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A DIAGNOSTIC TEST FOR STRUCTURAL CHANGE  
IN COINTEGRATED REGRESSION MODELS

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# A DIAGNOSTIC TEST FOR STRUCTURAL CHANGE IN COINTEGRATED REGRESSION MODELS

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In this paper we derive the asymptotic distribution of the OLS based CUSUM test in the context of cointegrated regression models and tabulate its critical values. It is also found that the test has non-trivial local power irrespective of the particular type of structural change.

Keywords: structural change, cointegration, fully modified ordinary least squares, CUSUM test.

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## 1. INTRODUCTION

Recently Hansen (1993) examined the problem of testing for structural instability in the context of cointegrated regression models. Making use of the fully modified OLS (FM) estimation method of Phillips and Hansen (1990), Hansen (1993) derived the asymptotic distribution of various test statistics against different alternatives of interest. At the same time, Quintos and Phillips (1993) proposed a Lagrange Multiplier (LM) test against the random walk alternative which corresponds to Hansen's approach.

Both Quintos and Phillips (1993) and Hansen's (1993) approaches precisely specify the form of structural break under the alternative hypothesis. However, it is sometimes desirable to investigate generally the stability properties of the cointegrating model without specifying the possible alternatives. In this case, testing for structural change can be more or less regarded as a diagnostic test. In this paper we consider a test designed for this purpose. It is the CUSUM test with OLS residuals which was originally suggested by Ploberger and Krämer (1992) for stationary regressors. In addition to deriving the asymptotic distribution of the OLS based CUSUM test, we also investigate its local power performance. An interesting finding is that the test has non-trivial local power irrespective of the particular type of structural change. Considering that the OLS based CUSUM test can be easily calculated from the residuals, it is therefore recommended as a routine test for structural change in the context of cointegrated regression models.

The organization of this paper is as follows. Section 2 sets up the structure of the model. Section 3 investigates the limiting distribution of the test statistic. Section 4 examines the local power performance of the test. Concluding remarks are given in section 5.

## 2. THE COINTEGRATED REGRESSION MODEL

Consider the cointegrated regression model

$$y_t = Ax_t + u_t, \quad t = 1, \dots, T \quad (1)$$

where the process  $x_t = (x'_{1t}, x'_{2t})'$  is determined by the equations

$$x_{1t} = k_{1t}$$

$$x_{2t} = \Pi_1 k_{1t} + \Pi_2 k_{2t} + x_{2t}^0$$

$$x_{2t}^0 = x_{2t-1}^0 + v_t.$$

Define

$$\zeta_t = (u_t, v_t'), \quad k'_t = (k'_{1t}, k'_{2t})$$

where  $\zeta_t$  is a  $k+1$  dimensional sequence of stationary disturbances with mean zero, while the elements of  $k'_t$  are non-negative integer powers of time of order  $p$ . That is,  $k'_t = (1, t, t^2, \dots, t^p)$ .

This model is fairly general and encompasses a wide range of applications; see Hansen (1993) for details. It also turns out to be especially convenient for developing the asymptotic distribution of our test statistic.

Suppose  $\zeta_t = [u_t, v_t']$  satisfies the multivariate invariance principle as set out by Phillips and Durlauf (1986). Let  $R_T(r) = R_{[Tr]} = \sum_{t=1}^{Tr} \zeta_t$ , then  $T^{-1/2}R_{[Tr]} \xrightarrow{d} W(r) = (W_0(r), W_1(r))$ , where  $W(r)$  is a  $k+1$  dimensional Brownian Motion and is partitioned in conformity with  $\zeta_t$ , with covariance matrix

$$\Psi = \begin{bmatrix} \omega_0^2 & \Psi_{01} \\ \Psi_{10} & \Psi_1 \end{bmatrix} = \lim_{T \rightarrow \infty} T^{-1} E \left( \sum_{t=1}^T \zeta_t \right) \left( \sum_{t=1}^T \zeta_t \right)' = \Sigma + \Lambda + \Lambda',$$

where

$$\Sigma = \lim_{T \rightarrow \infty} T^{-1} E \left( \sum_{t=1}^T \zeta_t \zeta_t' \right) = \begin{bmatrix} \sigma_0^2 & \Sigma_{01} \\ \Sigma_{10} & \Sigma_1 \end{bmatrix},$$

$$\Lambda = \lim_{T \rightarrow \infty} T^{-1} \sum_{t=2}^T \sum_{j=1}^{t-1} E(\zeta_j \zeta_t') = \begin{bmatrix} \lambda_0^2 & \Lambda_{01} \\ \Lambda_{10} & \Lambda_1 \end{bmatrix}$$

where  $\Psi$ ,  $\Sigma$  and  $\Lambda$  are partitioned in conformity with  $\zeta_t$ . Also define

$$\Delta = \Sigma + \Lambda = \begin{bmatrix} \Delta_0 & \Delta_{01} \\ \Delta_{10} & \Delta_1 \end{bmatrix}.$$

Denote consistent estimators of  $\Psi$  and  $\Delta$  as  $\hat{\Psi}$  and  $\hat{\Delta}$ , respectively.

Partition  $\hat{\Psi}$  and  $\hat{\Delta}$  as  $\hat{\Psi}$  and  $\hat{\Delta}$ . Set

$$\hat{\omega}_{0.1}^2 = \hat{\omega}_0^2 - \hat{\Psi}_{01} \hat{\Psi}_1^{-1} \hat{\Psi}_{10}, \quad \hat{\Delta}_{10}^+ = \hat{\Delta}_{10} - \hat{\Delta}_1 \hat{\Psi}_1^{-1} \hat{\Psi}_{10}.$$

Define the transformed dependent variable

$$y_t^+ = y_t - \hat{\Psi}_{01}' \hat{\Psi}_1^{-1} v_t.$$

The cointegrated regression model (1) can be written as

$$y_t^+ = A x_t + u_t^+ \quad (2)$$

with the FM estimator of  $A$  developed by Phillips and Hansen (1990) given by

$$\hat{A}^+ = \left[ \sum_1^T y_t^+ x_t' - T(0, \hat{\Delta}_{10}^+) \right] \left( \sum_1^T x_t x_t' \right)^{-1}, \quad (3)$$

with the FM residuals

$$\hat{u}_t^+ = y_t^+ - \hat{A}^+ x_t.$$

Before going to the next section, we modify model (1) to incorporate possible parameter instability by allowing  $\gamma$  to depend on time.

$$y_t = A_t x_t + u_t \quad (4)$$

The null hypothesis can be formulated as

$$H_0: A_1 = A_2 = \dots = A_T = A,$$

with the alternative being  $H_1$ : At least one equality does not hold.

### 3. ASYMPTOTIC DISTRIBUTION OF THE TEST STATISTIC

Let  $[.]$  denote "integer part", and define the FM OLS based CUSUM test statistic



$$B^{(T)}(\tau) = \frac{1}{\hat{\omega}_{0.1} \sqrt{T}} \sum_{t=1}^{[T\tau]} \hat{u}_t^{+(T)}. \quad (5)$$

We reject  $H_0$  for large values of  $\sup_{0 < \tau < 1} |B^{(T)}(\tau)|$ .  $B^{(T)}(\tau)$  simply comprises standardized partial sums of the FM OLS residuals. This is identical to the OLS based CUSUM test proposed by Ploberger and Krämer (1992), except that OLS residuals are replaced by FM OLS residuals and the estimated error variance is replaced by the long run variance estimate  $\hat{\omega}_{0.1}^2$ .

For convenience, denote  $S_T(r) = S_{[Tr]} = \sum_1^{[Tr]} (x_t \hat{u}_t^+ - \begin{pmatrix} 0 \\ \hat{\Delta}_{10}^+ \end{pmatrix})$ , and  $k(r) = (1, r, \dots, r^p)'$ . Also denote  $X(r) = \begin{pmatrix} k(r) \\ B_1(r) \end{pmatrix}$ ,  $S(r) = \int_0^r X dB_{0.1}$  and  $M(r) = \int_0^r XX'$  where  $B_{0.1}(r)$  and  $B_1(r)$  are independent standard Brownian Motions with dimensions 1 and  $k$ , respectively. Then the asymptotic distribution of the FM OLS based CUSUM test is found as a direct consequence of Theorem 2 of Hansen (1993) in which he sets up the asymptotic distribution of  $S_T(r)$ . Observe that  $\sum_1^{[Tr]} \hat{u}_t^+$  is simply the first element of the vector  $S_T(r)$ . Therefore by Theorem 2 of Hansen (1993), we have

$$B^{(T)}(\tau) = \frac{1}{\hat{\omega}_{0.1} \sqrt{T}} \sum_{t=1}^{[T\tau]} \hat{u}_t^{+(T)} \xrightarrow{d} S_1^*(\tau) \quad (6)$$

where  $S_1^*(\tau)$  is the first element of vector

$$S^*(\tau) = S(\tau) - M(\tau)M(1)^{-1}S(1)$$

which is free of any nuisance parameters.

**Theorem 1.** The FM OLS based CUSUM statistic (5) for parameter constancy in model (4) has asymptotic distribution given by  $S_1^*(\tau)$ .

A couple of special cases are worth considering. In particular, if  $k_{1t} = 1$  and  $k_{2t} = 0$ , then (4) can be expressed as



$$\begin{aligned} y_t &= \alpha_t + \beta_t x_t + u_t \\ x_t &= x_{t-1} + v_t \end{aligned} \quad t = 1, \dots, T. \quad (7)$$

In this case,  $x_t$  is  $I(1)$  without drift. To get  $S_1^*(\tau)$ , denote

$$P(r) = 1 - \int_0^1 B_1' (\int_0^1 B_1 B_1')^{-1} B_1, \quad Q(r) = B_1 - \int_0^1 B_1.$$

Observe that

$$\begin{aligned} M(\tau) &= \begin{bmatrix} \tau & \int_0^\tau B_1' \\ \int_0^\tau B_1' & \int_0^\tau B_1 B_1' \end{bmatrix}, \\ M(1)^{-1} &= \begin{bmatrix} (\int_0^1 P)^{-1} & -(\int_0^1 P)^{-1} \int_0^1 B_1' (\int_0^1 B_1 B_1')^{-1} \\ -(\int_0^1 Q Q')^{-1} \int_0^1 B_1 & (\int_0^1 Q Q')^{-1} \end{bmatrix}. \end{aligned}$$

Thus  $B^{(T)}(\tau) \xrightarrow{d} S_1^*(\tau)$

$$\begin{aligned} &= B_{0.1}(\tau) - \tau (\int_0^1 P)^{-1} \int_0^1 [1 - \int_0^1 B_1' (\int_0^1 B_1 B_1')^{-1} B_1] dB_{0.1} \\ &\quad - \int_0^\tau B_1' (\int_0^1 Q Q')^{-1} (\int_0^1 B_1 dB_{0.1} - \int_0^1 B_1 \int_0^1 dB_{0.1}) \\ &= B_{0.1}(\tau) - \tau (\int_0^1 P)^{-1} \int_0^1 P dB_{0.1} - \int_0^\tau B_1' (\int_0^1 Q Q')^{-1} \int_0^1 Q dB_{0.1} \end{aligned} \quad (8)$$

On the other hand, if  $k_{1t} = 1$  and  $k_{2t} = t$ , then (4) can be expressed as

$$\begin{aligned} y_t &= \alpha_t + \beta_t x_t + u_t \\ x_t &= \mu + x_{t-1} + v_t \end{aligned} \quad t = 1, \dots, T. \quad (9)$$

In this case,  $x_t$  is  $I(1)$  with drift. Again, to get  $S_1^*(\tau)$ , denote

$$P(r) = 1 - \int_0^1 (r, B_1') \left[ \int_0^1 \begin{pmatrix} r \\ B_1 \end{pmatrix} (r, B_1') \right]^{-1} \begin{pmatrix} r \\ B_1 \end{pmatrix}, \quad Q(r) = \begin{pmatrix} r \\ B_1 \end{pmatrix} - \int_0^1 \begin{pmatrix} r \\ B_1 \end{pmatrix}.$$

Observe that

$$\begin{aligned} M(\tau) &= \begin{bmatrix} \tau & \int_0^\tau (r, B_1') \\ \int_0^\tau \begin{pmatrix} r \\ B_1 \end{pmatrix} & \int_0^\tau \begin{pmatrix} r \\ B_1 \end{pmatrix} (r, B_1') \end{bmatrix}, \\ M(1)^{-1} &= \begin{bmatrix} (\int_0^1 P)^{-1} & -(\int_0^1 P)^{-1} \int_0^1 (r, B_1') \left[ \int_0^1 \begin{pmatrix} r \\ B_1 \end{pmatrix} (r, B_1') \right]^{-1} \begin{pmatrix} r \\ B_1 \end{pmatrix} \\ -(\int_0^1 Q Q')^{-1} \int_0^1 \begin{pmatrix} r \\ B_1 \end{pmatrix} & (\int_0^1 Q Q')^{-1} \end{bmatrix}. \end{aligned}$$

Thus  $B^{(T)}(\tau) \xrightarrow{d} S_1^*(\tau)$

$$\begin{aligned}
&= B_{0.1}(\tau) - \tau(\int_0^1 P)^{-1} \int_0^1 \left\{ 1 - \int_0^1 (r, B_1') \left[ \int_0^1 (B_1) (r, B_1') \right]^{-1} (B_1) \right\} dB_{0.1} \\
&\quad - \int_0^1 (r, B_1') (\int_0^1 Q Q')^{-1} \left[ \int_0^1 (B_1) dB_{0.1} - \int_0^1 (B_1) \int_0^1 dB_{0.1} \right] \\
&= B_{0.1}(\tau) - \tau(\int_0^1 P)^{-1} \int_0^1 P dB_{0.1} - \left( \frac{1}{2} \tau^2, \int_0^1 B_1' \right) (\int_0^1 Q Q')^{-1} \int_0^1 Q dB_{0.1} \quad (10)
\end{aligned}$$

Since the asymptotic distribution of  $B^{(T)}(\tau)$  does not depend on any nuisance parameters, we can obtain the critical values of the FM OLS based CUSUM test by calculating  $\sup |B^{(T)}(\tau)|$  for  $0 < \tau < 1$ . In particular, we tabulate the critical values for models (7) and (9), respectively. They are found by simulation using a GAUSS program with a sample size of 1000 and 10000 replications for one to five explanatory variables. The results are given in Table 1 and Table 2, respectively.

#### 4. LOCAL POWER PERFORMANCE

For convenience, we consider the local power of the test under the condition that no constant term is included in the regression and  $\{x_t\}$  are strictly exogenous. In this case, only OLS estimation is required to establish valid asymptotic distributions of  $B^{(T)}(r)$ . It is easy to show that under  $H_0$  and according to Theorem 3.1 of Park and Phillips (1988)

$$\begin{aligned}
B^{(T)}(\tau) &= \frac{1}{\hat{\omega}_0 \sqrt{T}} \sum_{t=1}^{[T\tau]} \hat{u}_t = \frac{1}{\hat{\omega}_0 \sqrt{T}} \left[ \sum_{t=1}^{[T\tau]} u_t - \sum_{t=1}^{[T\tau]} (\hat{\beta} - \beta) x_t \right] \\
&\xrightarrow{d} B^*(\tau) = B_{0.1}(\tau) - \int_0^1 B_1' (\int_0^1 B_1 B_1')^{-1} \int_0^1 B_1 dB_{0.1} \quad (11)
\end{aligned}$$

where  $\hat{u}_t$  are the OLS residuals.

Suppose  $\beta_{t,T} = \beta_0 + T^{-1}g(r)$ , where  $r = t/T$  and  $g(r)$  can be any step function or uniform limit of step functions. Denote  $y_t = \beta_{t,T} x_t + u_t$ . Under  $H_0$ ,  $\beta$  is estimated by

$$\begin{aligned}
\hat{\beta} &= \sum_{t=1}^T y_t x_t' (\sum_{t=1}^T x_t x_t')^{-1} \text{ with residuals } \hat{u}_t = y_t - \hat{\beta} x_t. \text{ Thus we have} \\
B^{(T)}(\tau) &= \frac{1}{\hat{\omega}_0 \sqrt{T}} \sum_{t=1}^{[T\tau]} [u_t + \beta_0 x_t + T^{-1} g(r) x_t - \hat{\beta}^{(T)} x_t] \\
&= \frac{1}{\hat{\omega}_0 \sqrt{T}} \sum_{t=1}^{[T\tau]} [u_t + T^{-1} g(r) x_t - (g(r) \sum_{t=1}^T \frac{x_t x_t'}{T} + \sum_{t=1}^T u_t x_t' (\sum_{t=1}^T x_t x_t')^{-1} x_t)] \\
&\stackrel{d}{\Rightarrow} \frac{1}{\omega_0} [W_0(\tau) + \int_0^\tau g(r) W_1 - (\int_0^1 g(r) W_1 W_1' + \int_0^1 W_1 dW_0') \int_0^\tau (\int_0^1 W_1 W_1')^{-1} W_1] \\
&= B^*(\tau) + \frac{1}{\omega_0} \Sigma_1^{1/2} [\int_0^\tau g(r) B_1 - \int_0^1 g(r) B_1 B_1' \int_0^\tau (\int_0^1 B_1 B_1')^{-1} B_1] \quad (12)
\end{aligned}$$

When  $H_0$  is true,  $\beta_{t,T} = \beta_0$ , we must have  $g(r) = 0$  for  $r \in (0, 1)$ . The OLS based CUSUM test converges to its null distribution. Under  $H_1$ ,  $g(r) \neq 0$ , (10) is well defined and distinct from the null distribution. The test thus has non-trivial power irrespective of the particular type of structural change. This conclusion is significant since it is in contrast to the result derived by Ploberger and Krämer (1992) in which they found that when the regressors are stationary, the OLS based CUSUM test has only trivial local power against structural changes that are orthogonal to the mean regressor.

## 5. AN APPLICATION

In his paper, Hansen (1993) investigated the stability property of three postwar U.S. interest rates. In particular, he examined the cointegrated regression model between the federal funds rate (FF) and the 90-day treasury bill rate (TB3) and found some evidence of structural change using his supF, meanF and  $L_c$  tests. Here we apply the FM OLS based CUSUM test to test the same model. We obtain the following regression result

$$\begin{aligned}
TB3_t &= 0.487 + 0.830 FF_t, \\
(.14) \quad & (.02)
\end{aligned}$$

$$\sup_{0 < \tau < 1} |B^{(T)}(\tau)| = 1.130, \text{ p-value} = 0.063.$$

Our result supports Hansen's conclusion. The test rejects the null hypothesis of no structural change at the 10% level. Although the test is not significant at the 5% level, its p-value is close to 0.05. Such a result is quite similar to that of the  $L_c$  test of Hansen (1993), while the FM OLS based CUSUM test requires much less computational effort.

## 6. CONCLUDING REMARKS

In this paper we have examined the OLS based CUSUM test in the context of the cointegrated regression model. In addition to deriving its asymptotic distribution, which is nuisance parameter free, we also show that it has non-trivial local power irrespective of the form of structural change. This significantly contrasts with its performance under stationary regression models, where it has trivial local power in certain cases.

Since the associated residuals are simply a natural by-product of FM estimation of cointegrated regression models, the FM OLS based CUSUM test can be very easily conducted. We thus recommend it as a routine test for structural change in the context of cointegrated regression models.

Table 1

Asymptotic Critical Values of the FM OLS based CUSUM Test for Model (7)

Number of regressors (Excluding Constant)	10%	5%	1%
1	1.0477	1.1684	1.4255
2	0.9342	1.0413	1.2561
3	0.8381	0.9336	1.1782
4	0.7687	0.8475	1.0383
5	0.7122	0.7890	0.9680

Table 2

Asymptotic Critical Values of the FM OLS based CUSUM Test for Model (9)

Number of regressors (Excluding Constant)	10%	5%	1%
1	0.7678	0.8332	0.9694
2	0.7214	0.7855	0.9162
3	0.6796	0.7385	0.8761
4	0.6421	0.6951	0.8270
5	0.6173	0.6681	0.7787

## REFERENCES

- Hansen, B. (1993): "Tests for parameter instability in regressions with  $I(1)$  processes," *Journal of Business and Economic Statistics*, 10, 321 - 355.
- Park, J. Y., and P. C. B. Phillips (1988): "Statistical inference in regressions with Integrated processes: Part 1," *Econometric Theory*, 4, 468 - 497.
- Phillips, P. C. B., and S. N. Durlauf (1986): "Multiple time series regression with integrated processes," *Review of Economic Studies*, 53, 473 - 496.
- Phillips, P. C. B., and B. Hansen (1990): "Statistical inference in instrumental variables regression with  $I(1)$  processes," *Review of Economic Studies*, 57, 99 - 125.
- Ploberger, W., and W. Krämer (1992): "The CUSUM test with OLS residuals," *Econometrica*, 60, 271 - 285.
- Quintos, C. E., and P. C. B. Phillips (1993): "Parameter constancy in cointegrating regressions," *Empirical Economics*, 18, 675 - 706.

