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TESTING HILDRETH-HOUCK AGAINST RETURN TO NORMALCY RANDOM REGRESSION COEFFICIENTS

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ABSTRACT

In the context of the linear regression model, this paper considers testing for a single Hildreth-Houck random coefficient against the alternative that the coefficient follows the return to normalcy model. We attempt to construct a point-optimal invariant test but find that we have to resort to the class of approximate point-optimal invariant (APOI) tests introduced by King (1987). Empirical power calculations show that these tests have good small-sample properties compared to the likelihood ratio and Wald tests. A particular APOI test is recommended and is found to be remarkably robust to nonnormality.

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1. INTRODUCTION

The standard assumption that regression coefficients are fixed constants has been seen by many as being restrictive and, in some cases, unrealistic. There is now a rich literature on this topic with a variety of varying coefficient models being proposed. Published surveys include the book by Raj and Ullah (1981) and the more recent work by Swamy, Conway and Le Blanc (1988a, 1988b, 1989). There is also an associated literature on testing the constant coefficient model against particular varying coefficient alternatives, see for example Breusch and Pagan (1979), Watson and Engle (1985) and Shively (1988a, 1988b).

A key area of empirical work using varying coefficient models is in the context of the market model, which hypothesises that the return on a company's shares is a linear function of the return on the market as a whole. The theory that the systematic risk, which is the slope coefficient in the linear model, is constant over time has been challenged for a variety of microeconomic and macroeconomic reasons. Some investigators have suggested that these factors can be proxied by treating the systematic risk as following a Hildreth-Houck (1968) formulation. Examples of the use of this model may be found in Fabozzi and Francis (1978), Francis and Fabozzi (1980) and Fabozzi, Francis and Lee (1982). This has been criticised by others who claim systematic risk is autocorrelated and therefore should be modelled using Rosenberg's (1973) formulation. Examples of this approach may be found in Sunder (1980), Bos and Newbold (1984) and Faff, Lee and Fry (1992). A possible explanation for this diversity of results is that tests for the presence of particular varying coefficient models have been found to have good power against a range of alternative models (see Watson and Engle (1985) and Brooks (1993b)). In addition the Hildreth-Houck (1968) random coefficient model is a special case of the Rosenberg (1973) return to normalcy random coefficient model. An obvious question is which model best fits the data? In their empirical investigation of systematic risk in the market model, Bos and Newbold (1984), tested the Hildreth-Houck model against the return to normalcy alternative. They used the Wald and likelihood ratio tests which have an asymptotic justification and argued that, despite having 120 observations, their tests lacked power.

In this paper, we consider the problem of testing a single Hildreth-Houck random coefficient against the alternative that the coefficient follows the return to normalcy model. In order to construct a test with good small-sample power properties, we adopt a point-optimal testing approach. Point-optimal tests have been found to have excellent small-sample power properties when testing for the presence of varying coefficients (see for example, King (1987), Shively (1988a, 1988b) and Brooks (1993a)). That they also provide exact tests in small samples is a further advantage. Unfortunately, the point-optimal approach relies on the assumption of normally distributed errors. A feature of econometric modelling based on financial data is the possibility of nonnormal disturbances (see for example Fama (1965) and Harris (1987)). Given the market model based motivation for this testing problem, nonnormal disturbances are a strong possibility. This means that robustness of point-optimal tests to nonnormality is an important issue. In this paper we compare the small-sample size and power properties of the point-optimal tests with the Wald and likelihood ratio tests. Comparisons are made for both normal and nonnormal disturbances.

The plan of the paper is as follows. Section 2 sets out the testing problem. Section 3 then discusses the point-optimal and approximate point-optimal solutions. Attention is given to the choice of parameter values for use in the approximate point-optimal invariant (APOI) test. Sections 4 and 5 report the empirical power comparison of the APOI tests with the likelihood ratio and Wald tests under normality and nonnormality respectively. Section 6 concludes with a discussion of the use of the recommended test in the context of the market model.

2. THE TESTING PROBLEM

Consider the linear regression model with a single varying coefficient, α_t, namely

$$y_t = x_t\alpha_t + z_t'\beta + \epsilon_t, \qquad t = 1, ..., n, \tag{1}$$

where x_t is a known scalar regressor, z_t is a kx1 vector of observations on k nonstochastic regressors, β is a kx1 vector of unknown constant coefficients and ϵ_t is a disturbance term such that

$$\epsilon_t \sim IN(0, \sigma^2), \qquad t = 1, ..., n. \tag{2}$$

If α_t follows the Hildreth-Houck random coefficient model then

$$\alpha_t = \bar{\alpha} + a_t \tag{3}$$

where

$$a_t \sim IN(0, \lambda\sigma^2), \qquad t = 1, ..., n, \tag{4}$$

and a_t is independent of ϵ_t. Alternatively, if α_t follows Rosenberg's return to normalcy process then

$$\alpha_t - \bar{\alpha} = \phi(\alpha_{t-1} - \bar{\alpha}) + a_t \tag{5}$$

where a_t is generated as (4) and is independent of ϵ_t. For (5) to be a stationary process, ϕ must be such that $| \phi | < 1$. However, the rationale for this model is that the varying coefficient changes slowly over time implying that ϕ is non-negative. Collins, Ledolter and Rayburn (1987), for example, argued that applied researchers should expect the smooth changes in α associated with a positive ϕ, and not the oscillations in α that would be produced by a negative value. Also a negative value for ϕ causes considerable difficulties in interpretation particularly if the period between observations is changed. For these reasons we assume $0 \leq \phi < 1$.

Under either (3) or (5), the model (1), (2) and (4) can be written as

$$y_t = x_t\bar{\alpha} + z_t'\beta + v_t, \qquad t = 1, ..., n, \tag{6}$$

in which v_t is normally distributed with mean zero and variance-covariances that are determined by the α_t and ϵ_t processes.

When α_t is generated by (3) and (4) then

$$var(v_t) = \sigma^2(1 + \lambda x_t^2), \qquad t = 1, ..., n,$$

and

$$cov(v_t v_s) = 0, \qquad for \quad t \neq s.$$

On the other hand, when α_t is determined by (4) and (5) then

$$var(v_t) = \sigma^2(1 + \lambda x_t^2/(1 - \phi^2)), \qquad t = 1, ..., n,$$

and

$$cov(v_t v_s) = \sigma^2 \lambda x_t x_s \phi^{|t-s|}/(1-\phi^2), \quad for \quad t \neq s.$$

In other words, under (4) and (5)

$$v \sim N[0, \sigma^2(I_n + \lambda\Omega(\phi))] \tag{7}$$

where $\Omega(\phi)$ is an $n \times n$ matrix whose typical element is

$$\Omega(\phi)_{st} = x_s x_t \phi^{|t-s|}/(1-\phi^2).$$

Note that $\Omega(0)$, which we will denote by Ω_0, is given by

$$\Omega_0 = diag(x_1^2, ..., x_n^2)$$

and under (3) and (4)

$$v \sim N[0, \sigma^2(I_n + \lambda\Omega_0)].$$

Our problem of interest is therefore one of testing

$$H_0 : \phi = 0$$

against

$$H_a : \phi > 0$$

with respect to (6) and (7). This involves testing whether the disturbance covariance matrix of a standard linear regression model can be parameterised (up to a scalar constant) as $I_n + \lambda\Omega_0$ or as $I_n + \lambda\Omega(\phi)$. Observe that λ is a nuisance parameter. Furthermore, if $\lambda = 0$ then ϕ is not identified. However, this is not an issue as $\lambda = 0$ implies α_t is constant under either (3) or (5). We assume that α_t is not constant which implies λ must be non-negative.

3. AN APPROXIMATE POINT-OPTIMAL TEST

The problem of testing H_0 against H_a is invariant with respect to transformations of the form

$$y_t \rightarrow \delta y_t + \gamma_0 x_t + z_t'\gamma, \qquad t = 1, ..., n,$$

where δ is a positive scalar, γ_0 is a scalar and γ is a $k \times 1$ vector. Following King (1987), it may be possible to construct the critical region for a point-optimal invariant (POI) test with optimal power at $(\lambda, \phi) = (\lambda_1, \phi_1)$, in which $\lambda_1 \geq 0$ and $\phi_1 \geq 0$, based on rejecting H_0 for

$$s(\lambda_0, \lambda_1, \phi_1) = \tilde{u}'(I_n + \lambda_1 \Omega(\phi_1))^{-1} \tilde{u} / \hat{u}'(I_n + \lambda_0 \Omega_0)^{-1} \hat{u} < c \tag{8}$$

where $\tilde{u}$ and $\hat{u}$ are the generalised least squares residual vectors from (6) assuming covariance matrices $I_n + \lambda_1 \Omega(\phi_1)$ and $I_n + \lambda_0 \Omega_0$, respectively. Critical regions of the form of (8) are most powerful invariant for the simpler problem of testing H_0': $(\lambda, \phi) = (\lambda_0, 0)$ against H_a': $(\lambda, \phi) = (\lambda_1, \phi_1)$. They are also POI for the wider problem of testing H_0 against H_a provided the critical value appropriate under H_0' remains unchanged when the null hypothesis is widened to H_0. Of course, λ_0 is a parameter we are free to choose. Thus the existence of a POI test of this form requires λ_0 and the critical value c to be chosen such that

$$Pr[s(\lambda_0, \lambda_1, \phi_1) < c \quad | \quad u \sim N(0, I_n + \lambda_0 \Omega_0)] = \eta \tag{9}$$

and

$$Pr[s(\lambda_0, \lambda_1, \phi_1) < c \quad | \quad u \sim N(0, I_n + \lambda \Omega_0), 0 < \lambda] \leq \eta \tag{10}$$

where η is the desired level of significance. When λ_0 and c cannot be chosen to solve (9) and (10) simultaneously, King (1987) recommends the use of an APOI test. Such tests have critical regions of the form of (8) with c chosen so that (10) holds and λ_0 is chosen to make the LHS of (9) as close to η as possible.

APOI tests have been found to have good small-sample power properties when testing for a simple AR(4) disturbance process in the presence of an AR(1) process (see King (1989)). Notice the similarity of this testing problem to that currently under consideration. Both involve one parameter tests of the disturbance covariance matrix in the presence of a second nuisance parameter that cannot be eliminated by invariance arguments.

Our experience with the X matrices and η values of the experiment reported below is that it is not possible to find c and λ_0 values to solve (9) and (10) simultaneously. This is because local maxima of the LHS of (10) always occur at the two extremes of the range of λ values under consideration. When λ_0 is chosen to be one such extreme value, the global maximum of the LHS of (10) always occurs at the other boundary value. This makes it impossible to jump from H_0' to H_0 without changing the critical value. We therefore shall turn our attention to the class of APOI tests which involves choosing c and λ_0 such that (10) holds and

$$\eta - Pr[s(\lambda_0, \lambda_1, \phi_1) < c \quad | \quad u \sim N(0, I_n + \lambda_0 \Omega_0)] \tag{11}$$

is minimised. Ensuring that (10) holds requires a search over the λ parameter space. We can simplify this task by restricting the range of λ values. Unfortunately, this is not as straightforward as it may seem. Although λ is the ratio of the random coefficient disturbance variance to the regression disturbance variance, its contribution to the variance of the composite disturbance v_t, depends on the scale of x_t^2. In the case of the Hildreth-Houck model, this can be seen from

$$\sigma_t^2 = \sigma^2(1 + \lambda x_t^2).$$

In other words, λ increases the variance of v_t by $\sigma^2 \lambda x_t^2$ or, relative to σ^2, by

$$q = \lambda x_t^2.$$

We therefore feel it is appropriate to consider bounds on q and following Evans and King (1985, 1988), we suggest 10 as a reasonable upper bound. This gives an implied upper bound on λ as:

$$\lambda^u = 10/max(x_t^2).$$

Zero is an obvious lower bound although $\lambda = 0$ must give a test whose power must equal its size given that ϕ is not identified in this case. Therefore the range of possible λ values is given by $0 \leq \lambda \leq \lambda^u$.

Through numerical experimentation, we found that for any choice of λ_0, λ_1 and ϕ_1 values, the LHS of (10), as a function of λ, first decreases, then increases and finally flattens out well before $\lambda = \lambda^u$. For different λ_0, λ_1 and ϕ_1 values, the maximum of the LHS of (10), within the range $0 \leq \lambda \leq \lambda^u$, occurs at either endpoint. Hence, the particular λ value from this range that gives equality in (10) and therefore determines c is either $\lambda = 0$ or $\lambda = \lambda^u$. Just as King (1989) found when testing for seasonal autocorrelation in the presence of an AR(1) error process. we found that choosing λ_0 to be that value which results in equality in (10) at both endpoints, minimises (11) as required.

Thus for any given values of λ_1, ϕ_1 and η, an APOI test can be constructed by the following iterative process:

(i) Guess a possible value for λ_0 denoted λ_0^*.

(ii) Solve

$$Pr[s(\lambda_0^*, \lambda_1, \phi_1) < c^* \quad | \quad u \sim N(0, I_n)] = \eta \tag{12}$$

for c^* and evaluate

$$Pr[s(\lambda_0^*, \lambda_1, \phi_1) < c^* \quad | \quad u \sim N(0, I_n + \lambda^u \Omega_0)]. \tag{13}$$

(iii) If (13) equals η then the desired λ_0 and c values have been found. If (13) is less (greater) than η, make λ_0^* smaller (larger) and repeat (ii) and (iii).

Note that numerical methods analogous to those outlined in King (1989) can be used to calculate the LHS of (12) and (13). The modifications of these methods required for the calculation of sizes and powers are outlined in the appendix. We will denote the test statistic based on this choice of λ_0 value by $s(\lambda_0^*, \lambda_1, \phi_1)$.

Finally, applications of the test require choices to be made for λ_1 and ϕ_1. A simple but instructive way to view these choices is that a particular power surface is being chosen in preference to other power surfaces. Hence in making our choices, we should address the issue of power. Because ϕ is the parameter of interest, the point-optimal test literature (see for example King (1987)) suggests that ϕ_1 should be chosen so that the power of the test at $\phi = \phi_1$ has some desired level. A difficulty in our application is that power at $\phi = \phi_1$ is a function of λ. An obvious solution is to choose ϕ_1 such that the level of power at $\phi = \phi_1$ and $\lambda = \lambda_1$ has a desired level. We denote the resultant test statistic by $s(\lambda_0^*, \lambda_1, \phi_1^*)$.

This then leaves the choice of λ_1 value. Our choice of λ_1 must be related to the scale of x_t. At present we only consider two possible choices for λ_1, namely

$$\lambda_1^u = \lambda^u/2$$

and

$$\lambda_1^b = \lambda^u/4.$$

These correspond to Evans and King's (1985) choices of values for λ_1 when testing constant coefficients against Hildreth-Houck random coefficients. It should be noted that their preferred choice is that of λ_1^a. Finally for the purposes of comparison we include a test with a purely arbitrary choice of λ_0, λ_1 and ϕ_1. This test chooses $\lambda_0^a = \lambda_1^a$. For this test the critical value is chosen to control the size at the bound at which size is a maximum.

4. EMPIRICAL POWER CALCULATIONS

In order to assess the small-sample properties of APOI tests, the sizes and powers of the $s(\lambda_0^a, \lambda_1^a, \phi_1)$, $s(\lambda_0^*, \lambda_1^a, \phi_1)$, $s(\lambda_0^*, \lambda_1^a, \phi_1^*)$ and $s(\lambda_0^*, \lambda_1^b, \phi_1^*)$ tests were calculated using the methodology outlined in the appendix. The model is the regression (1) with nonstochastic regressors in which the coefficient of one regressor is generated as (3) and (4) under H_0 and (5) and (4) under H_a. The Monte Carlo method was used to calculate sizes and powers of the likelihood ratio and Wald tests. Details of these tests may be found in Collins, Ledolter and Rayburn (1987). In the following discussion and tables the $s(\lambda_0^a, \lambda_1^a, \phi_1)$, $s(\lambda_0^*, \lambda_1^a, \phi_1)$, $s(\lambda_0^*, \lambda_1^a, \phi_1^*)$, $s(\lambda_0^*, \lambda_1^b, \phi_1^*)$, likelihood ratio and Wald tests are respectively denoted as s_1, s_2, s_3, s_4, lr and t For each APOI test, exact critical values that ensure that the probability of a type I error is less than or equal to five per cent over the range $0 \leq \lambda \leq \lambda^u$, were calculated. These critical values and their associated λ_0, λ_1 and ϕ_1 values are given in table 1. Powers of the six tests were calculated at q = 1.0, 2.0, 5.0, 10.0 and 25.0 and ϕ_1 = 0.0. 0.2, 0.5. 0.7 and 0.9 for the following regressor sets:

X1: n=31. A constant and a linear time trend. The time trend is the regressor with the varying coefficient.

X2: n=41. The first 41 observations of Durbin and Watson's (1951, p.159) consumption of spirits example. Log of annual UK income is the regressor with the varying coefficient.

X3: n=41. A constant, quarterly Australian household disposable income and private consumption expenditure series commencing 1959(4). The consumption series is lagged one quarter and is the regressor with the varying coefficient.

X4: n=60. A constant, quarterly Australian household disposable income and private consumption expenditure series commencing 1959(4) and a full complement of quarterly seasonal dummies. The consumption series is lagged one quarter and is the regressor with the varying coefficient.

X5: n=30. A constant, quarterly Australian private capital movements and quarterly Australian government capital movements commencing 1968(1). The latter is the regressor with the varying coefficient.

X6: n=60. A constant and the monthly continuously compounded return on the equally weighted market index commencing 1978(1) from Faff, Lee and Fry (1992). The market returns data is the regressor with the varying coefficient.

Calculated powers and sizes of the tests are given in tables 2, 3 and 4. These tables include a column of q values equal to 25.0, in order to assess the sizes for each APOI test outside the range of allowed λ values. For the tests which choose λ_0^*, the size is never greater than 0.054, suggesting the insensitivity of the tests to this range. On the other hand for the test with the arbitrarily chosen λ_0 the size in the case of $X5$ is 0.072 and in the case of $X6$ is 0.078. This may be viewed with concern. Of much greater concern is the unreliability of the critical values of the asymptotic tests. The lr test is typically undersized with maximum sizes of 0.016, 0.009, 0.007, 0.020, 0.018 and 0.154 for the $X1$ to $X6$ matrices respectively. On the other hand the Wald test is typically oversized with maximum respective sizes of 0.232, 0.055, 0.067, 0.153, 0.400 and 0.448 respectively. It is interesting to note that the largest size occurs for the largest sample size (n=60) and only a single non-constant regressor suggesting that asymptotic critical values for this test should be used with extreme caution.

The powers of all tests increase as either ϕ increases or λ (or equivalently q) increases, *ceteris paribus*. Calculated powers below the nominal size of 0.05 occurred occasionally for the s_1 test ($X5$ and $X6$ only) and frequently for the lr test when ϕ and λ are small. Clearly it is difficult to compare the asymptotic tests with the APOI tests because the former tests have sizes significantly different from 0.05. The best we can do is look for situations in which one test has lower size and higher power than the other tests. We see this for $X6$ and the lr test which has higher size and lower power than the s_2, s_3 and s_4 tests. We conclude from this that the latter tests have superior power relative to the lr test. For $X2$ and $X3$ the Wald test has higher size and slightly lower power than the s_2, s_3 and s_4 tests particularly for high λ values. We are left to conclude that when the asymptotic critical values are accurate, the Wald test has power nearly comparable with that of the s_2, s_3 and s_4 tests.

Our results indicate that the s_2 and s_3 tests have extremely similar powers. The only noticeable differences occur for $X5$ and favour the former test when $\phi \leq 0.5$ and the latter test when $\phi \geq 0.7$. This leads us to conclude that there is little to be gained from choosing ϕ_1 in an optimal manner. When comparing the s_3 and s_4 tests, one observes that the latter test performs better for low λ values and the former test has higher power for high values of λ. There also appears to be a slight tendency for relative powers to shift in favour of the s_4 test as ϕ increases. As a result, it is extremely difficult to choose between these two tests. We marginally prefer the s_3 test because of its better overall power when $\phi = 0.2$ and 0.5, although the results show a general insensitivity of the testing problem to the choice of λ_1 value.

The s_1 test is typically inferior to the other three APOI tests. It often has a power advantage when $q = 25.0$ (possibly because of its higher size for $q = 25.0$) but is distinctly less powerful when $q = 1.0, 2.0, 5.0$. Only for $X2$ is the s_1 test identical in power to the s_2 test. We therefore feel there is a case for choosing λ_0 optimally despite the extra computation involved. Accordingly we recommend the use of the s_2 test on the basis of its small-sample properties.

5. ROBUSTNESS TO NONNORMALITY

In this section, we investigate the effects of nonnormality on the six tests. The Monte Carlo method was used to simulate sizes and powers of the tests under selected nonnormal disturbance distributions. This was achieved by generating independent pseudo-random values from the same nonnormal distribution for ϵ_t in (1) and $a_t/\sqrt{\lambda}$ in (3) or (5) as required. Because the resultant disturbance term, v_t, is made up of two independent components, central limit theorem considerations suggest that the distribution of v_t will be closer to normality than the original distribution of ϵ_t and $a_t/\sqrt{\lambda}$.

The nonnormal pseudo-random variates were generated by the algorithm proposed by Ramberg and Schmeiser (1972, 1974) and used in the robustness studies of Evans (1992), Lee (1992) and Brooks (1993b). The algorithm is:

$$r(p) = \theta_1 + (p^{\theta_3} - (1-p)^{\theta_4})/\theta_2, \qquad 0 \le p \le 1$$

where $r(p)$ is the generated pseudo-random variate, p is a uniform pseudo-random variate, θ_1 is a location parameter, θ_2 is a scale parameter and θ_3 and θ_4 are shape parameters. Tables of θ values which allow for a wide variety of distributions are provided in Ramberg, Tadikamalla, Dudewicz and Mykytka (1979).

Following Lee (1992) and Brooks (1993b) the nonnormal distributions considered are: (i) the distribution with no skewness and kurtosis of six, θ=(0, -0.1686, -0.0802, -0.0802)'; (ii) the distribution with no skewness and kurtosis of nine, θ=(0, -0.3203, -0.1359, -0.1359)'; and (iii) the distribution with skewness of one and kurtosis of six, θ=(-0.379, -0.0562, -0.0187, -0.0388)'. As the tests' sizes and powers are invariant to the values of β, $\bar{\alpha}$ and σ^2, these were set equal to 0, 0 and 1, respectively. One thousand replications were used.

Selected results of the Monte Carlo experiment are presented in table 5 for sizes and table 6 for powers. The following discussion is based on all the results. The results indicate that the APOI tests and our recommended test in particular are remarkably robust to nonnormality under the null hypothesis. This is similar to Evans' (1992) finding with respect to point-optimal tests for autocorrelation and in mild contrast to the conclusions of Evans (1992) and Brooks (1993b) regarding point-optimal tests for heteroscedasticity. This may be a reflection of the fact that our testing problem involves an autocorrelated error component. We also find that the power advantage of the s_2, s_3 and s_4 tests over the lr and Wald tests becomes greater as one moves from normality to nonnormality.

The effects of nonnormality on the power of our recommended test are also slight over all cases considered, with absolute differences in power rarely being greater than 0.02 and never greater than 0.05. In fact for the 144 cases where powers have been calculated there are only nine instances where the power under nonnormality differs from the power under normality by greater than 0.02. Overall the s_2 test appears to be very robust to nonnormality.

6. CONCLUSIONS

Our recommended test has been applied by Brooks, Faff and Lee (1992) to address the issue of the form of time variation of systematic risk in the Australian equity market. They find that for those cases where systematic risk is found to be time varying, application of the recommended test is unable to reject the Hildreth-Houck (1968) model as the appropriate model of time variation. This result is consistent with that of Bos and Newbold (1984) who attributed their finding to the use of an asymptotic test believed to lack power in finite samples. We find the lr test does lack power mostly because it is undersized. It is not clear however that the Wald test lacks power and indeed we found it was typically oversized. The use of our recommended test by Brooks, Faff and Lee (1992) does not have these deficiencies and is therefore a strong corroboration of the Bos and Newbold (1984) result. Such evidence is likely to be of assistance in portfolio selection.

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Table 1 - Parameter values and critical values for the four APOI tests.				
	λ_0	λ_1	ϕ_1	c
$X1$				
$s(\lambda_0^a, \lambda_1^a, \phi_1)$	0.52029	0.52029	0.50000	0.90510
$s(\lambda_0^*, \lambda_1^a, \phi_1)$	0.56076	0.52029	0.50000	0.94315
$s(\lambda_0^*, \lambda_1^a, \phi_1^*)$	0.54937	0.52029	0.54898	0.92998
$s(\lambda_0^*, \lambda_1^b, \phi_1^*)$	0.30085	0.26015	0.66131	0.93764
$X2$				
$s(\lambda_0^a, \lambda_1^a, \phi_1)$	1.22488	1.22488	0.50000	0.98225
$s(\lambda_0^*, \lambda_1^a, \phi_1)$	1.41531	1.22488	0.50000	1.10692
$s(\lambda_0^*, \lambda_1^a, \phi_1^*)$	1.53346	1.22488	0.35111	1.15869
$s(\lambda_0^*, \lambda_1^b, \phi_1^*)$	0.78581	0.61757	0.40761	1.12701
$X3$				
$s(\lambda_0^a, \lambda_1^a, \phi_1)$	0.25340	0.25340	0.50000	0.93752
$s(\lambda_0^*, \lambda_1^a, \phi_1)$	0.18259	0.25340	0.50000	0.76177
$s(\lambda_0^*, \lambda_1^a, \phi_1^*)$	0.20321	0.25340	0.40258	0.81345
$s(\lambda_0^*, \lambda_1^b, \phi_1^*)$	0.11520	0.12670	0.49217	0.87868
$X4$				
$s(\lambda_0^a, \lambda_1^a, \phi_1)$	0.07754	0.07754	0.50000	0.92009
$s(\lambda_0^*, \lambda_1^a, \phi_1)$	0.06799	0.07754	0.50000	0.86760
$s(\lambda_0^*, \lambda_1^a, \varphi_1^*)$	0.07236	0.07754	0.42002	0.90170
$s(\lambda_0^*, \lambda_1^b, \phi_1^*)$	0.03982	0.03877	0.53879	0.92465
$X5$				
$s(\lambda_0^a, \lambda_1^a, \phi_1)$	0.00052	0.00052	0.50000	0.87493
$s(\lambda_0^*, \lambda_1^a, \phi_1)$	0.00073	0.00052	0.50000	1.00251
$s(\lambda_0^*, \lambda_1^a, \phi_1^*)$	0.00084	0.00052	0.70909	0.99937
$s(\lambda_0^*, \lambda_1^b, \phi_1^*)$	0.00052	0.00026	0.81406	0.99980
$X6$				
$s(\lambda_0^a, \lambda_1^a, \phi_1)$	0.02243	0.02243	0.50000	0.90541
$s(\lambda_0^*, \lambda_1^a, \phi_1)$	0.02913	0.02243	0.50000	0.99778
$s(\lambda_0^*, \lambda_1^a, \phi_1^*)$	0.02993	0.02243	0.55057	0.99619
$s(\lambda_0^*, \lambda_1^b, \phi_1^*)$	0.01793	0.01122	0.66580	0.99632

Table 2 - Sizes and powers of the six tests for X1 and X2 under normality

q	1.0		2.0		5.0		10.0		25.0	
	X1	X2	X1	X2	X1	X2	X1	X2	X1	X2
					$\phi = 0.0$					
s_1	.036	.050	.038	.050	.044	.050	.050	.050	.056	.050
s_2	.046	.050	.045	.050	.047	.050	.050	.050	.053	.050
s_3	.046	.050	.045	.050	.047	.050	.050	.050	.054	.050
s_4	.046	.050	.046	.050	.048	.050	.050	.050	.052	.050
lr	.007	.003	.009	.001	.010	.006	.013	.007	.016	.009
t	.232	.055	.169	.053	.128	.054	.095	.051	.070	.055
					$\phi = 0.2$					
s_1	.061	.136	.083	.182	.129	.238	.167	.268	.204	.289
s_2	.074	.136	.095	.182	.135	.238	.166	.268	.196	.289
s_3	.074	.137	.094	.183	.135	.239	.166	.269	.196	.291
s_4	.077	.137	.097	.183	.133	.238	.159	.267	.182	.289
lr	.011	.027	.020	.039	.034	.056	.062	.066	.076	.075
t	.289	.140	.287	.182	.294	.226	.235	.256	.220	.268
					$\phi = 0.5$					
s_1	.145	.426	.248	.597	.426	.750	.540	.809	.632	.845
s_2	.168	.426	.269	.597	.437	.750	.541	.809	.624	.845
s_3	.167	.424	.268	.595	.436	.749	.541	.808	.624	.845
s_4	.180	.429	.280	.598	.430	.749	.518	.808	.586	.845
lr	.042	.174	.096	.312	.214	.485	.307	.567	.418	.628
t	.454	.424	.507	.585	.598	.719	.626	.773	.688	.802
					$\phi = 0.7$					
s_1	.274	.680	.452	.840	.678	.934	.787	.960	.857	.972
s_2	.302	.680	.475	.840	.688	.934	.789	.960	.854	.972
s_3	.302	.674	.476	.836	.689	.932	.790	.959	.855	.972
s_4	.328	.682	.494	.840	.684	.933	.771	.958	.829	.971
lr	.118	.407	.258	.624	.452	.805	.609	.862	.745	.894
t	.574	.676	.688	.826	.798	.925	.860	.952	.903	.963
					$\phi = 0.9$					
s_1	.476	.859	.674	.949	.854	.985	.919	.993	.954	.996
s_2	.505	.859	.694	.949	.861	.985	.921	.993	.954	.996
s_3	.507	.851	.696	.945	.862	.984	.922	.992	.955	.995
s_4	.539	.858	.713	.948	.860	.985	.913	.992	.944	.995
lr	.296	.674	.500	.858	.725	.953	.836	.969	.914	.981
t	.728	.856	.823	.937	.909	.984	.958	.991	.974	.995

Table 3 - Sizes and powers of the six tests for X3 and X4 under normality

q	1.0		2.0		5.0		10.0		25.0	
	X3	X4	X3	X4	X3	X4	X3	X4	X3	X4
					$\phi = 0.0$					
s_1	.045	.044	.044	.041	.042	.039	.042	.039	.042	.040
s_2	.048	.047	.048	.046	.049	.048	.050	.050	.051	.053
s_3	.048	.047	.048	.046	.049	.048	.050	.050	.051	.053
s_4	.048	.047	.048	.046	.049	.048	.050	.050	.051	.053
lr	.005	.011	.002	.015	.004	.020	.006	.018	.007	.016
t	.063	.153	.067	.143	.060	.131	.058	.102	.061	.077
					$\phi = 0.2$					
s_1	.094	.077	.126	.102	.179	.157	.214	.206	.243	.258
s_2	.100	.083	.137	.113	.198	.181	.238	241	.272	.303
s_3	.100	.083	.137	.114	.198	.182	.239	.242	.273	.304
s_4	.101	.085	.138	.115	.195	.178	.232	.228	.263	.277
lr	.016	.016	.022	.040	.045	.070	.053	.101	.072	.151
t	.113	.219	.160	.273	.223	.306	.250	.328	.260	.373
					$\phi = 0.5$					
s_1	.271	.197	.432	.346	.638	.605	.736	.754	.802	.853
s_2	.283	.209	.451	.368	.660	.636	.758	.784	.822	.878
s_3	.282	.208	.449	.368	.659	.636	.758	.784	.822	.878
s_4	.292	.222	.457	.381	.656	.627	.748	.758	.808	.845
lr	.079	.071	.182	.188	.383	.405	.506	.586	.599	.760
t	.309	.363	.462	.517	.639	.733	.731	.831	.792	.924
					$\phi = 0.7$					
s_1	.501	.390	.713	.638	.883	.882	.936	.955	.963	.984
s_2	.514	.405	.726	.656	.893	.895	.943	.962	.967	.987
s_3	.509	.401	.723	.652	.891	.893	.942	.962	.967	.987
s_4	.531	.432	.735	.673	.891	.891	.939	.954	.963	.981
lr	.239	.201	.443	.424	.715	.731	.827	.900	.892	.968
t	.525	.525	.710	.738	.872	.924	.933	.970	.952	.988
					$\phi = 0.9$					
s_1	.751	.685	.896	.883	.972	.981	.988	.995	.994	.999
s_2	.759	.696	.902	.890	.974	.983	.989	.996	.995	.999
s_3	.753	.688	.899	.886	.973	.981	.989	.995	.995	.999
s_4	.774	.722	.907	.899	.974	.982	.988	.995	.994	.999
lr	.518	.455	.753	.748	.914	.940	.955	.981	.970	.996
t	.753	.737	.893	.904	.965	.977	.985	.995	.991	.999

Table 4 - Sizes and powers of the six tests for X5 and X6 under normality										
q	1.0		2.0		5.0		10.0		25.0	
	X5	X6	X5	X6	X5	X6	X5	X6	X5	X6
					$\phi = 0.0$					
s_1	.017	.013	.022	.018	.035	.032	.050	.050	.072	.078
s_2	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050
s_3	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050
s_4	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050
lr	.004	.077	.007	.122	.007	.154	.013	.066	.018	.022
t	.400	.448	.342	.375	.247	.262	.184	.200	.125	.152
					$\phi = 0.2$					
s_1	.023	.020	.035	.035	.072	.088	.117	.161	.182	.268
s_2	.063	.069	.073	.086	.095	.123	.115	.158	.137	.199
s_3	.062	.069	.074	.086	.093	.122	.112	.157	.133	.198
s_4	.062	.069	.070	.085	.090	.118	.106	.148	.123	.180
lr	.009	.093	.009	.141	.026	.180	.037	.120	.059	.111
t	.419	.511	.396	.466	.334	.423	.303	.419	.294	.419
					$\phi = 0.5$					
s_1	.045	.053	.091	.129	.222	.361	.359	.581	.515	.774
s_2	.103	.138	.153	.227	.258	.420	.351	.573	.448	.710
s_3	.102	.137	.150	.227	.255	.420	.346	.573	.442	.710
s_4	.104	.142	.151	.231	.244	.409	.321	.544	.397	.662
lr	.016	.144	.042	.247	.105	.335	.185	.454	.323	.622
t	.489	.620	.500	.653	.549	.753	.604	.824	.667	.884
					$\phi = 0.7$					
s_1	.098	.147	.208	.350	.444	.709	.623	.881	.774	.963
s_2	.177	.275	.286	.472	.481	.751	.615	.879	.728	.949
s_3	.179	.277	.291	.475	.487	.754	.620	.881	.731	.949
s_4	.189	.296	.298	.493	.476	.750	.591	.866	.687	.932
lr	.049	.243	.115	.451	.274	.669	.439	.804	.616	.926
t	.557	.726	.608	.822	.712	.923	.791	.958	.866	.986
					$\phi = 0.9$					
s_1	.278	.522	.475	.783	.728	.955	.852	.989	.930	.998
s_2	.371	.644	.548	.843	.754	.964	.351	.989	.915	.997
s_3	.389	.652	.567	.848	.768	.966	.861	.990	.921	.997
s_4	.417	.690	.585	.866	.765	.967	.848	.989	.902	.996
lr	.169	.590	.332	.785	.571	.931	.715	.976	.864	.995
t	.658	.892	.765	.950	.852	.984	.922	.988	.963	.999

Table 5 - Size performance of the six tests for X2, X3 and X6 under nonnormality

	Kurtosis = 6			Kurtosis = 9			Skewness = 1, Kurtosis = 6		
	X2	X3	X6	X2	X3	X6	X2	X3	X6
					$q = 0.0$				
s_1	.053	.049	.011	.048	.046	.012	.058	.055	.013
s_2	.053	.045	.049	.049	.046	.049	.059	.050	.055
s_3	.054	.046	.050	.051	.047	.050	.062	.053	.054
s_4	.053	.043	.040	.049	.043	.044	.057	.048	.052
lr	.008	.006	.042	.007	.006	.053	.009	.007	.046
t	.057	.044	.584	.052	.040	.576	.049	.052	.575
					$q = 2.0$				
s_1	.039	.034	.021	.039	.031	.025	.041	.033	.023
s_2	.039	.046	.057	.037	.042	.058	.041	.036	.065
s_3	.040	.044	.056	.041	.040	.058	.041	.040	.063
s_4	.043	.042	.051	.041	.038	.049	.040	.035	.062
lr	.002	.003	.106	.003	.003	.098	.004	.004	.112
t	.047	.053	.391	.047	.048	.395	.049	.060	.373
					$q = 5.0$				
s_1	.036	.030	.034	.035	.031	.034	.042	.030	.039
s_2	.036	.037	.053	.035	.035	.050	.042	.037	.053
s_3	.037	.039	.052	.036	.036	.053	.044	.036	.054
s_4	.040	.039	.046	.040	.036	.047	.045	.035	.059
lr	.004	.003	.119	.002	.003	.100	.006	.004	.113
t	.053	.057	.268	.050	.054	.280	.049	.054	.268
					$q = 10.0$				
s_1	.044	.030	.051	.040	.030	.052	.047	.034	.057
s_2	.044	.035	.056	.040	.036	.055	.048	.038	.050
s_3	.045	.037	.057	.041	.037	.055	.048	.040	.051
s_4	.046	.037	.051	.043	.037	.051	.049	.041	.056
lr	.008	.005	.089	.005	.005	.079	.006	.004	.085
t	.047	.055	.210	.046	.054	.212	.047	.049	.211

Table 6 - Power performance of the six tests for X2, X3 and X6 under nonnormality									
	Kurtosis = 6			Kurtosis = 9			Skewness = 1, Kurtosis = 6		
	X2	X3	X6	X2	X3	X6	X2	X3	X6
				$q = 1.0, \phi = 0.2$					
s_1	.128	.094	.027	.129	.093	.028	.133	.086	.030
s_2	.129	.095	.081	.128	.100	.082	.132	.092	.086
s_3	.128	.100	.081	.124	.100	.084	.137	.096	.086
s_4	.131	.092	.083	.129	.094	.084	.128	.092	.085
lr	.029	.014	.091	.026	.012	.089	.021	.009	.106
t	.127	.107	.514	.123	.098	.507	.132	.114	.491
				$q = 10.0, \phi = 0.9$					
s_1	.993	.989	.983	.992	.990	.983	.991	.985	.984
s_2	.993	.993	.983	.992	.993	.981	.991	.988	.985
s_3	.993	.992	.985	.992	.993	.981	.990	.988	.986
s_4	.993	.994	.987	.993	.994	.983	.990	.988	.986
lr	.972	.949	.973	.971	.950	.969	.967	.949	.973
t	.990	.983	.998	.991	.981	.995	.986	.979	.999
				$q = 2.0, \phi = 0.7$					
s_1	.835	.711	.348	.831	.713	.351	.831	.708	.343
s_2	.836	.721	.482	.832	.716	.489	.830	.722	.483
s_3	.827	.716	.483	.825	.714	.491	.826	.717	.486
s_4	.831	.728	.500	.827	.723	.505	.832	.733	.499
lr	.627	.445	.391	.611	.437	.389	.624	.423	.402
t	.805	.675	.812	.799	.673	.802	.809	.673	.800
				$q = 5.0, \phi = 0.5$					
s_1	.761	.648	.364	.766	.641	.366	.765	.662	.353
s_2	.761	.668	.420	.766	.660	.417	.766	.681	.419
s_3	.758	.664	.418	.759	.662	.418	.767	.679	.420
s_4	.761	.663	.406	.761	.653	.403	.768	.669	.399
lr	.469	.369	.345	.469	.364	.335	.476	.356	.340
t	.707	.608	.756	.705	.605	.752	.714	.617	.742

APPENDIX

Critical values, sizes and powers of the APOI tests require calculation of probablilities of the form,

$$Pr[s(\lambda_0^*, \lambda_1, \phi_1) < c^* \quad | \quad u \sim N(0, \Sigma)]$$

$$= Pr[\Sigma_{i=1}^m \varphi_i \xi_i^2 < 0], \tag{14}$$

in which, ξ_i^2 are independent χ_1^2 random variables and φ_i are the non-zero eigenvalues of,

$$(\Sigma^{1/2})'(\Delta^{-1} - \Delta^{-1}X(X'\Delta^{-1}X)^{-1}X'\Delta^{-1} - c^*(\Theta^{-1} - \Theta^{-1}X(X'\Theta^{-1}X)^{-1}X'\Theta^{-1}))\Sigma^{1/2},$$

where $\Delta = (I_n + \lambda_1\Omega(\phi_1))$ and $\Theta = (I_n + \lambda_0^*\Omega_0)$. The RHS of (14) can be calculated using Koerts and Abrahamse's (1969) FQUAD subroutine or Davies (1980) algorithm. Calculation of critical values require c^* and λ_0^* to be found that set (14) equal to the desired nominal size at $\Sigma = I_n$ and $\Sigma = I_n + \lambda_0^u\Omega_0$. This can be done iteratively as outlined in section 3. Sizes and powers can be calculated from (14) by setting $\Sigma = I_n + \lambda_0\Omega_0$ and $\Sigma = I_n + \lambda_1\Omega(\phi_1)$, respectively.