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SMALL-SAMPLE POWER OF TESTS FOR INEQUALITY RESTRICTIONS: THE CASE OF QUARTER-DEPENDENT REGRESSION ERRORS*

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ABSTRACT

Testing for inequality restricted hypotheses has obtained increasing attention in recent years in econometrics. While many tests have been proposed for these testing problems, little is available on the power of these tests. In this paper, we examine the power of two tests in the literature, the locally most mean powerful invariant test and the Kuhn-Tucker test, in the case of testing for quarter-dependent simple AR(4) errors in linear regressions.

JEL classification: C12

Keywords: inequality restrictions, locally most mean powerful test, Kuhn-Tucker test, size and power, bounds of critical values.

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1. INTRODUCTION

Economic theory and stylized statistical facts often suggest that the range of certain parameters in a model are inequality restricted. For example, we often expect the estimates of regression coefficients to have "right" signs. Also in regression models based on undifferenced time series economic data, one typically expects positive autocorrelation in the errors caused by omitted variables that are positively correlated. Functional considerations such as variances never being negative also lead to restrictions on parameters.

It is well recognized that the incorporation of non-sample information such as one-sided inequality restrictions on parameters can improve power. In recent years, many multivariate one-sided tests have been proposed. For example, Gouriéroux, Holly and Monfort (1980, 1982) proposed one-sided versions of the conventional likelihood ratio test, Wald test and Lagrange multiplier (LM) test in a general framework and in the case of testing regression coefficients. The one-sided LM test is also called the Kuhn-Tucker (KT) test. These tests' statistics are asymptotically distributed under the null hypothesis as a probability mixture of independent chi-squared distributions with different degrees of freedom. When the number of parameters under test is large, calculation of the probability weights is very complicated. As an alternative approach, King and Wu (1990) proposed the locally most mean powerful (LMMP) test which maximizes the average slope of the power curve at the null hypothesis. A great variety of other tests have been proposed. A detailed review of this literature is given by Wu and King (1994).

While each of the above tests has a theoretical justification, our knowledge of their small-sample properties is quite limited. The purpose of this paper is to investigate the performance of the KT test and the LMMP invariant (LMMPI) test in the case of testing for quarter-dependent simple AR(4) errors in the linear regression model. In the next section, we introduce the two tests. We also present the point optimal invariant (POI) test whose power can be used as a benchmark in the assessment of the power of the KT and LMMPI tests. Section 3 describes the set-up and results of the experiment. Section 4 contains final remarks.

2. THE MODEL AND THE TESTS

Consider the linear regression model

$$y = X\beta + u \tag{1}$$

where X is an $n \times k$ nonstochastic matrix of rank $k < n$, and u is an $n \times 1$ error vector. Through the work of Thomas and Wallis (1971) and Wallis (1972) among others, the simple AR(4) process has become an accepted way of modelling seasonal autocorrelation in quarterly data. In its general form, it can be viewed as four separate simple AR(4) processes, one for each quarter. These processes can be expressed as

$$u_t = \rho_i u_{t-4} + \epsilon_t, \qquad \text{for t from quarter i, i = 1, 2, 3, 4,} \tag{2}$$

where $\epsilon_t \sim IN(0, \sigma^2)$. In applications based on undifferenced quarterly data, any correlation is typically expected to be positive. This leads to the restrictions $0 \le \rho_i < 1$ for $i = 1, \cdots, 4$, which we will assume for the remainder of the paper. Largely for parsimonious reasons, the ρ_i in (2) are typically assumed to have the same value so $\rho_i = \rho$ for all i. Clearly it may be more realistic to allow ρ_i to vary with i as implied by (2). Tests of the parsimonious model have been proposed by Thomas and Wallis (1971), Wallis (1972), Vinod (1973), King and Giles (1978) and King (1984).

Let $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)'$. Our interest is in testing

$$H_0: \quad \rho = 0 \qquad \text{against} \qquad H_a: \quad 0 \le \rho < 1,\ \rho \ne 0. \tag{3}$$

Let $G(\rho)$ be the $n \times n$ matrix whose (i,j)-th element is

$$g_{ij} = \begin{cases} (1-\rho_i^2)^{1/2} & \text{for } i = j = 1, 2, 3, 4, \\ 1 & i = j > 5, \\ -\rho_q & i = 4p + q,\ j = 4(p-1) + q, \\ & \text{with } p = 1, 2, \cdots,\ q = 1, 2, 3, 4, \\ 0 & \text{other } i \text{ and } j. \end{cases}$$

Then (2) can be written in the vector form $G(\rho)u = \epsilon$. The covariance matrix of the error is therefore $V(u) = \sigma^2\Omega(\rho)$ with

$$\Omega(\rho) = G(\rho)^{-1}G(\rho)'^{-1}. \tag{4}$$

The log-likelihood function for model (1) is

$$L(\beta,\sigma^2,\rho) = -\frac{n}{2}\, ln(2\pi\sigma^2) - \frac{1}{2}\, ln|\Omega(\rho)| - \frac{1}{2\sigma^2}\,(y-X\beta)'\Omega(\rho)^{-1}(y-X\beta). \tag{5}$$

Then, for $i = 1, 2, 3, 4$,

$$\frac{\partial L}{\partial \rho_i} = -\frac{1}{2}\,|\Omega(\rho)^{-1}|\frac{\partial|\Omega(\rho)|}{\partial \rho_i} - \frac{1}{2\sigma^2}\,(y-X\beta)'\frac{\partial \Omega(\rho)^{-1}}{\partial \rho_i}(y-X\beta). \tag{6}$$

Let $\psi = (\beta,\ \sigma^2,\ \rho_1,\ \rho_2,\ \rho_3,\ \rho_4)'$. It is easy to check that the information matrix $I = -E(\partial^2 L/\partial\psi\partial\psi')$ under H_0 is block diagonal among the elements of ψ. The inverse of the block corresponding to $\rho = (\rho_1,\ \rho_2,\ \rho_3,\ \rho_4)'$ is

$$I_0^{\rho} = \text{diag}(1/m,\ 1/m,\ 1/m,\ 1/m) \tag{7}$$

where $m = [(n-4)/4]$, and $[x]$ denotes the integer part of x.

From King and Wu (1990) (also see Wu and King (1994)), the general form of the LMMPI test for testing $\theta = 0$ against $\theta \ge 0$ with $\theta \ne 0$ in the context of (1) with error covariance matrix $V(u) = \sigma^2\Omega(\theta)$ is to reject $\theta = 0$ for large values of $s = \hat{u}'A\hat{u}/\hat{u}'\hat{u}$ where $A = \sum_{i=1}^{p} \partial\Omega(\theta)/\partial\theta_i \big|_{\theta=0}$, p is the dimension of θ and

$\hat{u}$ is the ordinary least squares residual vector from (1). Applying this to (4), it can be shown that the LMMPI test statistic is

$$s = \frac{\hat{u}'A\hat{u}}{\hat{u}'\hat{u}} = \sum_{i=5}^{n} \hat{u}_i \hat{u}_{i-4} / \sum_{i=1}^{n} \hat{u}_i^2 \tag{8}$$

where A is the matrix with elements on the fourth diagonal lines being -1 and other elements being zero. This test is equivalent to the modified Wallis test investigated by King (1984). Consequently, Wallis's test for simple AR(4) disturbances is an approximately LMMPI test of H_0 against H_a.

A general form of the KT test of $\theta = 0$ against $\theta \geq 0$ with $\theta \neq 0$ is to reject $\theta = 0$ for large values of

$$w = (\partial L/\partial\theta|_{\theta=0} - \partial L/\partial\theta|_{\theta=\tilde{\theta}})' I^{\theta}|_{\theta=0} (\partial L/\partial\theta|_{\theta=0} - \partial L/\partial\theta|_{\theta=\tilde{\theta}})$$

where I^{θ} is the inverse of the block in the information matrix that corresponds to θ, and $\tilde{\theta}$ is the inequality restricted ($\theta \geq 0$, $\theta \neq 0$) maximum likelihood (ML) estimate of θ. (See Gouriéroux *et al.* 1980 or Wu and King (1994)). Note that if the $\partial L/\partial\theta|_{\theta=\tilde{\theta}}$ term is removed, w becomes the conventional LM test statistic. So the KT test modifies the LM test by adding the inequality restricted score $\partial L/\partial\theta|_{\theta=\tilde{\theta}}$ to take account of the one-sided nature of the testing problem. In the case of (3), it can be shown, from (6) and (7) that

$$w = \frac{1}{m} \sum_{i=1}^{4} \left(-\frac{n}{2} \frac{\hat{u}'A_i\hat{u}}{\hat{u}'\hat{u}} - \frac{\partial L}{\partial \rho_i}\Big|_{\rho=\tilde{\rho}} \right)^2 \tag{9}$$

where A_i is the matrix whose elements on its fourth diagonal lines and (i+4p)-th rows, p = 0, 1, 2, ···, are -1, and all other elements are zero.

One way to assess the performance of a test is to check how close its power is to the power envelope (PE), i.e., the maximum attainable power over the alternative parameter space. In testing the error covariance matrix V(u), the PE for the class of invariant tests can be constructed from the point optimal invariant (POI) tests (see King (1987b)). To test for (3), the point optimal test that gives the power ceiling at ρ^* rejects H_0 for large values of

$$s(\rho^*) = \frac{\hat{u}'\Delta\hat{u}}{\hat{u}'\hat{u}} \tag{10}$$

where $\Delta = \Omega(\rho^*)^{-1} - \Omega(\rho^*)^{-1}X(X'\Omega(\rho^*)^{-1}X)^{-1}X'\Omega(\rho^*)^{-1}$. The powers of the $s(\rho^*)$ test at $\rho = \rho^*$ calculated at different points of ρ^* constitute the PE.

3. AN EMPIRICAL POWER STUDY

In order to compare the small-sample power of the KT and LMMPI tests with the PE, we conducted an empirical power comparison. The form of the LMMPI test statistic (8) and the POI test statistic (10) are similar to that of the familiar Durbin-Watson (DW) test statistic. Thus their powers can be calculated using the methodology developed for the DW test (see King (1987a)). For our calculations, we used a modified version of Koerts and Abrahamse's (1969) FQUAD subroutine.

Calculating power of the KT test is complicated as it involves finding the inequality restricted ML estimate $\tilde{\rho}$. We used the following procedure based on King (1986) to find $\tilde{\rho}$. Let $\Omega(\rho) = QQ'$, $y^* = Q^{-1}y$ and $X^* = Q^{-1}X$. Then (5) can be written as

$$L(\beta,\sigma^2,\rho) = c - \frac{n}{2}\ln\sigma^2 - \ln|Q| - \frac{1}{2\sigma^2}(y^*-X^*\beta)'(y^*-X^*\beta) \tag{11}$$

where c, as well as c_1 below, are constants. Concentrating out β and σ^2, we have

$$\hat{\beta} = (X^{*\prime}X^*)^{-1}X^{*\prime}y^*,$$

$$\hat{\sigma}^2 = \hat{\eta}^{*\prime}\hat{\eta}^*/n, \qquad \hat{\eta}^* = y^* - X^*\hat{\beta},$$

so (11) becomes

$$L(\rho) = c_1 - \frac{n}{2} \, ln \, \hat{\eta}^{*\prime}\hat{\eta}^{*} - \ln|Q|$$

$$= c_1 - \frac{n}{2} \, ln \, \hat{\eta}^{**\prime}\hat{\eta}^{**}, \qquad (12)$$

where $\hat{\eta}^{**} = \hat{\eta}^{*}|Q|^{1/n}$. Thus $\tilde{\rho}$ can be found by minimizing $\hat{\eta}^{**\prime}\hat{\eta}^{**}$ subject to H_a in (3). We utilized the IMSL MATH/LIBRARY (1989) subroutine DBCONG to carry out the minimization. We used $\tilde{\rho}_i = \max(0, \hat{\rho}_i = \sum_{t(i)} \hat{u}_t\hat{u}_{t-4} / \sum_{t=1}^{n} \hat{u}_t^2)$ as the starting point of the iterations, where $\sum_{t(i)}$ indicates summation over those t that belong to the i-th quarter, i = 1, 2, 3, 4. Once $\tilde{\rho}$ is found, (6) is used, with β, σ^2 and ρ being replaced by $\hat{\beta}$, $\hat{\sigma}^2$ and $\tilde{\rho}$, respectively, to find $\partial L/\partial \rho_i|_{\rho=\tilde{\rho}}$.

Four data sets were used in the empirical comparisons. They are:

X1 (n×3). A constant dummy, the quarterly Australian Consumer Price Index (ACPI) commencing from 1959(1), and the ACPI lagged one quarter.

X2 (n×5). X1, plus the ACPI lagged two quarters and three quarters.

X3 (n×5). The full set of quarterly seasonal dummy variables and quarterly Australian retail trade commencing from 1968(1).

X4 (n×4). A constant dummy, quarterly Australian private capital movements, government capital movements and retail trade. All commence from 1968(1).

These X matrices represent a range of situations. The ACPI reveals a slow smooth trend and a small seasonal pattern. X4 is a choppy data set because the two capital movements series fluctuate wildly and are strongly seasonal.

Powers of the tests were calculated at points of combinations of ρ_i = 0.0, 0.2, 0.4, 0.6, i = 1, 2, 3, 4, for n = 20, 60. We used 1000 repetitions to calculate the power of the KT test at each point.

In order to avoid calculation of the probability weights, Kodde and Palm (1986) provided bounds denoted c_L and c_U for the asymptotical critical values of the KT test. Table 1 summarizes the performance of the bounds at the 5% significance level and also gives the exact 5% critical values obtained by simulation for our data sets. Observe that using the asymptotic critical values for n = 20 in the case of X3 leads to an actual size smaller than 0.016. With this exception, all the other figures are as expected. Our estimated probabilities of the KT test statistic falling in the inconclusive region under H_0 are all below 0.17. Unlike the DW test, this probability shows a tendency to widen as n increases. We also note that the size of a test based on c_U tends to increase as n increases. This suggests there may be some merits in a conservative test that uses c_U as the critical value when n is large.

We now turn our attention to the power of the LMMPI and KT tests. For a fair comparison, exact or simulated critical values were used at the 5% level of significance. The results are in table 2. Overall, the LMMPI test performs very well and is always more powerful than the KT test except on the boundary where one or more ρ_i value is zero. For example, except on the boundary of the parameter space, the LMMPI test has at least 77% of the PE power for X2 and n = 20, and 90% of the PE power for X2 and n = 60. The smallest power difference for both sample sizes is 0.002 at $\rho = (0.2,0.2,0.2,0.2)'$, while the largest differences are 0.086 for n = 20 at $\rho = (0.2,0.6,0.6,0.6)'$, and 0.076 at $\rho = (0.2,0.2,0.2,0.6)'$ for n = 60. On average, when the sample size is large, the LMMPI test has power at a higher percentage of the PE. These results suggest that the power of the LMMPI test converges to the PE as the sample size becomes larger. These patterns carry over to X1, X3 and X4. In particular, we note that the LMMPI test always has power closest to the PE at

$\rho = (0.2,0.2,0.2,0.2)'$. This reflects the fact that the LMMPI test is also the locally best invariant test along the mid direction which in our case is $\rho_1 = \rho_2 = \rho_3 = \rho_4 > 0$ (see Wu and King (1994)). In fact our LMMPI test is identical to the modified Wallis test discussed by King (1984).

The KT test is less powerful than the LMMPI test in general. Its power advantage is greatest on the boundary of the parameter space, particularly for the larger sample size. For X2 and 15 non-boundary points, the power of the KT test is no less than 69% of the PE when n = 20 and 86% of the PE when n = 60 (recall the corresponding percentages for the LMMPI test are 77% and 90%). It is noticeable that as n increases, the power difference between the two tests diminishes. At the three points on the boundary, both tests lack power relative to the PE.

5. FINAL REMARKS

Limited though they are, the empirical sizes and powers reported above suggest the following conclusions. Away from the boundary of the parameter space, the LMMPI test dominates the KT test in terms of power. There is evidence of this dominance declining as the sample size increases. This together with its better performance on the boundary might lead one to have faith in the KT test in very large samples. While Kodde and Palm's (1986) bounds provide a partial solution to the problem of finding critical values of the KT test, we recommend the use of simulation to find p-values or critical values. Because the form of the LMMPI test is similar to that of the DW test, either exact or approximate critical values can be found using the methodology for the DW test (see King (1987a) for a survey).

Another issue addressed in this paper is that of modelling regression error seasonality with quarter-dependent simple AR(4) processes. The normal practice is to assume a common autoregressive parameter for each quarter. It turns out that the LMMPI test for the former is the locally best invariant test for the latter model. Hence our results suggest the power gains tend to outweigh the losses when testing for the presence of the more parsimonious model, particularly in small samples. We suspect there may be many other situations in which parsimony in the parameters under test will yield similar results.

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Table 1: Simulated critical values of the KT test and sizes of the KT test based on Kodde and Palm's bounds* at 5% level

Design matrix	n = 20			n = 60		
	Simulated critical value	Size		Simulated critical value	Size	
		c_L	c_U		c_L	c_U
X1	5.228	.165	.010	5.258	.179	.010
X2	5.074	.156	.007	4.944	.164	.012
X3	1.374	.016	.000	3.986	.107	.005
X4	4.156	.096	.007	4.800	.152	.009

* $c_L = 2.706$, $c_U = 8.761$.

Table 2: Calculated powers of the LMMPI and KT tests and the power envelope against quarter-dependent simple AR(4) errors at 5% level

					n = 20				n = 60			
ρ_1	ρ_2	ρ_3	ρ_4	Test	X1	X2	X3	X4	X1	X2	X3	X4
.0	.0	.0	.4	LMMPI	.109	.087	.077	.086	.207	.200	.179	.199
				KT	.199	.077	.075	.078	.258	.270	.205	.244
				PE	.176	.145	.104	.134	.413	.397	.362	.396
.0	.0	.4	.4	LMMPI	.194	.138	.119	.170	.447	.430	.390	.437
				KT	.193	.120	.112	.158	.482	.499	.407	.465
				PE	.254	.198	.155	.224	.635	.614	.577	.622
.0	.4	.4	.4	LMMPI	.279	.205	.175	.235	.697	.670	.630	.681
				KT	.266	.169	.167	.209	.686	.685	.601	.666
				PE	.322	.255	.204	.275	.788	.765	.732	.774
.2	.2	.2	.2	LMMPI	.167	.142	.127	.151	.410	.390	.379	.404
				KT	.158	.131	.115	.127	.335	.339	.297	.350
				PE	.168	.144	.127	.152	.411	.392	.379	.405
.4	.4	.4	.4	LMMPI	.399	.316	.247	.338	.880	.859	.832	.873
				KT	.354	.268	.224	.289	.822	.823	.733	.817
				PE	.407	.334	.252	.350	.884	.866	.836	.876
.6	.6	.6	.6	LMMPI	.685	.555	.391	.577	.994	.991	.982	.993
				KT	.608	.475	.330	.497	.981	.979	.953	.980
				PE	.709	.619	.406	.623	.995	.993	.984	.994
.2	.2	.2	.4	LMMPI	.220	.174	.149	.183	.567	.536	.508	.549
				KT	.205	.156	.135	.151	.488	.490	.417	.472
				PE	.236	.192	.154	.194	.594	.570	.540	.581
.2	.2	.4	.4	LMMPI	.281	.212	.178	.243	.693	.667	.633	.681
				KT	.269	.180	.151	.207	.628	.623	.545	.609
				PE	.298	.235	.187	.258	.726	.703	.668	.715

Table 2 (continued)

.2	.4	.4	.4	LMMPI	.333	.253	.211	.282	.801	.774	.742	.788
				KT	.311	.214	.195	.238	.739	.735	.651	.719
				PE	.349	.279	.219	.297	.820	.798	.764	.808
.2	.2	.2	.6	LMMPI	.304	.225	.172	.225	.724	.701	.639	.708
				KT	.287	.215	.153	.187	.704	.699	.592	.682
				PE	.359	.293	.190	.271	.798	.777	.721	.784
.2	.2	.6	.6	LMMPI	.451	.320	.235	.375	.898	.882	.830	.889
				KT	.420	.277	.203	.336	.890	.876	.789	.875
				PE	.501	.402	.265	.427	.937	.926	.888	.930
.2	.6	.6	.6	LMMPI	.565	.422	.307	.460	.971	.962	.936	.966
				KT	.513	.358	.266	.400	.955	.939	.893	.950
				PE	.608	.508	.337	.517	.982	.977	.957	.978
.2	.2	.4	.6	LMMPI	.365	.263	.204	.286	.819	.798	.744	.806
				KT	.342	.233	.173	.244	.790	.774	.690	.773
				PE	.409	.327	.222	.327	.867	.849	.803	.856
.2	.4	.4	.6	LMMPI	.417	.306	.238	.326	.890	.871	.829	.878
				KT	.383	.261	.215	.279	.850	.854	.767	.834
				PE	.453	.366	.255	.362	.915	.900	.863	.905
.2	.4	.6	.6	LMMPI	.501	.363	.272	.415	.942	.929	.893	.934
				KT	.454	.302	.236	.356	.916	.909	.832	.918
				PE	.540	.438	.296	.458	.960	.952	.923	.955
.4	.4	.4	.6	LMMPI	.481	.368	.277	.383	.938	.924	.895	.932
				KT	.425	.311	.242	.325	.900	.890	.817	.895
				PE	.503	.415	.286	.412	.946	.934	.905	.939
.4	.4	.6	.6	LMMPI	.561	.425	.313	.469	.970	.961	.938	.966
				KT	.491	.342	.263	.414	.942	.936	.875	.947
				PE	.584	.483	.327	.502	.975	.969	.947	.971
.4	.6	.6	.6	LMMPI	.621	.481	.350	.514	.986	.981	.965	.983
				KT	.546	.402	.298	.438	.971	.955	.924	.966
				PE	.649	.551	.367	.558	.989	.985	.971	.986