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A Simple Nested Test of AIDS.

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Abstract: The MPIGLOG specification of an indirect utility function gives rise to Cooper and McLaren's (1992) Modified AIDS (MAIDS) specification, which nests AIDS. Following the 'combined' approach outlined by Fry, Fry and McLaren (1993), we transform our deterministic equations to logratio form for estimation. This procedure not only restricts the shares implied by the model to the unit simplex, but also provides a transparent representation of the restriction implied by AIDS. We estimate MAIDS (with and without the AIDS restriction imposed) using the 'combined' approach and proceed to test the AIDS restriction.

Keywords: Compositional data, MAIDS, AIDS, stochastic specification, unit simplex.

J. E. L. Classification: D12, C51, C30.

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1. Introduction.

Regularity has long been recognised as an important issue in the specification of demand systems. In particular, share equations based on the Almost Ideal Demand System (AIDS) frequently violate curvature restrictions. Indeed, this problem was noted by Deaton and Muellbauer (1980) in their initial estimation of AIDS. Recently, Cooper and McLaren (1992) have noted that violation of regularity conditions is due to the fact that the PIGLOG cost function that underlies AIDS is not globally regular. This revelation led Cooper and McLaren to modify the PIGLOG cost function (thus deriving MPIGLOG) and hence to develop Modified AIDS (MAIDS) as the 'MPIGLOG counterpart' to AIDS (as derived using the PIGLOG cost function). This new demand system is attractive, as it preserves regularity in a much wider region of price-expenditure space.

The next step after specification of the demand system is estimation. The equations are generally estimated assuming that the disturbances adhere to the multivariate normal distribution. However, Fry, Fry and McLaren (1993) point out that the nature of this stochastic specification implies a non-zero probability of shares outside the unit simplex, *even if* the deterministic specification is 'regular' (MPIGLOG). Their solution to this problem is to apply the 'additive logratio transformation' as outlined in Aitchison (1986) and model the resulting logratios as multivariate normal.

In this paper, we apply the additive logratio transformation to MAIDS. As a result, we find that we have a transparent representation of the restriction implied by AIDS and a very simple test of this restriction. The outline of the rest of this paper is as follows: in section 2 we outline the specification of the MAIDS share equations and how they are derived. In section 3 we discuss the specification of the stochastic component in the share equations and outline Fry *et al*'s solution to the problem of shares outside the (0,1) interval. Empirical results for MAIDS are given in section 4 and we test the AIDS restriction. Finally, in section 5 we give some concluding remarks.

2. The Demand Systems.

Define the N-vector of commodity demands $\mathbf{q} = (q_1, \dots, q_N)'$, with corresponding price vector $\mathbf{p} = (p_1, \dots, p_N)'$, and define expenditure, or cost, by $c = \mathbf{p}'\mathbf{q}$. Using the convention that upper case letters represent functions explaining their lower case values, demand system estimation requires specification of the system of demand functions:

$$Q_i(c, \mathbf{p}) \qquad (i = 1, \ldots, N)$$

where the vector of functions $Q_i(.)$ is consistent with the theory of demand. For estimation purposes, it is convenient to work with the translation to budget shares:

$$w_i = p_i q_i / c, \qquad W_i(c, \mathbf{p}) = p_i Q_i(c, \mathbf{p})/c.$$

Then a complete model consists of a specification for the explanation of observed data:

$$w_i = W_i(c, \mathbf{p}) + u_i \qquad (i = 1, \ldots, N)$$

in terms of the functional form of $W_i(.)$ and the stochastic specification of the error u_i.

In order to specify the $W_i(.)$, duality theory (see Diewert (1982)) suggests the specification of an indirect utility function $U(c, \mathbf{p})$. If the resulting system of shares, $W_i(.)$, is to be regular (consistent with the paradigm of maximising utility subject to a budget constraint), then U(.) should satisfy the following regularity conditions:

RU1 U is homogeneous of degree zero (HDO) in (c, **p**),
RU2 U is non-increasing in **p**,
RU3 U is non-decreasing in c,
RU4 U is quasi-convex in **p**.

If U(.) satisfies these properties for all values of (c, **p**) in the N+1 dimensional non-negative orthant, U is said to be globally regular. Otherwise U(.) and the corresponding demand equations are regular over a region. It is convenient to parameterise U in terms of measures of real expenditure, defined in terms of price indices P(**p**). Consider the specification:

$$U(c, \mathbf{p}) = [\ln(c / P1)](c^{\eta} / P2) \qquad (1)$$

where η is a parameter (in the following, Greek letters represent parameters), and P1(**p**), P2(**p**), which act as deflators, are parameterised as:

$$\ln P1 = \ln \kappa + \sum_{i=1}^{N} \alpha_i \ln p_i + (\tfrac{1}{2}) \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij}^{*} \ln p_i \ln p_j$$

$$\ln P2 = \sum_{i=1}^{N} \beta_i \ln p_i .$$

By the definition of P1, only $\gamma_{ij} = \frac{1}{2}\left(\gamma^*_{ij} + \gamma^*_{ji}\right)$ can be identified, hence $\gamma_{ij} = \gamma_{ji}$ is imposed. Homogeneity of U requires the restrictions:

$$\sum_{i=1}^{N}\alpha_i = 1, \quad \sum_{i=1}^{N}\beta_i = \eta, \quad \sum_{j=1}^{N}\gamma_{ij} = 0, \qquad (i = 1, \dots, N)$$

which coincide with the requirement that P1 and P2 be homogeneous of degree one and η, respectively. Let $\overline{P}$ be the set of prices over which P1(**p**) and P2(**p**) are non-decreasing and concave. Then it is shown in Cooper and McLaren (1992) that, provided $0 \leq \eta \leq 1$, the corresponding U function satisfies the monotonicity and curvature properties RU2 - RU4 over the set $S = \{(c, \mathbf{p}): \mathbf{p} \in \overline{P}, \ln(c/P1(\mathbf{p})) \geq 1\}$. Since this corresponds to all values of real expenditure (as measured by $(c/P1(\mathbf{p}))$) greater than a minimum value, it is convenient to identify this minimum value with the lower limit in the sample, by normalising expenditure and prices to unity at this point, and imposing κ=1.

Application of Roy's identity to U as specified in (1) gives the system of demand equations:

$$W_i = \frac{\alpha_i + \sum_{j=1}^{N}\gamma_{ij}\ln p_j + \beta_i R}{1+\eta R} \qquad (i = 1, \dots, N) \tag{2}$$

where R = ln(c/P1). When $\eta = 0$, this becomes the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980). Allowing η to take values greater than zero generates a system of demand equations that is regular over a larger region. For example, the monotonicity of P2 requires that the β_i be ≥ 0, and this is clearly impossible for all β_i (for other than the trivial case) if they are required to sum to zero. Appending an error term (u_i), our estimating equations become:

$$w_i = W_i(\cdot) + u_i \qquad (i = 1, \dots, N) \tag{3}$$

where $W_i(\cdot)$ is the deterministic component as specified in (2) and u_i is the stochastic component.

If, as is usual, the u_i are specified as multivariate normal, the resulting empirical model, which nests AIDS, was denoted MAIDS by Cooper and McLaren. Their estimates showed that the resulting model is indeed regular over a wider region than is the restricted AIDS model. However, the nesting of AIDS within MAIDS as above is not particularly attractive, because of the non-linearities introduced when $\eta \neq 0$. In the next section, a more appropriate specification of the stochastic error term is introduced. Under this specification, a far more natural nesting of AIDS within MAIDS is possible, allowing a transparent representation of the restrictions implied by AIDS.

3. The Stochastic Specification of the Share Equations.

The application of an additive stochastic component that follows the multivariate normal distribution has often been recognised as inappropriate in the context of demand share equations, as this leads to a non-zero probability of shares outside the unit simplex (for various solutions to this problem, see *inter alia* Woodland (1979), Aitchison (1986), Chavas and Sergerson (1987), McElroy (1987), Ronning (1988), Fry *et al* (1993), Grunwald, Raftery and Guttorp (1993)).

The solution that we shall adopt involves the alternative distributional assumption that is used by Fry *et al* - additive logistic normality. This particular distributional assumption is quite an attractive choice for two reasons. Firstly, since we can transform the equations and shares and carry out estimation using multivariate normality, the additive logistic normal distribution is easy to work with. Secondly, application of this transformation to MAIDS provides a more natural nesting of AIDS and results in a transparent representation of the restrictions implied by AIDS. For these reasons, we believe that this solution is, for our purposes, the most appropriate.

The additive logratio transformation takes us from S^n (the n dimensional unit simplex, where $n = N - 1$) to R^n and is defined as:

$$y_i = \log\left(\frac{w_i}{w_N}\right) \qquad (i = 1, \dots, n)$$

with Jacobian: $(w_1 \times w_2 \times \dots \times w_N)^{-1}$. These new logratio variables have logratio covariance matrix Σ given by:

$$\Sigma = \operatorname{cov}\left(\log\left(\frac{w_i}{w_N}\right), \log\left(\frac{w_j}{w_N}\right)\right) \qquad (i, j = 1, \dots, n).$$

If necessary, we can recover the shares (w_i) by applying the inverse transformation: the additive logistic transformation. This transformation takes us from R^n to S^n and is defined as:

$$w_i = \frac{\exp(y_i)}{[\exp(y_1) + \exp(y_2) + \ \dots \ + \exp(y_n) \ + 1]}$$

$$w_N = 1 - w_1 - w_2 - \ \dots \ - w_n$$

and if we assume that the logratios are multivariate normal, then the shares will be additive logistic normal (for details and proofs, see *inter alia* Aitchison (1986), Fry *et al* (1993)).

We can then form a standard multivariate regression model in which the deterministic component is derived from microeconomic theory and the stochastic component is derived from statistical theory. Thus our general functional form for estimation is:

$$y_i = \log\left(\frac{w_i}{w_N}\right) = \log\left(\frac{W_i(Z,\beta)}{W_N(Z,\beta)}\right) + u_i \quad (i = 1, \dots, n). \tag{4}$$

Directly applying this methodology to the MAIDS share equations (3) and cancelling out the denominators, we find our equations become:

$$y_i = \log\left(\frac{w_i}{w_N}\right) = \log\left(\frac{\alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln p_j + \beta_i R}{\alpha_N + \sum_{j=1}^{N} \gamma_{Nj} \ln p_j + \beta_N R}\right) + u_i \tag{5}$$

where $\sum_{i=1}^{N} \alpha_i = 1, \qquad \sum_{i=1}^{N} \gamma_{ij} = \sum_{j=1}^{N} \gamma_{ij} = 0, \qquad \gamma_{ij} = \gamma_{ji}, \qquad \sum_{i=1}^{N} \beta_i = \eta$

and the u_i follow the multivariate normal distribution. Necessary conditions for regularity for all R > (c/P1) are:

$$0 < \eta \leq 1, \qquad \beta_i \geq 0 \qquad \forall \, i = 1, \dots, N.$$

The corresponding specification for AIDS is *exactly* the same, except that we have the additional restriction that $\eta = 0$ which requires that the β_i be unconstrained. Thus the specification we have adopted (via the transformation) shows the AIDS restriction to be very simple to impose. In the next section, we estimate the MAIDS and nested AIDS equations using Australian data and test the AIDS restriction.

4. Empirical Example.

4.1 Models.

The models of sections 2 and 3 relate to individuals (or households). When using aggregate time series data, serial correlation may be introduced if our equations are estimated as specified in (3) using per capita, rather than representative, expenditure and income distribution has been changing systematically over the sample period. Thus we need to incorporate variables that will take account of such changes to correct for the serial correlation arising as a consequence of misspecification. Unfortunately, appropriate data on income distribution was not available, so we introduced proxy variables to capture these effects. Since the primary purpose of including these variables is to remove serial correlation, we have chosen to include them as part of the stochastic component, rather than as part of the mean (deterministic component). As such, they are appended 'post-transformation' - in the same manner as the error term (u_i) and our estimating equations (5) become:

$$y_i = \log\left(\frac{w_i}{w_N}\right) = \log\left(\frac{\alpha_i + \sum_{j=1}^{N}\gamma_{ij}\ln p_j + \beta_i R}{\alpha_N + \sum_{j=1}^{N}\gamma_{Nj}\ln p_j + \beta_N R}\right) + \mu_i' X + u_i \qquad (6)$$

where X is a vector of ('macro') explanatory variables capturing the change in distribution of real income over the sample period, μ_i are parameters satisfying $\sum_{i=1}^{N}\mu_i = 0$ and all other terms are defined as previously. Incorporating $\mu_i' X$ as part of the stochastic term and applying the additive logratio transformation ensures that the shares implied by the 'modified' model are restricted to the unit simplex - a property that is neither guaranteed, nor likely to hold, under the (traditional) 'macro' specification.

4.2 Data.

The MAIDS and nested AIDS models were estimated using annual Australian data over the period 1954/5 to 1991/92. Our data is based on that of Chung and Powell (1987) and Cooper and McLaren (1992), updating the sample from 1988/89 using the methodology adopted by Cooper and McLaren. The categories we have used are: Food (F), Cigarettes, Tobacco and Alcoholic Drinks (T), Clothing (C), and All Other Expenditure (O). The rent and durables categories (although available) are problematic, as noted by Cooper and McLaren, and are thus excluded from the analysis.

The macro variables used to capture the change in the distribution of real income over the sample were: the inflation rate (I), the unemployment rate (U) and the participation rate (P). These proxies were required as specific data on income distribution was not available over the sample period. Obviously, the characteristics of the error term (u_i) will be sensitive to the adequacy of these variables as proxies. All estimation was carried out using the LSQ option in TSP386 (version 4.2B), which is well suited to the estimation of systems with complex cross-equation restrictions.

4.3 Results.

Table 1 gives the results for MAIDS with homogeneity and symmetry imposed.[2] The ★ shows that all parameter estimates for the 'other' category were derived from aggregation restrictions and thus not directly estimated. The • denotes a parameter that has been constrained by the symmetry restrictions. We should note that κ was set to unity (as outlined in section 2) and prices and total expenditure normalised to unity at the beginning of the sample.

We have only included results for MAIDS under the 'combined' approach. The regularity conditions (RU1 - RU4 as outlined in section 2) that hold under the traditional approach (for details see Cooper and McLaren (1992)) should also hold in this case, since the freely estimated β_i's are all negative, although η is marginally above unity. We should note in passing that the Durbin-Watson statistics are quite low, but this is probably a reflection of the sub-optimal performance of our macro variables as proxies for the changes in income distribution over the sample.

Our final task is then to formally test the AIDS restriction. This was done using a standard likelihood ratio test. Using the results in table 1, we construct our Likelihood Ratio test statistic as:

$$2\ \ln(L_1 - L_0),$$

where L_1 and L_0 are the unrestricted and restricted values of the system log-likelihoods, respectively, we have a calculated value of 18.312. The critical value for $\chi^2_{(1)}$ at the 1% level is 6.63 and thus we have a clear rejection of the AIDS restriction and MAIDS is the more appropriate model for this data. So we have found a 'nice' way of nesting AIDS within MAIDS that has the added advantage of restricting the shares implied by the model

[2]The results for the Nested AIDS model are available from the authors on request.

to the unit simplex *even if* we include variables to capture the effects of changes in the distribution of real income.

5. Conclusion.

In this paper, we applied the additive logratio transformation to the MAIDS share equations and found that they neatly nest the AIDS share equations. In addition to providing a transparent representation of the AIDS restriction, this methodology also ensured that the individual shares implied by the model could not violate the (0,1) interval. The MAIDS and Nested AIDS models were estimated using annual Australian expenditure data from 1954/55 to 1991/92 for four categories and the AIDS restriction was, in this case, decisively rejected. This implies that the Modified AIDS specification for demand systems, together with the 'combined' approach to estimation, is a more appropriate way to estimate demand systems.

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Table 1: Maximum Likelihood Estimates for MAIDS

(Homogeneity and Symmetry Imposed).

	Food	Tobacco & Alcohol	Clothing & Footwear	Other★
α_i	0.317 (0.004)	0.118 (0.004)	0.146 (0.005)	0.419 (0.007)
β_i	0.124 (0.055)	0.021 (0.028)	0.018 (0.029)	0.926 (0.147)
γ_{iF}	0.189 (0.018)	-0.072 (0.016)	-0.029 (0.016)	-0.087 (0.025)
γ_{iT}	•	-0.016 (0.019)	-0.019 (0.014)	0.107 (0.032)
γ_{iC}	•	•	0.026 (0.020)	0.022 (0.029)
γ_{iO}	•	•	•	-0.042 (0.067)
μ_{iI}	0.144 (0.134)	0.781 (0.270)	0.770 (0.242)	-1.695 (0.550)
μ_{iU}	-0.477 (0.359)	-3.211 (0.547)	-2.747 (0.602)	6.435 (1.319)
μ_{iP}	-1.702 (0.521)	-2.525 (0.988)	-2.674 (0.970)	6.902 (2.142)
R^2	0.995	0.977	0.987	—
D.W.	0.761	1.228	1.150	—

L (system) = 260.000

$\eta = 1.089\ (0.239)$

AIDS Restriction Test: $2(L_1 - L_0) = 18.312$

1% Critical Value: $\chi^2_{(1)} = 6.63$

Note: Standard Errors in Parentheses.