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INVENTORY CONTROL:

BACK TO THE MOLEHILLS

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Inventory Control: Back to the Molehills

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Abstract

It is argued in this paper that conventional inventory control theory in the form that it is implemented and widely used today, is largely the product of an era when most businesses operated with primitive mechanical calculators rather than computers. The methods devised at the time reflect the fact that calculations had to be undertaken by hand and therefore could be neither complicated nor burdensome. There was, as a consequence, an extensive reliance on analytical methods. This enabled the development and use of tables and nomographs to simplify and streamline the associated calculations.

In recent years we have witnessed changes of revolutionary proportions with the development and widespread penetration of cheap, powerful computational technologies into most aspects of business activity. The proposition put forward and elaborated in this paper is that it is timely to review current practices in inventory control, and determine whether the new technologies provide opportunities for approaches possessing a greater reliance on numerical methods in place of those with an analytical orientation. As a consequence new possibilities for periodic review order-up-to and reorder level inventory systems are explored together with adaptations which allow for growth and seasonal effects in demand. A common feature of the proposed approaches is that they largely bypass the statistical forecasting methods commonly used in conjunction with computerised inventory control systems. Furthermore, they provide a mechanism for coping with problems of uncertainty without recourse to formal probability theory. It is argued that, as a consequence, they are better suited for use in most business settings where those delegated to control inventories usually lack the formal mathematical skills and knowledge to fully understand and to make effective use of the classical methods.

Keywords: Inventory control, business computing, spreadsheets, seasonal inventories

1. Introduction

The methods of inventory control commonly in use today had their origins in an era when most businesses had access to slow rudimentary mechanical calculating machines, limited in their scope to the four basic arithmetic operations of addition, subtraction, multiplication and division. All calculations were undertaken by hand. Because few devices possessed a programming capacity, the opportunity for the automation of routine, repetitive procedures was limited. By necessity, stock control had to rely on methods (Whitin, 1957; Brown, 1959, 1967; Lampkin & Flowerdew, 1963) designed to have low and relatively simple computational overheads. This was achieved by recourse to analytical methods from the theory of probability and statistics. Inventories, however, were controlled by clerks who typically possessed little or no formal training in mathematics beyond elementary arithmetic. To be viable, therefore, the more complex aspects of the theory had to be disguised. This was usually done with the aid of devices such as lookup tables and nomograms to represent the more complex relationships. Such tactics also facilitated the streamlining of computational loads. Thus, although the clerks typically had little understanding of the logic of the routines governing their daily work, the implementation of theory in this way proved to be viable and effective when judged against the constraints imposed by the computational technologies of those times.

The subsequent emergence of reliable mainframe computers provided business and industry with new capacities to automate, in a cost effective way, many of the older manual systems based on repetition and routine. Inventory control was one primary area of reform. The automation of the old calculation processes for inventory control reduced the need for clerks. It simultaneously eliminated a traditional constraint on expansion, providing business with a capacity to control much larger ranges of products and parts.

Mainframe computers also had a considerable impact on mathematical research, spawning an interest in algorithms as an approach to problem solving. This trend was reflected in inventory theory, where particular emphasis was placed on the dynamic modelling of inventory systems (Karlin, 1958; Galliher, Morse & Simond, 1959; Scarf, 1960; Iglehart & Karlin, 1962; Iglehart, 1963; Snyder, 1975). Such attempts heightened our understanding of the traditional methods because under static conditions they often emerged as the steady state forms of dynamic algorithms. Although inherently more general than the older methods, the new methods never gained widespread acceptance in business. Despite an opportunity for fundamental change, the old methods of inventory control prevailed in practice.

In more recent times we have observed the emergence of desktop computers largely as a response to the need for better word processing facilities. Desktop computers achieved much higher levels of penetration than mainframe computers. Their success, however, was achieved only after considerable innovation in and simplification of user interfaces. This was dictated by the fact that most of the potential users of word processing software had little prior knowledge of computing and possessed few, if any, programming skills. This quest for simplicity spilled over into the area of business modelling with the successful implementation of the spreadsheet paradigm. The big change from this development is that business modelling is no longer the sole preserve of those with programming skills.

The old methods of inventory control made extensive use of analytical methods to minimise computational loads. Nowadays, with desktop computers, computation is no longer an issue. The time is ripe to reconsider inventory problems, to explore new possibilities with a greater reliance on numerical methods. It is this theme that is addressed in this paper.

The general framework adopted is that of the spreadsheet. No doubt spreadsheets have been used for teaching inventory management. They have also sometimes been used to control inventories in small businesses. To the extent that they have been implemented correctly, such applications have relied on the old methods. In this paper, however, the intention is to use the spreadsheet as a framework to foster new ways of thinking about the old problems. This is not to imply, nor do we necessarily advocate, the ultimate implementation of the methods we devise in spreadsheet form. The spreadsheet, in this paper is merely a convenient vehicle for conceptualisation.

The new methods are outlined in the next section of the paper. The basic strategy adopted here is to work directly with historical demand data for an item and examine the retrospective performance of the system under different trial values of the ordering parameters. The idea is to seek those values of the ordering parameters that would have achieved the goals and objectives of the company had they been used in the past. It is these values that are then employed in the future. Variations on this theme to cope with growth and seasonal effects are also considered. In section 3 the traditional methods of inventory management are reviewed. Often little understood implementation problems associated with them are presented. A comparison with the proposed approach is then made. Finally, in section 4, an application involving the development of a spreadsheet prototype of the new approach for a car parts distributor is described.

2. Retrospective Simulation Method

2.1. Simple Example

Table 1 shows a spreadsheet reminiscent of the stock control cards commonly used in the old clerical systems for controlling inventories. It contains a record of demands for an item and depicts the associated inventory situation over a 10 week period under a so-called periodic review, order-up-to level inventory policy. Reviews are made at the beginning of each week. In this example enough is ordered at each review to raise the stock to the order-up-to-level (OUL) of 80, an 'ideal' level for the stock. A distinction is made between the concepts of stock and supply. Stock is defined as the quantity of an item actually available in the store. It can never be negative. The concept of supply is similar in that it also represents the quantity available in the store when it is positive. Unlike stock, however, it can take negative values. It then reflects the quantity backlogged during stockouts.

The logic of the system is illustrated by focussing on the row of Table 1 corresponding to week 2. Because the closing supply in period 1 is -52, the deficit of 132 below the OUL is eliminated by the placement of an order for the same amount. To simplify matters in this particular example, it is assumed that orders are delivered immediately. The quantity delivered then corresponds precisely to the quantity ordered in the same period. The stock level therefore always equals the OUL at the start of each period. Closing supply is found by subtracting demand from the opening stock. It is negative because demand during the week exceeds the initial supply. The actual quantity in the store is shown in the next column labelled *Closing Stock*. In this case it is zero, a refection of the stockout situation. Excess demand is that part of a weeks demand that cannot be satisfied during stockouts. The existence or absence of a stockout is also indicated by a 1 or 0 in the *Shortage Indicator* column. The *Average Stock* column contains the average for individual weeks stock defined by

$$AvgStock(t)=\begin{cases}\left(OpeningStock(t)+\text{Closing}Stock(t)\right)/2 & \text{if Closing}Supply(t)\geq 0\\ OpeningStock(t)^2/\left(2*Demand(t)\right) & \text{if Closing}Supply(t)<0\end{cases}. \tag{1}$$

Totals and weekly averages for the 10-week period are shown in the bottom rows. The fill rate, a measure of performance, is calculated using the formula

$$FillRate=1-\frac{\sum_{t=1}^{n}ExcessDemand(t)}{\sum_{t=1}^{n}Demand(t)} \tag{2}$$

t being a time index and n the number of periods covered by the spreadsheet. In Table 1 the fill rate of 71 percent means that 71 percent of demand was satisfied immediately from stock when the OUL was 80.

The proportion of weeks short is another potential measure of performance defined by

$$P_{out}=\frac{\sum_{t=1}^{n}ShortageIndicator(t)}{n} \tag{3}$$

Under the scenario depicted in Table 1 a shortage would have been experienced at the end of each week, a situation reflected by a P_{out} of 100 percent.

A spreadsheet environment is ideal to explore the effect of changes in the OUL on fill rate and the proportion of weeks short. The cell containing the order-up-to level of 80 can be overwritten with other hypothetical values and the consequences observed. By doing this, we effectively simulate what would have happened to the stock system for different trial values of the order-up-to-level. The approach is therefore ideally called *retrospective simulation*. It should be distinguished from the more commonly used Monte Carlo simulation method where, in an inventory context, demands are generated from probability distributions. In retrospective simulation, actual historical values of demand are employed instead of synthetic samples from random number generators.

Matters can be streamlined by specifying the desired fill rate and using the goal seeking tool commonly provided with modern spreadsheets to find the corresponding OUL. Table 2 shows the result for a specified fill rate of 95 percent. It illustrates the point that OUL's can be determined directly from historical demand data without the use of conventional forecasting methods nor the use of classical stochastic inventory models.

A more detailed examination of Table 2 indicates that it would have been necessary to increase the OUL to 118 to achieve the 95 percent fill rate. The proportion of weeks short would have dropped from 100 percent to 40 percent. The latter figure is still deceptively high despite the 95 percent fill rate. The proportion of weeks short is the sample analogue of the right hand tail of a probability distribution. This illustrates the point emphasised in Snyder (1980) that fill rates provide a more meaningful basis for inventory control than common tail probabilities.

2.2. Basic Control Systems

The method can be adapted to incorporate a fixed delivery lead time L. Given that orders may then be outstanding at each review, it is necessary to account for them in addition to supply, before placing a new order. The combination of both these quantities will be referred to as *total supply.*

An enhanced version of the spreadsheet model shown in Table 1 can be constructed based on the following stock-flow relationships:

$$TotalSupply(t) = ClosingSupply(t-1) + OnOrder(t-1) \tag{4}$$
$$Order(t) = OUL - TotalSupply(t) \tag{5}$$
$$Delivery(t) = Order(t-L) \tag{6}$$
$$OnOrder(t) = OnOrder(t-1) + Order(t) - Delivery(t) \tag{7}$$
$$OpeningSupply(t) = ClosingSupply(t-1) + Delivery(t) \tag{8}$$
$$OpeningStock(t) = \mathrm{Max}(OpeningSupply(t),0) \tag{9}$$
$$OpeningBacklog(t) = \mathrm{Max}(-OpeningSupply(t),0) \tag{10}$$
$$ClosingSupply(t) = OpeningSupply(t) - Demand(t) \tag{11}$$
$$ClosingStock(t) = \mathrm{Max}(ClosingSupply(t),0) \tag{12}$$
$$ClosingBacklog(t) = \mathrm{Max}(-ClosingSupply(t),0) \tag{13}$$
$$ExcessDemand(t) = ClosingBacklog(t) - OpeningBacklog(t) \tag{14}$$

Because it takes a lead time for the system to run-in, all averaging and calculations of performance measures must be undertaken with respect to the time span from $L+1$ to n.

It is also important to have the capacity to model those periodic review inventory systems where orders are delayed until the total supply drops below a critical point called the reorder level (ROL). In those cases where orders are always for a constant quantity, denoted by ROQ, (5) can be replaced with:

$$Order(t) = ROQ \text{ if } TotalSupply(t) \le ROL. \tag{15}$$

The size of an order is usually determined independently of the reorder level. It might be dictated by packing or transportation considerations. It might, where appropriate, be determined by economic considerations such as those associated with the classical economic order quantity formula (Harris, 1915; Snyder, 1973). Here only the reorder level required for the system to achieve a specified fill rate would be determined by the goal seeking method.

A variation of the reorder level system involves the decision rule

$$Order(t) = OUL - TotalSupply(t) \text{ if } TotalSupply(t) \leq ROL. \tag{16}$$

where

$$OUL = ROL + ROQ. \tag{17}$$

ROQ then represents the minimum size of the order quantity. The rule corresponds to the so-called (S,s) policy (Scarf, 1960).

2.3. Growth and Seasonal Effects

Retrospective simulation, as presented so far in the paper, is implicitly based on the assumption that underlying conditions remain unchanged over time. To the extent that demand used in the simulation extends over many years, it may be necessary to make adjustments for systematic change such as growth and seasonal effects. The traditional approach is to rely on statistical models to estimate growth rates and seasonal indexes eg Winters (1960) method of forecasting. The retrospective simulation approach outlined here represents a more direct approach to the problem.

Assuming, by way of illustration, that the review periods correspond to months, we define 12 seasonal factors denoted by $\{a(k), k=1, 12\}$ and a growth rate b. The global OUL is replaced by one that varies over time according to the linear formula:

$$OUL(t) = a(k) + b * t, \tag{18}$$

k being the month corresponding to period t. Because the order-up-to level is based on total lead time demand, the seasonal factors and the growth rate reference a time period equal to the lead time rather than the review period of a month.

The values of the $a(k)$ and b prior to the retrospective simulation are unknown. With the exception of the expression (1) for average stock, however, all the relationships associated with this problem are linear with non negative restrictions on some of the variables. The definition of average stock in those periods without a stockout may be extended to those periods in which a stockout is experienced. This approximation means that the entire model is then linear and that the retrospective simulation may then be stated as a linear programming problem. The aim would be to select the seasonal constants and growth rate to minimise total average stock subject to the constraints(4-14), together with the fill rate restriction (2). The resulting problem would normally be too large to be solved by the standard solver that accompanies modern spreadsheet programs. The problem, however, is still of moderate size on modern standards and is amenable to solution with third party linear programming add-ins. Although the associated constraint matrix is not uni modular, its special structural characteristics help to expedite the associated computations. Additional computational savings are possible with piecewise linear programming (Snyder, 1984; Fourer, 1985) although commercial software for this more specialised technique is not currently available.

3. Current Methods

A sales forecasting module would normally be found in most modern computerised inventory control systems. A good module would normally rely on more than one forecasting method, the choice often including simple exponential smoothing (Brown, 1959), trend corrected exponential smoothing (Holt, 1957), and the Winters(1960) method. It would contain a mechanism for estimating parameters. It would also contain procedures for automatically selecting the best method, usually done with respect to a method's capacity to predict sales on a reserved section of a sales sample. The module would possess the capacity to monitor prediction errors to facilitate the identification of problem inventories (Trigg, 1964). It would also possess manual overrides on the automated parts of the process.

The sales predictions and associated measures of error would be fed to a stock control module. The latter would be commonly based on stochastic models of the inventory system. Models with a fill rate orientation (Brown,1959; Lampkin & Flowerdew, 1963) or those with a cost minimisation orientation (Whitin, 1957; Hadley & Whitin, 1963) are used extensively. The fill rate approach for a periodic review order-up-to system, for example, is based on the equation:

$$\frac{\int_{OUL}^{\infty}(\xi - OUL)\phi^{(L+1)}(\xi)d\xi - \int_{OUL}^{\infty}(\xi - OUL)\phi^{(L)}(\xi)d\xi}{\int_0^{\infty}\xi\phi(\xi)d\xi} = 1 - FillRate \tag{21}$$

$\phi(\bullet)$ denoting the probability density function of demand over a review period, $\phi^{(L)}(\bullet)$ and $\phi^{(L+1)}(\bullet)$ denoting the densities of total demand over the lead time L and the period $L+1$. The integrals in the numerator are the mean backlogs at the end and beginning of a typical inventory cycle respectively. Their difference is the mean excess demand for a cycle. The denominator is the mean demand in a cycle so that the ratio measures the proportion of demand, on average, occurring during a stockout. Equation (21) is the stochastic analogue of (2). Because it is usual for any backlog to be eliminated in its entirety immediately following a delivery, the second term in the numerator is usually quite small. It is often dropped in expositions of inventory theory (eg. See Brown, 1959) to simplify the method employed to find the OUL. So-called partial expectation tables for standard probability distributions such as the normal (Brown, 1959) and gamma distributions (Snyder, 1984) may then be used to reduce the task of determining the OUL to basic arithmetical operations.

In practice the following difficulties are often encountered with the forecasting module.

- Typically, companies store only about three years of sales data in the disaggregated form required for inventory management on main frame computers. The lack of data can be an impediment to good forecasting. More specifically, the sales for most products follow a distinct life cycle, the form of which can vary between products. Life cycles are usually difficult to gauge without a sales history spanning most of a product's life. Thus life cycle approaches are impossible to implement in short-term forecasting contexts. The alternative is to employ the exponential smoothing methods. These allow quantities such as levels and growth rates to adapt to fundamental changes caused by life cycle effects without the need for an explicit representation of the life cycle. Larger values of the smoothing parameters are associated with greater underlying change. The lack of data disguises the presence of life cycle effects in time series plots, suggesting a picture of greater structural stability than that which actually prevails. A consequence of this phenomena is that smoothing parameters chosen to minimise total error criteria such as the sum of squared errors exhibit a downward bias.

- The standard deviation of the lead time demand in most systems is estimated with the formula $\sqrt{L}s$, s being the estimate of the standard deviation of review period demand. This formula is based on the assumption that review period demands are identically and independently distributed. Yet, the models underpinning the exponential smoothing methods explicitly assume non stationary behaviour with inter temporal dependencies. The consequent errors can be quite large (Johnson & Harrison, 1986; Harvey & Snyder, 1990). There is a general tendency to significantly underestimate the amount of uncertainty and this leads to lower safety stocks than those required to maintain desired fill rates.
- When using trend corrected exponential smoothing a local trend can temporarily point downwards leading to the possibility of negative forecasts.
- With the down-sizing of middle management in recent years, companies often operate without the resources to investigate items thrown up by monitoring systems. Those staff still employed in the inventory management area also often lack the statistical expertise to make appropriate judgements about adjustments to forecasting methods required for items rejected by the monitoring process.

Problems with the inventory module may also be encountered.

- Inventory models in common use are based on the assumption that review period demands are independently and identically distributed over time. Yet the modules which incorporate them are usually fed predictions and variances from the exponential smoothing methods which presume that the time series under consideration are potentially non stationary. In other words the forecast and inventory control modules are usually based on incompatible methods. Past attempts (Karlin, 1960; Iglehart and Karlin, 1962) to break away from the stationary assumption have resulted in methods that are not commonly used.
- The typical executive involved in inventory management is usually insufficiently trained to understand stochastic inventory models with formulae like (21). They are ill-equipped to recognise and correct faults in inventory software.

It has already been seen that order-up-to levels and reorder levels consistent with the objectives of a business can be determined by retrospective simulation. This can be done without recourse to a forecasting module and the associated implementation problems. It can also be done without classical inventory control theory and its problems. Herein lies the strength of the retrospective simulation approach.

4. Application

A prototype spreadsheet application incorporating the retrospective simulation concept was developed for a car parts distributor. The parts were mainly sourced from Japan and then distributed throughout Australia. The company already had a mainframe package to handle their inventories en-masse. The package included a state-of-the-art forecasting module with the features mentioned earlier. It also had an inventory control module based on a stochastic model of the inventory process.

Only a few staff remained to control inventories after a recent and quite massive downsizing. Those left expressed the need for a PC tool to supplement the mainframe system. Most lacked the prerequisite training in mathematics and statistics to understand the methods employed in the mainframe control system. They wanted a tool that they could understand. At the same time they wanted a tool that empowered them to make effective decisions.

Given this background tt was decided to develop a prototype spreadsheet tool. It would run on a desktop PC connected to the mainframe computer through the company's computer network. The tool would have the capacity to down load data on requested items from the mainframe to the spreadsheet. Graphical displays of the data would be available in forms that were found to be useful. It would possess the capacity for staff to easily explore the available alternatives and to see the consequences of their decisions. And it would also highlight courses of action required to achieve specified fill rates

The resulting spreadsheet model, implemented with Microsoft Excel 4, was based on the method of retrospective simulation outlined in section 2. Monthly sales data for three years was available. It was considered that items involving seasonal effects were so few in number that it was not cost-effective to make special provision for them. Furthermore, growth rates were considered to be too unstable to make explicit allowances for them. Given a lead time from Japan of three months, the first three months of demand data in the sample was reserved for a "run-in" period.

The main control panel is illustrated in Figure 1. Through buttons and associated dialog boxes it is possible for users to:

- Load data on a particular item by pressing the *Part* button. In the prototype this was done with files. It was envisaged, however, that when fully implemented the spreadsheet would access data directly from the mainframe over the network.
- Directly enter order-up-to levels, referred to in company parlance as desired stock levels (DSL), and to examine their effect on stock levels with the chart of opening and closing stock over a 2.75 year period, together with performance indicators such as fill rates and average stock levels.
- To set desired fill rates and allow the computer to determine the appropriate desired stock levels (DSL).
- To also determine the DSL using the tail probability of a normal distribution.

It was found that most of the parts the staff wished to examine with the tool possessed unstable structural features. The example illustrated in Figure 1 is a case in point. It shows great discrepancies in the fill rate over time. Such items are not handled properly by automated mainframe inventory control systems which are implicitly based on assumptions of stability. The ultimate answer for these items was deemed to lie with human judgement based on intuition and experience. The control panel in Figure 1 was devised as a way to provide staff with the necessary information to facilitate the required judgements. This was complemented by the plot of demand shown in Figure 2, accessed from the control panel by pressing the *Demand Plot* button. Together, the stock and demand plots enabled staff to devise what they deemed to be appropriate responses for such difficult items.

5. Concluding Remarks

When demands are assumed to be generated by a stationary stochastic process the retrospective simulation approach outlined in this paper for inventory control is remarkably simple. In non stationary cases it is necessary to resort to the use of linear programming. To this extent, some of the simplicity originally sought for the approach is lost. Nevertheless, the general notion of minimising stocks subject to a fill rate restriction has considerable intuitive appeal to management. Furthermore, the logic of the approach is internally consistent, unlike traditional approaches where the inventory methods are usually based on assumptions that are incompatible with those underpinning the forecasting methods. In most applications, because of the educational backgrounds of staff involved with inventory management, it would be necessary to treat the optimisation routine as a black box. Nevertheless, it is conjectured that implementations along these lines would be more stable, and require less management intervention, than the traditional approaches.

The spreadsheet application of retrospective simulation to control car parts, as described in this paper, was designed to complement an existing main frame system based on more traditional methods. Yet there is no inherent reason why this approach should always be restricted to a spreadsheet environment. The equations associated with the method are readily coded in a traditional computer language. Thus it can be readily adapted for use on mainframe computers and used in place of the traditional approaches. Out would go the forecasting module and all the associated overheads. Out would go the stochastic inventory models that have bewildered generations of practitioners. In would come an approach that is not only easier to understand, but one that is viable in practice with far fewer simplifying assumptions.

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Table 1. Retrospective Simulation of Order-Up-To Level System

Summary	
Order-Up-To Level	80
FillRate	71%
P_Out	100%
Avg Stock	58

OUL	80

Week	Order	Delivery	Opening Stock	Demand	Closing Supply	Closing Stock	Excess Demand	Shortage Indicator	Average Stock
0					0				
1	80	80	80	132	-52	0	52	1	48
2	132	132	80	130	-50	0	50	1	49
3	130	130	80	96	-16	0	16	1	67
4	96	96	80	91	-11	0	11	1	70
5	91	91	80	113	-33	0	33	1	57
6	113	113	80	123	-43	0	43	1	52
7	123	123	80	111	-31	0	31	1	58
8	111	111	80	142	-62	0	62	1	45
9	142	142	80	108	-28	0	28	1	59
10	108	108	80	83	-3	0	3	1	77
Total	1126	1126	800	1129	-329	0	329	10	582
Avg	113	113	80	113	-33	0	33	100%	58

Table 2. Retrospective Simulation of Order-Up-To Level System

Summary	
Order-Up-To Level	118
FillRate	95%
P_Out	40%
Avg Stock	83

OUL	118

Week	Order	Delivery	Opening Stock	Demand	Closing Supply	Closing Stock	Excess Demand	Shortage Indicator	Average Stock
0					0				
1	118	118	118	132	-14	0	14	1	105
2	132	132	118	130	-12	0	12	1	106
3	130	130	118	96	22	22	0	0	70
4	96	96	118	91	27	27	0	0	72
5	91	91	118	113	5	5	0	0	61
6	113	113	118	123	-5	0	5	1	113
7	123	123	118	111	7	7	0	0	62
8	111	111	118	142	-24	0	24	1	97
9	142	142	118	108	10	10	0	0	64
10	108	108	118	83	35	35	0	0	76
Total	1164	1164	1176	1129	47	104	56	4	826
Avg	116	116	118	113	5	10	6	40%	83

Figure 1. Main Control Panel of Prototype Inventory Management Package

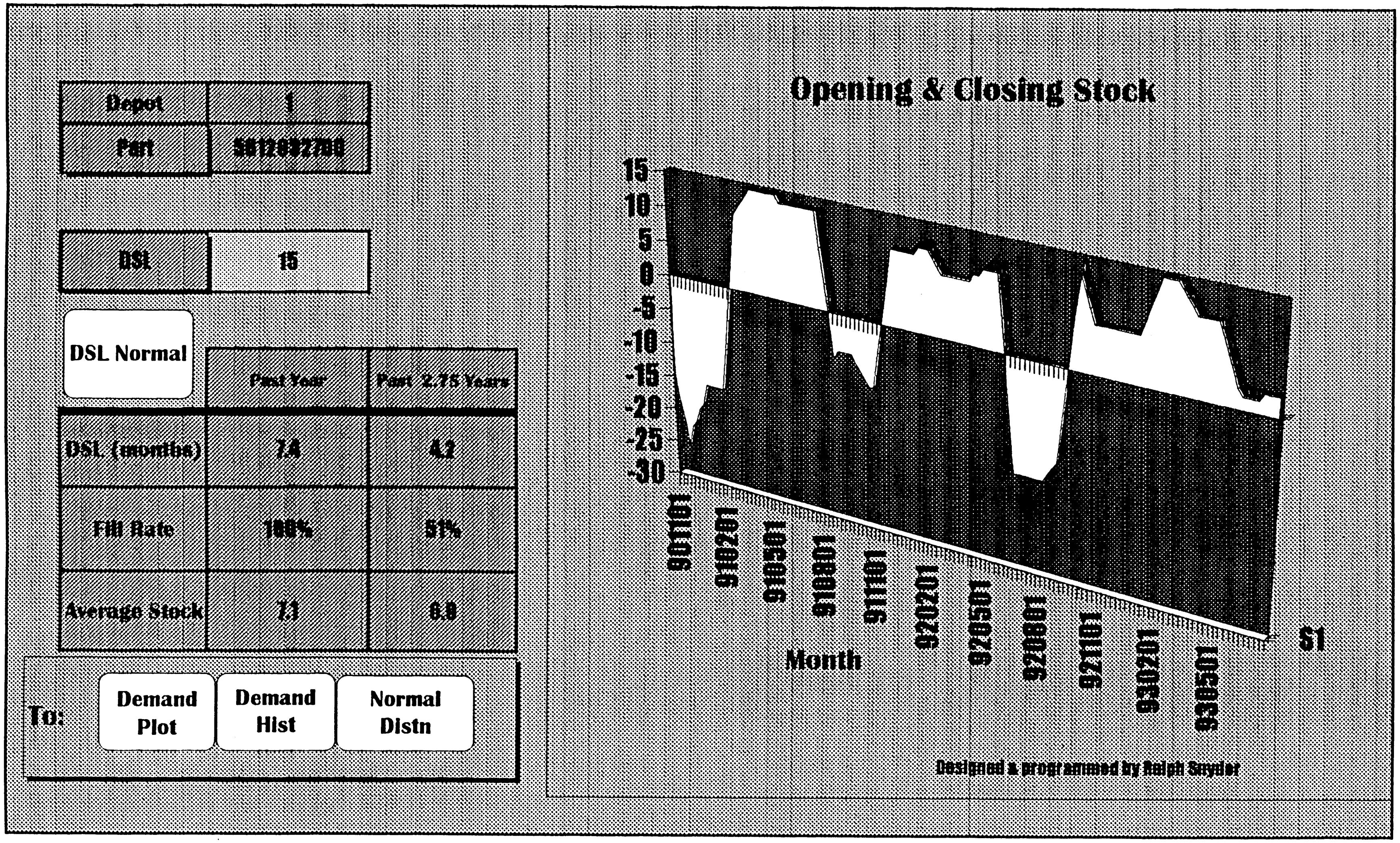

Figure 2. Monthly Demand Plot