagnostic tests as residual analysis, *Econometric Reviews* 2, 159-218.

Rahman, S. and King, M.L. (1993) Marginal likelihood score-based tests of regression disturbances in the presence of nuisance parameters. Mimeo, Monash University.

Rahman, S. and King, M.L. (1994) A comparison of marginal likelihood based and approximate point optimal tests for random regression coefficients in the presence of autocorrelation. *Pakistan Journal of Statistics*, 10, 375-394.

Rosenberg, B. (1973) The analysis of a cross-section of a time series by stochastically convergent parameter regression. *Annals of Economic and Social Measurement*, 2, 399-428.

Tunnicliffe Wilson, G. (1989) On the use of marginal likelihood in time series model estimation. *Journal of the Royal Statistical Society*, B, 51, 15-27.

van der Leeuw, J. (1994) The covariance matrix of ARMA-errors in closed form. *Journal of Econometrics*, 63, 397-405.

Wu, P.X. and King, M.L. (1994) One sided hypothesis testing in econometrics: A survey. *Pakistan Journal of Statistics*, 10, 261-300.

Table 1: Estimated **asymptotic sizes** of six tests of AR(4) disturbances in the presence of AR(1) disturbances using asymptotic critical values at the five percent level for design matrices **X1** and **X2**.

n	Test	θ_1									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
						X1					
30	LR	.105*	.101*	.104*	.101*	.101*	.102*	.104*	.109*	.105*	.107*
	MLR	.050	.051	.054	.054	.054	.053	.049	.049	.052	.056
	W	.168*	.169*	.172*	.171*	.179*	.189*	.187*	.190*	.197*	.208*
	MW	.074*	.079*	.077*	.076*	.073*	.073*	.075*	.078*	.081*	.077*
	LM	.049	.049	.051	.046	.054	.055	.055	.061	.075*	.087*
	MLM	.037	.037	.036	.037	.037	.039	.042	.046	.060	.073*
60	LR	.071*	.073*	.073*	.069*	.068*	.068*	.074*	.078*	.081*	.096*
	MLR	.046	.049	.049	.046	.045	.045	.044	.048	.050	.050
	W	.091*	.089*	.091*	.091*	.093*	.088*	.089*	.104*	.108*	.119*
	MW	.062	.064	.060	.059	.053	.056	.060	.054	.059	.064
	LM	.045	.044	.039	.044	.042	.048	.053	.058	.070*	.080*
	MLM	.040	.037	.038	.036	.034	.037	.041	.040	.042	.048
						X2					
30	LR	.116*	.116*	.115*	.114*	.119*	.127*	.123*	.123*	.130*	.156*
	MLR	.048	.052	.052	.052	.052	.053	.055	.057	.050	.057
	W	.205*	.198*	.204*	.212*	.213*	.212*	.211*	.209*	.228*	.241*
	MW	.089*	.091*	.087*	.085*	.087*	.085*	.087*	.088*	.084*	.098*
	LM	.047	.049	.052	.056	.055	.056	.066	.074*	.103*	.120*
	MLM	.039	.040	.038	.041	.043	.039	.045	.050	.048	.072*
60	LR	.078*	.076*	.073*	.076*	.071*	.077*	.083*	.081*	.078*	.085*
	MLR	.041	.040	.043	.040	.039	.044	.045	.043	.047	.053
	W	.109*	.101*	.102*	.103*	.102*	.108*	.112*	.114*	.110*	.116*
	MW	.055	.056	.053	.054	.057	.059	.058	.060	.056	.059
	LM	.042	.047	.047	.054	.050	.054	.057	.071*	.066	.076*
	MLM	.034	.038	.030*	.031*	.031*	.031*	.031*	.037	.042	.051

*** denotes significantly different from 0.05 at one percent level.**

Table 2: Estimated **asymptotic sizes** of six tests of AR(4) disturbances in the presence of AR(1) disturbances using asymptotic critical values at the five percent level for design matrices **X3** and **X4**.

n	Test	θ_1									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
						X3					
30	LR	.133*	.134*	.131*	.134*	.128*	.131*	.125*	.132*	.126*	.128*
	MLR	.054	.054	.055	.049	.049	.042	.045	.041	.048	.049
	W	.258*	.258*	.264*	.257*	.266*	.269*	.264*	.260*	.275*	.275*
	MW	.089*	.090*	.090*	.096*	.093*	.090*	.093*	.093*	.094*	.096*
	LM	.050	.047	.049	.051	.046	.046	.048	.061	.070*	.084*
	MLM	.036	.034	.034	.034	.033	.032*	.032*	.033	.041	.049
60	LR	.092*	.090*	.092*	.092*	.092*	.084*	.079*	.083*	.085*	.091*
	MLR	.051	.049	.051	.051	.045	.042	.047	.046	.051	.059
	W	.124*	.134*	.131*	.124*	.128*	.126*	.128*	.130*	.130*	.143*
	MW	.074*	.076*	.070*	.074*	.059	.063	.065	.061	.062	.064
	LM	.052	.049	.050	.050	.050	.048	.051	.057	.069*	.075*
	MLM	.044	.042	.039	.035	.039	.038	.040	.046	.043	.048
						X4					
30	LR	.219*	.214*	.213*	.207*	.205*	.200*	.200*	.194*	.195*	.198*
	MLR	.066	.063	.061	.063	.068*	.069*	.070*	.067	.060	.059
	W	.460*	.470*	.469*	.468*	.469*	.462*	.467*	.457*	.446*	.452*
	MW	.162*	.170*	.168*	.173*	.176*	.174*	.171*	.175*	.175*	.162*
	LM	.037	.039	.042	.041	.046	.052	.055	.054	.056	.079*
	MLM	.040	.040	.040	.043	.046	.044	.048	.052	.051	.064
60	LR	.107*	.107*	.110*	.107*	.109*	.106*	.111*	.104*	.098*	.102*
	MLR	.057	.059	.061	.057	.050	.056	.047	.051	.057	.054
	W	.171*	.173*	.168*	.171*	.176*	.177*	.170*	.168*	.170*	.163*
	MW	.074*	.080*	.081*	.074*	.077*	.077*	.079*	.078*	.071*	.074*
	LM	.046	.048	.048	.054	.061	.056	.060	.064	.070*	.067
	MLM	.039	.037	.035	.036	.039	.043	.048	.047	.053	.058

* denotes significantly different from 0.05 at one percent level.

Table 3: Estimated **asymptotic sizes** of eight tests for a Rosenberg coefficient in presence of a Hildreth-Houck random coefficient using asymptotic critical values at the five percent level for design matrices **X1** and **X3**.

n	Test	θ_1									
		.001	.05	.20	.50	1	3	7	30	100	200
						X1					
30	LR	.160*	.017*	.020*	.022*	.025*	.028*	.029*	.029*	.028*	.028*
	MLR	.047	.045	.056	.056	.056	.063	.062	.063	.063	.063
	W	.017*	.017*	.017*	.017*	.019*	.018*	.017*	.017*	.017*	.017*
	MW	.029*	.032*	.028*	.028*	.028*	.027*	.026*	.025*	.025*	.026*
	LM	.036	.036	.039	.050	.050	.050	.050	.051	.051	.051
	MLM	.039	.042	.042	.044	.045	.051	.051	.050	.050	.050
	ALMMP	.017*	.019*	.019*	.021*	.020*	.024*	.024*	.023*	.023*	.023*
	MALMMP	.051	.052	.050	.052	.052	.056	.056	.056	.056	.056
60	LR	.021*	.023*	.028*	.030*	.028*	.035	.036	.037	.037	.037
	MLR	.038	.048	.061	.067	.066	.069*	.068*	.067	.067	.067
	W	.022*	.023*	.027*	.023*	.022*	.021*	.021*	.017*	.017*	.017*
	MW	.036	.044	.054	.050	.047	.040	.042	.042	.041	.041
	LM	.048	.049	.054	.058	.060	.061	.058	.058	.057	.057
	MLM	.042	.044	.050	.054	.061	.059	.059	.058	.058	.058
	ALMMP	.022*	.022*	.025*	.030*	.031*	.034	.034	.033	.033	.034
	MALMMP	.051	.050	.055	.059	.059	.061	.060	.060	.059	.059
						X3					
30	LR	.015*	.016*	.019*	.021*	.023*	.022*	.024*	.024*	.024*	.024*
	MLR	.037	.041	.048	.056	.059	.058	.061	.062	.063	.063
	W	.016*	.014*	.016*	.018*	.021*	.021*	.020*	.020*	.019*	.019*
	MW	.027*	.026*	.024*	.026*	.028*	.031*	.029*	.030*	.030*	.029*
	LM	.039	.043	.048	.048	.051	.051	.052	.054	.053	.053
	MLM	.032	.033	.036	.038	.045	.042	.044	.044	.044	.044
	ALMMP	.010*	.011*	.013*	.013*	.013*	.013*	.013*	.013*	.013*	.013*
	MALMMP	.051	.050	.049	.050	.048	.052	.053	.052	.052	.052
60	LR	.011*	.012*	.014*	.015*	.018*	.023*	.025*	.025*	.025*	.025*
	MLR	.041	.046	.050	.060	.060	.057	.062	.059	.059	.059
	W	.010*	.010*	.013*	.011*	.011*	.008*	.010*	.009*	.009*	.009*
	MW	.039	.039	.044	.044	.038	.034	.033	.032*	.032*	.031*
	LM	.048	.053	.059	.065	.068*	.072*	.070*	.073*	.074*	.074*
	MLM	.042	.044	.043	.047	.048	.053	.054	.055	.055	.055
	ALMMP	.014*	.012*	.015*	.016*	.017*	.020*	.021*	.022*	.022*	.022*
	MALMMP	.047	.046	.049	.047	.050	.050	.050	.051	.051	.051

* denotes significantly different from 0.05 at one percent level.

Table 4: Estimated **asymptotic sizes** of eight tests for a Rosenberg coefficient in presence of a Hildreth-Houck random coefficient using asymptotic critical values at the five percent level for design matrices **X4** and **X5**.

n	Test	θ_1									
		.001	.05	.20	.50	1	3	7	30	100	200
						X4					
30	LR	.044	.045	.052	.055	.055	.055	.056	.056	.056	.056
	MLR	.070*	.073*	.071*	.074*	.075*	.077*	.077*	.079*	.079*	.079*
	W	.033	.035	.036	.035	.032*	.036	.035	.034	.032*	.032*
	MW	.047	.050	.052	.055	.055	.056	.053	.052	.051	.051
	LM	.046	.048	.050	.054	.051	.054	.054	.052	.051	.051
	MLM	.059	.059	.060	.059	.060	.061	.063	.064	.064	.064
	ALMMP	.033	.034	.034	.034	.032*	.034	.034	.036	.036	.036
	MALMMP	.072*	.071*	.071*	.072*	.073*	.069*	.071*	.070*	.070*	.070*
60	LR	.018*	.024*	.028*	.029*	.033	.031*	.030*	.030*	.030*	.030*
	MLR	.049	.049	.052	.050	.051	.051	.051	.051	.050	.050
	W	.025*	.021*	.022*	.023*	.020*	.016*	.015*	.014*	.013*	.013*
	MW	.120*	.115*	.098*	.083*	.074*	.058	.050	.047	.042	.044
	LM	.046	.049	.049	.052	.055	.057	.059	.057	.057	.057
	MLM	.048	.050	.052	.051	.051	.053	.052	.051	.051	.051
	ALMMP	.022*	.023*	.024*	.025*	.025*	.025*	.026*	.026*	.026*	.026*
	MALMMP	.042	.042	.044	.043	.042	.042	.042	.041	.041	.041
						X5					
30	LR	.033	.036	.036	.039	.042	.043	.041	.041	.041	.041
	MLR	.048	.051	.054	.057	.058	.060	.060	.060	.060	.060
	W	.034	.035	.037	.037	.038	.036	.036	.035	.035	.035
	MW	.031*	.031*	.031*	.026*	.024*	.022*	.023*	.022*	.023*	.023*
	LM	.047	.051	.053	.051	.057	.059	.059	.060	.061	.061
	MLM	.035	.035	.041	.043	.044	.046	.048	.048	.047	.048
	ALMMP	.021*	.022*	.023*	.026*	.026*	.026*	.027*	.029*	.029*	.029*
	MALMMP	.048	.049	.050	.049	.052	.047	.047	.048	.048	.048
60	LR	.020*	.020*	.021*	.029*	.034	.036	.035	.035	.035	.035
	MLR	.044	.046	.055	.056	.058	.060	.057	.054	.055	.055
	W	.017*	.018*	.016*	.018*	.022*	.016*	.021*	.020*	.020*	.020*
	MW	.033	.036	.043	.039	.037	.034	.035	.032*	.031*	.032*
	LM	.051	.051	.057	.062	.068*	.069*	.068*	.068*	.068*	.068*
	MLM	.042	.042	.042	.045	.050	.052	.055	.059	.060	.061
	ALMMP	.023*	.023*	.023*	.025*	.028*	.029*	.031*	.031*	.030*	.031*
	MALMMP	.044	.044	.045	.047	.049	.050	.050	.050	.050	.051

* denotes significantly different from 0.05 at one percent level.

Table 5: **Estimated power** of six tests of AR(4) disturbances in the presence of AR(1) disturbances using simulated critical values at the five percent level for design matrices **X1** and **X2**.

		θ_1	0.0	0.0	0.2	0.4	0.3	0.2	0.1	0.1	3	3
n	Tests	θ_2	0.0	0.5	0.6	0.0	0.2	0.5	-0.3	0.4	0.2	0.2
		θ_3	0.0	0.0	0.0	0.0	0.2	0.1	0.0	-0.2	0	0.2
		θ_4	0.0	0.0	0.0	0.4	0.2	0.1	0.3	0.0	0	0
							X1					
30	LR		.037	.161	.241	.131	.026	.102	.508	.191	.030	.028
	MLR		.043	.368	.496	.247	.089	.312	.582	.335	.068	.092
	W		.034	.124	.196	.114	.021	.087	.482	.158	.028	.028
	MW		.041	.172	.203	.180	.021	.077	.564	.247	.046	.035
	LM		.021	.096	.179	.184	.025	.088	.589	.106	.015	.016
	MLM		.021	.311	.423	.221	.075	.288	.553	.231	.036	.053
60	LR		.041	.699	.871	.452	.302	.745	.841	.640	.087	.189
	MLR		.043	.829	.931	.665	.554	.860	.870	.739	.173	.386
	W		.040	.689	.865	.459	.310	.743	.838	.621	.083	.189
	MW		.041	.782	.791	.510	.146	.566	.874	.710	.132	.229
	LM		.020	.660	.845	.452	.218	.727	.864	.583	.068	.119
	MLM		.041	.825	.939	.675	.524	.862	.893	.729	.163	.335
							X2					
30	LR		.033	.204	.319	.063	.062	.214	.204	.151	.045	.072
	MLR		.039	.316	.476	.234	.209	.413	.372	.231	.068	.120
	W		.034	.159	.273	.054	.060	.191	.208	.132	.039	.068
	MW		.035	.229	.354	.147	.092	.254	.323	.171	.050	.085
	LM		.013	.105	.203	.073	.039	.141	.209	.061	.023	.037
	MLM		.026	.253	.386	.226	.127	.321	.413	.173	.044	.062
60	LR		.039	.764	.909	.407	.521	.866	.669	.619	.128	.337
	MLR		.035	.814	.938	.664	.711	.911	.796	.670	.166	.440
	W		.038	.762	.904	.425	.534	.859	.683	.609	.130	.328
	MW		.043	.810	.934	.651	.696	.904	.780	.662	.162	.424
	LM		.026	.728	.895	.391	.404	.839	.715	.569	.093	.248
	MLM		.034	.815	.927	.657	.657	.895	.832	.671	.146	.361

Table 6: **Estimated power** of six tests of AR(4) disturbances in presence of AR(1) disturbances using simulated critical values at the five percent level for design matrices **X3** and **X4**.

		θ_1	0.0	0.0	0.2	0.4	0.3	0.2	0.1	0.1	3	3
n	Tests	θ_2	0.0	0.5	0.6	0.0	0.2	0.5	-0.3	0.4	0.2	0.2
		θ_3	0.0	0.0	0.0	0.0	0.2	0.1	0.0	-0.2	0	0.2
		θ_4	0.0	0.0	0.0	0.4	0.2	0.1	0.3	0.0	0	0
							X3					
30	LR		.047	.135	.214	.152	.029	.091	.545	.182	.042	.034
	MLR		.047	.306	.429	.223	.083	.263	.564	.309	.064	.091
	W		.043	.114	.187	.128	.028	.088	.489	.156	.038	.038
	MW		.049	.166	.198	.177	.028	.071	.547	.228	.045	.047
	LM		.021	.053	.127	.182	.026	.053	.592	.077	.013	.013
	MLM		.037	.337	.468	.277	.121	.331	.615	.305	.061	.075
60	LR		.048	.625	.812	.400	.200	.645	.836	.599	.076	.132
	MLR		.047	.799	.910	.610	.497	.833	.872	.726	.163	.362
	W		.047	.626	.829	.426	.273	.691	.833	.589	.080	.150
	MW		.045	.748	.881	.530	.292	.783	.861	.691	.127	.228
	LM		.033	.602	.793	.459	.162	.630	.873	.552	.056	.099
	MLM		.046	.812	.907	.652	.477	.817	.886	.724	.167	.162
							X4					
30	LR		.050	.171	.269	.111	.043	.178	.457	.154	.034	.039
	MLR		.039	.280	.408	.207	.158	.361	.480	.242	.062	.106
	W		.042	.152	.251	.113	.062	.177	.379	.146	.054	.052
	MW		.043	.189	.305	.144	.061	.189	.436	.178	.048	.057
	LM		.021	.101	.176	.116	.035	.132	.465	.044	.020	.016
	MLM		.030	.266	.378	.185	.111	.315	.491	.181	.050	.064
60	LR		.049	.650	.845	.464	.397	.786	.807	.585	.087	.214
	MLR		.047	.773	.904	.643	.605	.862	.833	.673	.143	.362
	W		.048	.660	.853	.473	.429	.803	.800	.585	.101	.227
	MW		.043	.726	.894	.571	.546	.849	.816	.634	.122	.286
	LM		.030	.590	.803	.445	.260	.715	.827	.534	.067	.142
	MLM		.032	.757	.893	.606	.533	.844	.839	.656	.136	.291

Table 7: **Estimated power** of eight tests for a Rosenberg coefficient in presence of a Hildreth-Houck random coefficient using simulated critical values at the five percent level for design matrices **X1** and **X3**.

n	Test	θ_1	.5	.5	.5	3	3	3	30	30	30
		θ_2	.3	.5	.8	.3	.5	.8	.3	.5	.8
						X1					
30	LR		.148	.301	.663	.288	.604	.915	.353	.722	.949
	MLR		.156	.323	.691	.293	.619	.926	.349	.722	.948
	W		.147	.314	.671	.294	.620	.917	.359	.731	.953
	MW		.148	.308	.563	.279	.579	.748	.337	.670	.768
	LM		.048	.119	.408	.106	.329	.765	.135	.425	.837
	MLM		.114	.241	.594	.213	.525	.876	.274	.621	.915
	ALMMP		.147	.312	.658	.288	.592	.904	.338	.697	.944
	MALMMP		.148	.312	.661	.290	.596	.902	.339	.698	.944
60	LR		.256	.559	.952	.506	.888	.998	.612	.997	1.00
	MLR		.273	.575	.959	.501	.891	.997	.604	.946	1.00
	W		.247	.561	.955	.500	.892	.998	.616	.950	1.00
	MW		.207	.503	.945	.383	.821	.996	.467	.896	.999
	LM		.126	.374	.890	.317	.776	.990	.397	.869	.998
	MLM		.191	.482	.925	.418	.845	.998	.508	.917	.999
	ALMMP		.272	.592	.947	.519	.901	.998	.633	.946	.999
	MALMMP		.273	.592	.948	.518	.901	.990	.635	.947	.999
						X3					
30	LR		.140	.278	.566	.275	.557	.875	.345	.664	.930
	MLR		.144	.293	.607	.271	.560	.896	.325	.645	.931
	W		.130	.276	.566	.261	.551	.879	.326	.658	.930
	MW		.149	.295	.601	.266	.552	.884	.326	.652	.937
	LM		.030	.073	.256	.061	.205	.556	.079	.291	.662
	MLM		.105	.214	.486	.148	.467	.798	.246	.562	.872
	ALMMP		.153	.296	.565	.286	.566	.854	.345	.643	.905
	MALMMP		.153	.289	.566	.273	.554	.854	.334	.639	.913
60	LR		.206	.438	.861	.497	.862	.993	.628	.933	.998
	MLR		.229	.471	.903	.492	.875	.996	.621	.934	1.00
	W		.195	.437	.865	.499	.860	.992	.634	.935	.998
	MW		.186	.434	.885	.385	.810	.993	.500	.892	.999
	LM		.043	.182	.644	.183	.585	.946	.281	.749	.981
	MLM		.161	.359	.832	.371	.795	.985	.475	.892	.995
	ALMMP		.230	.471	.976	.481	.859	.990	.629	.919	.996
	MALMMP		.239	.482	.887	.485	.866	.990	.629	.921	.997

Table 8: **Estimated power** of eight tests for a Rosenberg coefficient in presence of a Hildreth-Houck random coefficients using simulated critical values at the five percent level for design matrices **X4** and **X5**.

n	Test	θ_1	.5	.5	.5	3	3	3	30	30	30
		θ_2	.3	.5	.8	.3	.5	.8	.3	.5	.8
						X4					
30	LR		.135	.289	.720	.240	.570	.925	.297	.665	.959
	MLR		.142	.290	.735	.248	.561	.917	.300	.653	.959
	W		.135	.266	.710	.219	.500	.912	.250	.595	.954
	MW		.122	.238	.656	.167	.388	.866	.197	.454	.912
	LM		.068	.149	.500	.139	.365	.823	.155	.456	.882
	MLM		.119	.227	.617	.195	.488	.882	.242	.581	.931
	ALMMP		.148	.304	.694	.270	.578	.917	.323	.673	.950
	MALMMP		.147	.303	.697	.267	.576	.914	.321	.670	.950
60	LR		.255	.556	.930	.557	.900	.997	.676	.949	.999
	MLR		.257	.580	.947	.549	.903	.997	.669	.955	.999
	W		.243	.553	.942	.547	.891	.998	.650	.947	.999
	MW		.118	.312	.862	.175	.563	.985	.210	.685	.993
	LM		.084	.278	.808	.259	.713	.988	.357	.839	.997
	MLM		.161	.419	.890	.403	.822	.995	.504	.917	.999
	ALMMP		.280	.573	.938	.569	.910	.997	.676	.950	1.00
	MALMMP		.270	.560	.940	.559	.904	.997	.664	.952	1.00
						X5					
30	LR		.132	.132	.561	.250	.539	.866	.302	.632	.917
	MLR		.143	.143	.588	.245	.522	.867	.289	.619	.918
	W		.148	.148	.583	.235	.531	.873	.292	.630	.923
	MW		.154	.154	.571	.242	.501	.859	.284	.585	.916
	LM		.045	.086	.291	.078	.236	.587	.104	.331	.685
	MLM		.084	.177	.440	.162	.408	.754	.199	.504	.839
	ALMMP		.132	.269	.526	.258	.526	.824	.310	.605	.889
	MALMMP		.139	.283	.536	.263	.528	.836	.312	.609	.893
60	LR		.225	.473	.872	.508	.868	.993	.634	.936	1.00
	MLR		.227	.490	.897	.482	.873	.996	.601	.932	1.00
	W		.204	.466	.875	.490	.866	.993	.619	.935	.999
	MW		.183	.447	.878	.383	.796	.993	.468	.891	1.00
	LM		.065	.224	.700	.232	.639	.958	.328	.794	.980
	MLM		.150	.353	.811	.361	.778	.977	.462	.884	.995
	ALMMP		.224	.459	.871	.479	.861	.988	.607	.912	.997
	MALMMP		.243	.481	.879	.491	.869	.990	.617	.921	.998