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MARGINAL LIKELIHOOD SCORE-BASED TESTS OF REGRESSION DISTURBANCES IN THE PRESENCE OF NUISANCE PARAMETERS

by

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ABSTRACT

This paper is concerned with tests of the covariance matrix of the disturbances in the linear regression model that involve nuisance parameters which cannot be eliminated by usual invariance arguments. Score-based tests, namely Lagrange multiplier (LM) and locally most mean powerful (LMMP) tests are derived from the marginal likelihood. Applications considered include (i) testing for random regression coefficients; (ii) testing for second-order autoregressive (AR(2)) disturbances in the presence of AR(1) disturbances; and (iii) testing for ARMA(1,1) disturbances; each in the presence of AR(1) disturbances. An empirical size and power comparison shows that typically the new tests have more accurate asymptotic critical values and slightly more power than their respective conventional counterparts.

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1. Introduction

There is a large literature on testing the form of the covariance matrix of the disturbance term in the linear regression model. Reviews of various aspects of this literature have been written by Godfrey (1988), Judge et al. (1985), King (1987a, 1987b), Pagan and Hall (1983) and Pagan (1984), among others. The vast majority of this literature is concerned with testing the null hypothesis of white noise regression disturbances. However, it is reasonably unlikely that a given application of the linear regression model to economic data should have white noise disturbances. In fact, the large literature on testing regression disturbances acknowledges this very point. Therefore, it is highly desirable to be able to perform diagnostic tests of the regression disturbances in the presence of autocorrelation and/or heteroscedasticity.

An obvious candidate for such a role is the Lagrange multiplier (LM) test. Unfortunately, there is some doubt about the accuracy of the asymptotic critical values of this test, particularly when testing for heteroscedasticity. For example, Monte Carlo experiments reported by Breusch and Pagan (1979), Godfrey (1978), Griffiths and Surekha (1986) and Honda (1988) show that the LM test for heteroscedasticity rejects the null hypothesis less frequently than it should. Also, Moulton and Randolph (1989) report a similar problem with the LM test for error components in regressions with up to 506 observations.

An alternative approach to constructing LM tests of the covariance matrix of regression disturbances has been advocated by Ara and King (1993). Their approach involves treating a maximal invariant statistic as the observed data and using the density of the maximal invariant as the likelihood function. They showed that this is also equivalent to basing the inference on the marginal likelihood function. Use of the marginal likelihood function was first suggested by Kalbfleisch and Sprott (1970). Others to have proposed its use, particularly in the context of estimation, include Levenbach (1972, 1973), Patterson and Thompson (1975), Cooper and Thompson (1977) and Tunnicliffe Wilson (1989). The main theme of this literature is that the use of the marginal likelihood helps reduce bias. Results reported by Corduas (1986) and Ara and King (1993) show that the likelihood ratio and LM tests based on marginal likelihood functions are clearly more accurate than their conventional counterparts.

When more than one parameter is being tested, the LM test is a two-sided procedure. Natural one-sided testing problems often arise through economic theory and functional considerations such as variances being positive. Recently, King and Wu (1993) suggested an alternative form of the LM test for such problems. Their test statistic is based on the sum of scores. In the absence of nuisance parameters, this test is locally most mean powerful (LMMP) as it maximizes the mean slope of the power hypersurface in the neighbourhood of the null hypothesis.

In this paper we consider the construction of LM and asymptotic LMMP (ALMMP) tests using the marginal likelihood function. We extend the work of Ara and King (1993) by dealing with testing problems involving nuisance parameters which cannot be eliminated by the use of invariance arguments or marginal likelihood methods. There are two reasons for expecting this approach to be superior to that based on the classical likelihood function. The first is the direct elimination of certain nuisance parameters which, from the evidence reported by Ara and King (1993), can be expected to improve the accuracy of asymptotic critical values. The second potential improvement comes from the use of maximum marginal likelihood estimates of those parameters which cannot be directly eliminated. Such estimates are expected to be less biased than their classical counterparts and as we shall see, this has the potential to improve power. Applications include (i) testing for random regression coefficients in the presence of first-order autoregressive (AR(1)) disturbances; (ii) testing for AR(2) disturbances in the presence of AR(1) disturbances; and (iii) testing for ARMA(1,1) disturbances in the presence of AR(1) disturbances.

The plan of the paper is as follows. Section 2 considers the theory of constructing marginal-likelihood-based LM and ALMMP tests in the context of testing regression disturbances. The three applications are discussed in section 3. Section 4 reports a Monte Carlo experiment which compares the small-sample sizes and powers of the conventional and marginal-likelihood-based LM and ALMMP tests of Hildreth-Houck random coefficients in the presence of AR(1) regression disturbances. Some concluding remarks are made in the final section.

2. Theory

Consider the linear regression model with non-spherical disturbances

$$y = X\beta + u ; \qquad u \sim N(0,\sigma^2\Omega(\theta)) , \tag{1}$$

where y is $n\times1$, X is $n\times k$, nonstochastic and of rank $k < n$, and $\Omega(\theta)$ is a symmetric $n\times n$ matrix that is positive definite for θ ($p\times1$) in a subset of R^p which is of interest. The vectors β and θ and the scalar σ^2 are unknown. Suppose θ is partitioned as

$$\theta' = (\omega',\mu')$$

where $\omega' = (\theta_1,\ldots,\theta_q)$ and $\mu' = (\theta_{q+1},\ldots,\theta_p)$ are $q\times1$ and $(p-q)\times1$, respectively.

We are interested in testing $H_0 : \omega = 0$ against either

$$H_a : \omega \neq 0$$

or

$$H_a^+ : \omega > 0$$

where, in this context, > denotes ≥ for each component with a strict inequality for at least one component. This testing problem is invariant with respect to transformations of the form

$$y \longrightarrow \eta_0 y + X\eta \tag{2}$$

where η_0 is a positive scalar and η is a k×1 vector.

Let $m = n-k$, $M = I_n - X(X'X)^{-1}X'$, $z = My$ be the ordinary least squares (OLS) residual vector from (1) and P be an m×n matrix such that $PP' = I_m$ and $P'P = M$. The m×1 vector

$$v = Pz \,/\, (z'P'Pz)^{1/2}$$

is a maximal invariant under the group of transformations given by (2). Its probability density function is

$$f(v;\theta)dv = \tfrac{1}{2}\Gamma(m/2)\pi^{-m/2}|P\Omega(\theta)P'|^{-1/2}a^{-m/2}dv \tag{3}$$

where

$$\begin{aligned} a &= v'(P\Omega(\theta)P')^{-1}v \\ &= \hat{u}'\Omega^{-1}(\theta)\hat{u}/z'z , \end{aligned}$$

$\hat{u}$ is the generalized least squares (GLS) residual vector assuming covariance matrix $\sigma^2\Omega(\theta)$ and dv denotes the uniform measure on the surface of the unit m-sphere.

We wish to consider only invariant tests. The principle of invariance implies that we can treat v as the observed random vector and (3) as its density function. As Ara and King (1993) point out, we can therefore treat (3) as a likelihood function for θ and derive standard tests such as the likelihood ratio, Wald and LM tests. Ara and King also observed that this approach is equivalent to using the marginal likelihood for θ because (3) is directly proportional to the marginal likelihood for θ given by Tunnicliffe Wilson (1989). Thus by invariance or, equivalently, by the adoption of marginal likelihood methods, we have reduced the testing problem to one only involving θ. The subvector of θ, μ, is a (p-q)×1 vector of nuisance parameters.

The log likelihood function implied by (3) is

$$L(\theta) = \text{constant} - \frac{1}{2}\log\left|P\Omega(\theta)P'\right| - \frac{m}{2}\log\left[v'(P\Omega(\theta)P')^{-1}v\right] .$$

Ara and King (1993) have shown that the scores can be written as

$$\frac{\partial L(\theta)}{\partial \theta_i} = -\frac{m}{2}\left[\hat{u}'\,\frac{\partial \Omega^{-1}(\theta)}{\partial \theta_i}\,\hat{u} \Big/ \hat{u}'\Omega^{-1}(\theta)\hat{u}\right] - \frac{1}{2}\,\mathrm{tr}\left[\Delta(\theta)\,\frac{\partial \Omega(\theta)}{\partial \theta_i}\right]$$

and that the i,j^{th} element of the information matrix of L(θ) is

$$E\left\{-\frac{\partial^2 L(\theta)}{\partial \theta_i \partial \theta_j}\right\} = m\,\mathrm{tr}\left[\Delta(\theta)\,\frac{\partial \Omega(\theta)}{\partial \theta_i}\,\Delta(\theta)\,\frac{\partial \Omega(\theta)}{\partial \theta_j}\right] \Big/ (2m+4)$$

$$- \left\{\mathrm{tr}\left[\Delta(\theta)\,\frac{\partial \Omega(\theta)}{\partial \theta_i}\right]\mathrm{tr}\left[\Delta(\theta)\,\frac{\partial \Omega(\theta)}{\partial \theta_j}\right]\right\} \Big/ (2m+4)$$

where $\hat{u}$ is the GLS residual vector assuming covariance matrix Ω(θ) and

$$\Delta(\theta) = \Omega^{-1}(\theta) - \Omega^{-1}(\theta)X(X'\Omega^{-1}(\theta)X)^{-1}X'\Omega^{-1}(\theta)$$

$$= P'(P\Omega(\theta)P')^{-1}P .$$

In order to construct the LM test of H_0 against H_a based on (3), we need the scores with respect to $\omega = (\theta_1,\ldots,\theta_q)'$ and the information matrix evaluated under H_0. Let $\hat{\mu}$ denote the values which maximize the marginal likelihood (3) when $\omega = 0$; i.e., under H_0, and let $\hat{\theta} = (0',\hat{\mu}')'$. Define

$$A_i(\hat{\mu}) = \left.\frac{\partial\Omega(\theta)}{\partial\theta_i}\right|_{\theta = \hat{\theta}}$$

and note that

$$\left.\frac{\partial\Omega^{-1}(\theta)}{\partial\theta_i}\right|_{\theta = \hat{\theta}} = -\Omega^{-1}(\hat{\theta})A_i(\hat{\mu})\Omega^{-1}(\hat{\theta}) .$$

Let $\hat{s}$ denote the q×1 vector of scores with respect to the elements of ω evaluated at $\omega = 0$ and $\mu = \hat{\mu}$. Thus

$$\hat{s}_i = \frac{m}{2}\left[\hat{u}'\Omega^{-1}(\hat{\theta})A_i(\hat{\mu})\Omega^{-1}(\hat{\theta})\hat{u}/\hat{u}'\Omega^{-1}(\hat{\theta})\hat{u}\right] - tr\left[\Delta(\hat{\theta})A_i(\hat{\mu})\right]/2$$

$$= \frac{m}{2}\left[e'\bar{A}_i(\hat{\mu})e/e'e\right] - tr\left[M_*\bar{A}_i(\hat{\mu})\right]/2$$

where

$$M_* = I_n - X_*(X_*'X_*)^{-1}X_*' ,$$

$$X_* = \Omega(\hat{\theta})^{-1/2}X,$$

$$\bar{A}_i(\hat{\mu}) = \Omega(\hat{\theta})^{-1/2}A_i(\hat{\mu})\{\Omega(\hat{\theta})^{-1/2}\}',$$

$$e = \Omega(\hat{\theta})^{-1/2}\,\hat{u} .$$

is the OLS residual vector from the regression

$$\Omega(\hat{\theta})^{-1/2}y = \Omega(\hat{\theta})^{-1/2} X\beta + \Omega(\hat{\theta})^{-1/2} u, \tag{4}$$

i.e., (1) after transformation by premultiplication by $\Omega(\hat{\theta})^{-1/2}$, and $\hat{u}$ is the GLS residual vector assuming covariance matrix $\Omega(\hat{\theta})$. Let $\hat{I}$ denote the p×p information matrix evaluated at $\omega = 0$ and $\mu = \hat{\mu}$. Its i,j^{th} element is given by

$$\begin{aligned}\hat{I}(i,j) &= m \,\mathrm{tr}\Big[\Delta(\hat{\theta})A_i(\hat{\mu})\Delta(\hat{\theta})A_j(\hat{\mu})\Big] \;/\; (2m + 4) \\ &\quad - \Big\{\mathrm{tr}\Big[\Delta(\hat{\theta})A_i(\hat{\mu})\Big]\; \mathrm{tr}\Big[\Delta(\hat{\theta})A_j(\hat{\mu})\Big]\Big\} \;/\; (2m + 4) \\ &= m \,\mathrm{tr}\Big[M_*\bar{A}_i(\hat{\mu})M_*\bar{A}_j(\hat{\mu})\Big] \;/\; (2m + 4) \\ &\quad - \Big\{\mathrm{tr}\Big[M_*\bar{A}_i(\hat{\mu})\Big]\mathrm{tr}\Big[M_*\bar{A}_j(\hat{\mu})\Big]\Big\} \;/\; (2m + 4).\end{aligned}$$

Partition $\hat{I}$ as

$$\hat{I} = \begin{bmatrix}\hat{I}_{11} & \hat{I}_{12} \\ \hat{I}_{21} & \hat{I}_{22}\end{bmatrix}$$

where $\hat{I}_{11}$ and $\hat{I}_{22}$ are q×q and (p − q)×(p − q), respectively.

The LM test of H_0 against H_a: $\omega \neq 0$ rejects H_0 for large values of

$$r = \hat{s}'\Big[\hat{I}_{11} - \hat{I}_{12}\,\hat{I}_{22}^{-1}\,\hat{I}_{21}\Big]^{-1}\hat{s} \tag{5}$$

assuming an asymptotic chi-squared distribution with q degrees of freedom under H_0. If the information matrix evaluated at $\theta = \hat{\theta}$ is block diagonal, i.e., $\hat{I}_{12} = \hat{I}_{21} = 0$, then (5) simplifies to

$$r = \hat{s}'\hat{I}_{11}^{-1}\,\hat{s}.$$

In the special case of q = 1, (5) becomes

$$r = \left\{\frac{m}{2}\, e'\bar{A}_1(\hat{\mu})e/e'e - \mathrm{tr}\left[M_*\bar{A}_1(\hat{\mu})\right] / 2\right\}^2 / \left[\ \hat{I}_{11} - \hat{I}_{12}\,\hat{I}_{22}^{-1}\,\hat{I}_{21}\right]$$

and a one-sided LM test of H_0 against H_a^+ : $\omega > 0$ involves rejecting H_0 for large values of

$$\left\{\frac{m}{2}\, e'\bar{A}_1(\hat{\mu})e/e'e - \mathrm{tr}\left[M_*\bar{A}_1(\hat{\mu})\right] / 2\right\} / \left[\ \hat{I}_{11} - \hat{I}_{12}\,\hat{I}_{22}^{-1}\,\hat{I}_{21}\right]^{1/2} \qquad (6)$$

which has an asymptotic standard normal distribution under H_0.

If as well as q = 1, the information matrix is block diagonal, the one-sided LM test statistic (6) is of the form

$$\left\{\frac{m}{2}\, e'\bar{A}_1(\hat{\mu})e/e'e - \mathrm{tr}\left[M_*\bar{A}_1(\hat{\mu})\right]/2\right\}/$$

$$\left\{m\ \mathrm{tr}\left[M_*\bar{A}_1(\hat{\mu})\right]^2 / (2m + 4) - \left(\mathrm{tr}\left[M_*\bar{A}_1(\hat{\mu})\right]\right)^2 / (2m + 4)\right\}^{1/2} . \qquad (7)$$

It is noteworthy that this test statistic is of the form of King and Hillier's (1985) LBI test applied to (4) assuming $\mu = \hat{\mu}$ and using the standard two-moment normal approximation (see Evans and King, 1985) to obtain critical values.

In the case of testing H_0 against H_a^+ when q > 1, we can construct an ALMMP test by applying the results of King and Wu (1993) to the density function of v given by (3). This test rejects H_0 for large values of

$$\tau = \sum_{i=1}^{q} \hat{s}_i \,/ \left\{\ell'\hat{I}_{11}\ell - \ell'\hat{I}_{12}\hat{I}_{22}^{-1}\,\hat{I}_{21}\ell\right\}^{1/2}$$

$$= 1/2\left\{m\ e'\bar{A}(\hat{\mu})e/e'e - \mathrm{tr}\left[M_*\bar{A}(\hat{\mu})\right]\right\} / \left\{\ell'\hat{I}_{11}\ell - \ell'\hat{I}_{12}\hat{I}_{22}^{-1}\hat{I}_{21}\ell\right\}^{1/2} \qquad (8)$$

where $\bar{A}(\hat{\mu}) = \sum_{i=1}^{q} \bar{A}_i(\hat{\mu})$ and ℓ is the $q \times 1$ vector of ones. This statistic has a standard normal asymptotic distribution under H_0. If the information matrix evaluated at $\theta = \hat{\theta}$ is block diagonal, then the denominator of (8) becomes the square root of

$$\sum_{i=1}^{q} \sum_{j=1}^{q} m \, \mathrm{tr}\left[M_* \bar{A}_i(\hat{\mu}) M_* \bar{A}_j(\hat{\mu})\right] / (2m + 4)$$

$$- \left\{ \mathrm{tr}\left[M_* \bar{A}_i(\hat{\mu})\right] \; \mathrm{tr}\left[M_* \bar{A}_j(\hat{\mu})\right] \right\} / (2m + 4)$$

$$= \left\{ m \, \mathrm{tr}\left[M_* \bar{A}(\hat{\mu})\right]^2 - \left(\mathrm{tr}\left[M_* \bar{A}(\hat{\mu})\right] \right)^2 \right\} / (2m + 4) .$$

In other words, τ is of the form of (7) with

$$\bar{A}_1(\hat{\mu}) = \bar{A}(\hat{\mu}).$$

3. Applications

This section demonstrates the application of the above theory to (i) testing for Hildreth-Houck (1968) random regression coefficients in the presence of AR(1) disturbances, (ii) testing for AR(2) disturbances in the presence of AR(1) disturbances, and (iii) testing for ARMA(1,1) disturbances in the presence of AR(1) disturbances.

3.1 Testing for Hildreth-Houck random coefficients in the presence of AR(1) disturbances

Assuming the first column of X is a column of ones, write (1) as

$$y_t = \beta_1 + \sum_{j=2}^{k} \beta_{tj} x_{tj} + u_t, \qquad t = 1,\ldots,n. \qquad (9)$$

The Hildreth-Houck model assumes the regression coefficients β_{tj}, $j = 2,\ldots,k$, at time t are generated as

$$\beta_{tj} = \bar{\beta}_j + v_{tj}, \qquad t = 1,\ldots,n, \qquad (10)$$

where $v_{tj} \sim IN(0,\sigma_j^2)$, $j = 2,\ldots,k$. The disturbance term u_t is assumed to be generated by the stationary AR(1) process

$$u_t = \rho u_{t-1} + \varepsilon_t, \qquad |\rho| < 1$$

in which $\varepsilon_t \sim IN(0,\sigma_\varepsilon^2)$ and $var(u_t) = \sigma_u^2 = \sigma_\varepsilon^2/(1-\rho^2)$. Substituting (10) into (9), the model can be written as

$$y_t = \beta_1 + \sum_{j=2}^{k} \bar{\beta}_j x_{tj} + w_t, \qquad t = 1,\ldots,n,$$

where

$$w_t = u_t + \sum_{j=2}^{k} x_{tj} v_{tj} .$$

Assuming mutual independence between ε_t and v_{tj}, $j = 2,\ldots,k$, the covariance matrix of $w = (w_1,\ldots,w_n)'$ can be written as

$$Var(w) = \sigma_u^2 \begin{bmatrix} (1+\sum\lambda_j x_{1j}^2) & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & (1+\sum\lambda_j x_{2j}^2) & \rho & & \vdots \\ \rho^2 & \rho & (1+\sum\lambda_j x_{3j}^2) & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ \rho^{n-1} & \cdots & \cdots & \cdots & (1+\sum\lambda_j x_{nj}^2) \end{bmatrix}$$

$$= \sigma_u^2 \Omega(\theta)$$

where $\lambda_j = \sigma_j^2/\sigma_u^2$, $j = 2,\ldots,k$, $\theta' = (\lambda_2,\lambda_3,\ldots,\lambda_k,\rho)$ and summation is from $j = 2$ to k. Let $\lambda' = (\lambda_2,\ldots,\lambda_k)$. Our testing problem is one of testing

$$H_0\colon \lambda = 0$$

against

$$H_a^+\colon \lambda > 0$$

in the context of (1).

In order to construct the marginal-likelihood-based LM and ALMMP test statistics, r and τ, respectively, note that $p = k$, $q = k-1$,

$$\Omega(\hat{\theta})^{-1/2} = (1-\hat{\rho}^2)^{-1/2} \begin{bmatrix} (1-\hat{\rho}^2)^{1/2} & 0 & 0 & \cdots & 0 \\ -\hat{\rho} & 1 & 0 & & 0 \\ 0 & -\hat{\rho} & 1 & & \vdots \\ \vdots & & & \ddots & \vdots \\ & & & 1 & 0 \\ 0 & 0 & \cdots & -\hat{\rho} & 1 \end{bmatrix}, \qquad (11)$$

$$\frac{\partial\Omega(\theta)}{\partial\lambda_j} = \mathrm{diag}(x_{1j}^2,\ldots,x_{nj}^2),$$

and

$$A_k(\hat{\rho}) = \begin{bmatrix} 0 & 1 & 2\hat{\rho} & \cdots & (n-1)\hat{\rho}^{n-2} \\ 1 & 0 & 1 & & \vdots \\ 2\hat{\rho} & 1 & 0 & \ddots & \vdots \\ \vdots & & \ddots & & \vdots \\ & & & 0 & 1 \\ (n-1)\hat{\rho}^{n-2} & \cdots & \cdots & 1 & 0 \end{bmatrix} \qquad (12)$$

where $\hat{\rho}$ is the value of ρ which maximizes the log marginal likelihood function $L(\theta)$ with $\theta' = (0,0,\ldots 0,\rho)$. The residual vector e is $(1 - \hat{\rho}^2)^{-1/2}$ times the OLS residual vector from the transformed model

$$(1 - \hat{\rho}^2)^{1/2} y_1 = (1 - \hat{\rho}^2)^{1/2} x_1'\beta + (1 - \hat{\rho}^2)^{1/2} u_1 \tag{13}$$

$$y_t - \hat{\rho} y_{t-1} = (x_t - \hat{\rho} x_{t-1})'\beta + u_t - \hat{\rho} u_{t-1}, \quad t = 2,\ldots,n, \tag{14}$$

where $x_t' = (1, x_{t2},\ldots,x_{tk})$. The n×n matrix $\bar{A}_{j-1}(\hat{\rho})$ for $j = 2,\ldots,k$ is tridiagonal and symmetric with main diagonal

$$\left(x_{1j}^2, (x_{2j}^2 + \hat{\rho}^2 x_{1j}^2)/(1 - \hat{\rho}^2), \ldots, (x_{nj}^2 + \hat{\rho}^2 x_{n-1j}^2)/(1 - \hat{\rho}^2)\right)$$

and main off-diagonal

$$\left(-\hat{\rho}(1 - \hat{\rho}^2)^{-1/2} x_{1j}^2, -\hat{\rho} x_{2j}^2/(1 - \hat{\rho}^2), \ldots, -\hat{\rho} x_{n-1j}^2/(1 - \hat{\rho}^2)\right).$$

The n×n matrix $\bar{A}(\hat{\rho})$ used in the ALMMP test has the same form. Its main diagonal is

$$\left(\sum x_{1j}^2, \sum(x_{2j}^2 + \hat{\rho}^2 x_{1j}^2)/(1 - \hat{\rho}^2), \ldots, \sum(x_{nj}^2 + \hat{\rho}^2 x_{n-1j}^2)/(1 - \hat{\rho}^2)\right)$$

and its main off-diagonal is

$$\left(-\hat{\rho}(1 - \hat{\rho}^2)^{-1/2} \sum x_{1j}^2, -\hat{\rho}\sum x_{2j}^2/(1 - \hat{\rho}^2), \ldots, -\hat{\rho}\sum x_{n-1j}^2/(1 - \hat{\rho}^2)\right),$$

where again summation is from j = 2 to k.

3.2 Testing for AR(2) disturbances in the presence of AR(1) disturbances

Consider the linear regression model (1) in which the disturbances are generated by the stationary AR(2) process

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$$

where $\varepsilon_t \sim IN(0,\sigma_\varepsilon^2)$. In this model

$$\mathrm{Var}(u) = \sigma_u^2 \begin{bmatrix} 1 & \gamma_1 & \gamma_2 & \cdots & \gamma_{n-1} \\ \gamma_1 & 1 & \gamma_1 & & \vdots \\ \gamma_2 & \gamma_1 & 1 & \ddots & \vdots \\ \vdots & & & \ddots & \vdots \\ \vdots & & & 1 & \gamma_1 \\ \gamma_{n-1} & \cdots & \cdots & \gamma_1 & 1 \end{bmatrix}$$

$$= \sigma_u^2 \Omega(\theta) \qquad (15)$$

where

$$\sigma_u^2 = \frac{(1-\rho_2)\sigma_\varepsilon^2}{(1+\rho_2)[(1-\rho_2)^2 - \rho_1^2]}$$

$$\gamma_1 = \rho_1/(1-\rho_2)$$

$$\gamma_2 = \rho_2 + \rho_1^2/(1-\rho_2)$$

$$\gamma_j = \rho_1\gamma_{j-1} + \rho_2\gamma_{j-2}, \qquad j = 3, \ldots, n-1,$$

and $\theta = (\rho_2, \rho_1)'$. In this case we are interested in testing

$$H_0 : \rho_2 = 0$$

against

$$H_a : \rho_2 \neq 0$$

in the context of (1). Because $q = 1$, our interest is only in constructing the LM test statistic (5).

Observe that $p = 2$. The matrix $\Omega(\hat{\theta})^{-1/2}$ is given by (11) and $A_2(\hat{\rho})$ is given by (12), where again $\hat{\rho}$ is the value of ρ_1 which maximizes the log marginal likelihood function $L(\theta)$ with $\theta' = (0,\rho_1)$. Also the residual vector e is identical to that for the previous application, i.e., $(1-\hat{\rho})^{-1/2}$ times the OLS residual vector from the transformed model (13) and (14). Now

$$A_1(\hat{\rho}) = \begin{bmatrix} 0 & \hat{\rho} & \hat{\rho}^2+1 & \hat{\rho}^3+2\hat{\rho} & \cdots & \hat{\rho}^{n-1}+(n-2)\hat{\rho}^{n-3} \\ \hat{\rho} & 0 & \hat{\rho} & \hat{\rho}^2+1 & & \vdots \\ \hat{\rho}^2+1 & \hat{\rho} & 0 & \hat{\rho} & & \vdots \\ \hat{\rho}^3+2\hat{\rho} & \hat{\rho}^2+1 & \hat{\rho} & 0 & & \vdots \\ \vdots & & & & \ddots & \vdots \\ \vdots & & & & 0 & \hat{\rho} \\ \hat{\rho}^{n-1}+(n-2)\hat{\rho}^{n-3} & \cdots & \cdots & \cdots & \hat{\rho} & 0 \end{bmatrix}$$

and

$$\bar{A}_1(\hat{\rho}) = I +$$

$$\frac{1}{(1-\hat{\rho}^2)}\begin{bmatrix} -(1-\hat{\rho}^2) & \hat{\rho}(1-\hat{\rho}^2)^{1/2} & (1-\hat{\rho}^2)^{1/2} & \hat{\rho}(1-\hat{\rho}^2)^{1/2} & \cdots & \hat{\rho}^{n-3}(1-\hat{\rho}^2)^{1/2} \\ \hat{\rho}(1-\hat{\rho}^2)^{1/2} & -(1+\hat{\rho}^2) & 0 & 1-\hat{\rho}^2 & & (\hat{\rho}^{n-4}-\hat{\rho}^{n-2}) \\ (1-\hat{\rho}^2)^{1/2} & 0 & -(1+\hat{\rho}^2) & 0 & & \vdots \\ \hat{\rho}(1-\hat{\rho}^2)^{1/2} & 1-\hat{\rho}^2 & 0 & -(1+\hat{\rho}^2) & \ddots & \vdots \\ \vdots & & & & -(1+\hat{\rho}^2) & 0 \\ \hat{\rho}^{n-3}(1-\hat{\rho}^2)^{1/2} & \hat{\rho}^{n-4}-\hat{\rho}^{n-2} & \cdots & \cdots & 0 & -(1+\hat{\rho}^2) \end{bmatrix}.$$

3.3 Testing for ARMA(1,1) disturbances in the presence of AR(1) disturbances

Here we consider the linear regression model (1) in which the disturbances are generated by the stationary ARMA(1,1) process

$$u_t = \rho u_{t-1} + \varepsilon_t + \gamma\varepsilon_{t-1}, \qquad t = 1, \ldots, n,$$

where $\varepsilon_t \sim IN(0,\sigma_\varepsilon^2)$. In this model, $Var(u) = \sigma_u^2\Omega(\theta)$ is given by (15) where

$$\sigma_u^2 = \frac{1 + 2\rho\gamma + \gamma^2}{1-\rho^2}\,\sigma_\varepsilon^2$$

$$\gamma_1 = \frac{(\rho+\gamma)(1+\rho\gamma)}{1 + 2\rho\gamma + \gamma^2}$$

$$\gamma_j = \rho\gamma_{j-1}, \qquad j = 2, \ldots, n-1$$

and $\theta = (\gamma,\rho)$. We are interested in testing

$$H_0 : \gamma = 0$$

against

$$H_a : \gamma \neq 0$$

in the context of (1).

Note that as for testing for AR(2) disturbances in the presence of AR(1) disturbances, p = 2, q = 1, the matrix $\Omega(\hat{\theta})^{-1/2}$ is given by (11), $A_2(\hat{\rho})$ is given by (12) and the residual vector e is $(1-\hat{\rho}^2)^{-1/2}$ times the OLS residual vector from the transformed model (13) and (14). Now,

$$A_1(\hat{\rho}) = (1-\hat{\rho})^2 \begin{bmatrix} 0 & 1 & \hat{\rho} & \cdot & \cdot & \cdot & \hat{\rho}^{n-2} \\ 1 & 0 & 1 & & & & \\ \hat{\rho} & 1 & 0 & \cdot & & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & & \cdot & \\ \cdot & & & & & 0 & 1 \\ \hat{\rho}^{n-2} & \cdot & \cdot & \cdot & \cdot & 1 & 0 \end{bmatrix}$$

and

$$\bar{A}_1(\hat{\rho}) = \frac{(1-\hat{\rho})}{(1+\hat{\rho})} \begin{bmatrix} 0 & (1-\hat{\rho}^2)^{1/2} & 0 & & 0 \\ (1-\hat{\rho}^2)^{1/2} & -2\hat{\rho} & 1 & & 0 \\ 0 & 1 & -2\hat{\rho} & & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & & & -2\hat{\rho} & 1 \\ 0 & 0 & \cdot\;\cdot\;\cdot & 1 & -2\hat{\rho} \end{bmatrix}.$$

Thus

$$e'\bar{A}_1(\hat{\rho})e = 2(1-\hat{\rho})\left\{(1-\hat{\rho}^2)^{1/2}e_1e_2 + \sum_{i=3}^{n} e_ie_{i-1} - \hat{\rho}\sum_{i=2}^{n} e_i^2\right\}\Big/(1+\hat{\rho}).$$

4. Monte Carlo Experiment

In order to explore the small-sample properties of the LM and ALMMP tests constructed using the marginal likelihood function, we conducted a Monte Carlo experiment. The experiment concentrated on the problem of testing for Hildreth-Houck random coefficients in the presence of AR(1) disturbances in (1) as outlined in section 3.1. Estimated sizes and powers of the two marginal-likelihood-based tests which we denote by MLM and MALMMP were compared with those of the corresponding LM and ALMMP tests constructed using the standard likelihood approach.

4.1 Experimental Design

The Monte Carlo method was used to estimate sizes and powers for the following $n \times 3$ X matrices with $n = 20$ and $n = 60$.

X1: A constant dummy plus two independent trending regressors generated as

$$x_{ti} = z_{ti} + 0.25t$$

where z_{ti}, $t = 1, \ldots, n$, $i = 2, 3$ are independent AR(1) time series generated from

$$z_{ti} = 0.5z_{t-1i} + \eta_{ti}$$

and

$$\eta_{ti} \sim IN(0,1), \quad t = 1, \ldots, n, \quad i = 2, 3.$$

X2: A constant plus quarterly Australian total private capital movements and Australian total Government capital movements.

X3: The first n observations of Durbin and Watson's (1951) consumption of spirits example.

X4: A constant dummy plus quarterly seasonally adjusted Australian household disposable income and private final consumption expenditure commencing 1959(4).

The regressors in X1 were constructed with an obvious time trend while those in X2 have a seasonal component and fluctuate widely. X3 is comprised of annual data while X2 and X4 use quarterly data. In all cases p = 3 and we test $H_0 : \lambda_2 = \lambda_3 = 0$ against $H_a^+ : (\lambda_2, \lambda_3)' > 0$.

The experiment was conducted in two parts. The first involved using asymptotic critical values for all tests. In the second part, the Monte Carlo method was used to estimate appropriate critical values which were then used to provide a more meaningful comparison of powers. Because the true size of each test varies with ρ, the required critical value must result in sizes less than or equal to the significance level. This value was estimated as follows. The Monte Carlo method was used to calculate appropriate critical values at each of $\rho = 0, 0.1, 0.2, \ldots, 0.9$ and, for each test, the largest value was used as the critical value. These critical values were then used to calculate the second round of sizes and powers. A nominal significance level of five percent and 2000 iterations were used throughout.

From

$$\text{var}(w_t) = \sigma_u^2\left(1 + \sum_{j=2}^{3} \lambda_j x_{tj}^2\right)$$

it is clear that the relative contribution at time t of the stochastic part of the j^{th} regression coefficient to the variance of the composite error term, w_t, is $\lambda_j x_{tj}^2$. Thus what constitutes a large value of λ_j depends on the magnitude of x_{tj}^2. We took the view that it was unlikely that β_{tj} would contribute more than 10 times the variance of u_t to the composite error term. We therefore set

$$\lambda_j = \bar{\lambda}_j \lambda_j^u$$

where

$$\lambda_j^u = 10 \Big/ \max_t \left(x_{tj}^2 \right) , \quad j = 2,3 ,$$

and calculated sizes and powers at all combinations of ρ = 0, 0.3, 0.6, 0.9 and $\bar{\lambda}_j$ = 0, 0.02, 0.2, 1; j = 2,3.

All the tests required estimates of ρ under the null hypothesis of AR(1) disturbances. For the tests based on the conventional likelihood function, ρ was estimated using Beach and MacKinnon's (1978) maximum likelihood algorithm. For the marginal likelihood based tests, ρ was estimated by maximizing the marginal likelihood under H_0 which following Tunnicliffe Wilson (1989) can be written as

$$f(\rho|u) = \left(1-\rho^2\right)^{1/2} \left| X^*(\rho)'X^*(\rho) \right|^{-1/2} \left(z^*(\rho)'z^*(\rho) \right)^{-m/2}$$

where $X^*(\rho)$ and $z^*(\rho)$ are the n×k X matrix and the OLS residual vector, respectively, from the transformed regression (13) and (14) in which ρ replaces $\hat{\rho}$. We used the IMSL (1989) nonlinear maximization subroutines DIVMIF, DLFTRG and DLFDRG to solve this optimization problem. The error variance σ_u^2 and the components of the parameter vector $\bar{\beta}$ were all set to unity in the simulations. Note that the sizes and powers of each of the four tests are invariant to the values of $\bar{\beta}$ and σ_u^2.

4.2 The Results

Selected estimated sizes based on asymptotic critical values at the nominal level of 5% are reported in Table 1.

The conventional LM test sizes are typically different from 0.05 especially when ρ is large. While for X1 and X4 there did seem to be a tendency for sizes to get closer to 0.05 as n increases, this was not evident for X2 and X3. In contrast, almost all estimated sizes of the MLM test were not significantly different (at the 1% level) from 0.05 and all exceptions occur for the smaller sample size of n = 20. Estimated sizes for the ALMMP test are almost all below 0.05, most being significantly so. All estimated sizes are particularly small when ρ is large. There is evidence of an improvement in these sizes as n increases. The estimated sizes of the MALMMP test are typically much closer to 0.05 than its conventional counterpart, especially when n = 60.

Selected estimated sizes and powers based on critical values found by simulation for X1, X2 and X4 are presented in tables 2, 3 and 4, respectively. Because almost all the powers for X3 are less than 0.1, we have omitted them to save space. The variation in the regressors of X3 is such that it is very difficult to detect even very large contributions to the composite error term from stochastic coefficients. Also omitted are the powers when $\bar{\lambda}_j = 0.02$, j = 2,3, which are typically very similar to those for $\bar{\lambda}_j = 0$, except when $\rho = 0.9$ and n = 60. The following discussion is an analysis of all the results for X1, X2 and X4.

Typically powers of all tests increase as n increases. Exceptions are minor and occur most frequently when $\rho = 0$ and/or $\bar{\lambda}_j = 0.02$. There is a very clear tendency for powers to increase as ρ increases, *ceteris paribus*, particularly when n = 60. As expected, powers generally increase as either $\bar{\lambda}_2$ or $\bar{\lambda}_3$ increases although there are a surprising number of exceptions for X2 (ALMMP and MALMMP tests) and X4 (n = 20).

A comparison of the LM and MLM powers reveals that the marginal-likelihood-based test (MLM) is typically more powerful than the conventional LM test. The only exceptions occur for X2 and n = 60 when the two power curves cross, although the comparison still favours the MLM test overall. Power improvements of greater than 0.1 are not uncommon when the MLM test is used in place of the LM test.

The estimated power curves of the ALMMP and MALMMP tests are very similar, although there is a clear tendency for the MALMMP test to be more powerful for larger values of ρ. For the analogous testing problem without ρ as a nuisance parameter which can be viewed as a special case of our problem in which ρ is unknown, Ara and King (1993) report that these two tests have identical power curves when exact small sample critical values are used. This suggests that the modest improvement in power for larger ρ values is purely a consequence of using maximum marginal likelihood estimates of ρ which are less biased than their conventional counterparts.

For X1 and X4, the ALMMP test is always more powerful than the LM test as might be expected. For X2 and n = 20, a similar pattern occurs except on the boundary ($\bar{\lambda}_2 = 0$ or $\bar{\lambda}_3 = 0$) where there are examples of the LM test being more powerful. For X2 and n = 60, there are very few situations where the ALMMP test is more powerful than the LM test. In fact the former test has relatively poor power whenever $\bar{\lambda}_2$ or $\bar{\lambda}_3$ is one. Similar conclusions can be drawn from a comparison of the powers of the MALMMP and MLM tests although the dominance of the MALMMP test is less pronounced for X1 and X4 and the comparison favours the MLM test more strongly in the case of X2.

In summary, the use of marginal-likelihood-based tests rather than their traditional counterparts does typically result in a more accurate test in terms of both size and power. These improvements are very clear cut in the case of the LM test and are less obvious for the ALMMP test. The use of an ALMMP test in place of the equivalent LM test does improve power as expected for X1 and X4. For X2, particularly when n = 60, there is evidence of a serious power loss. Wu (1991) observed that LMMP tests can suffer a loss of power when the scores being summed are negatively correlated. This appears to explain the poor results for X2 given that the two regressors are negatively correlated.

5. Concluding Remarks

This paper outlines the construction of marginal-likelihood-based LM and ALMMP tests of regression disturbances in the presence of nuisance parameters which cannot be eliminated by the use of the usual invariance arguments. This extends the work of Ara and King (1993) who reported an improvement in the accuracy of asymptotic critical values in the case of no nuisance parameters. The Monte Carlo experiment we report clearly shows that the use of marginal-likelihood-based tests can improve both sizes and powers, particularly of the LM test. This appears to be a higher level of improvement than that reported by Ara and King. In our case, additional accuracy seems to come from the use of maximum marginal likelihood estimates of the nuisance parameters. On the basis of these findings, we conjecture that similar improvements in accuracy exist for likelihood ratio and Wald tests of regression disturbances in the presence of nuisance parameters.

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Table 1: Estimated sizes of the LM, MLM, ALMMP and MALMMP tests for random coefficients in the presence of AR(1) disturbances using asymptotic critical values at the 5% nominal level.

ρ	n = 20				n = 60			
	LM	MLM	ALMMP	MALMMP	LM	MLM	ALMMP	MALMMP
				X1				
0.0	.024	.062	.038	.060	.046	.054	.045	.051
0.2	.024	.064	.036	.059	.042	.050	.050	.058
0.4	.024	.057	.036	.056	.036	.049	.036	.052
0.6	.020	.056	.025	.050	.030	.046	.032	.051
0.8	.015	.050	.018	.044	.032	.050	.025	.051
0.9	.011	.046	.012	.044	.034	.052	.018	.051
				X2				
0.0	.022	.050	.036	.046	.022	.054	.016	.065
0.2	.020	.048	.027	.045	.020	.057	.017	.056
0.4	.021	.047	.020	.043	.023	.056	.016	.056
0.6	.027	.040	.014	.041	.018	.057	.013	.062
0.8	.060	.038	.009	.046	.015	.058	.018	.066
0.9	.092	.039	.006	.048	.018	.062	.022	.067
				X3				
0.0	.050	.057	.012	.040	.044	.050	.042	.054
0.2	.046	.054	.011	.033	.046	.051	.034	.050
0.4	.049	.046	.009	.031	.050	.048	.026	.047
0.6	.062	.044	.006	.024	.056	.052	.013	.045
0.8	.070	.042	.003	.021	.084	.052	.010	.037
0.9	.071	.044	.001	.017	.114	.048	.004	.029
				X4				
0.0	.054	.050	.017	.048	.037	.050	.039	.060
0.2	.051	.052	.018	.050	.039	.052	.043	.060
0.4	.070	.044	.016	.039	.036	.052	.040	.062
0.6	.096	.039	.010	.033	.040	.052	.035	.062
0.8	.122	.036	.009	.028	.048	.058	.028	.060
0.9	.131	.040	.008	.024	.068	.054	.022	.057

Table 2: Estimated sizes and powers for X1 of the LM, MLM, ALMMP and MALMMP tests for random coefficients in the presence of AR(1) disturbances using empirical critical values at the 5% level.

$\bar{\lambda}_1$	$\bar{\lambda}_2$	n = 20				n = 60			
		LM	MLM	ALMMP	MALMMP	LM	MLM	ALMMP	MALMMP
					ρ = 0.0				
0	0	.042	.050	.050	.050	.050	.049	.045	.047
	.2	.086	.100	.117	.110	.178	.190	.282	.284
	1	.224	.252	.304	.308	.718	.731	.814	.820
.2	0	.084	.098	.116	.112	.162	.170	.251	.246
	.2	.126	.142	.176	.172	.344	.352	.478	.484
	1	.237	.262	.329	.320	.762	.776	.852	.859
1	0	.202	.230	.276	.281	.658	.669	.784	.788
	.2	.218	.248	.306	.312	.727	.745	.839	.844
	1	.280	.294	.388	.376	.864	.872	.928	.934
					ρ = 0.3				
0	0	.044	.048	.045	.050	.042	.048	.047	.044
	.2	.100	.113	.132	.130	.224	.232	.329	.334
	1	.252	.268	.326	.334	.753	.767	.850	.854
.2	0	.090	.106	.129	.130	.198	.213	.292	.296
	.2	.137	.158	.196	.198	.406	.419	.550	.556
	1	.256	.284	.350	.354	.796	.804	.885	.890
1	0	.220	.250	.294	.305	.708	.728	.834	.835
	.2	.236	.271	.322	.332	.770	.786	.873	.879
	1	.291	.318	.401	.409	.880	.892	.940	.944
					ρ = 0.6				
0	0	.031	.046	.034	.042	.032	.043	.032	.045
	.2	.126	.178	.160	.202	.402	.441	.529	.568
	1	.294	.334	.367	.394	.844	.858	.908	.920
.2	0	.104	.152	.154	.186	.356	.404	.486	.526
	.2	.170	.211	.240	.278	.596	.633	.745	.768
	1	.298	.338	.390	.408	.867	.880	.928	.934
1	0	.257	.308	.340	.362	.823	.842	.912	.918
	.2	.272	.316	.364	.388	.850	.866	.929	.936
	1	.318	.360	.426	.446	.914	.924	.960	.963
					ρ = 0.9				
0	0	.020	.038	.020	.038	.036	.050	.018	.045
	.2	.242	.328	.310	.390	.820	.889	.890	.928
	1	.390	.411	.479	.498	.938	.954	.970	.974
.2	0	.206	.308	.287	.364	.792	.871	.886	.934
	.2	.274	.346	.376	.420	.888	.925	.946	.961
	1	.380	.404	.482	.494	.942	.954	.976	.979
1	0	.336	.390	.440	.466	.936	.950	.980	.983
	.2	.341	.386	.455	.480	.941	.954	.982	.982
	1	.375	.404	.500	.512	.956	.962	.984	.988

Table 3: Estimated sizes and powers for X2 of the LM, MLM, ALMMP and MALMMP tests for random coefficients in the presence of AR(1) disturbances using empirical critical values at the 5% level.

$\bar{\lambda}_1$	$\bar{\lambda}_2$	n = 20				n = 60			
		LM	MLM	ALMMP	MALMMP	LM	MLM	ALMMP	MALMMP
					ρ = 0.0				
0	0	.016	.046	.046	.046	.049	.046	.050	.047
	.2	.025	.097	.047	.045	.208	.181	.056	.054
	1	.072	.210	.046	.046	.681	.650	.106	.098
.2	0	.017	.068	.110	.111	.098	.098	.108	.110
	.2	.021	.089	.106	.097	.230	.222	.125	.118
	1	.052	.181	.084	.078	.689	.663	.156	.153
1	0	.038	.106	.246	.242	.254	.302	.350	.336
	.2	.036	.116	.226	.226	.358	.391	.341	.329
	1	.044	.150	.184	.175	.722	.714	.328	.316
					ρ = 0.3				
0	0	.013	.045	.044	.044	.044	.050	.044	.043
	.2	.026	.100	.042	.042	.252	.220	.062	.060
	1	.084	.237	.046	.044	.758	.728	.115	.110
.2	0	.016	.071	.123	.125	.096	.110	.128	.122
	.2	.019	.094	.104	.106	.278	.270	.138	.134
	1	.057	.192	.086	.078	.766	.740	.164	.156
1	0	.043	.118	.262	.251	.289	.334	.388	.376
	.2	.040	.123	.238	.230	.424	.446	.378	.367
	1	.043	.148	.184	.184	.788	.785	.351	.336
					ρ = 0.6				
0	0	.012	.040	.026	.041	.040	.046	.044	.046
	.2	.030	.130	.030	.041	.400	.385	.073	.071
	1	.096	.276	.038	.040	.882	.864	.138	.138
.2	0	.013	.078	.132	.168	.108	.148	.168	.181
	.2	.018	.108	.114	.133	.434	.441	.180	.181
	1	.068	.222	.092	.088	.881	.866	.208	.210
1	0	.046	.132	.284	.295	.372	.466	.504	.518
	.2	.045	.133	.258	.268	.576	.624	.472	.482
	1	.050	.164	.200	.198	.893	.896	.419	.414
					ρ = 0.9				
0	0	.050	.039	.010	.050	.034	.050	.046	.048
	.2	.066	.282	.027	.059	.819	.844	.119	.140
	1	.146	.383	.046	.062	.980	.983	.211	.226
.2	0	.028	.145	.210	.306	.280	.396	.394	.456
	.2	.034	.166	.154	.198	.845	.888	.360	.402
	1	.090	.284	.100	.112	.982	.982	.308	.324
1	0	.069	.163	.352	.356	.696	.813	.796	.848
	.2	.062	.164	.314	.320	.888	.930	.710	.754
	1	.054	.182	.223	.224	.984	.985	.547	.560

Table 4: Estimated sizes and powers for X4 of the LM, MLM, ALMMP and MALMMP tests for random coefficients in the presence of AR(1) disturbances using empirical critical values at the 5% level.

$\bar{\lambda}_1$	$\bar{\lambda}_2$	n = 20				n = 60			
		LM	MLM	ALMMP	MALMMP	LM	MLM	ALMMP	MALMMP
					ρ = 0.0				
0	0	.016	.046	.044	.044	.027	.041	.042	.050
	.2	.020	.044	.058	.054	.090	.128	.172	.180
	1	.020	.060	.066	.070	.357	.462	.548	.566
.2	0	.021	.048	.060	.056	.086	.127	.174	.180
	.2	.020	.052	.060	.062	.176	.240	.308	.313
	1	.021	.064	.069	.073	.415	.516	.594	.622
1	0	.022	.064	.073	.076	.378	.475	.572	.586
	.2	.021	.067	.074	.078	.430	.522	.620	.638
	1	.023	.066	.080	.079	.548	.646	.716	.728
					ρ = 0.3				
0	0	.022	.043	.046	.040	.024	.042	.047	.047
	.2	.021	.048	.062	.060	.104	.158	.212	.219
	1	.022	.058	.069	.074	.405	.503	.592	.602
.2	0	.022	.054	.065	.062	.106	.157	.215	.222
	.2	.021	.054	.067	.070	.214	.286	.364	.368
	1	.026	.061	.072	.075	.456	.546	.638	.644
1	0	.023	.063	.073	.079	.426	.524	.621	.633
	.2	.026	.064	.074	.080	.472	.564	.656	.664
	1	.025	.064	.078	.079	.576	.680	.737	.742
					ρ = 0.6				
0	0	.035	.037	.028	.032	.028	.046	.039	.050
	.2	.023	.064	.073	.080	.197	.295	.345	.370
	1	.026	.061	.078	.082	.508	.621	.676	.688
.2	0	.024	.069	.078	.084	.208	.304	.354	.378
	.2	.022	.064	.078	.087	.338	.450	.524	.540
	1	.025	.062	.078	.080	.541	.656	.702	.712
1	0	.026	.066	.087	.090	.538	.651	.714	.726
	.2	.026	.066	.085	.088	.564	.678	.731	.740
	1	.026	.068	.086	.084	.648	.732	.776	.783
					ρ = 0.9				
0	0	.050	.038	.016	.020	.050	.047	.027	.045
	.2	.018	.064	.088	.102	.550	.707	.694	.744
	1	.024	.067	.084	.081	.690	.778	.794	.810
.2	0	.018	.070	.098	.110	.580	.726	.727	.770
	.2	.022	.068	.088	.096	.658	.770	.782	.810
	1	.024	.067	.086	.085	.702	.788	.809	.822
1	0	.026	.071	.092	.091	.736	.819	.842	.856
	.2	.024	.071	.092	.090	.730	.816	.841	.852
	1	.022	.071	.088	.088	.737	.814	.841	.850