MONASH 5/94

MONASH UNIVERSITY

AUSTRALIA

HYPOTHESIS TESTING OF VARYING COEFFICIENT REGRESSION MODELS: PROCEDURES AND APPLICATIONS

Robert D. Brooks and Maxwell L. King

Working Paper No. 5/94

May 1994

DEPARTMENT OF ECONOMETRICS

ISSN 1032-3813

ISBN 0 7326 0394 3

HYPOTHESIS TESTING OF VARYING COEFFICIENT REGRESSION MODELS: PROCEDURES AND APPLICATIONS

Robert D. Brooks and Maxwell L. King

Working Paper No. 5/94

May 1994

DEPARTMENT OF ECONOMETRICS

MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

HYPOTHESIS TESTING OF VARYING COEFFICIENT REGRESSION MODELS: PROCEDURES AND APPLICATIONS

Robert D. Brooks

Department of Economics and Finance

Royal Melbourne Institute of Technology, GPO Box 2476V, Melbourne, Victoria 3001

Australia

and

Maxwell L. King

Department of Econometrics

Monash University, Clayton, Victoria 3168

Australia

Abstract

There is a considerable literature in econometrics on varying coefficient regression models. Some of the proposed models are simple and parsimonious. However given that even the simplest varying coefficient model is more complex than the constant coefficient model, researchers need to be able to test the adequacy of the constant coefficient simplification. This paper surveys the literature on testing for the presence of varying regression coefficients. We outline the wide variety of tests that have been proposed and look in detail at comparisons of the properties of different tests. In general, the literature indicates that tests that take into account the one-sided nature of the testing problem do best. Therefore tests such as point optimal tests and locally most mean powerful tests typically have superior power properties relative to other available tests. The paper concludes with a review of the application of this methodology in two areas of empirical finance. These involve testing the time constancy of systematic risk in the market model and testing the unbiased prediction hypothesis in forward and futures markets.

Key Words

Hildreth-Houck random coefficients; Lagrange multiplier tests; point optimal tests; small sample power; return to normalcy random coefficients; testing forward pricing models; testing the market model.

1. Introduction

The literature on varying coefficient regression models dates from the early days of econometrics, with Keynes' (1939) critique of the use of constant coefficient models by Tinbergen (1939). Despite this early advocacy of the use of such models, their popularity was not enhanced until the development of simple parametric forms of varying coefficient models such as those suggested by Hildreth and Houck (1968), Rosenberg (1973) and Cooley and Prescott (1973a), and Lucas' (1976) econometric policy evaluation critique based on the rational expectations revolution in macroeconomics. Since then, the use of varying coefficient models has taken off with published surveys on the topic including Raj and Ullah (1981), Beck (1983), Chow (1984), Nicholls and Pagan (1985) and Swamy, Conway and LeBlanc (1988a, 1988b, 1989). Accordingly, applied econometric work has begun to take interest in the varying coefficient regression model as a serious alternative to traditional fixed coefficient modelling. Of particular recent interest has been Granger's (1993) suggestion that varying coefficient models are likely to provide an adequate approximation to non-linear models.

The aim of this paper is to review the literature on hypothesis testing of varying coefficient models. Accordingly, Section 2 describes the main varying coefficient models in which the coefficient varies stochastically over time. While limited consideration is given to models of discrete regime shifts in Section 3.4, the main focus of this paper is stochastic varying coefficient models where the parameters of interest evolve continuously over time as opposed to changing discretely. Section 3 then reviews the literature on testing for the presence of varying coefficients in the context of the linear regression model. It finds a variety of testing strategies have been used and that the evidence on power considerations favours tests that take into account the one-sided nature of the testing problem such as point optimal tests (see King 1987a) and locally most mean powerful tests (see King and

Wu 1990 and Wu and King 1994). Section 4 reviews the theory of point optimal testing and its application to testing varying coefficient models. Section 5 discusses some of the applied work using varying coefficient models, focusing on two main applications in empirical finance. These applications are the time constancy of systematic risk in the market model, and the unbiased prediction hypothesis in forward and futures markets. In general, both of these applications have provided some evidence in favour of the presence of varying coefficient regression models. Section 6 contains some concluding remarks.

2. Varying Coefficient Models

Of interest is the linear regression model with a single time varying coefficient,

$$y_t = x_t\beta_t + z_t'\alpha + \epsilon_t, \tag{1}$$

in which y_t is the dependent variable, x_t is the non-stochastic regressor with the single varying coefficient β_t, z_t is a $k \times 1$ vector of non-stochastic explanatory variables with fixed coefficient vector α, $\epsilon_t \sim IN(0, \sigma^2)$ and $t = 1, 2, ..., n$.

2.1 The Hildreth-Houck Random Coefficient Model

The Hildreth and Houck (1968) random coefficient model states that the single varying coefficient β_t from (1) follows the process,

$$\beta_t = \bar{\beta} + u_t, \tag{2}$$

in which $u_t \sim IN(0, \lambda_0\sigma^2)$ and is independent of ϵ_t.

If β_t follows this process then by substitution, the model (1) becomes,

$$y_t = x_t\bar{\beta} + z_t'\alpha + v_t, \tag{3}$$

in which $v_t = \epsilon_t + x_t u_t$. The properties of v_t are that it is normally distributed with,

$$E(v_t) = 0,$$

$$Var(v_t) = \sigma^2(1 + \lambda_0 x_t^2),$$

and

$$Cov(v_t, v_s) = 0, \quad t \neq s.$$

Therefore, if the single varying coefficient β_t follows the Hildreth-Houck random coefficient model, then the disturbance term in the reparameterised model (3) will be heteroscedastic, making ordinary least squares (OLS) estimates of the parameters of the model inefficient, confidence intervals based on those estimates misleading and forecasts generated from the estimated model inefficient. The Hildreth-Houck random coefficient model collapses back to the constant coefficient model when $\lambda_0 = 0$, making this the key parameter for the model.

From an economic perspective, the Hildreth-Houck model has an instantaneous mean reversion property. When the coefficient β_t is shocked away from its mean, $\bar{\beta}$, it always instantaneously reverts back to $\bar{\beta}$. Therefore the effect of any shock is merely transitory and confined to the period in which it occurs.

2.2 Rosenberg's Return to Normalcy Random Coefficient Model

The Rosenberg (1973) return to normalcy random coefficient model states that the single time varying coefficient β_t from (1) follows the process,

$$\beta_t = \phi\beta_{t-1} + (1-\phi)\bar{\beta} + a_t, \tag{4}$$

in which $a_t \sim IN(0, \lambda_1\sigma^2)$ and is independent of ϵ_t. For (4) to be a stationary process, it is required that $| \phi |< 1$. However a more economically meaningful restriction is $0 \leq \phi < 1$, as it produces a smooth evolution of the coefficient over time as opposed to the oscillations which would be associated with a negative ϕ value. As Collins, Ledolter and Rayburn (1987) point out, a smooth evolution of the coefficient over time is what applied researchers would expect to find. Further, a negative ϕ value poses difficulties in interpretation and analysis if the period between observations is changed.

Under this process, the model (1) transforms to,

$$y_t = x_t\bar{\beta} + z_t'\alpha + v_t, \tag{5}$$

in which $v_t = x_t(\beta_t - \bar{\beta}) + \epsilon_t$. The properties of v_t are that it is normally distributed with,

$$E(v_t) = 0,$$

$$Var(v_t) = \sigma^2(1 + x_t^2\lambda_1/(1-\phi^2)),$$

and

$$Cov(v_t, v_s) = (\lambda_1\sigma^2 x_t x_s \phi^{|t-s|})/(1-\phi^2), \qquad t \neq s.$$

Therefore, when the single varying coefficient β_t from the model (1) follows the Rosenberg return to normalcy random coefficient process, the disturbance term in the reparameterised model (5), will be both autocorrelated and heteroscedastic. This makes OLS an inefficient method by which to estimate this model. It again leads to misleading confidence intervals on the estimated parameters and inefficient forecasts from the estimated model. The model collapses back to the constant coefficient model when $\lambda_1 = 0$ leaving ϕ unidentified and unnecessary in the case of the constant coefficient model.

From an economic perspective, Rosenberg's model still possesses a mean reversion property. However, unlike the Hildreth-Houck model this mean reversion is not instantaneous. The speed of mean reversion depends on the value of the AR(1) parameter ϕ. The speed of mean reversion is greater the smaller is the value of ϕ. In the limiting case of $\phi = 0$, the Rosenberg model collapses back to the Hildreth-Houck model, and mean reversion becomes instantaneous. In the more general setting of non-zero ϕ, any shocks do have a persistent effect, the duration of that effect being dependent on ϕ. However, these shocks eventually die out, returning the process to its mean.

2.3 Other Models

There exist a variety of other varying coefficient models. A popular alternative is the random walk coefficient model introduced by Cooley and Prescott (1973a, 1973b), where β_t from (1) follows the process,

$$\beta_t = \beta_{t-1} + b_t, \tag{6}$$

in which $b_t \sim IN(0, \lambda_2\sigma^2)$ and is independent of ϵ_t.

This random walk coefficient model can be viewed as a limiting case of the Rosenberg model, in which $\phi = 1$. In this case, the coefficient has no mean reversion properties and the effects of any shocks are permanent. It is this property which makes this model unappealing in most economic situations. Typically economic theory, will provide likely mean values for the coefficients, therefore implying that any suggested varying coefficient process should allow the coefficients to return to this mean.

This model was extended to the ARIMA(0,1,1) case by Cooley and Prescott (1976) where β_t from (1) follows the process,

$$\beta_t = \beta_{t-1} + p_t - \kappa p_{t-1}, \tag{7}$$

in which $p_t \sim IN(0, \lambda_3\sigma^2)$ and is independent of ϵ_t. The unfortunate feature of this model, given that it is an extension of the random walk coefficient model, is that it also does not have a mean reversion property. For this reason, Ashley (1984) claimed that the Rosenberg model should be preferred to the Cooley-Prescott model as the mean of a coefficient will typically be a value of some economic significance.

A different extension of the random walk coefficient model is the ARIMA(1,1,0) model introduced by Shively (1988a). This model has β_t from (1) following the process,

$$\beta_t = \beta_{t-1} + \mu(\beta_{t-1} - \beta_{t-2}) + c_t. \tag{8}$$

in which $c_t \sim IN(0, \lambda_4\sigma^2)$ and is independent of ϵ_t. The ARIMA(1,1,0) coefficient model collapses back to the random walk coefficient model in the case where $\mu = 0$. The ARIMA(1,1,0) coefficient model also has no mean reversion property, and hence may prove unappealing in most economic modelling situations.

The possibility of the varying coefficient following a more general process, such as an ARIMA(1,0,1) process has been suggested by Ohlson and Rosenberg (1982) and Collins, Ledolter and Rayburn (1987). This would give the process for the single time varying coefficient β_t from (1) as,

$$\beta_t = \rho\beta_{t-1} + (1-\rho)\bar{\beta} + \nu_t + \theta\nu_{t-1}. \tag{9}$$

in which $\nu_t \sim IN(0, \lambda_5\sigma^2)$ and is independent of ϵ_t.

Finally, in the regime of stochastic parameter regression models, Nicholls and Pagan (1985) suggested a more general view of an ARIMA(p,d,q) model. In this case,

$$\rho(L)(\beta_t - \bar{\beta}) = \theta(L)\nu_t. \tag{10}$$

where L is the lag operator and $\rho(L)$ and $\theta(L)$ are polynomials in the lag operator.

The difficulty with a general process such as this, lies in identification of the appropriate model. In the context of identifying a process for the coefficient, one has even greater difficulty than in the case of identification of an ARIMA model for a time series. This is because while the researcher is able to directly observe a time series, no such luxury is present with a time varying coefficient.

In general, these other models are not as satisfactory as either the Hildreth-Houck or Rosenberg models. The two main reasons for this are the absence of mean reversion properties in the integrated models and the lack of parsimony in the more general ARMA(p,q) models. The mean reversion property of the varying coefficient model is of critical importance. Frequently economic theory will suggest testable restrictions on the mean response of the coefficient. If the varying coefficient model chosen does not possess a mean reversion property then one is unable to test these theoretically significant restrictions. The need to restrict attention to simple parsimonious models with a mean reversion property is on the grounds of making the estimation of the model simple. As such, estimation of even the simple models is difficult (see Swamy, Conway and Le Blanc (1988b)).

2.4 Multiple Time Varying Coefficients

An obvious generalisation of the single time varying coefficient model (1) is to allow multiple time varying coefficients so that (1) becomes,

$$y_t = x_t'\beta_t + z_t'\alpha + \epsilon_t, \tag{11}$$

in which x_t is a $p \times 1$ vector of regressors and β_t is a $p \times 1$ vector of time varying coefficients. A special case of (11) is,

$$y_t = x_t'\beta_t + \epsilon_t, \tag{12}$$

in which all the coefficients vary and $\alpha = 0$.

For Hildreth-Houck random coefficients,

$$\beta_t = \bar{\beta} + u_t, \tag{13}$$

where now $\bar{\beta}$ is a $p \times 1$ vector of constants and u_t is a $p \times 1$ disturbance vector such that $u_t \sim N(0, \sigma^2 \Omega)$. The disturbances u_t and ϵ_t are assumed independent and often Ω is assumed diagonal. The analogous model to (3) is,

$$y_t = x_t' \bar{\beta} + z_t' \alpha + v_t, \tag{14}$$

where $v_t \sim IN(0, \sigma^2(1 + x_t' \Omega x_t))$.

In the case of Rosenberg's return to normalcy model, the most general coefficient process is for $\beta_t - \bar{\beta}$ to follow a general first-order vector autoregressive (VAR(1)) process,

$$(\beta_t - \bar{\beta}) = \Delta(\beta_{t-1} - \bar{\beta}) + a_t, \tag{15}$$

where Δ is a $p \times p$ matrix of coefficients and $a_t \sim N(0, \sigma^2 \Omega)$. The disturbances a_t and ϵ_t are assumed independent and for (15) to be a stationary process the eigenvalues of Δ must be within the unit circle. As before, Δ and Ω are often assumed to be diagonal. The analogous model to (5) is now (14) in which,

$$E(v_t) = 0,$$

$$Var(v_t) = \sigma^2(1 + x_t'(\Omega + \Delta \Omega \Delta' + \Delta^2 \Omega (\Delta')^2 + ...)x_t),$$

and,

$$Cov(v_t, v_s) = \sigma^2 x_t' \Delta^{t-s}(\Omega + \Delta \Omega \Delta' + \Delta^2 \Omega (\Delta')^2 + ...)x_s), \quad \text{for} \quad t > s.$$

The random walk coefficient model generalises to β_t following the process,

$$\beta_t = \beta_{t-1} + b_t, \tag{16}$$

where β_t is a $p \times 1$ vector, $b_t \sim N(0, \sigma^2 \Omega)$, and b_t is assumed independent of ϵ_t. Then (11) can be written as,

$$y_t = x_t' \beta_0 + z_t' \alpha + v_t,$$

where now,

$$v_t = \epsilon_t + \Sigma_{j=1}^{t} x_t' b_j,$$

so that,

$$E(v_t) = 0,$$

$$Var(v_t) = \sigma^2(1 + t x_t' \Omega x_t),$$

and

$$Cov(v_t, v_s) = \sigma^2 \min(s,t) x_s' \Omega x_t'.$$

3. Testing For Varying Coefficient Models

3.1 Constant versus Hildreth-Houck Random Coefficients

The problem of testing the constant coefficient model against the single Hildreth-Houck (1968) random coefficient model (1) and (2) can be viewed as testing the constant coefficient model (3) for a special case of the more general form of heteroscedasticity,

$$Var(v_t) = \sigma^2(1 + \lambda_0 q_t^*)^d,$$

in which,

$$q_t^* = (q_t - q_{min})/(q_{max} - q_{min}).$$

The Hildreth-Houck model is the special case of this where $d = 1$, and $q_t = x_t^2$. The key parameter in this testing problem is λ_0, with the testing problem of interest being,

$$H_0 : \lambda_0 = 0 \quad \text{against} \quad H_a : \lambda_0 > 0.$$

The testing problem is one-sided, because λ_0 is a ratio of variances and therefore must be strictly non-negative.

Tests for this problem can therefore be divided into two main categories. We will first consider general tests for additive heteroscedasticity, and then tests specifically designed for detecting the Hildreth-Houck random coefficient model. An early test for additive heteroscedasticity is that suggested by Goldfeld and Quandt (1965).

Goldfeld and Quandt's test first requires the observations to be ordered according to the magnitude of the additive heteroscedasticity. Then c central observations are omitted, and seperate regressions run on the first $(n-c)/2$ observations and the last $(n-c)/2$ observations. The test statistic is

$$F = SSE_2/SSE_1,$$

in which SSE_1 is the sum of squared residuals from the regression involving the first $(n-c)/2$ observations and SSE_2 is the sum of squared residuals from the regression involving the last $(n-c)/2$ observations. The statistic is distributed $F((n-c-2k-2)/2, (n-c-2k-2)/2)$ under the null hypothesis and rejects the null hypothesis for large values of the test statistic.

Szroeter (1978) suggested a test for heteroscedasticity based on the Durbin-Watson (1950, 1951) bounds. He suggested the statistic,

$$\tilde{d} = \Sigma_{i=1}^{n} d_i \hat{\epsilon}_i^2 / \Sigma_{i=1}^{n} \hat{\epsilon}_i^2,$$

in which the observations are ordered in a manner consistent with an increasing disturbance variance, $\hat{\epsilon}$ is the OLS residual vector from the regression (3) and,

$$d_i = 2(1 - cos(i\pi/n+1)).$$

The test rejects the null hypothesis of homoscedasticity when

$$\tilde{d} > 4 - d_l(n+1, k+2),$$

where $d_l(n+1, k+2)$ is the Durbin-Watson lower bound for $n+1$ observations and $k+2$ regressors including the constant dummy. The test fails to reject the null hypothesis when

$$\tilde{d} < 4 - d_u(n+1, k+2),$$

where $d_u(n+1, k+2)$ is the Durbin-Watson upper bound for $n+1$ observations and $k+2$ regressors including the constant dummy. Otherwise the test is inconclusive. Harrison (1980) examined the small sample properties of this test and found it to be subject to a high degree of inconclusiveness. As a consequence he proposed two supplementary procedures for circumventing this problem. King (1981) tabulated tighter bounds for Szroeter's test.

Harrison and McCabe (1979) suggested a bounds test based on the statistic

$$b = \hat{\epsilon}' A \hat{\epsilon} / \hat{\epsilon}' \hat{\epsilon},$$

where A is a matrix with m ones on its diagonal and zeros everywhere else. Critical values and bounds for this statistic can be derived in an identical manner to that for the Durbin-Watson test. Harrison and McCabe found their test to have similar power to the Goldfeld-Quandt F test.

King (1982) (also see Evans and King 1988) suggested an alternative bounds test which is approximately locally best invariant (LBI) (see King and Hillier 1985) in testing for additive heteroscedasticity. His test statistic is

$$s = (\Sigma_{t=1}^{n} \hat{\epsilon}_t^2 (t-1)/n) / \Sigma_{t=1}^{n} \hat{\epsilon}_t^2.$$

Bounds on the critical value for the test statistic are provided by King (1982). He compared the power of his test and the Goldfeld-Quandt F test, and found his test to have better small sample power properties.

A problem with the above tests is that they require the observations to be ordered according to the heteroscedasticity. This makes them difficult to apply, a problem which is amplified if one is dealing with multiple varying coefficients. Accordingly consideration is now given to tests designed with specific recognition given to the problem of testing for the Hildreth-Houck random coefficient model. An early test specifically designed for the single parameter Hildreth-Houck model is the t test introduced by Theil (1971). This test is based on OLS residuals from (3). Working with residuals, one can write:

$$\hat{\epsilon}_t^2 = \sigma^2 m_{tt} + \lambda_0 \sigma^2 \Sigma_{i=1}^{n} m_{ti}^2 x_t^2 + w_t,$$

in which m_{ij} is the $(i,j)^{th}$ element of the M matrix from the regression (3), i.e., $I_n - X(X'X)^{-1}X'$, where the t^{th} row of X is (z_t', x_t). This relationship can be interpreted as a regression relationship with a disturbance term w_t and, after estimation by OLS, a

t test can be conducted on λ_0. There exist three main problems with this test. First, the properties of w_t are such that generalised least squares (GLS) has to be applied, and the GLS estimator is a function of the unknown λ_0. Second, estimation is typically unconstrained and a large number of negative estimates are obtained for λ_0, a result which is not consistent with theory given the non-negativity restriction on λ_0 as it is a ratio of variances. Negative estimates of λ_0 are not unexpected with unconstrained estimation because for case when $\lambda_0 = 0$, unconstrained estimation is likely to generate some insignificantly negative estimates. Third, the size and power properties of this test are largely unknown. This t test can be generalised to testing for multiple Hildreth-Houck random coefficients by running a regression on x_{1t}^2, x_{2t}^2, through to x_{kt}^2 and then conducting an F test. All of the problems are amplified in this setting.

A classic asymptotic test for this testing problem is the Lagrange multiplier (LM) test suggested by Breusch and Pagan (1979). Their test is designed as a test for general heteroscedasticity of the form $h(1+w_t'\delta)$, where w_t is a vector of known exogenous variables and δ is a vector of unknown parameters. The null hypothesis is $\delta = 0$. A special case is the multiple Hildreth-Houck random coefficient model (11) and (13) in which,

$$\Omega = \text{diag}(\delta_1, ..., \delta_p), \tag{17}$$

so that $w_t' = (x_{t1}^2, ..., x_{tp}^2)$ and $h(.) = \sigma^2(.)$. The LM test statistic for this problem is calculated by forming the variable,

$$g_t = \hat{\epsilon}_t^2/\hat{\sigma}^2 - 1,$$

where $\hat{\epsilon}_t$ are the OLS residuals from (14) and $\hat{\sigma}^2 = \Sigma_{t=1}^n \hat{\epsilon}_t^2/n$, and then regressing g_t on w_t supplemented with an intercept term. The calculated value of the LM test statistic is then half of the explained sum of squares from this regression or,

$$g'W(W'W)^{-1}W'g/2, \tag{18}$$

where W is an $n \times (p+1)$ matrix whose t^{th} row is $(1, w_t')$. Under $H_0 : \delta = 0$ this statistic has the conventional asymptotic χ^2 distribution with p degrees of freedom.

Breusch and Pagan's LM test was constructed assuming normal disturbances. Koenker (1981) observed that its size and power can be affected by non-normality and suggested the modified statistic,

$$ng'W(W'W)^{-1}W'g/g'g = nR^2, \qquad (19)$$

where R^2 is the squared multiple correlation between g and W. Both Koenker (1981) and Evans (1992) have found this modified version of the LM test to be more robust to non-normality.

Because for our testing problem, δ is a vector of ratios of variances, its elements can only take non-negative values. Breusch and Pagan argued that the LM test is to be preferred to the likelihood ratio (LR) test, because the parameters of interest (λ_0 or Ω from (17)) fall on the boundary of the parameter space under H_0, meaning that the LR test no longer has its conventional asymptotic distribution. They also found that the use of asymptotic critical values for the LM test leads to a test which is undersized in small samples. This result has been confirmed in simulation studies by Griffiths and Surekha (1986), Honda (1988) and more recently Ara and King (1993). Honda suggested what appears to be a reliable method of size correcting the test so that it has actual size closer to the nominal level.

Ara and King have shown that this problem of lower than nominal size can be overcome by the use of marginal likelihoods in place of conventional likelihoods when the LM test is constructed. In the case of testing $H_0 : \delta = 0$ where δ is defined by (17) in the context of (11) and (13), let X be the $n \times (p+k)$ matrix of observations on the regressors in (13), $m = n - p - k$, $\tilde{\sigma}^2 = \Sigma_{t=1}^{n} \hat{\epsilon}^2/m$ and m_{ij} be the $(i,j)^{th}$ element of the projection matrix $I_n - X(X'X)^{-1}X'$. The marginal likelihood based LM test statistic is of the form,

$$s(0)'I(0)^{-1}s(0), \qquad (20)$$

where $s(0)$ is the $p \times 1$ score vector whose i^{th} element is,

$$s_i(0) = \Sigma_{t=1}^{n} x_{ti}^2(\hat{\epsilon}_t^2/\tilde{\sigma}^2 - m_{tt})/2$$

and $I(0)$ is the $p \times p$ matrix with $(i,j)^{th}$ element,

$$m(\Sigma_{t=1}^{n} x_{tj}^2 \Sigma_{s=1}^{n} x_{si}^2 m_{ts}^2)/(2m+4) - (\Sigma_{t=1}^{n} m_{tt} x_{ti}^2)(\Sigma_{t=1}^{n} m_{tt} x_{tj}^2)/(2m+4).$$

Under H_0, (20) has an asymptotic χ^2 distribution with p degrees of freedom.

An obvious weakness of these three forms of the LM test is that they are two-sided tests while our testing problem is one-sided in nature because variances cannot be negative. Clearly the one-sided information should be exploited to improve power. This is most easily done in the single parameter case, i.e. in the context of (1) and (2). A one-sided LM test can be constructed by taking the square root of the LM statistic (18) and giving it the sign of the score evaluated at H_0 estimates, namely the sign of $\Sigma_{t=1}^{n} x_t^2 g_t$. This also applies to Koenker's robust LM test statistic given by (19). The one-sided marginal likelihood based LM test statistic corresponding to (20) is,

$$a^{-1/2}\Sigma_{t=1}^{n} x_t^2(\hat{\epsilon}_t^2/\tilde{\sigma}^2 - m_{tt}), \tag{21}$$

where,

$$a = 2(m[\Sigma_{t=1}^{n} x_t^2 \Sigma_{s=1}^{n} x_s^2 m_{ts}^2] - [\Sigma_{t=1}^{n} m_{tt} x_t^2]^2)/(m+2).$$

In all three cases the test statistics have standard normal asymptotic distributions under H_0 and the tests reject H_0 for large values of their statistics. As Ara and King note, the one-sided LM test based on (21) is equivalent to King and Hillier's (1985) one-sided LBI test using the standard normal approximation to obtain critical values (see Evans and King 1985c).

Another class of tests which successfully exploit the one-sided nature of the testing problem are point optimal tests (see King 1987a for a survey). A point optimal invariant (POI) test for a general form of heteroscedasticity and the special case of a single Hildreth-Houck random coefficient has been developed by Evans and King (1985a, 1988). Their test involves rejecting H_0 for small values of :

$$g(\lambda_0^*) = \ddot{\epsilon}'(I + \lambda_0^* \ddot{\Sigma})^{-1} \ddot{\epsilon}/\hat{\epsilon}'\hat{\epsilon}$$

in which $\ddot{\Sigma}$ is a diagonal matrix with typical element q_t^*, $\ddot{\epsilon}$ is the GLS residual vector from the estimation of (1) assuming covariance matrix $(I + \lambda_0^* \ddot{\Sigma})$ and $\hat{\epsilon}$ is the OLS residual vector.

To make the test operational, a value for λ_0^* must be chosen. Evans and King (1985a, 1988) experimented with values for λ_0^* of 2.5, 5.0, 7.5, 10.0, with increasing λ_0^* values reflecting more severe heteroscedasticity. They compared these four different versions of the point optimal test with Goldfeld and Quandt's F test, the bounds tests suggested by Szroeter (1978), Harrison and McCabe (1979) and King (1982) and Breusch and Pagan's LM test. They found the point optimal test to have superior power properties and advocated a choice of λ_0^* equal to 5.0.

Evans and King (1985a) also suggested an alternative method by which a value for λ_0^* can be chosen. They advocated choosing λ_0^* so that the coefficient of variation of the variance of the residuals from (3) is 0.5. Milan (1984) extended their result to develop a POI test for multiple Hildreth-Houck random coefficients. He then showed the power superiority of his POI test to the bounds tests advocated by Szroeter and Harrison and McCabe and Breusch and Pagan's LM test.

Brooks (1993b) proposed an alternative version of the point optimal test for the presence of a single Hildreth-Houck random coefficient. His version of the test rejects the constant coefficient model for small values of the statistic,

$$g(\lambda_0^*) = \dot{\epsilon}'(I + \lambda_0^*\dot{\Sigma})^{-1}\dot{\epsilon}/\hat{\epsilon}'\hat{\epsilon}$$

in which $\dot{\Sigma}$ is a diagonal matrix with typical element x_t^2, $\dot{\epsilon}$ is the GLS residual vector from the estimation of (1) assuming covariance matrix $(I + \lambda_0^*\dot{\Sigma})$ and $\hat{\epsilon}$ is the OLS residual vector. He suggested choosing λ_0^* so that the power of the test equalled some desired value for that particular λ_0 value. The basis for this suggestion was the fact that Shively (1988a, 1988b) had recommended such a choice for the λ value in testing for the Rosenberg random coefficient model and the random walk coefficient model. No power comparison has been conducted between these different versions of the point optimal test.

An alternative form of the LM test for one-sided testing problems has recently been suggested by King and Wu (1990). Their test statistic is based on the sum of scores. If there are no nuisance parameters, this test is locally most mean powerful (LMMP) as it maximises the mean slope of the power hypersurface at the null hypothesis. Just as for the regular LM test, the information matrix can be used to construct a test statistic based on the sum of scores which has an asymptotic standard normal distribution under H_0.

Wu (1991) and Ara and King (1993) have investigated the application of these tests to testing for the presence of multiple Hildreth-Houck random coefficients in the context of (11), (13) and (17). The asymptotic LMMP test derived from the standard likelihood function is based on rejecting H_0 for large values of,

$$\Sigma_{t=1}^{n} g_t \Sigma_{i=1}^{p} x_{ti}^2 / (2\Sigma_{t=1}^{n}[\Sigma_{i=1}^{p} x_{ti}^2 - \Sigma_{t=1}^{n}\Sigma_{i=1}^{p} x_{ti}^2/n])^{1/2},$$

which has an asymptotic standard normal distribution under H_0. The corresponding asymptotic LMMP test statistic derived from the marginal likelihood is of the form of (21) but with x_t^2 (and x_s^2) replaced by $\Sigma_{i=1}^{p} x_{ti}^2$ (and $\Sigma_{i=1}^{p} x_{si}^2$) wherever they appear. Wu compared powers of the LMMPI test with the power envelope while Ara and King compared powers of the asymptotic LMMP test with those of Breusch and Pagan's LM test and their marginal likelihood counterparts. In most circumstances the LMMP based tests are more powerful than the two-sided LM tests, particularly when the squared regressors corresponding to the coefficients under test are positively correlated. If these regressors are uncorrelated or negatively correlated, the two-sided LM tests seem to be generally more reliable in terms of power. Rahman and King (1993) have extended Ara and King's study to the problem of testing for multiple Hildreth-Houck coefficients in the presence of AR(1) regression disturbances.

3.2 Constant versus Rosenberg Random Coefficients

In the context of testing for the presence of a single Rosenberg (1973) coefficient, the key parameter that leads to a departure from the constant coefficient model is λ_1. Therefore the problem of interest is testing,

$$H_0 : \lambda_1 = 0 \quad \text{against} \quad H_a : \lambda_1 > 0.$$

This testing problem is one-sided because λ_1 is a ratio of variances which by definition must be non-negative. An interesting feature is the presence of the nuisance parameter ϕ only under the alternative hypothesis. The consequence of this is that classical large sample tests are invalid.

An early test for this problem is the t test suggested by Sunder (1980). This test is based on OLS residuals from (5). Working with these residuals one can write:

$$\hat{\epsilon}_t^2 = \sigma^2 m_{tt} + \lambda_1 \sigma^2 \Sigma_{i=1}^n \Sigma_{j=1}^n m_{ti} m_{tj} x_i x_j \psi_{ij} + w_t,$$

where,

$$\psi_{ij} = \phi^{i+j}/(1-\phi^2) + \phi^{|i-j|}\Sigma_{k=1}^{l} \phi^{2k},$$

in which l is the minimum of i and j.

By choosing a value for the unknown ϕ and treating w_t as a disturbance term, one can interpret this as a regression equation and then carry out a t test on the estimate of λ_1. There are four main problems with this test. The first of these is the choice of the unknown ϕ value. Sunder provides no evidence on the sensitivity of the test to the choice of ϕ value. The remaining difficulties are identical to those of Theil's (1971) t test for the Hildreth-Houck model. These are the need for GLS estimation because of the properties of w_t, the fact that the GLS estimator is a function of the unknown λ_1, the possibility of negative estimates of λ_1 and the lack of evidence on the size and power properties of the test. As is the case with the t test for the Hildreth-Houck model, the t test for the Rosenberg model also generalises to an F test in the case of testing for multiple varying coefficients.

The key difficulty with this testing problem is that ϕ is not identified under H_0. To overcome this, Watson and Engle (1985) suggested a different approach to testing. They noted that the size of any test for this problem will be uneffected by ϕ. They therefore suggested a Davies (1977) test based on the application of Roy's (1953) union-intersection principle. Unfortunately for this problem there is no closed form solution to the Davies test, so they suggested an approximation to that test.

Their approximation produces the statistic,

$$S(\phi) = (S_1 + S_2(\phi))/S_3(\phi),$$

where,

$$S_1 = (\Sigma_{t=1}^n x_t^2((\hat{\epsilon}_t^2/\hat{r}) - 1))/2,$$

$$S_2(\phi) = (\Sigma_{t=2}^{n}\hat{\epsilon}_t x_t \Sigma_{i=1}^{t-1}\hat{\epsilon}_i x_i \phi^{t-i})/\hat{r},$$

$$S_3(\phi) = ((\Sigma_{t=1}^{n} x_t^4 + \Sigma_{t=2}^{n} x_t^2 \Sigma_{i=1}^{t-1} x_i^2 \phi^{2(t-i)})/2 - ((\Sigma_{t=1}^{n} x_t^2)^2)/2n)^{1/2}$$

and $\hat{r}$ is the sample correlation between x_t and $\hat{\epsilon}_t$. The test depends upon the unknown ϕ, which Watson and Engle choose by conducting a grid search for ϕ over the range (-1,+1) and choosing the ϕ value that maximises $S(\phi)$. The explanation of this test is that the S_1 component of the statistic checks for heteroscedasticity, while the $S_2(\phi)$ component checks for autocorrelation.

Watson and Engle compared their test to both the Breusch-Pagan LM test for the Hildreth-Houck random coefficient model, and the Durbin-Watson bounds test for AR(1) disturbances. Not surprisingly they found their test to have superior power properties to both of these tests in detecting the Rosenberg coefficient model.

King (1987b) criticised the Watson and Engle test on the grounds that both its exact and asymptotic distributions are unknown. King therefore suggested the use of a LBI test. His test also turns out to be LBI against the random walk coefficient alternative. The test statistic is given by:

$$s = 2\Sigma_{i=2}^{n}(\Sigma_{t=i}^{n} x_t \hat{\epsilon}_t)^2 / \Sigma_{t=1}^{n}\hat{\epsilon}_t^2.$$

The test rejects the null hypothesis of the constant coefficient model for large values of the test statistic. An advantage of this test is that it can be written as a ratio of quadratic forms in normal variables. Therefore exact critical values for the statistic can be calculated using analogous methods to those for the Durbin-Watson test.

Shively (1988a) suggested the use of a POI test for this problem, arguing that an appropriate choice of points would produce a test which has good power over a wide range of the parameter space. A POI test rejects H_0 for small values of

$$T(\lambda_1^*, \phi_1) = \tilde{\epsilon}'(I + \lambda_1^*\Omega(\phi_1))^{-1}\tilde{\epsilon}/\hat{\epsilon}'\hat{\epsilon}$$

in which $\tilde{\epsilon}$ is the GLS residual vector from the estimation of (1) assuming covariance matrix $(I + \lambda_1^*\Omega(\phi_1))$ and $\hat{\epsilon}$ is the OLS residual vector. $\sigma^2(I + \lambda_1\Omega(\phi))$ is the covariance matrix for the return to normalcy process, and $\Omega(\phi)$ has a typical element of

$$\Omega(\phi)_{st} = x_s x_t \phi^{|s-t|}/(1 - \phi^2).$$

To make the test operational, values must be chosen for λ_1^* and ϕ_1. On the basis of an empirical power comparison, Shively recommended that one set $\phi_1 = 0.7$. He proposed that λ_1^* be chosen so that the optimised power of the test is 0.5 against that λ_1 value when $\phi_1 = 0.7$. This choice of λ_1^* is denoted as $\bar{\lambda}_1^*$. The point optimal test is therefore based on $T(\bar{\lambda}_1^*, 0.7)$.

Shively conducted a Monte Carlo power comparison of his test and those suggested by Watson and Engle and King. The difficulty in making such a comparison is that for the Watson and Engle test its exact and asymptotic distributions are unknown. Therefore Shively derived a small sample equivalent of the Watson and Engle test for which approximate small sample critical values can be found. Including this version of the Watson and Engle test in his Monte Carlo power comparison, he found that for all three regressor sets he considered, his POI test is always superior to King's LBI test. His test is also nearly always superior to the small sample equivalent version of Watson and Engle's test. From this, one concludes that on the basis of power considerations, Shively's POI test is to be preferred when testing the constant coefficient model against the alternative of the presence of a single return to normalcy random regression coefficient.

Brooks (1993a) suggested a modification to Shively's POI test by advocating a different mechanism for choosing a value for the nuisance parameter ϕ_1. Instead of an arbitrary choice of $\phi_1 = 0.7$, he advocated choosing ϕ_1 to maximise the average power of the POI test over a grid of ϕ points. After experimenting with a variety of different grids he found that a power gain over the arbitrary choice could be obtained by this method.

King and Shively (1993) have suggested that the problem associated with the fact that ϕ cannot be identified under the null hypothesis can be overcome by reparameterisation. They reparameterised the testing problem in terms of polar co-ordinates leading to a one-sided multiparameter testing problem in terms of the length of a vector (the λ component) and the angle that vector makes with the horizontal axis (the ϕ component). They solved this reparameterised testing problem by the application of King and Wu's (1990) LMMPI test. Their suggested test was found to have mildly superior power properties relative to the Watson and Engle test, but they did not investigate their test's properties relative to the point optimal test, which does not have a problem with ϕ being unidentified when the null hypothesis is true.

3.3 Constant versus Other Forms of Varying Coefficients

There is also a wide literature on testing the constant coefficient model against other forms of varying coefficient model. In cases where the point optimal test has been considered it has usually been found to have superior power properties to alternative tests.

In the context of testing the constant coefficient model against the random walk coefficient alternative, classical asymptotic tests have been proposed by Garbade (1977) and Pagan and Tanaka (1979). Garbade proposed the use of the LR test for this problem. Despite noting the fact that the distribution of the test statistic is not the usual asymptotic χ^2 distribution, because the null hypothesis of constant coefficients falls on the boundary of the parameter space, he preferred this test in a power comparison over the CUSUM and CUSUMSQ tests suggested by Brown, Durbin and Evans (1975). Pagan and Tanaka criticised Garbade's advocacy of the LR test because of the problems with its null distribution. They advocated the LM test because its null distribution is the conventional asymptotic χ^2 distribution. They then conducted an empirical power comparison between the LM and LR tests, and found the LM test to have superior power properties close to the null hypothesis, and the LR test to have superior power properties further from the null hypothesis. Tanaka (1983) has considered the LM test in the context of testing for the presence of a single random walk coefficient and derived an asymptotically standard normal one-sided version of the LM test.

Sunder (1980) proposed a t test for this problem similar to the t test he proposed for the Rosenberg case and the t test proposed by Theil (1971) for the Hildreth-Houck case. The test is again based on a secondary regression involving OLS residuals of the form:

$$\hat{\epsilon}_t^2 = \sigma^2 m_{tt} + \lambda\sigma^2 \Sigma_i \Sigma_j m_{ti} m_{tj} x_i x_j \min(i,j) + w_t,$$

where the notation is identical to that used for the t tests discussed in the case of the Hildreth-Houck and Rosenberg models. The test is carried out by running a regression of this form and then carrying out a t test on the estimate of λ.

The problems with this test are the same as in the cases of the Hildreth-Houck and Rosenberg models. The properties of w_t are such that the regression needs to be estimated by GLS, but the GLS estimator is a function of the unknown λ. Estimation is typically unconstrained so again there is no guarantee of a non-negative λ estimate. The test also generalises to the case of multiple varying coefficients.

LaMotte and McWhorter (1978) proposed an exact F test for the presence of multiple random walk regression coefficients, which includes the testing problem of a single random walk regression coefficient as a special case. In their context of (11) and (16) their test is as follows. Let V be the matrix whose $(i,j)^{th}$ element is,

$$V_{ij} = \min(i,j)x_i'\Omega x_j,$$

where Ω is assumed diagonal. Let X be the $n \times (p+k)$ matrix of observations on the regressors in (13), $m = n - p - k$ and let P be an $m \times n$ matrix such that $P'P = I_m$ and $PP' = I_n - X(X'X)^{-1}X'$. Suppose $q_1, ..., q_d$ are the distinct eigenvalues for $P'VP$ with multiplicities $r_1, ..., r_d$, respectively. Corresponding to q_i, let H_i be an $m \times r_i$ matrix whose columns are orthonormal eigenvectors of $P'VP$ and define,

$$Q_i = y'PH_iH_i'P'y'.$$

LaMotte and McWhorter's test statistic is given by,

$$F_g = (S_g/n_g)/((SSE - S_g)/(m - n_g)),$$

where, SSE is the sum of squared residuals from an OLS regression of (14),

$$S_g = \Sigma_{i=1}^{g} Q_i,$$

in which g is an integer such that $0 < g < d$, and,

$$n_g = \Sigma_{i=1}^{g} r_i.$$

The statistic F_g is distributed $F(n_g, m - n_g)$ under H_0. To make the test operational, a value for g has to be chosen. This value of g will have no effect on the size of the test but will effect the power. LaMotte and McWhorter suggested that g be chosen to minimise

$$s^* = ((n_g \Sigma_{i=g+1}^{d} r_i q_i)/((m - n_g)\Sigma_{i=1}^{g} r_i q_i))l_a,$$

where, q_i are the eigenvalues of the covariance matrix of the residuals under the alternative hypothesis, and l_a is the $(1-a)100$ percentile of the $F(n_g, m-n_g)$ distribution.

LaMotte and McWhorter (1979) presented an empirical power comparison between their F_g test, Garbade's LR test and the Brown, Durbin and Evans CUSUM test. They found that for all points considered, the F_g test has superior power properties to the LR and CUSUM tests. LaMotte and McWhorter and Simonds, LaMotte and McWhorter (1986) compared the power of the F_g test and Sunder's t test. Both studies found that close to the null hypothesis of constant coefficients, the t test has better power properties than the F_g test, a finding both studies attributed to the t test being oversized. Further away from the null hypothesis both studies found the F_g test to have superior power properties relative to the t test.

Nyblom and Makelainen (1983) proposed the use of an LBI test for the problem of testing for a single random walk coefficient. As King (1987b) has shown, this test is equivalent to the LBI test for the Rosenberg model. In the case of normally distributed errors small sample critical values for this test can be found exactly as the test can be written as a ratio of quadratic forms in normal variables. The asymptotic distribution of this test statistic has been derived by Nyblom and Makelainen, Nabeya and Tanaka (1988) and Leybourne and McCabe (1989a). The usefulness of these results is that asymptotically the same critical values hold for almost any parent distribtuion which meets the regularity conditions.

Nyblom and Makelainen compared the power performance of their LBI test and the F_g test suggested by LaMotte and McWhorter (1978). They found that close to the null hypothesis of a constant coefficient model that the LBI test has superior power properties, but that further away from the null hypothesis, the F_g test has superior power properties. Nyblom (1989) and Jandhyala and MacNeill (1992) have also suggested the use of the LBI test as a test for a structural break point. Interpreting the test in this manner, they found it has good power when a structural break occurs at either end of the sample but not when the structural break occurs in the middle of the sample. The Nyblom and Makelainen version of the LBI test is conditional on the starting value for β_t being fixed. Leybourne and McCabe (1989b) have derived the LBI test for the case where the starting value for

β_t is a normally distributed random variable.

Leybourne and McCabe (1992) have also derived score-based tests of constant parameters in nonlinear regression models against the alternative that the parameters follow a random walk vector process. In McCabe and Leybourne (1993), they constructed and evaluated score-based tests against Hildreth-Houck parameter processes in nonlinear regression models.

Shively (1988b) suggested the use of a POI test for a random walk coefficient in the linear regression model, arguing that an appropriate choice of point would produce a test which has good power over a wide range of the parameter space. A POI test rejects H_0 for small values of

$$T(\lambda^*) = \acute{\epsilon}'(I + \lambda^*\Delta)^{-1}\acute{\epsilon}/\hat{\epsilon}'\hat{\epsilon}$$

in which $\acute{\epsilon}$ is the GLS residual vector from the estimation of (1) assuming covariance matrix $(I + \lambda^*\Delta)$ and $\hat{\epsilon}$ is the OLS residual vector. $\sigma^2(I + \lambda\Delta)$ is the covariance matrix for the random walk coefficient model. The $\lambda\Delta$ component has a typical element of

$$\lambda\Delta_{st} = x_t^2\lambda t, \quad t = s,$$

and,

$$\lambda\Delta_{st} = \lambda\Delta_{ts} = x_t x_s \lambda t, \quad t < s.$$

To make the test operational, a value for λ^* must be chosen. Shively recommended that λ^* be chosen so that the power of the test against that specific λ value is 0.5. He found the point optimal test to have superior power properties relative to LaMotte and McWhorter's F_g test and Nyblom and Makelainen's LBI test for the three regressor sets in his empirical power comparison.

King and Shively (1993) have considered the problem of testing for multiple random walk coefficients. They tackled this problem in a manner analogous to their solution of the problem of testing for Rosenberg coefficient variation.

In the context of the ARIMA(1,1,0) coefficient model, Shively (1988a) attempted to derive an approximate Davies (1977) test, but found the process too computationally burdensome, and so no further consideration was given to that test. Shively derived an LBI test for this model but did not consider its empirical power properties. He also derived the POI test which in this context involves rejecting the null hypothesis for small values of,

$$T(\lambda^*, \mu^*) = \grave{\epsilon}'(I + \lambda^* \Upsilon(\mu^*))^{-1} \grave{\epsilon} / \hat{\epsilon}' \hat{\epsilon}$$

in which $\grave{\epsilon}$ is the GLS residual vector from the estimation of (1) assuming covariance matrix $(I + \lambda^* \Upsilon(\mu^*))$ and $\hat{\epsilon}$ is the OLS residual vector. $\sigma^2(I + \lambda \Upsilon(\mu))$ is the covariance matrix for the ARIMA(1,1,0) coefficient model. The $\Upsilon(\mu)$ component has a typical element of

$$\Upsilon(\mu)_{st} = x_s x_t cov(\Sigma_{i=1}^{s} \gamma_i \Sigma_{j=1}^{t} \gamma_j),$$

where,

$$\gamma_t = \beta_t - \beta_{t-1}.$$

The POI test was then compared to the power envelope and Shively found that, except for the case of large negative μ values, the POI test's power is always close to the power envelope.

In the context of the ARMA(1,1) coefficient model, both Ohlson and Rosenberg (1982) and Collins, Ledolter and Rayburn (1987) have advocated the use of the LR test. At present there exist no alternatives to this classical large sample test for this problem. Further, the small sample performance of this test remains uninvestigated.

In general, when testing for the constant coefficient model against a particular varying coefficient alternative, it would appear that point optimal testing is the best strategy. The key issues in the application of point optimal testing are the methods to be used to select the points for use in the test, and the robustness of such tests to departures from the ideal settings for which they have been designed. These are discussed in Section 4.

3.4 Testing for a Structural Break Point

While parametric varying coefficient models frequently have a solid economic rationale, a large number of applied researchers have tested for coefficient variation by testing for a structural break point. Therefore, this section considers the main tests for a structural break point which have been used in the empirical finance literature that is discussed in Section 5.

An early test for a structural break point is the LR test suggested by Quandt (1960). This test is based on

$$L = \hat{\sigma}_1^i \hat{\sigma}_2^{n-i} / \hat{\sigma}^n,$$

in which $\hat{\sigma}_1$ is the estimate of the standard deviation for the model up to and including period i and $\hat{\sigma}_2$ is the estimate of the standard deviation of the model from period $i+1$ onwards. Choosing i to minimise L is used to detect the observation at which the break point occurred. The problem with this test statistic is that $-2\log L$ does not have the conventional asymptotic χ^2 distribution under the null hypothesis. Quandt confirmed this empirically. For details of the actual distribution theory in this case, see Kim and Siegmund (1989) and Andrews (1993).

Chow (1960) suggested the use of an F test. Assuming the sample can be split into two sub-periods, one up to the structural break and one after the structural break, the F test is given by,

$$F = \big(n_1 + n_2 - 2(k+1)\big)\big(SSE - (SSE_1 + SSE_2)\big)/\big((k+1)(SSE_1 + SSE_2)\big),$$

in which the subscript 1 refers to the period before the structural break, and the subscript 2 refers to the period after the structural break. The key problem with the application of this test is knowing where a potential structural break occurs to be able to split the sample.

Farley and Hinich (1970) considered a special case of Quandt's LR test. They assumed that the possible shift point is uniformly distributed over the sample and derived the LR test in this context. They found the test to have good power properties where the structural break occurs near the middle of the sample but to perform poorly when the structural break occurs at either end of the sample. Farley, Hinich and McGuire (1975) have investigated the relative power properties of the Quandt LR, Chow and Farley and Hinich tests. In their comparison they applied the Chow test by assuming the break point was in the middle of the sample. In terms of the power of the tests when the true break point was in the middle of the sample, they found the Chow test to perform best followed by the Farley and Hinich test and the Quandt test. When the true break point occurred elsewhere in the sample they found the Farley and Hinich test to perform best followed by the Chow test and the Quandt test.

Ashley (1984) generalised Chow's test to the case where there are multiple structural breaks in the sample. In the context of a regression, this requires a series of dummy variables which become active every time there is a structural break. Ashley called his test the stabilogram test. The problem with this test is the determination of the number of structural breaks in the sample. Ashley compared the power of two different versions of the stabilogram test to Chow's F test, the Brown, Durbin and Evans (1975) CUSUM test and Garbade's (1977) LR test for the random walk coefficient model. For the case of β following a random walk coefficient model or the Rosenberg return to normalcy model, he found Garbade's LR test to perform best, followed by the stabilogram test, with the other two tests performing relatively poorly. For the case of a discrete jump in β, the stabilogram test and the Chow F test performed best. On the basis of this, Ashley recommended the use of the stabilogram test.

To detect the timing of a structural break point, Brown, Durbin and Evans (1975) advocated the use of the CUSUM and CUSUMSQ tests. Both of these tests are based on recursive residuals which can be defined in the context of (3) as:

$$e_t = (y_t - X_t' \mathrm{B}_{t-1})/(1 + \mathrm{X}_t'(\mathrm{X}_{t-1}'\mathrm{X}_{t-1})^{-1}\mathrm{X}_t)^{1/2},$$

in which, $X_t = (z_t', x_t)'$, $\mathrm{B} = (\alpha', \bar{\beta})'$ and B_{t-1} is the OLS estimate of B based on the first $t-1$ observations. The CUSUM is defined as:

$$W_r = (1/\hat{\sigma})\Sigma_{t=k+2}^{r} e_t$$

which can be plotted over time. If a structural break occurs in the sample, then this plot should be significantly different from the expected theoretical plot under the hypothesis of no structural break. To determine whether the plot is significantly different, significance lines can be added to the plot, and when the CUSUM crosses either of these lines that is evidence of a structural break at or before that point. The exact positioning of the lines is dependent on the level at which the researcher wishes to conduct the test.

An alternative test is the CUSUMSQ test which is based on the quantities

$$S_r = \Sigma_{t=k+2}^{r} e_t^2 / \Sigma_{t=k+2}^{n} e_t^2.$$

As was the case with the CUSUM, these quantities can be plotted as a function of time, r, along with associated significance lines. Once the CUSUMSQ crosses either of these lines it is evidence of a structural break having occurred at or before that point.

The CUSUMSQ procedure was extended to the case of OLS residuals by McCabe and Harrison (1980). Given that their approach is based on OLS residuals, exact significance lines cannot be found and therefore bounds for such lines are provided. They found the CUSUMSQ test based on recursive residuals to have superior power properties to their test based on OLS residuals. The CUSUM procedure was extended to the case of OLS residuals by Ploberger and Kramer (1992). They found the power of the CUSUM test based on recursive residuals and that based on OLS residuals to be similar, except for late in the sample when the test based on OLS residuals is found to have superior power properties. McCabe (1988) suggests an exact finite sample procedure which maximises the probability of finding a change point if it exists. McCabe's procedure is based on the maximum of weighted CUSUMS of OLS residuals, and shows the incompatibility of CUSUMS and recursive residuals in this context.

The problem of testing for a structural break point of unknown timing has been intensely researched in the recent econometric literature. For instance the problem has been considered by Andrews (1993), Andrews and Ploberger (1994), and Hansen (1991a, 1991b, 1991c). Interestingly, the problem of testing for a structural break point is similar to that of testing for the presence of a Rosenberg coefficient, in that it produces a testing problem in which there is a nuisance parameter present only under the alternative hypothesis. Accordingly the reparameterisation approach used by King and Shively (1993) in testing for the presence of a return to normalcy regression coefficient can be adapted to the problem of testing for a structural break point. This suggestion has been followed up by Tan and King (1992) and Tan (1994).

A more conventional solution to this problem of a nuisance parameter present only under the alternative is to construct test statistics (such as LR, Wald or LM test statistics) assuming the troublesome nuisance parameter is known. Then the test statistic is maximised with respect to this parameter which in our case is the timing of the structural break. This method of test construction may be justified by Roy's (1953) Type 1 principle or its generalisation known as the union-intersection principle (see Roy, Gnanadesikan and Srivastava, (1971), pp. 36-46). The main difficulty with this as a test statistic is in finding its null distribution. Davies (1977, 1987) provides some help but the main thrust has come through the work of Andrews (1993) and Hansen (1991a, 1991b, 1991c). In particular Andrews found that the asymptotic null distribution of the maximised test statistics in the case of LR, Wald and LM tests are given by the supremum of a standardised tied-down Bessel process whose order is at least one. Related papers which also look at the small-sample properties of these tests include Andrews and Ploberger (1994) and Andrews, Lee and Ploberger (1994).

3.5 Testing for the Form of Coefficient Variation

As Section 2 has shown, there exist in the literature a large number of possible varying coefficient models. Therefore a researcher having rejected the constant coefficient model in the direction of a particular departure is faced with the problem of which varying coefficient model to choose. This problem is non-trivial given that tests aimed at detecting a particular varying coefficient departure are likely to have reasonable power at detecting other varying coefficient departures from the constant coefficient model. Just as the Durbin-Watson test can have good power against many different autocorrelated error processes (see King and Evans (1988) and Kariya (1988)), so too do tests for the presence of a particular varying coefficient model have good power against other varying coefficient models. Furthermore, King's (1987b) test is known to be LBI against both the Rosenberg coefficient alternative and the random walk coefficient alternative. In addition, Brooks (1993b) found that POI tests designed for the Hildreth-Houck model and Rosenberg model respectively have good power in detecting the other model.

In choosing a varying coefficient model in an economic context, a researcher is likely to be guided by economic theory, which may restrict attention to a limited subset of varying coefficient models, say those with mean reversion properties. Despite this, tests are still needed to discriminate between the particular alternatives in this restricted subset. However the econometric literature contains few tests designed for such a problem.

The key exceptions are the market model papers by Bos and Newbold (1984) who considered testing the Hildreth-Houck against the Rosenberg model, and Collins, Ledolter and Rayburn (1987) who considered testing the Hildreth-Houck model against the special case ARMA(1,1) model suggested by Ohlson and Rosenberg (1982). Both these studies apply classical Wald and LR tests to this problem, but conjectured that they lack power in finite samples.

Brooks and King (1994) have considered this problem and designed an approximately POI (APOI) test for the null hypothesis of the Hildreth-Houck model with the alternative model being the Rosenberg model. Their test rejects the null hypothesis for small values of the statistic,

$$s(\lambda_0^*, \lambda_1^*, \phi_1) = \tilde{\epsilon}'(I_n + \lambda_1^*\Omega(\phi_1))^{-1}\tilde{\epsilon}/\dot{\epsilon}'(I_n + \lambda_0^*\dot{\Sigma})^{-1}\dot{\epsilon}$$

where $\tilde{\epsilon}$ and $\dot{\epsilon}$ are the GLS residual vectors from the estimation of (1) assuming covariance matrices $I_n + \lambda_1^*\Omega(\phi_1)$ and $I_n + \lambda_0^*\dot{\Sigma}$, respectively.

Brooks and King found that the size properties of the APOI tests are superior to the size properties of the asymptotically valid Wald and LR tests. For cases where the tests had similar sizes the APOI tests were found to have better power properties. The APOI tests are made operational by a choice of λ_0^*, λ_1^* and ϕ_1 values. On the basis of an empirical power comparison they recommended choosing λ_0^* in an optimal manner to control the size properties of the test. They recommended that λ_1^* and ϕ_1 be arbitrarily chosen at mid-range values.

An interesting feature of testing for the form of coefficient variation is that typically such a test will only be carried out after an initial test gives rise to rejection of the constant coefficient model. Therefore testing for the form of coefficient variation is likely to be done in a pre-testing context. This is likely to have consequences for model selection, estimation, hypothesis testing and forecasting using varying coefficient models. Brooks (1993c) analysed the performance of a strategy using the Brooks (1993a) POI test and the Brooks and King APOI test in selecting between the constant coefficient model, the Hildreth-Houck random coefficient model and the Rosenberg model. He found that in the majority of cases the correct model was selected.

4. Point Optimal Testing

As Section 3 has shown, where point optimal tests have been developed for testing for particular varying coefficient models, they generally have superior power properties relative to other tests in the literature. This has been demonstrated for the Hildreth-Houck (1968) random coefficient case by Evans and King (1985a, 1988) and Milan (1984), for the Rosenberg (1973) return to normalcy coefficient case by Shively (1988a) and for the random walk coefficient case by Shively (1988b). Further in the related area of testing in structural time series models, Franzini and Harvey (1983) and Nyblom (1986) have found POI tests perform well. Given this, it is worthwhile to briefly review the literature on point optimal testing, noting that a detailed survey is provided by King (1987a).

Applications of point optimal testing have typically focused on testing the disturbance covariance matrix of a linear regression model. By invariance arguments, most test statistics for these problems reject the relevant null hypothesis for small values of the test statistic,

$$s = \tilde{\epsilon}'\Gamma_1^{-1}\tilde{\epsilon}/\hat{\epsilon}'\Gamma_0^{-1}\hat{\epsilon},$$

in which $\tilde{\epsilon}$ is the GLS residual vector from the estimation of (1) assuming a particular covariance matrix denoted by Γ_1 and $\hat{\epsilon}$ is the GLS residual vector assuming covariance matrix Γ_0. In many cases the null hypothesis is of well behaved disturbances, so Γ_0 is an identity matrix making $\hat{\epsilon}$ the OLS residual vector. The issue of the calculation of critical values and significance levels for such test statistics is discussed in the appendix.

In most applications Γ_1 will depend upon some unknown parameters, the exact parameters and structure of Γ_1 depending on the relevant testing problem. The problem in making the point optimal test operational is the choice of values for these unknown parameters. Given the choice of parameters, the test will be most powerful invariant for testing the null hypothesis that $\sigma^2\Gamma_0$ is the covariance matrix against the alternative hypothesis that $\sigma^2\Gamma_1$ is the covariance matrix when Γ_1 is determined by the chosen parameter values.

It is anticipated that with well chosen parameter values the test will have good power over a wide range of the parameter space.

For the problem of testing for an AR(1) disturbance process, King (1985a) recommended that the unknown parameter of interest be set to a representative middle range value. King (1984) in testing for a simple AR(4) process, and Evans and King (1985b) generalising to higher order simple AR disturbance processes, also make a similar recommendation. King (1983) considering an MA(1) disturbance process also recommended a representative choice of parameter value. As noted earlier, Evans and King (1985a, 1988) also make this recommendation in testing for additive heteroscedasticity and the special case of the Hildreth-Houck random coefficient model.

An alternative to the above arbitrary middle range choice of parameter values is to choose the parameters with respect to some optimality criteria. King (1985b) extended his work on testing for MA(1) disturbances to a case where the value of the unknown parameter is chosen to make the power of the test some designated value for that choice of parameter. Brooks (1993b) advocated a similar procedure when testing for the Hildreth-Houck random coefficient model. Shively (1988a) used this approach when testing for the Rosenberg return to normalcy coefficient model and the ARIMA(1,1,0) model and Shively (1988b) also used this strategy when testing for the random walk coefficient model. Brooks (1993a) extended Shively's (1988a) work on testing for the Rosenberg model recommending that both the parameter of interest and nuisance parameters be chosen to give the test desired power properties. Milan (1984) advocated a choice of parameter value based on the coefficient of variation of the implied disturbance variances for the case of the Hildreth-Houck random coefficient model.

All of the testing problems considered above are cases where the null model is a linear regression model giving an identity matrix for Γ_0. It is worthwhile considering cases where the null model has a disturbance process that is not so well behaved. King (1989) considered the problem of testing for AR(4) disturbances in the presence of AR(1) disturbances. He found that a point optimal test could not be constructed because of the presence of a nuisance parameter under the null hypothesis. However he was able to find an approximately point optimal invariant test for this problem. The resultant test is approximately point optimal because a modification of the distribution under the null hypothesis produces a test which is point optimal. He recommended that all values of test parameters be chosen by optimality criteria associated with power. Silvapulle and King (1991) found a similar result when applying an approximately point optimal invariant test to the non-nested problem of testing MA(1) disturbances against AR(1) disturbances. Brooks and King (1994) also found a similar result when testing the Hildreth-Houck random coefficient model against the Rosenberg random coefficient model.

Accordingly, an important but unresolved issue in the point optimal testing literature is the choice of values for the parameters required to make the tests operational. Another important issue is the robustness of point optimal tests to departures from the ideal situations for which they are designed. In particular, point optimal testing relies upon the normality of disturbances. There is a literature suggesting that certain economic time series, particularly financial data, are better characterised by non-normal distributions. Evidence for the stock market is provided by Fama (1965), Teichmoeller (1971), Fielitz and Smith (1972), Praetz (1972), Officer (1972), Blattberg and Gonedes (1974), Osborn (1974), Praetz and Wilson (1978) and Harris (1987). Evidence for the foreign exchange market is documented in Westerfield (1977), Boothe and Glassman (1987), Hsieh (1988), Baillie and Bollerslev (1989), Engle and Gonzalez-Rivera (1991) and Lye and Martin (1994). Evidence for futures markets is contained in Clark (1973) and Rainbow and Praetz (1986). The robustness of point optimal tests to non-normality has not been widely investigated. The key study is Evans (1992) who examined the robustness properties of the size of point optimal tests for autocorrelation and heteroscedasticity to non-normality. She found that the size of the point optimal tests for both AR(1) disturbances and AR(4) disturbances are robust to departures from normality. However point optimal tests for heteroscedasticity while robust to skewness were found to have their size effected by kurtosis. Because most varying coefficient models are special cases of heteroscedasticity, the robustness properties of point optimal tests for such varying coefficient models are of interest.

Brooks (1993b) studied the robustness of the POI test for the Rosenberg random coefficient model. He found a similar result to Evans, in that, the size and power properties of the test were not effected by skewness but were effected by kurtosis. The effects of increasing kurtosis was to make the test oversized and to reduce the power of the test for mid-range parameter values. Brooks and King (1994) analysed the robustness of the APOI test for testing the Hildreth-Houck model against the Rosenberg model. They found their APOI test to be remarkably robust to non-normality. This is consistent with the findings of Evans on tests for autocorrelation, as the APOI test is testing the autocorrelation parameter in that problem. Rahman (1993) investigated the robustness of two APOI tests for multiple Hildreth-Houck coefficients in the presence of autocorrelation and compared their performance with those of the LM test and the asymptotic LMMP (ALMMP) test. He concluded that in terms of size the APOI and ALMMP tests are rather robust. The power results confirmed that all tests performed adequately under nonnormality.

5. Applications of Varying Coefficient Models

5.1 Market Model Applications

Consider the market model,

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}, \tag{22}$$

in which R_{it} is the return on asset i, R_{mt} is the return on the market portfolio, ϵ_{it} is assumed to be distributed $IN(0, \sigma^2)$ and α_i and β_i are unknown asset specific parameters.

The key parameter to be estimated in (22) is β_i, which measures the systematic risk of holding asset i. In the context of the capital asset pricing model (CAPM), this is the key risk measure as all other risk can be diversified away through holding the market portfolio. The traditional approach to estimation of the market model assumes β_i is constant over time thus allowing the application of OLS to (22). Despite this, recent literature has suggested that β_i may in fact be time varying. This would invalidate the application of OLS to (22), and help to provide evidence as to the risk profile of assets over time.

While it would be appealing to derive the time variation in systematic risk directly along with the derivation of the market model, this is typically not the approach taken in the literature. Such literature has generally attempted to overcome supposed deficiencies in the derivation of the market model.

A number of authors have argued that time variation may be due to microeconomic factors at the level of the firm. Fabozzi and Francis (1978) suggested the reasons of alterations to the product mix or changes in leverage or dividend policy as giving rise to time variation in systematic risk. Bos and Newbold (1984) claimed that changes in the operational structure of the firm may be the cause of time variation in systematic risk. Dielman and Nantell (1982) argued that the key operational change is likely to be merger activity. Turnbull (1977) identified maturity and growth of the firm as important determinants of systematic risk. Therefore, as the firm matures and its growth rate fluctuates through time, then so too may its β risk change. Time variation in systematic risk due to microeconomic factors is also consistent with some of the arguments provided by Blume (1975). For example, he suggested that when firms engage in any project which is risky, the risk of the project may tend to be less extreme over time.

Alternatively, macroeconomic factors may lead to time variation in the systematic risk. Both Fabozzi and Francis and Bos and Newbold claim that business cycle factors such as inflation and unemployment may account for the time variation in systematic risk. Another possibility is to attribute time variation in systematic risk to the behaviour of portfolio managers as is done by Alexander, Benson and Eger (1982).

Other authors have suggested that it is the unrealistic nature of some of the assumptions underlying the market model which gives rise to time variation in systematic risk. Fabozzi, Francis and Lee (1982) argued that the assumption underlying the CAPM of no capital market imperfections is unrealistic, and that the presence of such imperfections may lead to time variation in systematic risk. Fabozzi, Francis and Lee also questioned the CAPM assumption of a distribution of asset returns which is normal. Any non-normality in this distribution they claimed could also lead to time variation in systematic risk. The final criticism they made of CAPM is in the difficulties in accurately measuring the market portfolio. The impossibility of obtaining data on some components particularly the human capital component, may also produce time variation in systematic risk according to Fabozzi, Francis and Lee.

Despite the desirability of actually modelling the factors that lead to time variation of systematic risk, these factors are typically unobservable and so cannot be directly analysed.

Accordingly, most authors have modelled the time variation in systematic risk with simple parametric models.

A number of authors have suggested that β be modelled by the Hildreth-Houck (1968) random coefficient model. Fabozzi and Francis (1978) and Fabozzi, Francis and Lee (1982) have tested for this model using data for the New York Stock Exchange (NYSE), while Francis and Fabozzi (1980) tested for this alternative using mutual funds data. In all of these papers, evidence was found in favour of this alternative. The test used in these papers to detect the random coefficient model is Theil's (1971) t test from a secondary regression. The use of such a simplistic test procedure may cast doubt on the reliability of such results. In fact, Alexander and Benson (1982) claimed that the above results overstate the case for β following the Hildreth-Houck random coefficient model due to inefficiencies in the t test procedure.

Another possibility is to consider a more general alternative. Such an alternative is Rosenberg's (1973) AR(1) or return to normalcy coefficient model. This alternative has been suggested by Bos and Newbold (1984) for NYSE data and Faff, Lee and Fry (1992) for Australian data. Bos and Newbold made use of Watson and Engle's (1985) test and found evidence of β variation. Faff, Lee and Fry made use of a Burr (1942) approximation to the distribution of King's (1987b) LBI test for this problem. Both of these studies found evidence in favour of β variation. Brooks, Faff and Lee (1992, 1994) have corroborated the finding of Faff, Lee and Fry using Brooks' (1993a) POI test. Despite the fact that the POI test is known to have superior power properties relative to the LBI test, the results across these studies are almost identical.

Other authors have proposed some of the other possible varying coefficient models for β. Sunder (1980), Garbade and Rentzler (1981), Alexander, Benson and Eger (1982) and Simonds, La Motte and McWhorter (1986) proposed a random walk coefficient model as most appropriate. Sunder used a t test from a secondary regression and found evidence in favour of β variation for NYSE data. Garbade and Rentzler used Garbade's (1977) LR test for the random walk coefficient model and also found evidence of β variation for NYSE data. Alexander, Benson and Eger tested for β variation in mutual funds data using both LaMotte and McWhorter's (1978) F_g test and Sunder's t test. Simonds, LaMotte and

McWhorter (1986) used LaMotte and McWhorter's F_g test and found a greater degree of evidence of β non-constancy in NYSE data than Sunder had found.

These results are of interest because if β does follow the random walk coefficient model, then it has no mean reversion property. This is inconsistent with finance theory notions of market efficiency and suggestions of the existence of some mean value for systematic risk, such as Blume's (1975) notion of the existence of a grand mean for β. However it does appear that tests for one form of coefficient variation, have power against other forms of coefficient variation. Therefore, these results are perfectly consistent with another model, probably with mean reversion tendencies providing the model for β variation.

Ohlson and Rosenberg (1982) attempted to generalise the AR(1) model for β by considering an AR(2) process for β. They found no evidence to support extending the AR(1) model in this manner. A more successful general alternative for β variation considered is the ARMA(1,1) model suggested by Ohlson and Rosenberg and Collins, Ledolter and Rayburn (1987). These studies tested for this alternative with NYSE data using the LR test and found evidence of β variation.

Given the existence of a number of alternative models for β variation the obvious question is which model is the best. This issue has only received attention for US markets in the papers by Bos and Newbold (1984) and Collins, Ledolter and Rayburn (1987), and for Australian markets in Brooks, Faff and Lee (1992, 1994). The findings of these studies are mixed; Bos and Newbold found in favour of purely random variation in β (such as the Hildreth-Houck model), while Collins, Ledolter and Rayburn found some evidence in favour of sequential variation models for β (such as the ARMA(1,1) coefficient model). For Australian data, Brooks, Faff and Lee have corroborated the Bos and Newbold finding. This is of significance given that the APOI test used in the Australian studies is known to have superior power properties relative to the asymptotic tests used in the US studies.

5.2 Forward Pricing Applications

Futures markets perform a variety of functions including risk transference, information processing and forward pricing. The performance of the forward pricing function of futures markets is necessary for forward contract negotiations and the hedging decisions of agents. The historical development of the forward pricing function is due to Working (1942, 1949). The performance of the forward pricing function gives rise to the property that the futures price should be an unbiased predictor of the future spot price. For this to be true, futures prices must fully reflect all relevant information and the following conditions must hold:

(1) $F_{t-k,t} = E(S_t|\Phi_{t-k})$,

(2) $S_t = E(S_t|\Phi_{t-k}) + \epsilon_t$

and are sufficient to imply

(3) $S_t = F_{t-k,t} + \epsilon_t$,

in which $F_{t-k,t}$ is the futures price formed at time $t-k$ for delivery at time t, S_t is the spot price at time t, $E(S_t|\Phi_{t-k})$ is the expectation of the spot price conditional on the set of publicly available information at time $t-k$, i.e., Φ_{t-k}, and ϵ_t represents shocks to the economy which are assumed to be distributed $IN(0, \sigma^2)$.

Proposition (1) states that the futures price is an expectation of the future spot price. Proposition (2) is a statement that the expectations of agents are rational. Together these two propositions imply (3) which is that the futures price is an unbiased predictor of the future spot price.

Given this relationship, the testing of the unbiased prediction hypothesis is commonly carried out by forming the OLS regression equation of the form:

$$S_t = \alpha + \beta F_{t-k,t} + \epsilon_t, \tag{23}$$

in which α and β are unknown constant parameters and ϵ_t is assumed to be distributed $IN(0, \sigma^2)$. The unbiased prediction hypothesis is tested by testing:

$$H_0 : \beta = 1 \quad \text{against} \quad H_a : \beta \neq 1.$$

An early example of this approach is that of Tomek and Gray (1970), and the literature on this approach is surveyed in Goss (1986). However there may be valid reasons why a constant coefficient model is inappropriate in this context. In the closely related literature on testing the efficiency of forward foreign exchange markets, Chiang (1988) and Barnhart and Szakmary (1991) have found evidence in favour of varying coefficient models. Further notions such as agents engaging in learning processes as they move towards equilibrium are likely to produce varying coefficient models. Such an idea is that of asymptotically rational expectations introduced by Stein (1986) in the case of commodity futures markets and then extended by Stein (1992) to the case of financial futures markets.

Both Chiang and Barnhart and Szakmary tested the efficiency of the forward foreign exchange market. Chiang considered the following exchange rates - USA/France, USA/Canada, USA/W. Germany and USA/UK. Barnhart and Szakmary considered all of these except for the USA/France case, but added the USA/Japan exchange rate to their analysis. In both studies the authors make use of one month forward contracts.

Chiang conducted a series of tests on the stability of β in (23). He used Quandt's (1960) LR test to identify the likely break point and then assuming that is the break point used Chow's (1960) F test. The Brown, Durbin and Evans (1975) CUSUMSQ test was also used to provide evidence of where a likely break point occurs. The combination of these tests suggested time variation in the coefficients. To determine whether the form of time variation is a single break point or better modelled by a stochastic coefficient model, Chiang ran a series of rolling regressions, and then attempted to identify an ARIMA model for the estimated rolling regression coefficients. He found a degree of persistence in the coefficients and an apparent absence of mean reversion properties when fitting ARIMA models to the estimated coefficients from the rolling regressions. His fitted models typically have a simple structure depending purely on relatively distant lags of greater than three months. This absence of mean reversion properties is unusual given the strong theoretical restriction to be tested on the mean of β. It may be a reflection of the fact that typically forward foreign exchange markets do not produce results consistent with the unbiased prediction hypothesis. This evidence on the failure of forward foreign exchange markets to satisfy the unbiased prediction hypothesis has been surveyed in Hodrick (1987) and Goodhart (1988). Recent Australian evidence supporting this proposition is provided by Smith and Gruen (1989).

Barnhart and Szakmary also conducted a series of tests for parameter stability. They made use of both Quandt's LR test and its extension by Farley and Hinich (1970) as well as Chow's F test and its extension by Ashley (1984). The combination of these tests also found evidence of parameter variation.

It is of interest to see if these results for forward foreign exchange markets carry across to futures markets. Brooks (1993d) has considered this problem for Australian financial futures by restricting the possible set of varying coefficient models to the Hildreth-Houck or the Rosenberg model in futures markets. Both of these models are parsimonious special cases of the ARIMA models suggested by Chiang and, further, they also provide evidence on the learning procedures of agents in such markets. This is clearly of primary interest. Further to this, Ashley has suggested that the Rosenberg model with its mean reversion properties is an economically plausible model. Given that the testing of forward efficiency is a test of the mean value of the key parameter β in (23), it is difficult to see why one would advocate varying coefficient models which exhibit no mean reversion tendencies. Goss (1986) surveys the evidence in futures markets and found evidence consistent with the unbiased prediction hypothesis in contrast to the inconsistent evidence provided in the case of forward foreign exchange markets.

In terms of whether β varies over time, the findings of Brooks are mixed. For Australian interest rate futures, such as ninety day bank accepted bills and ten year government bonds, no evidence of coefficient variation could be found using Brooks' (1993a) POI test. For share price index futures the application of the same test found strong evidence of coefficient variation at all lengths to maturity. The issue of which varying coefficient model is best for share price index futures was then addressed by the application of Brooks and King's (1994) APOI test. Close to maturity the evidence favours the Hildreth-Houck model, while further from maturity the evidence favours the Rosenberg model.

6. Conclusion

This paper has surveyed the literature on varying coefficient regression models examining both the testing for such models and their application in two areas of finance. In the context of testing for the presence of a single varying coefficient, a strong theme in the literature is that the point optimal approach is the preferred method of test construction, particularly with respect to power. There still exist cases for which point optimal tests have not been developed and also point optimal testing has not been compared to other new approaches such as reparameterisation of the testing problem. A further area which requires work is in the area of testing multiple varying coefficients. For the Hildreth-Houck (1968) case, King and Wu (1990) have suggested a multiparameter one-sided test and found it to have superior power properties relative to other available tests, although not for all circumstances. For these and other cases further research is still needed. This work is likely to be of importance as a greater range of potential applications lie in the field of multiple varying coefficients.

Acknowledgements

This paper is based on material in the first authors Ph.D. thesis in the Department of Econometrics, Monash University. This research was supported in part by an ARC grant. We are grateful to Shahidur Rahman, Swee Liang Tan, Tom Shively and Brendan McCabe for constructive comments on previous versions of this paper.

Appendix

Calculation Of Critical Values And Significance Levels Of Point Optimal Tests

As stated in Section 4, the general version of the point optimal test rejects H_0 for small values of the statistic,

$$s = \tilde{\epsilon}'\Gamma_1^{-1}\tilde{\epsilon}/\hat{\epsilon}'\Gamma_0^{-1}\hat{\epsilon}.$$

Here we discuss the calculation of exact critical values and p-values. For further detail, see King (1987a).

Consider first the case where $\Gamma_0 = I$ and

$$s = \tilde{\epsilon}'\Gamma_1^{-1}\tilde{\epsilon}/\hat{\epsilon}'\hat{\epsilon}.$$

Critical values and p-values can be derived from knowledge of

$$Pr(s < s^* \quad | \quad \epsilon \sim IN(0,\sigma^2)),$$

or equivalently

$$Pr[\Sigma_{i=1}^{m}(\tau_i - s^*)\xi_i^2 < 0] = \omega, \tag{24}$$

in which, ξ_i^2 are independent χ_1^2 random variables and τ_i are the non-zero eigenvalues of

$$\Gamma_1^{-1} - \Gamma_1^{-1}X(X'\Gamma_1^{-1}X)^{-1}X'\Gamma_1^{-1}.$$

To solve for critical values one fixes ω and solves for s^*, while solving for p-values requires fixing s^* and solving for ω.

This can be done iteratively using Koerts and Abrahamse's (1969) FQUAD subroutine, Farebrother's (1980) PAN procedure or Davies' (1980) algorithm to evaluate the left-hand-side of (24). All of these methods require the calculation of eigenvalues and can therefore be costly in large samples. Consequently researchers may wish to consider methods that do not require the calculation of eigenvalues such as Palm and Sneek's (1984) Householder transformation approach or Shively, Ansley and Kohn's (1990) modified Kalman filter approach. Shively, Ansley and Kohn present evidence on the computational cost savings from their approach, finding that such savings can be substantial in large samples.

Now consider the more complex case where $\Gamma_0 \neq I$. Here one is no longer dealing with a simple null hypothesis as the structure of Γ_0 will now depend on nuisance parameters. Let us assume that the structure of Γ_0 depends on a single nuisance parameter, say γ_0. This is the structure of the problem considered by King (1989), Silvapulle and King (1991) and Brooks and King (1994). In all of these cases the authors find that to control the size of the test, the possible values of γ_0 must be bounded and a value for γ_0 must be optimally chosen as say, γ_0^*.

Accordingly H_0 is now rejected for small values of the statistic,

$$s(\gamma_0^*) = \tilde{\epsilon}'\Gamma_1^{-1}\tilde{\epsilon}/\hat{\epsilon}'\Gamma_0^{*\,-1}\hat{\epsilon}.$$

To determine what constitutes a small value of this statistic critical values must be found, which requires evaluating

$$Pr(s(\gamma_0^*) < s^{**} \quad | \quad v \sim N(0, \lambda_0\sigma^2\Gamma_0))$$

with the significance level of the test being the supremum of these probabilities with respect to γ_0. This can be done by computing,

$$Pr[\Sigma_{i=1}^{m}\varphi_i\xi_i^2 < 0] = \omega,$$

in which, ξ_i^2 are independent χ_1^2 random variables and φ_i are the non-zero eigenvalues of,

$$(\Gamma_0^{1/2})'(\Gamma_1^{-1} - \Gamma_1^{-1}X(X'\Gamma_1^{-1}X)^{-1}X'\Gamma_1^{-1} - s^{**}(\Gamma_0^{*-1} - \Gamma_0^{*-1}X(X'\Gamma_0^{*-1}X)^{-1}X'\Gamma_0^{*-1}))\Gamma_0^{1/2},$$

using the methods for computing (24) outlined above.

References

1. Alexander, G.J. and Benson, P.G. (1982). More on beta as a random coefficient. *J. Fin. Quant. Anal.* 17, 27-36.

2. Alexander, G.J., Benson, P.G. and Eger, C.E. (1982). Timing decisions and the behaviour of mutual fund systematic risk. *J. Fin. Quant. Anal.* 17, 579-602.

3. Andrews, D.W.K. (1993). Tests for parameter instability and structural change with unknown change point. *Econometrica* 61, 821-856.

4. Andrews, D.W.K., Lee, I. and Ploberger, W. (1994). Optimal changepoint tests for linear regression. *J. Econometrics*, forthcoming.

5. Andrews, D.W.K. and Ploberger, W. (1994). Optimal tests when a nuisance parameter is present only under the alternative. *Econometrica*, forthcoming.

6. Ashley, R. (1984). A simple test for regression parameter instability. *Econ. Inq.* 22, 253-268.

7. Ara, I. and King, M.L. (1993). Marginal likelihood based tests of regression disturbances. Paper presented at the 1993 Australasian Meeting of the Econometric Society, Sydney.

8. Baillie, R.T. and Bollerslev, T. (1989). The message in daily exchange rates: A conditional variance tale. *J. Bus. Econ. Statist.* 7, 297-305.

9. Barnhart, S.W. and Szakmary, A.C. (1991). Testing the unbiased forward rate hypothesis: Evidence on unit roots, cointegration and stochastic coefficients. *J. Fin. Quant. Anal.* 26, 245-267.

10. Beck, N. (1983). Time-varying parameter regression models. *Amer. J. Pol. Sci.* 27, 557-600.

11. Blattberg, R.C. and Gonedes, N.J. (1974). A comparison of the stable and Student distributions as statistical models for stock prices. *J. Bus.* 47, 244-280.

12. Blume, M.E. (1975). Betas and their regression tendencies. *J. Fin.* 30, 785-795.

13. Boothe, P. and Glassman, D. (1987). The statistical distribution of exchange rates: Empirical evidence and economic implications. *J. Inter. Econ.* 22, 297-319.

14. Bos, T. and Newbold, P. (1984). An empirical investigation of the possibility of stochastic systematic risk in the market model. *J. Bus.* 57, 35-41.

15. Breusch, T.S. and Pagan, A.R. (1979). A simple test for heteroscedasticty and random coefficient variation. *Econometrica* 47, 1287-1294.

16. Brooks, R.D. (1993a). Alternative point optimal tests for regression coefficient stability. *J. Econometrics* 57, 365-376.

17. Brooks, R.D. (1993b). The robustness of point optimal testing for Rosenberg random regression coefficients. *Econometric Rev.*, forthcoming.

18. Brooks, R.D. (1993c). Using a sequence of point optimal tests to select a varying coefficient model. Mimeo, RMIT.

19. Brooks, R.D. (1993d). The unbiased prediction hypothesis in futures markets: A varying coefficient approach. Mimeo, RMIT.

20. Brooks, R.D., Faff, R.W. and Lee, J.H.H. (1992). The form of time variation of systematic risk: Some Australian evidence. *App. Fin. Econ.* 2, 191-198.

21. Brooks, R.D., Faff, R.W. and Lee, J.H.H. (1994). Beta stability and portfolio formation. *Pac. Bas. Fin. J.*, forthcoming.

22. Brooks, R.D. and King, M.L. (1994). Testing Hildreth-Houck against return to normalcy random regression coefficients. *J. Quant. Econ.*, forthcoming.

23. Brown, R.L., Durbin, J. and Evans, J.M. (1975). Techniques for testing the constancy of regression relationships over time. *J. Roy. Statist. Soc.* B 37, 149-163.

24. Burr, I.W. (1942). Cumulative frequency functions. *Ann. Math. Statist.* 13, 215-232.

25. Chiang, T.C. (1988). The forward rate as a predictor of the future spot rate - A stochastic coefficient approach. *J. Mon. Cred. Bank.* 20, 212-232.

26. Chow, G.C. (1960). Tests of equality between sets of coefficients in two linear regressions. *Econometrica* 28, 591-605.

27. Chow, G.C. (1984). Random and changing coefficient models. Chapter 21 in Z. Griliches and M.D. Intriligator (eds.), *Handbook of Econometrics.* Elsevier Science, Amsterdam.

28. Clark, P.K. (1973). A subordinated stochastic process with finite variance for speculative prices. *Econometrica* 41, 135-155.

29. Collins, D.W., Ledolter, J. and Rayburn, J. (1987). Some further evidence on the stochastic properties of systematic risk. *J. Bus.* 60, 425-448.

30. Cooley, T.F. and Prescott, E.C. (1973a). An adaptive regression model. *Inter. Econ. Rev.* 14, 364-371.

31. Cooley, T.F. and Prescott, E.C. (1973b). Tests of an adaptive regression model. *Rev. Econ. Statist.* 55, 248-256.

32. Cooley, T.F. and Prescott, E.C. (1976). Estimation in the presence of stochastic parameter variation. *Econometrica* 44, 167-184.

33. Davies, R.B. (1977). Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* 64, 247-254.

34. Davies, R.B. (1980). Algorithm AS155: The distribution of a linear combination of χ^2 random variables. *Appl. Statist.* 29, 323-333.

35. Davies, R.B. (1987). Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* 74, 33-43.

36. Dielman, T.E. and Nantell, T.J. (1982). Tender offer mergers and shareholders wealth: A random coefficient regression approach. *Proc. Bus. Econ. Statist. Sect. Amer. Statist. Assoc.* 334-339.

37. Durbin, J. and Watson, G.S. (1950). Testing for serial correlation in least squares regression I. *Biometrika* 37, 409-428.

38. Durbin, J. and Watson, G.S. (1951). Testing for serial correlation in least squares regression II. *Biometrika* 38, 159-178.

39. Engle, R.F. and Gonzalez-Rivera, G. (1991). Semiparametric ARCH models. *J. Bus. Econ. Statist.* 9, 345-359.

40. Evans, M.A. (1992). Robustness of size of tests of autocorrelation and heteroscedasticity to nonnormality. *J. Econometrics* 51, 7-24.

41. Evans, M.A. and King, M.L. (1985a). A point optimal test for heteroscedastic disturbances. *J. Econometrics* 27, 163-178.

42. Evans, M.A. and King, M.L. (1985b). Higher order generalisations of first order autoregressive tests. *Comm. Statist. Theory Meth.* 14, 2907-2918.

43. Evans, M.A. and King, M.L. (1985c). Critical value approximations for tests of linear regression disturbances. *Austral. J. Statist.* 27, 68-83.

44. Evans, M.A. and King, M.L. (1988). A further class of tests for heteroscedasticity. *J. Econometrics* 37, 265-276.

45. Fabozzi, F.J. and Francis, J.C. (1978). Beta as a random coefficient. *J. Fin. Quant. Anal.* 13, 101-115.

46. Fabozzi, F.J., Francis, J.C. and Lee, C.F. (1982). Specification error, random coefficient and the risk-return relationship. *Quart. Rev. Econ. Bus.* 22, 23-30.

47. Faff, R.W., Lee, J.H.H. and Fry, T.R.L. (1992). Time stationarity of systematic risk: Some Australian evidence. *J. Bus. Fin. Acc.* 19, 253-270.

48. Fama, E.F. (1965). The behaviour of stock market prices. *J. Bus.* 38, 34-105.

49. Farebrother, R.W. (1980). Algorithm AS153: Pan's procedure for the tail probabilities of the Durbin-Watson statistic. *Appl. Statist.* 29, 224-227 and 30, 189.

50. Farley, J.U. and Hinich, M.J. (1970). A test for a shifting slope coefficient in a linear model. *J. Amer. Statist. Assoc.* 65, 1320-1329.

51. Farley, J.U., Hinich, M.J. and McGuire, T.W. (1975). Some comparisons of tests for a shift in the slopes of a multivariate linear time series model. *J. Econometrics* 3, 297-318.

52. Fielitz, B.D. and Smith, E.W. (1972). Asymmetric stable distributions of stock price changes. *J. Amer. Statist. Assoc.* 67, 813-814.

53. Francis, J.C. and Fabozzi, F.J. (1980). Stability of mutual fund systematic risk statistics. *J. Bus. Res.* 8, 263-275.

54. Franzini, L. and Harvey, A.C. (1983). Testing for deterministic trend and seasonal components in time series models. *Biometrika* 70, 673-682.

55. Garbade, K. (1977). Two methods for examining the stability of regression coefficients. *J. Amer. Statist. Assoc.* 72, 54-63.

56. Garbade, K. and Rentzler, J. (1981). Testing the hypothesis of beta stationarity. *Inter. Econ. Rev.* 22, 577-586.

57. Goldfeld, S.M. and Quandt, R.E. (1965). Some tests for homoscedasticity. *J. Amer. Statist. Assoc.* 60, 539-547.

58. Goodhart, C. (1988). The foreign exchange market: A random walk with a dragging anchor. *Economica* 55, 437-460.

59. Goss, B.A. (1986). The forward pricing function of the London Metal Exchange. Chapter 6 in B.A. Goss (ed.), *Futures Markets: Their Establishment and Performance.* Croom Helm, London.

60. Granger, C.W.J. (1993). Strategies for modelling nonlinear time series relationships. *Econ. Rec.* 69, 233-238.

61. Griffiths, W.E. and Surekha, K. (1986). A Monte Carlo evaluation of the power of some tests for heteroscedasticity. *J. Econometrics* 31, 219-231.

62. Hansen, B.E. (1991a). A comparison of tests for parameter instability: An examination of asymptotic local power. Mimeo, University of Rochester.

63. Hansen, B.E. (1991b). Inference when a nuisance parameter is not identified under the null hypothesis. Mimeo, University of Rochester.

64. Hansen, B.E. (1991c). Testing for parameter instability in linear models. Mimeo, University of Rochester.

65. Harris, L. (1987). Transaction data tests of the mixture of distributions hypothesis. *J. Fin. Quant. Anal.* 22, 127-141.

66. Harrison, M.J. (1980). The small sample performance of the Szroeter bounds test for heteroscedasticity and a simple test for use when Szroeter's test is inconclusive. *Oxford Bull. Econ. Statist.* 42, 235-250.

67. Harrison, M.J. and McCabe, B.P.M. (1979). A test for heteroscedasticity based on ordinary least squares residuals. *J. Amer. Statist. Assoc.* 74, 494-499.

68. Hildreth, C. and Houck, J.P. (1968). Some estimators for a linear model with random coefficients. *J. Amer. Statist. Assoc.* 63, 584-595.

69. Hodrick, R.J. (1987). *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets.* Harwood, Switzerland.

70. Honda, Y. (1988). A size correction to the Lagrange multiplier test for heteroscedasticity. *J. Econometrics* 38, 375-386.

71. Hsieh, D.A. (1988). The statistical properties of daily foreign exchange rates: 1974 - 1983. *J. Inter. Econ.* 24, 129-145.

72. Jandhyala, V.K. and MacNeill, I.B. (1992). On testing for the constancy of regression coefficients under random walk and change-point alternatives. *Econometric Theory* 8, 501-517.

73. Kariya, T. (1988). The class of models for which the Durbin-Watson test is locally optimal. *Inter. Econ. Rev.* 29, 167-176.

74. Keynes, J.M. (1939). Professor Tinbergen's method. *Econ. J.* 49, 559-568.

75. Kim, H.J. and Siegmund, D. (1989). The likelihood ratio test for a change-point in simple linear regression. *Biometrika* 76, 409-423.

76. King, M.L. (1981). A note on Szroeter's bounds test. *Oxford Bull. Econ. Statist.* 43, 315-321.

77. King, M.L. (1982). A bounds test for heteroscedasticity. Department of Econometrics and Operations Research Working Paper 5/82, Monash University.

78. King, M.L. (1983). Testing for moving average regression disturbances. *Austral. J. Statist.* 25, 23-34.

79. King, M.L. (1984). A new test for fourth-order autoregressive disturbances. *J. Econometrics* 24, 269-277.

80. King, M.L. (1985a). A point optimal test for autoregressive disturbances. *J. Econometrics* 27, 21-37.

81. King, M.L. (1985b). A point optimal test for moving average regression disturbances. *Econometric Theory* 1, 211-222.

82. King, M.L. (1987a). Towards a theory of point optimal testing. *Econometric Rev.* 6, 169-218.

83. King, M.L. (1987b). An alternative test for regression coefficient stability. *Rev. Econ. Statist.* 69, 379-381.

84. King, M.L. (1989). Testing for fourth-order autocorrelation in regression disturbances when first-order autocorrelation is present. *J. Econometrics* 41, 285-301.

85. King, M.L. and Evans, M.A. (1988). Locally optimal properties of the Durbin-Watson test. *Econometric Theory* 4, 509-516.

86. King, M.L. and Hillier, G.H. (1985). Locally best invariant tests of the error covariance matrix of the linear regression model. *J. Roy. Statist. Soc.* B 47, 98-102.

87. King, M.L. and Shively, T.S. (1993). Locally optimal testing when a nuisance parameter is present only under the alternative. *Rev. Econ. Statist.*, 75, 1-7.

88. King, M.L. and Wu, P.X. (1990). Locally optimal one-sided tests for multiparameter hypotheses. Department of Econometrics Working Paper 2/90, Monash University.

89. Koenker, R. (1981). A note on Studentising a test for heteroscedasticity. *J. Econometrics* 17, 107-112.

90. Koerts, J. and Abrahamse, A.P.J. (1969). *On the Theory and Application of the General Linear Model.* Rotterdam University Press, Rotterdam.

91. La Motte, L.R. and McWhorter, A. (1978). An exact test for the presence of random walk coefficients in a linear regression model. *J. Amer. Statist. Assoc.* 73, 816-820.

92. La Motte, L.R. and McWhorter, A. (1979). A comparison of tests for random walk regression coefficients. *Proc. Bus. Econ. Statist. Sect. Amer. Statist. Assoc.*, 596-600.

93. Leybourne, S.J. and McCabe, B.P.M. (1989a). On the distribution of some test statistics for coefficient constancy. *Biometrika* 76, 169-177.

94. Leybourne, S.J. and McCabe, B.P.M. (1989b). Testing for coefficient constancy in random walk models with particular reference to the initial value problem. *Emp. Econ.* 14, 105-112.

95. Leybourne, S.J. and McCabe, B.P.M. (1992). A simple test for parameter constancy in a nonlinear time series regression model. *Econ. Letters* 38, 157-162.

96. Lucas, R.E. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conf. Pub. Pol.* 1, 19-46.

97. Lye, J.N. and Martin, V.L. (1994). Towards a theory of nonlinear models. Chapter 5 in J. Creedy and V.L. Martin (eds.), *Chaos and Nonlinear Models in Economics: Theory and Applications.* Edward Elgar, London.

98. McCabe, B.P.M. (1988). A multiple decision theory analysis of structural stability in regression. *Econometric Theory* 4, 499-508.

99. McCabe, B.P.M. and Harrison, M.J. (1980). Testing the constancy of regression relationships over time using least squares residuals. *Appl. Statist.* 29, 142-148.

100. McCabe, B.P.M. and Leybourne, S.J. (1993). Testing for parameter constancy in nonlinear regression models. *J. Roy. Statist. Soc.* B 55, 133-144.

101. Milan, L. (1984). Testing for the Hildreth-Houck random coefficient model. Unpublished M.Ec. thesis, Monash University.

102. Nabeya, S. and Tanaka, K. (1988). Asymptotic theory of a test for the constancy of regression coefficients against the random walk alternative. *Ann. Statist.* 16, 218-235.

103. Nicholls, D.F. and Pagan, A.R. (1985). Varying coefficient regression. Chapter 16 in E.J. Hannan, P.R. Krishnaiah and M.M. Rao (eds.), *Handbook of Statistics* 5. Elsevier Science, Amsterdam.

104. Nyblom, J. (1986). Testing for deterministic linear trend in time series. *J. Amer. Statist. Assoc.* 81, 545-549.

105. Nyblom, J. (1989). Testing for the constancy of parameters over time. *J. Amer. Statist. Assoc.* 84, 223-230.

106. Nyblom, J. and Makelainen, T. (1983). Comparisons of tests for the presence of random walk coefficients in a simple linear model. *J. Amer. Statist. Assoc.* 78, 856-864.

107. Officer, R.R. (1972). The distribution of stock returns. *J. Amer. Statist. Assoc.* 67, 807-812.

108. Ohlson, J. and Rosenberg, B. (1982). Systematic risk of the CRSP equal-weighted common stock index: A history estimated by stochastic parameter regression. *J. Bus.* 55, 121-145.

109. Osborn, D.R. (1974). The distribution of price changes on the Sydney Stock Exchange. *Austral. J. Statist.* 16, 44-49.

110. Pagan, A.R. and Tanaka, K. (1979). A further test for assessing the stability of regression coefficients. Working Papers in Economics and Econometrics No. 016, Australian National University.

111. Palm, F.C. and Sneek, J.M. (1984). Significance tests and spurious correlation in regression models with autocorrelated errors. *Statist. Hefte* 25, 87-105.

112. Ploberger, W. and Kramer, W. (1992). The CUSUM test with OLS residuals. *Econometrica* 60, 271-285.

113. Praetz, P.D. (1972). The distribution of share price changes. *J. Bus.* 45, 49-55.

114. Praetz, P.D. and Wilson, E.J.G. (1978). The distribution of stock market returns: 1958-1973. *Austral. J. Man.* 3, 79-90.

115. Quandt, R.E. (1960). Tests of the hypothesis that a linear regression obeys two seperate regimes. *J. Amer. Statist. Assoc.* 55, 324-330.

116. Rahman, S. (1993). Aspects of diagnostic testing of disturbances in econometric regression models. Unpublished Ph.D. thesis, Monash University.

117. Rahman, S. and King, M.L. (1993). Marginal likelihood score-based tests of regression disturbances in the presence of nuisance parameters. Submitted to *J. Econometrics.*

118. Rainbow, K. and Praetz, P.D. (1986). The distribution of returns in Sydney wool futures. Chapter 9 in B.A. Goss (ed.), *Futures Markets: Their Establishment and Performance.* Croom Helm, London.

119. Raj, B. and Ullah, A. (1981). *Econometrics: A Varying Coefficients Approach*, Croom Helm, London.

120. Rosenberg, B. (1973). The analysis of a cross-section of time series by stochastically convergent parameter regression. *Ann. Econ. Social Meas.* 2, 399-428.

121. Roy, S.N. (1953). On a heuristic method of test construction and its use in multivariate analysis. *Ann. Math. Statist.* 24, 220-238.

122. Roy, S.N., Gnanadesikan, R. and Srivastava, J.N. (1971). *Analysis and Design of Certain Quantitative Multiresponse Experiments.* Pergamon Press, Oxford.

123. Shively, T.S. (1988a). An analysis of tests for regression coefficient stability. *J. Econometrics* 39, 367-386.

124. Shively, T.S. (1988b). An exact test for a stochastic coefficient in a time series regression model. *J. Time Ser. Anal.* 9, 81-88.

125. Shively, T.S., Ansley, C.F. and Kohn, R. (1990). Fast evaluation of the distribution of the Durbin-Watson and other invariant test statistics in time series regression. *J. Amer. Statist. Assoc.* 85, 676-685.

126. Silvapulle, P. and King, M.L. (1991). Testing moving average against autoregressive disturbances in the linear regression model. *J. Bus. Econ. Statist.* 9, 329-335.

127. Simonds, R.R., La Motte, L.R. and McWhorter, A. (1986). Testing for nonstationarity of market risk: An exact test and power considerations. *J. Fin. Quant. Anal.* 21, 209-220.

128. Smith, J. and Gruen, D.W.R. (1989). A random walk around the $A: Expectations, risk, interest rates and consequences for external imbalance. Reserve Bank of Australia Research Discussion Paper 8906.

129. Stein, J.L. (1986). *The Economics of Futures Markets.* Blackwell, New York.

130. Stein, J.L. (1992). Rational expectations and welfare in financial futures markets. Chapter 1 in B.A. Goss (ed.), *Rational Expectations and Efficiency in Futures Markets.* Routledge, London.

131. Sunder, S. (1980). Stationarity of market risk: Random coefficient tests for individual stocks. *J. Fin.* 35, 883-896.

132. Swamy, P.A.V.B., Conway, R.K. and Le Blanc, M.R. (1988a). The stochastic coefficients approach to econometric modelling, Part I: A critique of fixed coefficient models. *J. Agri. Econ. Res.* 40, 2-10.

133. Swamy, P.A.V.B., Conway, R.K. and Le Blanc, M.R. (1988b). The stochastic coefficients approach to econometric modelling, Part II: Description and motivation. *J. Agri. Econ. Res.* 40, 21-30.

134. Swamy, P.A.V.B., Conway, R.K. and Le Blanc, M.R. (1989). The stochastic coefficients approach to econometric modelling, Part III: Estimation, stability testing and prediction. *J. Agri. Econ. Res.* 41, 4-20.

135. Szroeter, J. (1978). A class of parametric tests for heteroscedasticity in linear econometric models. *Econometrica* 46, 1311-1327.

136. Tan, S.L. (1994). Aspects of testing for a structural break in the linear regression model. Unpublished Ph.D. thesis, Monash University.

137. Tan, S.L. and King, M.L. (1992). Testing for structural change when the change point is unknown. Mimeo, Monash University.

138. Tanaka, K. (1983). Non-normality of the Lagrange multiplier statistic for testing the constancy of regression coefficients. *Econometrica* 51, 1577-1582.

139. Teichmoeller, J. (1971). A note on the distribution of stock price changes. *J. Amer. Statist. Assoc.* 66, 282-284.

140. Theil, H. (1971). *Principles of Econometrics.* North-Holland, Amsterdam.

141. Tinbergen, J. (1939). *Statistical Testing of Business Cycle Theories Vol. I, A Method and Its Application to Investment Activity.* League of Nations, Geneva.

142. Tomek, W.G. and Gray, R.W. (1970). Temporal relationships among prices on commodity futures markets: Their allocative and stabilising roles. *Amer. J. Agri. Econ.* 52, 372-380.

143. Turnbull, S.M. (1977). Market value and systematic risk. *J. Fin.* 32, 1125-1142.

144. Watson, M.W. and Engle, R.F. (1985). Testing for regression coefficient stability with a stationary AR(1) alternative. *Rev. Econ. Statist.* 67, 341-346.

145. Westerfield, J. (1977). An examination of foreign exchange risk under fixed and floating rate regimes. *J. Inter. Econ.* 7, 181-200.

146. Working, H. (1942). Quotations on commodity futures as price forecasts. *Econometrica* 10, 39-52.

147. Working, H. (1949). The theory of price of storage. *Amer. Econ. Rev.* 39, 1254-1262.

148. Wu, P.X. (1991). One-sided and partially one-sided multiparameter hypothesis testing in econometrics. Unpublished Ph.D. thesis, Monash University.

149. Wu, P.X. and King, M.L. (1994). One-sided hypothesis testing in econometrics: A survey. *Pak. J. Statist.* This issue.