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AND APPROXIMATE POINT OPTIMAL TESTS

FOR RANDOM REGRESSION COEFFICIENTS

IN THE PRESENCE OF AUTOCORRELATION

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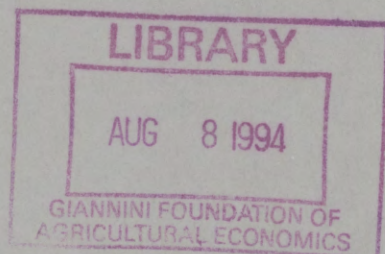MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

A COMPARISON OF MARGINAL LIKELIHOOD BASED AND APPROXIMATE POINT OPTIMAL TESTS FOR RANDOM REGRESSION COEFFICIENTS IN THE PRESENCE OF AUTOCORRELATION

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Abstract

With respect to testing linear regression disturbances, two methods of test construction have recently been found to work well. These are traditional asymptotic tests based on the marginal likelihood or equivalently the likelihood of the maximal invariant and point optimal or approximate point optimal (APO) tests. The former approach has been found to work well for testing for random regression coefficients in the presence of autocorrelated errors. This paper constructs APO invariant (APOI) tests for this testing problem and extends the previous Monte Carlo study to include APOI tests. We conclude that for this testing problem, the extra work required to apply APOI tests hardly seems worthwhile, particularly for larger sample sizes.

Key Words

Invariance; Lagrange multiplier test; locally most mean powerful tests; Monte Carlo study; power.

1. Introduction

When non-experimental data is used in regression analysis, the specification of the covariance matrix of the disturbance term is typically a matter for concern. This has given rise to a substantial literature on testing the form of this covariance matrix, particularly in econometric applications. Godfrey (1988), Judge *et al.* (1985), King (1987a, 1987b), Kramer and Sonnberger (1986), Pagan and Hall (1983) and Pagan (1984) provide reviews of various aspects of this literature which has at least two major strands. One strand advocates the use of classical asymptotic test procedures such as likelihood ratio (LR), Wald and Lagrange multiplier (LM) tests. Another suggests the use of tests that optimize power, either locally to the null hypothesis or, in the case of point optimal tests, at a predetermined point under the alternative hypothesis.

While classical asymptotic tests are relatively easy to apply, Monte Carlo studies of their small sample properties have led to doubts about the accuracy of their critical values. For example King (1987a, p.59), after reviewing the long literature on testing for autocorrelation in linear models concluded that 'the LR test is a particularly unreliable test.' Breusch and Pagan's (1979) LM test for heteroscedasticity has repeatedly been found to have true sizes in small samples well below its nominal size; see for example Griffiths and Surekha (1986), Honda (1988) and Ara and King (1993). A similar but possibly greater problem has been highlighted by Moulton and Randolph (1989) in the case of testing for error components in regression disturbances. It is also worth noting that all these testing problems are typically one-sided in nature. Unfortunately the classical asymptotic tests are nearly always applied without exploiting this knowledge. In contrast, the locally optimal tests are one-sided and usually are applied as exact tests although finding the appropriate critical value or p-value can be a cumbersome process. Most small-sample power comparisons have revealed that the extra computation is well worthwhile; for a survey of the point-optimal literature, see King (1987b).

Recently Ara and King (1993) conjectured that one explanation for the relative poor performance of the asymptotic tests in small samples, is that the presence of nuisance parameters causes biases in the estimates of key parameters in the test statistics. For tests of regression disturbances, the regression coefficients and other disturbance parameters not

2

under test are nuisance parameters. Ara and King suggested that invariance arguments be used to overcome this problem. This involves treating a maximal invariant statistic as the observed data and its density as the likelihood function. They demonstrated that this is equivalent to constructing tests based on the marginal likelihood function. Estimates based on the marginal likelihood function are known to be less biased than those based on the full likelihood functions; see Tunnicliffe Wilson (1989) and the references therein. Furthermore, Grose and King (1993) report distinctly less biased probabilities of selection when familiar model selection procedures are applied to marginal likelihoods rather than full likelihoods.

Ara and King's study suggests that the maximal invariant/marginal likelihood (MIML) approach produces more accurate asymptotic critical values for the LM and LR tests when the null hypothesis is independent identically normally distributed regression disturbances. The extension of these results to more general null hypotheses is the subject of Rahman and King (1993). They conducted a Monte Carlo size and power comparison of conventional and MIML asymptotic tests of Hildreth-Houck (1968) random coefficients in the presence of first-order autoregressive (AR(1)) errors. They concluded that MIML based LM tests have more accurate asymptotic critical values and slightly better powers than their conventional counterparts. The additional power seemed to be a direct consequence of the use of maximum MIML estimates (ie., maximum likelihood estimates using the density of the maximum invariant or equivalently the marginal likelihood as the likelihood function) of the nuisance parameters that cannot be eliminated by invariance.

These new results raise questions about whether locally optimal tests, particularly point optimal tests, continue to have enough of a power advantage to make the extra work required for their application worthwhile. Certainly point optimal tests have been found to have excellent small-sample power properties when testing for random regression coefficients as documented by King (1987b) and Brooks and King (1994). Unfortunately, the presence of nuisance parameters that cannot be eliminated through invariance (or similarity) arguments, considerably complicates the construction of point optimal tests. In fact, there is no guarantee that the method of construction outlined by King (1987b) will work. In such circumstances, an approximate point optimal (APO) test procedure is

suggested. An obvious question is whether the extra work required, results in a clearly superior test than the more convenient MIML based LM test.

The aim of this paper is to answer this question in the case of testing for Hildreth-Houck random regression coefficients in the presence of AR(1) disturbances. As we shall see, this is a testing problem for which we have difficulty constructing a truely point optimal invariant test and have to resort to the APO invariant (APOI) option. We extend the empirical size and power comparison reported in Rahman and King (1993) by calculating the sizes and powers of two APOI tests. Unfortunately our knowledge of the performance of APOI tests is somewhat limited. Previous empirical size and power calculations presented by King (1989), Silvapulle and King (1991) and Brooks and King (1994) have only involved tests of a single autocorrelation parameter. It therefore will be interesting to observe the small-sample performance of APOI tests when more than one parameter and heteroscedasticity rather than autocorrelation is under test.

The plan of this paper is as follows. Section 2 outlines the model and the testing problem. The class of APOI tests for this problem is introduced in Section 3. The Monte Carlo experiment which compares the small-sample sizes and powers of two APOI with two MIML based tests is reported in Section 4. Some concluding remarks are made in the final section.

2. The Model and the Testing Problem

Consider the regression model with Hildreth-Houck (1968) random regression coefficients and disturbances that follow a stationary AR(1) process:

$$y_t = \bar{\beta}_1 + \sum_{j=2}^{k} \beta_{tj} x_{tj} + u_t, \qquad t = 1, \ldots, n, \tag{1}$$

where y_t is the tth observation on the dependent variable, x_{tj} is the tth observation on the jth non-stochastic explanatory variable and $\bar{\beta}_1$ is an unknown constant. The Hildreth-Houck model assumes the regression coefficients, β_{tj}, $j = 2, \ldots, k$, at time t, are generated as

$$\beta_{tj} = \bar{\beta}_j + \nu_{tj}, \qquad t = 1, \ldots, n, \tag{2}$$

4

in which $\bar{\beta}_j$ is the mean response of the dependent variable to a unit change in the jth explanatory variable and ν_{tj} is an error term such that $\nu_{tj} \sim \text{IN}(0, \sigma_j^2), j = 2, \ldots, k$. The disturbance term u_t is assumed to be generated as

$$u_t = \rho u_{t-1} + \epsilon_t, \qquad |\rho| < 1, \qquad t = 1, \ldots, n,$$

in which $\epsilon_t \sim \text{IN}(0, \sigma_\epsilon^2)$ and $\text{var}(u_t) = \sigma_u^2 = \sigma_\epsilon^2/(1 - \rho^2)$.

By substituting (2) into (1), the model can be written as

$$y_t = \bar{\beta}_1 + \sum_{j=2}^{k} \bar{\beta}_j x_{tj} + w_t, \qquad t = 1, \ldots, n,$$

where

$$w_t = u_t + \sum_{j=2}^{k} x_{tj} \nu_{tj}.$$

For n observations, this model can be written in matrix notation as

$$y = X\bar{\beta} + w, \tag{3}$$

where y is $n \times 1, X$ is an $n \times k$ non-stochastic matrix of rank $k < n$, $\bar{\beta} = (\bar{\beta}_1, \ldots, \bar{\beta}_k)'$ is a $k \times 1$ vector of fixed parameters and $w = (w_1, \ldots, w_n)'$ is an $n \times 1$ disturbance vector. Assuming mutual independence between ϵ_t and $\nu_{tj}, j = 2, \ldots, k, t = 1, \ldots, n$; the covariance matrix of w may be expressed as $\sigma_u^2 \Omega(\lambda, \rho)$ where

$$\Omega(\lambda, \rho) = \begin{bmatrix} (1 + \sum_{j=2}^{k} \lambda_j x_{1j}^2) & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & (1 + \sum_{j=2}^{k} \lambda_j x_{2j}^2) & \rho & \cdots & \rho^{n-2} \\ \rho^2 & \rho & (1 + \sum_{j=2}^{k} \lambda_j x_{3j}^2) & \cdots & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \cdots & \cdots & (1 + \sum_{j=2}^{k} \lambda_j x_{nj}^2) \end{bmatrix} \tag{4}$$

in which $\lambda_j = \sigma_j^2/\sigma_u^2$ for $j = 2, \ldots, k$, and $\lambda = (\lambda_2, \ldots, \lambda_k)'$. Thus

$$w \sim \text{N}(0, \sigma_u^2 \Omega(\lambda, \rho)). \tag{5}$$

5

We also assume that for the λ and ρ values of interest, $\Omega(\lambda, \rho)$ is of full rank.

The testing problem we will consider is one of testing

$$H_0 : \lambda = 0$$

against

$$H_a : \lambda > 0$$

in the context of (3), (4) and (5). Observe that because λ is a $(k-1) \times 1$ vector of ratios of variances, its elements can never be negative. The notation $\lambda > 0$ signifies $\lambda_i \geq 0$ for $i = 2, \ldots, k$, with at least one strict inequality. Note that this testing problem is invariant to linear transformations of the form

$$y^* \rightarrow \eta_0 y + X\eta \tag{6}$$

where η_0 is a positive scalar and η is a $k \times 1$ vector. This allows us to restrict attention only to tests which are invariant to transformations of the form (6). The latter can be achieved by finding a maximal invariant with respect to these transformations and then treating this maximal invariant as the observed data.

When one assumes AR(1) disturbances in an econometric application of (1), one is frequently also willing to state the sign of the correlation coefficient. If such information is available it should be incorporated into the test. In recognition of this we will assume $0 \leq \rho < 1$ which for practical reasons we will approximate with $0 \leq \rho \leq 0.99999$. The following analysis can easily be amended to allow other assumptions about the range of ρ values.

3. An Approximate Point Optimal Invariant Test

It is well-known (see for example King (1987b)) that the $m \times 1$ vector

$$v = Pz/(z'P'Pz)^{1/2}$$

is a maximal invariant under the group of transformations (6) where $m = n - k$, $z = My$ is the ordinary least squares residual vector from (3), $M = I_n - X(X'X)^{-1}X'$ and P is an

6

$m \times n$ matrix such that $P'P = M$ and $PP' = I_m$. From King (1980), the density function of v under (3), (4) and (5) is

$$g(v, \lambda, \rho) = \frac{1}{2} \Gamma(m/2) \pi^{-m/2} \mid P\Omega(\lambda, \rho)P' \mid^{-1/2} [v'(P\Omega(\lambda, \rho)P')^{-1}v]^{-m/2} dv$$

where dv is the uniform measure on the m-dimensional unit sphere. Through invariance, therefore, our testing problem is one of testing

$$H_0 : v \text{ has density } g(v, 0, \rho), \ 0 \leq \rho \leq 0.99999;$$

against

$$H_a : v \text{ has density } g(v, \lambda, \rho), \ 0 \leq \rho \leq 0.99999.$$

Following King (1987b), it is possible that a point optimal invariant (POI) test with optimal power at $\lambda = \lambda^* > 0$, $\rho = \rho_1$ will have critical regions of the form

$$r(\rho_0, \lambda^*, \rho_1) = \hat{w}' \Omega^{-1}(\lambda^*, \rho_1) \hat{w} / \tilde{w}' \Omega^{-1}(0, \rho_0) \tilde{w} < c \tag{7}$$

where $\hat{w}$ and $\tilde{w}$ are the generalized least squares residual vectors from (3) assuming covariance matrices $\Omega(\lambda^*, \rho_1)$ and $\Omega(0, \rho_0)$, respectively, and c is an appropriate critical value. The existence of a POI test in the form of (7) requires c and the fixed parameter ρ_0 to be chosen such that

$$\Pr[r(\rho_0, \lambda^*, \rho_1) < c \mid w \sim \mathrm{N}(0, \Omega(0, \rho_0))] = \alpha \tag{8}$$

$$\Pr[r(\rho_0, \lambda^*, \rho_1) < c \mid w \sim \mathrm{N}(0, \Omega(0, \rho)), \ 0 \leq \rho \leq 0.99999] \leq \alpha \tag{9}$$

where α is the desired level of significance.

Probabilities of the form

$$\Pr[r(\rho_0, \lambda^*, \rho_1) < c \mid w \sim \mathrm{N}(0, \Sigma)]$$

where Σ is an $n \times n$ covariance matrix, can be written as

$$\Pr[u'(\Delta_1 - c\Delta_0)u < 0 \mid w \sim \mathrm{N}(0, \Sigma)] = \Pr\left[\sum_{i=1}^{n} \delta_i \xi_i^2 < 0\right] \tag{10}$$

where if $\Omega_0 = \Omega(0, \rho_0)$ and $\Omega_1 = \Omega(\lambda^*, \rho_1)$ then

$$\Delta_j = \Omega_j^{-1} - \Omega_j^{-1} X (X' \Omega_j^{-1} X)^{-1} X' \Omega_j^{-1}, \qquad j = 0, 1;$$

δ_i are the eigenvalues of $(\Delta_1 - c\Delta_0)\Sigma$ and ξ_i^2 are independent chi-squared random variables with one degree of freedom. Probabilities of the form of (10) can be evaluated using standard algorithms based on Imhof's (1961) algorithm, see King (1987b). For example, the SHAZAM computer package (White (1978)) enables the user to calculate (10) using Davies' (1980) algorithm.

At least for the models and choices of ρ_1 and λ^* values reported in the next section, we found it was never possible to find c and ρ_0 values that solve (8) and (9) simultaneously. This is because for fixed c and ρ_0, the left-hand-side of (9) first decreases and then increases as ρ goes from zero to 0.99999. Therefore local maxima occur at $\rho = 0$ and $\rho = 0.99999$ with one being the global maximum in this range. If ρ_0 is moved towards the global maximum, the height of this maximum decreases relative to that of the other local maximum until the global maximum switches from one endpoint to the other.

Thus for our testing problem, we need to turn our attention to the class of APOI tests introduced by King (1987b) and explored for various testing problems by King (1989), Silvapulle and King (1991) and Brooks and King (1994). An APOI test based on $r(\rho_0, \lambda^*, \rho_1)$ has ρ_0 and its critical value c^* chosen so that

$$\alpha - \Pr\left[r(\rho_0, \lambda^*, \rho_1) < c^* \mid w \sim N(0, \Omega_0)\right] \tag{11}$$

is as close to zero as possible. Clearly when (11) is zero the test is a true POI test. Numerical experiments by King (1989), Silvapulle and King (1991) and Brooks and King (1994) reveal that the recommended choice of ρ_0 value is that value which results in a global maxima at the end-points of the ρ space. We found this indeed to be the case for our problem. The value of ρ_0 that minimizes (11) is that value which results in a global maxima at both $\rho = 0$ and $\rho = 0.99999$.

An important issue is what values should λ^* and ρ_1 take in the test statistic. There are basically two approaches to this problem. Either they can be set arbitrarily to representative or middle values such as $\rho_1 = 0.5$ or a more systematic but computationally demanding approach can be adopted by taking into account the power of the resultant test. Examples of the latter approach are discussed by King (1989) and Brooks and King (1994). In the light of the Brooks and King finding that the extra computation delivers

8

very little improvement in overall power for a related random coefficient testing problem, we chose the representative value approach in our evaluation reported in the next section.

Given choices for λ^* and ρ_1, our APOI test procedure can be applied as follows:

(i) Guess a possible value for ρ_0 which is in the range $0 \leq \rho_0 \leq 0.99999$. (A middle value is typically a good starting point.)

(ii) Solve

$$\Pr\left[r(\rho_0, \lambda^*, \rho_1) < c \mid w \sim \mathrm{N}(0, \Omega(0,0))\right] = \alpha \tag{12}$$

for c. (Note $\Omega(0,0) = I_n$.)

(iii) Using these values of ρ_0 and c, evaluate the left-hand-side of (9) at $\rho = 0.99999$. If the resultant probability is below (above) α make ρ_0 smaller (larger) and repeat steps (ii) and (iii). Stop when the resultant probability is equal to α.

There are a range of numerical algorithms that can be helpful in solving (12) and determining appropriate step sizes for ρ_0 in this iterative process. We used the secant method, (see for example Conte (1965)).

4. Monte Carlo Experiment

In order to compare the small-sample properties of APOI tests with those of the MIML based tests explored by Rahman and King (1993), we extended the latter's Monte Carlo study to include two APOI tests. Two APOI tests were chosen to allow a minimal assessment of the sensitivity of the tests to the choice of λ^* and ρ_1 values.

4.1 Experimental Design

For completeness we will outline the experimental design employed by Rahman and King (1993). They used the Monte Carlo method to estimate the sizes and powers of the LM and King and Wu's (1993) asymptotic locally most mean powerful (ALMMP) tests using both the conventional likelihood and the MIML based approaches. Following Rahman and King, we will use LM and ALMMP to denote the conventional likelihood

based tests and MLM and MALMMP their MIML based counterparts. The study used the following $n \times 3$ X matrices with $n = 20$ and $n = 60$.

$X1$: A constant dummy plus two independent trending regressors generated as

$$x_{tj} = z_{tj} + 0.25t$$

where $z_{tj}, t = 1, \ldots, n; j = 2, 3$ are independent AR(1) time series generated from

$$z_{tj} = 0.5z_{t-1j} + \eta_{tj}$$

and

$$\eta_{tj} \sim \text{IN}(0,1), \ t = 1, \ldots, n, \ j = 2, 3.$$

$X2$: A constant plus quarterly Australian total private capital movements and Australian total Government capital movements.

$X3$: The first n observations of Durbin and Watson's (1951) consumption of spirits example.

$X4$: A constant dummy plus quarterly seasonally adjusted Australian household disposable income and private final consumption expenditure commencing 1959(4).

Because $k = 3$, all tests were of $H_0 : \lambda_2 = \lambda_3 = 0$ against $H_a : (\lambda_2, \lambda_3)' > 0$. A five percent nominal significance level and 2000 iterations were used throughout. Both asymptotic and simulated critical values were used by Rahman and King. The latter were obtained for each test and regressor matrix by taking the largest critical value from the set of estimated true critical values at $\rho = 0, 0.1, 0.2, \ldots, 0.9$ obtained via the Monte Carlo method.

As Rahman and King observed from

$$\text{var}(w_t) = \sigma^2 \left(1 + \sum_{j=2}^{3} \lambda_j x_{tj}^2 \right),$$

what makes a large value of λ_j depends on the magnitude of x_{tj}^2. As it seems unlikely that β_{tj} would contribute more than 10 times the variance of u_t to w_t, they set

$$\lambda_j = \bar{\lambda}_j \lambda_j^u$$

where

$$\lambda_j^u = 10/\max_t(x_{tj}^2), \ j = 2,3$$

provides an upper bound. They calculated powers at all combinations of $\rho = 0, 0.3, 0.6, 0.9$ and $\bar{\lambda}_j = 0, 0.01, 0.1, 0.5; j = 2, 3$.

The first APOI test used in the current study involved choosing λ^* such that $\bar{\lambda}_j = 0.01, j = 2, 3$ and $\rho_1 = 0.5$. We denote this test by r_1. The second APOI test, denoted r_2, involved the choice of $\bar{\lambda}_j = 0.1, j = 2, 3$ with $\rho_1 = 0.5$. For these tests, exact critical values were calculated as outlined in Section 3 using a modified version of Koerts and Abrahamse's (1969) FQUAD subroutine with maximum integration and truncation errors of 10^{-6}.

4.2 Results

Selected estimated sizes of the r_1 and r_2 tests are presented in Table 1. We see that both tests' sizes first decrease and then increase as ρ increases from zero. Note that the critical values are such that the true size (as opposed to the estimated size) is 0.05 at $\rho = 0.0$ and $\rho = 0.99999$. Sizes away from the endpoints are significantly below 0.05 and show a clear tendency to decrease as n increases. Table 1 also tabulates the value of ρ_0 used in the r_1 and r_2 test statistics. For these values it is clear that (11) is typically significantly different from zero, suggesting there is a question mark over whether these APOI tests are nearly optimizing power at the ρ_1 and λ^* values used in the test statistics. In contrast to these size results, Rahman and King (1993) report that almost all estimated sizes of the MLM test based on asymptotic critical values were not significantly different from 0.05. The asymptotic critical values of the MALMMP test also produce sizes reasonably close to the nominal size. We are left to conclude that MIML based asymptotic tests typically have actual sizes closer to 0.05 than do APOI tests.

Tables 2, 3 and 4 present estimated sizes and powers based on exact critical values for the r_1 and r_2 tests and simulated critical values for the MLM and MALMMP tests. The size and power values for these latter tests come from the Rahman and King (1993) study and are included here for ease of comparison. Following Rahman and King, the $X3$ results

11

are not included because these powers are very low, almost always less than 0.1. This appears to be caused by the relative smooth nature of the regressors in this case. The latter results in a pattern of heteroscedasticity in the disturbances, w_t, that is difficult to detect. Powers for $\bar{\lambda}_j = 0.01$ are also omitted because they are typically similar to those for $\bar{\lambda}_j = 0$ except when $\rho = 0.9$ and $n = 60$. The discussion that follows covers the results for $\bar{\lambda}_j = 0.01$ omitted from the tables but not those for $X3$.

Powers of almost all tests increase as n increases, *ceteris paribus*. For the $X1$ and $X4$ design matrices, the increases are relatively higher and more uniform across all tests. For the $X2$ design matrix, the increases are not as great for the MALMMP test as for the other tests. Away from the null hypothesis, there is a clear tendency for powers to increase as ρ increases, *ceteris paribus*. Not surprisingly, powers typically increase as either $\bar{\lambda}_2$ or $\bar{\lambda}_3$ increases although there are a number of exceptions, particularly for $X2$ and $X4$.

Of the APOI tests, the r_2 test is almost always more powerful than the r_1 test with power differences in favour of the r_2 test ranging up to 0.125. The performance of the r_2 test relative to the MLM and MALMMP tests depends very much on the sample size and the design matrix. With the exception of a few points near H_0, when $n = 20$ the r_2 test dominates the MIML based tests for $X1$ and to a lesser extent for $X4$. In the remaining cases, no one test dominates in terms of power. For $X1$ and $n = 60$, the r_2 test clearly has better overall power than the MLM test while the MALMMP test has a significant power advantage over the r_2 test for points closer to H_0 with the reverse being the case for points some distance from H_0. A similar crossing of powers occurs for $X2$ and the r_2 and MLM tests with the MLM test being on average 0.056 more powerful when $n = 60$. $X2$ is a data set for which the MALMMP test performs poorly, particularly on the boundary of the alternative hypothesis parameter space. The powers of the MIML based tests and r_2 also cross for $X4$ and $n = 60$ with points nearer H_0 favouring the former tests. In this case the MALMMP test has the highest average power followed by the r_2 test and then the MLM test so one could argue that the MALMMP test has the best overall power because its average power is highest.

5. Concluding Remarks

From the results reported above, it appears that MIML based tests are very competitive in terms of power with APOI tests when testing for random regression coefficients in the presence of AR(1) disturbances. These tests also appear to have more desirable sizes than APOI tests. The extra computation required to apply APOI tests does not seem to result in clear-cut improvements in either size or power, particularly for larger sample sizes. It appears that the lower than nominal sizes for middle values of the nuisance parameter, ρ, may be a cause of the APOI tests not outperforming the other tests. Unfortunately, neither of the two MIML based tests considered by Rahman and King (1993) dominates the other. Generally the MALMMP test is best but for certain data sets in which the component scores used in the test statistic are negatively correlated, the test can perform particularly poorly. For large samples, the MLM test is reasonably reliable but obviously can be improved upon because it does not exploit the one-sided nature of the testing problem. Exactly how this might be best achieved remains a topic for further research.

Acknowledgement

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Table 1: Estimated sizes of the r_1 and r_2 tests for random coefficients in the presence of AR(1) disturbances at the 5% level

Test	ρ_0	T	ρ 0.0	0.2	0.4	0.6	0.8	0.9
					$X1$			
r_1	0.4463	20	0.051	0.040	0.032	0.032	0.036	0.042
r_2	0.2957		0.050	0.037	0.032	0.034	0.038	0.041
r_1	0.4648	60	0.051	0.025	0.017	0.010	0.010	0.022
r_2	0.3547		0.046	0.029	0.014	0.012	0.017	0.034
					$X2$			
r_1	0.4668	20	0.052	0.036	0.022	0.018	0.018	0.028
r_2	0.3123		0.049	0.030	0.024	0.021	0.028	0.040
r_1	0.4929	60	0.052	0.037	0.024	0.016	0.011	0.014
r_2	0.4612		0.052	0.036	0.022	0.016	0.012	0.015
					$X3$			
r_1	0.4131	20	0.042	0.019	0.012	0.012	0.018	0.034
r_2	0.1768		0.044	0.021	0.013	0.012	0.022	0.033
r_1	0.4349	60	0.048	0.008	0.000	0.000	0.001	0.014
r_2	0.2242		0.046	0.006	0.001	0.000	0.003	0.018
					$X4$			
r_1	0.4263	20	0.044	0.020	0.013	0.010	0.022	0.034
r_2	0.2053		0.048	0.028	0.020	0.016	0.027	0.037
r_1	0.4735	60	0.050	0.036	0.020	0.012	0.014	0.025
r_2	0.3679		0.048	0.026	0.014	0.012	0.014	0.018

Table 2: Estimated sizes and powers for $X1$ of the r_1, r_2, MLM and MALMMP tests for random coefficients in the presence of AR(1) disturbances using exact or simulated critical values at the 5% level.

$\bar{\lambda}_1$	$\bar{\lambda}_2$	$n = 20$				$n = 60$			
		r_1	r_2	MLM	MALMMP	r_1	r_2	MLM	MALMMP
					$\rho = 0.0$				
0	0	.051	.050	.050	.050	.051	.046	.049	.047
	.1	.125	.135	.100	.110	.230	.260	.190	.284
	.5	.324	.376	.252	.308	.712	.832	.731	.820
.1	0	.113	.124	.098	.112	.199	.218	.170	.246
	.1	.178	.206	.142	.172	.388	.456	.352	.484
	.5	.339	.405	.262	.320	.750	.873	.776	.859
.5	0	.288	.340	.230	.281	.670	.771	.669	.788
	.1	.316	.384	.248	.312	.734	.834	.745	.844
	.5	.390	.488	.294	.376	.853	.948	.872	.934
					$\rho = 0.3$				
0	0	.036	.036	.048	.050	.021	.022	.048	.044
	.1	.132	.144	.113	.130	.248	.306	.232	.334
	.5	.352	.406	.268	.334	.786	.906	.767	.854
.1	0	.120	.124	.106	.130	.212	.240	.213	.296
	.1	.196	.223	.158	.198	.462	.542	.419	.556
	.5	.379	.438	.284	.354	.818	.932	.804	.890
.5	0	.322	.380	.250	.305	.752	.854	.728	.835
	.1	.354	.420	.271	.332	.806	.904	.786	.879
	.5	.430	.516	.318	.409	.896	.974	.892	.944
					$\rho = 0.6$				
0	0	.032	.034	.046	.042	.010	.012	.043	.045
	.1	.182	.190	.178	.202	.410	.478	.441	.568
	.5	.434	.494	.334	.394	.884	.966	.858	.920
.1	0	.162	.168	.152	.186	.350	.386	.404	.526
	.1	.277	.295	.211	.278	.644	.744	.633	.768
	.5	.454	.518	.338	.408	.899	.975	.880	.934
.5	0	.384	.448	.308	.362	.871	.946	.842	.918
	.1	.412	.492	.316	.388	.896	.968	.866	.936
	.5	.482	.583	.360	.446	.938	.991	.924	.963
					$\rho = 0.9$				
0	0	.042	.041	.038	.038	.022	.034	.050	.045
	.1	.412	.432	.328	.390	.855	.919	.889	.928
	.5	.552	.645	.411	.498	.963	.998	.954	.974
.1	0	.364	.396	.308	.364	.833	.867	.871	.934
	.1	.468	.530	.346	.420	.927	.980	.925	.961
	.5	.554	.658	.404	.494	.968	.998	.954	.979
.5	0	.494	.619	.390	.466	.970	.994	.950	.983
	.1	.516	.632	.386	.480	.972	.997	.954	.982
	.5	.552	.676	.404	.512	.974	.999	.962	.988

Table 3: Estimated sizes and powers for $X2$ of the r_1, r_2, MLM and MALMMP tests for random coefficients in the presence of AR(1) disturbances using exact or simulated critical values at the 5% level.

			$n = 20$					$n = 60$		
$\bar{\lambda}_1$	$\bar{\lambda}_2$	r_1	r_2	MLM	MALMMP		r_1	r_2	MLM	MALMMP
					$\rho = 0.0$					
0	0	.052	.049	.046	.046		.052	.052	.046	.047
	.1	.052	.058	.097	.045		.202	.196	.181	.054
	.5	.094	.104	.210	.046		.655	.683	.650	.098
.1	0	.089	.095	.068	.111		.058	.060	.098	.110
	.1	.091	.104	.089	.097		.218	.220	.222	.118
	.5	.114	.137	.181	.078		.664	.698	.663	.153
.5	0	.178	.208	.106	.242		.120	.153	.302	.336
	.1	.177	.211	.116	.226		.284	.317	.391	.329
	.5	.174	.211	.150	.175		.694	.740	.714	.316
					$\rho = 0.3$					
0	0	.026	.028	.045	.044		.029	.027	.050	.043
	.1	.034	.041	.100	.042		.218	.222	.220	.060
	.5	.084	.100	.237	.044		.742	.778	.728	.110
.1	0	.078	.085	.071	.125		.042	.044	.110	.122
	.1	.086	.098	.094	.106		.247	.258	.270	.134
	.5	.111	.130	.192	.078		.756	.788	.740	.156
.5	0	.184	.218	.118	.251		.124	.156	.334	.376
	.1	.182	.216	.123	.230		.332	.383	.446	.367
	.5	.178	.215	.148	.184		.788	.830	.785	.336
					$\rho = 0.6$					
0	0	.018	.021	.040	.041		.016	.016	.046	.046
	.1	.030	.042	.130	.041		.336	.328	.385	.071
	.5	.087	.116	.276	.040		.862	.890	.864	.138
.1	0	.091	.100	.078	.168		.030	.038	.148	.181
	.1	.098	.114	.108	.133		.373	.386	.441	.181
	.5	.130	.158	.222	.088		.870	.898	.866	.210
.5	0	.202	.239	.132	.295		.166	.220	.466	.518
	.1	.198	.240	.133	.268		.490	.544	.624	.482
	.5	.192	.242	.164	.198		.888	.922	.896	.414
					$\rho = 0.9$					
0	0	.028	.040	.039	.050		.014	.015	.050	.048
	.1	.082	.106	.282	.059		.736	.742	.844	.140
	.5	.150	.192	.383	.062		.979	.988	.983	.226
.1	0	.194	.215	.145	.306		.106	.128	.396	.456
	.1	.184	.216	.166	.198		.778	.800	.888	.402
	.5	.182	.228	.284	.112		.982	.988	.982	.324
.5	0	.257	.302	.163	.356		.432	.528	.813	.848
	.1	.249	.296	.164	.320		.844	.883	.930	.754
	.5	.231	.290	.182	.224		.982	.992	.985	.560

Table 4: Estimated sizes and powers for $X4$ of the r_1, r_2, MLM and MALMMP tests for random coefficients in the presence of AR(1) disturbances using exact or simulated critical values at the 5% level.

$\bar{\lambda}_1$	$\bar{\lambda}_2$	$n = 20$				$n = 60$			
		r_1	r_2	MLM	MALMMP	r_1	r_2	MLM	MALMMP
					$\rho = 0.0$				
0	0	.044	.048	.046	.044	.050	.048	.041	.050
	.1	.066	.072	.044	.054	.158	.166	.128	.180
	.5	.082	.086	.060	.070	.483	.561	.462	.566
.1	0	.068	.074	.048	.056	.158	.163	.127	.180
	.1	.078	.082	.052	.062	.278	.302	.240	.313
	.5	.084	.088	.064	.073	.518	.622	.516	.622
.5	0	.086	.094	.064	.076	.496	.573	.475	.586
	.1	.086	.092	.067	.078	.532	.629	.522	.638
	.5	.080	.096	.066	.079	.630	.734	.646	.728
					$\rho = 0.3$				
0	0	.016	.024	.043	.040	.028	.019	.042	.047
	.1	.047	.050	.048	.060	.152	.159	.158	.219
	.5	.067	.081	.058	.074	.529	.616	.503	.602
.1	0	.048	.052	.054	.062	.152	.156	.157	.222
	.1	.062	.071	.054	.070	.304	.314	.286	.368
	.5	.068	.082	.061	.075	.569	.660	.546	.644
.5	0	.070	.086	.063	.079	.552	.629	.524	.633
	.1	.074	.090	.064	.080	.585	.674	.564	.664
	.5	.078	.092	.064	.079	.664	.773	.680	.742
					$\rho = 0.6$				
0	0	.010	.016	.037	.032	.012	.012	.046	.050
	.1	.044	.050	.064	.080	.237	.227	.295	.370
	.5	.072	.086	.061	.082	.616	.704	.621	.688
.1	0	.047	.053	.069	.084	.240	.220	.304	.378
	.1	.058	.070	.064	.087	.420	.438	.450	.540
	.5	.074	.090	.062	.080	.647	.744	.656	.712
.5	0	.076	.092	.066	.090	.644	.724	.651	.726
	.1	.076	.093	.066	.088	.671	.761	.678	.740
	.5	.077	.098	.068	.084	.734	.834	.732	.783
					$\rho = 0.9$				
0	0	.034	.037	.038	.020	.025	.018	.047	.045
	.1	.074	.080	.064	.102	.583	.594	.707	.744
	.5	.078	.097	.067	.081	.770	.856	.778	.810
.1	0	.077	.087	.070	.110	.614	.610	.726	.770
	.1	.080	.098	.068	.096	.710	.768	.770	.810
	.5	.078	.098	.067	.085	.782	.867	.788	.822
.5	0	.084	.108	.071	.091	.803	.882	.819	.856
	.1	.084	.106	.071	.090	.806	.888	.816	.852
	.5	.084	.100	.071	.088	.810	.902	.814	.850