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Testing for Subblock Effects in Multi-stage Linear Regression Models*

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Abstract

Increasingly survey data is being used in regression analysis. If disturbance correlation arising from block and/or subblock random effects is ignored, it can lead to inefficient regression estimates and predictions as well as misleading inferences. This paper addresses the problem of testing for subblock effects in the presence of block effects in a three-stage sampling regression model. Point optimal invariant (POI) and one-sided and two-sided Lagrange multiplier (LM) tests are constructed. An empirical comparison of the sizes and powers of two versions of the POI tests and the two LM tests is reported. It reveals that the true size of the one-sided LM test is about one-half of its nominal size and that the two-sided LM test lacks power. The POI tests are found to have extremely desirable small-sample properties.

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1. Introduction

Increasingly, survey data is being used for regression analysis, particularly in economics and other social sciences. Often the survey design means that observations are gathered in clusters or blocks and therefore may not be independent. The possibility that the use of such data may lead to positive intrablock or intracluster correlation in regression disturbances has been pointed out by Holt, Smith and Winter (1980), Holt and Scott (1981) and Scott and Holt (1982). It is well known that ignoring such disturbance correlation can lead to inefficient estimates and predictions as well as misleading inferences from hypothesis tests and confidence intervals.

In the case of a two-stage design, i.e. random sampling from a range of different blocks or clusters, a number of tests for intrablock correlation in regression disturbances have been proposed. For example, Deaton and Irish (1983), King and Evans (1986) and Bhatti (1991) discussed and investigated the use of the Durbin-Watson test, the one-sided Lagrange multiplier (LM) test and King's (1987) point optimal invariant (POI) test.

Often there are more than two stages in a survey design. The first stage may be a choice of cities or states which, for geographical reasons, are typically very different and so intrablock correlation might well be expected. Sampling within the chosen cities or states may also be multi-staged with a second choice of clusters or blocks from which samples are taken. It is often not so obvious whether intrablock correlation within these second-stage blocks should be expected. Therefore there is a need to be able to test for such correlation in the presence of first-stage intra-block correlation. The main purpose of this paper is to extend the work of King and Evans (1986) and Bhatti (1991) to this specific testing problem.

This extension is not entirely straightforward because the first-stage intrablock correlation coefficient is an unwanted nuisance parameter.

The plan of the rest of this paper is as follows. The three-stage sampling regression model is introduced in the next section together with the testing problem under consideration. The class of POI tests for this problem and also one-sided and two-sided LM tests are constructed in Section 3. An empirical comparison of the sizes and powers of two versions of the POI tests and the two LM tests is reported in Section 4. Some concluding remarks are made in the final section.

2. The Model and Testing Problem

The following is an extension of the two-stage sampling regression model considered by King and Evans (1986) and Bhatti (1991) to a three-stage sampling regression (3SSR) model. Suppose the total of n observations are sampled from m first-stage blocks (or clusters), with m(i) second-stage subblocks from the i^{th} block and with m(i,j) third-stage observations from the j^{th} subblock of the i^{th} block, such that $n = \sum_{i=1}^{m}\sum_{j=1}^{m(i)} m(i,j)$. The 3SSR model can be written as

$$y_{ijk} = x'_{ijk}\beta + u_{ijk} \qquad (1)$$

for observations $k = 1,2,\ldots,m(i,j)$ from the j^{th} subblock, subblocks $j = 1,2,\ldots,m(i)$, from the i^{th} block, $i = 1,2,\ldots,m$, where y_{ijk} is the $(i,j,k)^{th}$ observation on the dependent variable, x_{ijk} is a $p\times1$ vector of the $(i,j,k)^{th}$ observations on p independent variables one of which may be a constant, and β is a $p\times1$ vector of regression coefficients. The error term u_{ijk} is assumed to be generated as

$$u_{ijk} = v_i + v_{ij} + v_{ijk} \qquad (2)$$

where v_i is the i^{th} block random effect, v_{ij} is the j^{th} subblock random effect in the i^{th} block and v_{ijk} is the random effect for the $(i,j,k)^{th}$ observation. These three error components are assumed to be mutually independent and normally distributed with

$$E(v_i) = E(v_{ij}) = E(v_{ijk}) = 0$$

and $\mathrm{var}(v_i) = \sigma_1^2$, $\mathrm{var}(v_{ij}) = \sigma_2^2$, $\mathrm{var}(v_{ijk}) = \sigma_3^2$.

This implies that

$$E(u_{ijk}) = 0 \tag{3}$$

and

$$E(u_{ijk}u_{rst}) = \begin{cases} 0 & \text{for } i \neq r \text{ and any } j,s,k,t, \\ \sigma_1^2 & \text{for } i = r,\ j \neq s \text{ and any } k,t, \\ \sigma_1^2 + \sigma_2^2 & \text{for } i = r,\ j = s \text{ and } k \neq t, \\ \sigma_1^2 + \sigma_2^2 + \sigma_3^2 & \text{for } i = r,\ j = s \text{ and } k = t; \end{cases} \tag{4}$$

for $k,t = 1,2,\ldots,m(i,j)$; $j,s = 1,2,\ldots,m(i)$ and $i,r = 1,2,\ldots,m$. (3) and (4) give rise to intra-block correlation with coefficient $\rho_1 = \sigma_1^2/\sigma^2$ and intra-subblock correlation with coefficient $\rho_2 = \sigma_2^2/\sigma^2$ where $\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$. Observe that $0 \leq \rho_1, \rho_2 \leq 1$ and $\rho_1 + \rho_2 \leq 1$.

The regression model (1) under (3) and (4) can be written more compactly in matrix notation as

$$y = X\beta + u \tag{5}$$

in which $y = \left(y_{111}, y_{112}, \ldots, y_{121}, y_{122}, \ldots, y_{m,m(m),m(m,m(m))}\right)'$ and u are $n\times1$ and X is $n\times p$. The disturbance vector is distributed as

$$u \sim N\left(0, \sigma^2 \Omega(\rho_1, \rho_2)\right) \tag{6}$$

where

$$\Omega(\rho_1, \rho_2) = \overset{m}{\underset{i=1}{\oplus}} \Omega_i(\rho_1, \rho_2) \tag{7}$$

is a block diagonal matrix with submatrices

$$\Omega_i(\rho_1, \rho_2) = (1 - \rho_1 - \rho_2) I_{m_i} + \rho_1 E_{m_i} + \rho_2 \overset{m(i)}{\underset{j=1}{\oplus}} E_{m(i,j)} \tag{8}$$

in which $m_i = \sum_{j=1}^{m(i)} m(i,j)$, I_{m_i} is an $m_i \times m_i$ identity matrix and E_{m_i} and $E_{m(i,j)}$ are square matrices of ones of dimension m_i and $m(i,j)$, respectively.

Although we can proceed with unbalanced data, i.e., different numbers of observations in each subblock and subblocks in each main block; for the convenience of the remaining discussion, we will assume balanced data. In this case let $T = m(i,j)$ and $s = m(i)$, $i = 1,\ldots,m$; $j = 1,\ldots,s$; so that T is the number of observations in each subblock and s is the number of subblocks in each block. Then (7) and (8) can be written in a more simplified form as

$$\Omega(\rho_1, \rho_2) = (1 - \rho_1 - \rho_2) I_n + \rho_1 D_1 + \rho_2 D_2 \tag{9}$$

where $D_1 = I_m \otimes E_{sT}$ and $D_2 = I_{ms} \otimes E_T$.

Our interest is in testing

$$H_0 : \rho_2 = 0, \quad \rho_1 > 0$$

against

$$H_a : \rho_2 > 0, \quad \rho_1 > 0$$

in the context of (5), (6) and (9). An important consideration is the possible range of ρ_1 values. One option is to assume $0 \le \rho_1 < 1$ but we will assume $0 \le \rho_1 \le 0.5$ given that ρ_1 and ρ_2 follow the constraint

$$0 \le \rho_1 + \rho_2 \le 1 .$$

This choice of range for ρ_1 turns out to have almost no effect in practice on the properties of the tests reported below.

3. The Tests

In terms of the analogous but simpler problem of testing for block effects in a two-stage sampling regression model, POI and one-sided LM tests have been found to have good properties (see King and Evans (1986) and Bhatti (1991)). This section considers the construction of POI and LM tests for the more complicated problem of testing H_0 against H_a.

3.1 Point-optimal invariant tests

The problem of testing H_0 against H_a is invariant to transformations of the form

$$y \longrightarrow \eta_0 y + X\eta ,$$

where η_0 is a positive scalar and η is a p×1 vector. Following King (1987, 1989), it may be that critical regions of a POI test with optimal power at $(\rho_1,\rho_2)' = (\rho_{11},\rho_{21})'$ are of the form

$$s\left(\rho_{10},\rho_{11},\rho_{21}\right) = \hat{u}'\Omega^{-1}(\rho_{11},\rho_{21})\,\hat{u} \Big/ \tilde{u}'\Omega^{-1}(\rho_{10},0)\,\tilde{u} < c$$

in which $\hat{u}$ and $\tilde{u}$ are the generalized least squares residual vectors from (5) assuming covariance matrices $\Omega(\rho_{11},\rho_{21})$ and $\Omega(\rho_{10},0)$, respectively. The existence of a POI test of this form requires the critical value c and the parameter ρ_{10} to be chosen such that

$$\Pr\left[s\left(\rho_{10},\rho_{11},\rho_{21}\right) < c \;\middle|\; u \sim N\left(0,\Omega(\rho_{10},0)\right)\right] = \alpha \tag{10}$$

and

$$\Pr\left[s\left(\rho_{10},\rho_{11},\rho_{21}\right) < c \;\middle|\; u \sim N\left(0,\Omega(\rho_1,0)\right),\; 0 \le \rho_1 \le 0.5\right] \le \alpha \tag{11}$$

where α is the desired level of significance.

Let $\Omega_0 = \Omega(\rho_{10},0)$ and $\Omega_1 = \Omega(\rho_{11},\rho_{21})$. Under H_0 for any given value of ρ_1, the left hand side of (11) can be calculated as

$$\Pr\left[u'\Gamma\left(\rho_{10},\rho_{11},\rho_{21}\right)u < 0\right] = \Pr\left[\sum_{i=1}^{n}\nu_i\xi_i^2 < 0\right] \tag{12}$$

where

$$\Gamma\left(\rho_{10},\rho_{11},\rho_{21}\right) = \Omega_1^{-1} - \Omega_1^{-1}X\left(X'\Omega_1^{-1}X\right)^{-1}X'\Omega_1^{-1}$$

$$- c\left\{\Omega_0^{-1} - \Omega_0^{-1}X\left(X'\Omega_0^{-1}X\right)^{-1}X'\Omega_0^{-1}\right\},$$

$\nu_1,\ldots,\nu_n$ are the eigenvalues of $\Gamma(\rho_{10},\rho_{11},\rho_{21})\Omega(\rho_1,0)$ and $\xi = (\xi_1,\ldots,\xi_n)' \sim N(0,I_n)$. Thus (12) can be evaluated using Imhof's (1961) algorithm, which can be implemented using Koerts and Abrahamse's (1969) FQUAD subroutine or Davies' (1980) algorithm. Shively, Ansley and Kohn (1990) have proposed an alternative method for calculating probabilities such as (12) which does not require the separate calculation of eigenvalues. Powers when $u \sim N(0,\sigma^2\Omega(\rho_1,\rho_2))$ can be calculated in an identical fashion with $\Omega(\rho_1,\rho_2)$ replacing $\Omega(\rho_1,0)$ in the matrix whose eigenvalues are required.

After some experimentation with the X matrices used in the Monte Carlo experiment described below, we found it was possible to solve (10) and (11) simultaneously for c and ρ_{10}. This can be done as follows:

(i) Pick a starting value for ρ_{10} and solve (10) for c.

(ii) Evaluate (11) at ρ_1 values around ρ_{10}. If (11) is a maximum at $\rho_1 = \rho_{10}$, we have the required ρ_{10} value.

(iii) Otherwise move ρ_{10} towards the ρ_1 value which maximizes (11) and solve (10) for c.

(iv) Beginning at (ii), repeat the process.

The resulting ρ_{10} value, which we denote as ρ_{10}^*, makes the test based on rejecting H_0 for

$$s\left(\rho_{10}^*, \rho_{11}, \rho_{21}\right) < c$$

a POI test which optimizes power at $(\rho_1, \rho_2)' = (\rho_{11}, \rho_{21})'$. For the test to be operational, this point at which power is to be optimized must be chosen. One approach, which is explored further below, is to choose sensible middle values for ρ_{11} and ρ_{21}. More complicated procedures which involve choosing $(\rho_{11}, \rho_{21})'$ so that the optimized power takes a predetermined value such as 0.5 have been suggested in the literature. See for example, King (1987, 1989) for further details.

Finally we note that there is no guarantee that the required ρ_{10}^* value can always be found. There may be some combinations of X, α, and $(\rho_{11}, \rho_{21})'$ for which it is impossible to solve (10) and (11) simultaneously. In such cases we recommend the approximate POI test outlined in King (1987, 1989).

3.2 Lagrange multiplier test

The LM test has proved to be very popular in econometrics and has been used in a wide range of testing situations. A good survey of the literature may be found in Godfrey (1988). Typically the LM test is constructed and applied as a two-sided test. When a single parameter is under test, a one-sided version can be constructed which, in the absence of nuisance parameters, is a locally best test. In this section we outline the construction of both the one-sided and two-sided versions of the LM test of H_0.

The log likelihood function of (5), (6) and (9) is

$$L = \frac{-n}{2} \ln(2\pi\sigma^2) - 1/2 \ln|\Omega(\rho_1,\rho_2)| - 1/2 (y - X\beta)'\Omega^{-1}(\rho_1,\rho_2)(y - X\beta)/\sigma^2.$$

The score with respect to the parameter under test evaluated under H_0 is

$$\left.\frac{\partial L}{\partial \rho_2}\right|_{H_0} = d_1 - 1/2(y - X\beta)'\Omega^{-1}(\rho_1,0)(I_n - D_2)\Omega^{-1}(\rho_1,0)(y - X\beta)/\sigma^2 \qquad (13)$$

where

$$d_1 = -1/2 \left.\frac{\partial|\Omega(\rho_1,\rho_2)|}{\partial \rho_2}|\Omega(\rho_1,\rho_2)|^{-1}\right|_{H_0}$$

$$= n(T - 1)b / 2$$

in which

$$b = -\rho_1(1 - \rho_1)^{-1}(1 - \rho_1 + sT\rho_1)^{-1}. \qquad (14)$$

From Magnus (1978, Theorem 3), the information matrix $\mathcal{I}$ of the linear regression model (5), (6) and (9) is of the form

$$\mathcal{I} = \begin{bmatrix} X'\Omega^{-1}(\rho_1,\rho_2)X/\sigma^2 & 0 \\ 0 & \psi(\theta) \end{bmatrix},$$

where θ is a 3×1 vector such that $\theta_1 = \sigma^2$, $\theta_2 = \rho_1$ and $\theta_3 = \rho_2$. $\psi(\theta)$ is a 3×3 symmetric matrix whose $(i,j)^{th}$ element is

$$\psi_{ij} = 1/2 \ \mathrm{tr}\left[\frac{\partial \Sigma^{-1}(\theta)}{\partial \theta_i} \Sigma(\theta) \frac{\partial \Sigma^{-1}(\theta)}{\partial \theta_j} \Sigma(\theta) \right]$$

where $\Sigma(\theta) = \sigma^2\Omega(\rho_1,\rho_2)$. Under H_0, these elements are

$$\psi_{11} = \frac{n}{2\sigma^4}, \qquad \psi_{12} = \frac{nb(sT-1)}{2\sigma^2}, \qquad \psi_{13} = \frac{nb(T-1)}{2\sigma^2},$$

$$\psi_{22} = \frac{n(1-sT)}{2}\left[-a^2 + b^2sT(1 - sT) + 2ab(1 - sT)\right],$$

$$\psi_{23} = \frac{n(1-T)}{2}\left[-a^2 + b^2sT(1 - sT) + 2ab(1 - sT)\right],$$

$$\psi_{33} = \frac{n(1-T)}{2}\left[-a^2 + b^2sT(1 - T) + 2ab(1 - T)\right],$$

where $a = (1 - \rho_1)^{-1}$ and b is defined by (14).

For the asymptotic variance of (13) under H_0, we need the bottom right element of $\psi^{-1}(\theta)$ under H_0. It is

$$\psi^{33} = \left\{ \psi_{33} - \left(\psi_{13},\psi_{23}\right) \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{12} & \psi_{22} \end{bmatrix}^{-1} \begin{bmatrix} \psi_{13} \\ \psi_{23} \end{bmatrix} \right\}^{-1}$$

which after much tedious algebra reduces to

$$\psi^{33} = \frac{2(sT-1)(1-\rho_1)^2}{nT(T-1)(s-1)} . \qquad (15)$$

In order to construct an LM test using (13) and (15), unknown nuisance parameters β, σ^2 and ρ_1 need to be replaced by their maximum likelihood estimates under H_0. We use $\hat{\beta}$, $\hat{\sigma}^2$ and $\hat{\rho}_1$ to denote these estimates. Observe that

$$\hat{\beta} = \left(X'\Omega^{-1}(\hat{\rho}_1,0)X\right)^{-1} X'\Omega^{-1}(\hat{\rho}_1,0)y \ ,$$

$$\hat{\sigma}^2 = y'\left(\Omega^{-1}(\hat{\rho}_1,0) - \Omega^{-1}(\hat{\rho}_1,0)X\left(X'\Omega^{-1}(\hat{\rho}_1,0)X\right)^{-1}X'\Omega^{-1}(\hat{\rho}_1,0)\right)y/n$$

and

$$\Omega^{-1}(\rho_1,0) = (1-\rho_1)^{-1}\left[I_n - \rho_1\left\{1 + (Ts-1)\rho_1\right\}^{-1}D_1\right].$$

Now (13) becomes

$$\left.\frac{\partial \hat{L}}{\partial \rho_2}\right|_{H_0} = \hat{d}_1 - 1/2(y-X\hat{\beta})'\ \Omega^{-1}(\hat{\rho}_1,0)\left(I_n-D_2\right)\Omega^{-1}(\hat{\rho}_1,0)(y-X\hat{\beta}) \,/\, \hat{\sigma}^2$$

$$= \hat{s}_1$$

say, where $\hat{d}_1$ is d_1 evaluated at $\rho_1 = \hat{\rho}_1$.

Let $\hat{\psi}^{33}$ denote ψ^{33} with ρ_1 replaced by $\hat{\rho}_1$. A one-sided LM test, which we denote by LM1, can be applied by rejecting H_0 for large values of

$$\hat{s}_1\left(\hat{\psi}^{33}\right)^{-1/2}$$

which has an asymptotic standard normal distribution under H_0. The two-sided version of this test, which we denote by LM2, involves rejecting H_0 for large values of

$$\hat{s}_1^2 \,/\, \hat{\psi}^{33}$$

which has an asymptotic chi-squared distribution with one degree of freedom under H_0.

4. An Empirical Comparison of Sizes and Powers

A Monte Carlo experiment was conducted in order to assess and compare the small-sample size and power performance of the LM1 and LM2 tests and two versions of the POI test, namely $s(\rho^*_{10},0.1,0.1)$ and $s(\rho^*_{10},0.25,0.25)$. We will denote the latter two tests as $s_{0.1}$ and $s_{0.25}$, respectively. The main objectives of the experiment were to assess the accuracy of the asymptotic critical values of the LM tests and to compare the powers of the LM tests with those of the POI tests.

4.1 Experimental design

The Monte Carlo experiment was divided into two parts. The first involved estimating sizes of the LM tests at ρ_1 = 0.0, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5 under H_0 using asymptotic critical values at the five percent nominal level. The second part of the experiment was conducted in two stages. The first stage consisted of the calculation of appropriate critical values in order to compare powers of tests at approximately the same true significance level. Critical values and ρ^*_{10} values for the POI tests were computed as outlined in section 3.2. For the LM1 and LM2 tests, the Monte Carlo method was used to estimate exact critical values at ρ_1 = 0.0, 0.05, 0.1, 0.2, 0.3, 0.4 and 0.5 under H_0. From each set of seven critical values, the largest was selected thus ensuring that at least at these chosen points, the size of the test does not exceed the nominal level which was set at five percent throughout. The second stage involved the calculation of powers of the four tests using these critical values.

The following design matrices were used in the study.

X1 : (n,p,m,s,T) = (24,3,2,3,4) and (48,3,2,3,8).

Three-stage sample of inputs into farm production in Bangladesh. The main blocks are samples from the Divisions of Khulna and Rajshahi.

Subblocks are districts within each of these divisions. Individual elements are farms. The regressors are a constant, biological-chemical input per acre and human labour in adult man-days. Further details may be found in Hoque (1988, 1991).

X2 : (n,p,m,s,T) = (24,3,2,3,4) and (64,3,2,4,8).

Cross-sectional Australian census data used by King and Evans (1986). Regressors are a constant, population and number of households in each of 64 demographic groups.

X3 : (n,p,m,s,T) = (24,4,2,3,4) and (72,4,2,3,12).

Artificially generated data. The regressors are a constant, a uniform random variable and two independent log-normal random variables.

These three data sets reflect a variety of economic phenomenon and also have been used in earlier empirical studies.

Sizes and powers were calculated for all combinations of $\rho_1 = \rho_2 = 0.0$, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5. For the POI tests, this was done as outlined in section 3.1 with the aid of a modified version of Koerts and Abrahamse's (1969) FQUAD subroutine with maximum integration and truncation errors of 10^{-6}. The Monte Carlo method with two thousand replications was used for the LM tests. Disturbances were generated based on (2) with pseudo-random normal variates generated as described by King and Giles (1984). The results of Breusch (1980) imply the sizes and powers of the LM tests are invariant to the values taken by β and σ^2. For the purpose of the Monte Carlo calculations, β_i, $i = 1,\ldots,p$, and σ^2 were all set to unity. By construction, the POI tests are also invariant to the values of β and σ^2.

The LM tests require maximum likelihood estimates of ρ_1 under H_0. This was done by adapting Ansley's (1979) method for estimating ARMA models to the problem in hand. The first step was to find an efficient method for transforming (5) by the inverse of the Cholesky decomposition of $\Omega(\rho_1,0)$ given ρ_1. This allows the concentrated likelihood function to be written as a sum of squares, thus reducing the estimation problem to minimizing a sum of squares. The latter was handled by the IMSL subroutine DBCLSF from the IMSL MATH/LIBRARY (1989) with the constraint that $|\rho_1| \leq 1$.

4.2 The Results

Table 1 reports the estimated sizes of the LM tests based on asymptotic critical values at the five percent significance level. Almost always the estimated sizes are below the nominal level, the only exceptions being nonsignificant and occurring for n = 24 and low ρ_1 values. All LM1 sizes are significantly below the nominal level, typically being less than 0.025. These sizes show a clear tendency to increase as n increases with the largest sizes occurring for the largest sample size, n = 72 for X3. On the other hand, the LM2 sizes show a disturbing tendency to decrease as n increases and for the larger samples, very few estimated sizes are not significantly different from 0.05. The LM2 sizes also tend to decrease as the nuisance parameter ρ_1 increases. In general the sizes at $\rho_1 = 0$ are acceptable while those at $\rho_1 = 0.5$ are not. In contrast, we see from Tables 3 - 5 that the sizes of the POI tests are 0.05 for all values of ρ_1.

Calculated values of ρ^*_{10} and c for the POI tests and estimated critical values for the LM1 and LM2 tests are given in Table 2. It is interesting to note the high degree of similarity in ρ^*_{10} values for the $s_{0.1}$ test.

Selected calculated powers of the four tests are presented in Tables 3 - 5. The powers of the POI tests all increase as n increases, as ρ_1 increases or ρ_2 increases, *ceteris paribus*. The LM1 test and to a lesser extent the LM2 test powers also follow this pattern with some minor exceptions when ρ_2 is small particularly for n = 24. The calculated powers of the POI and LM1 tests are always greater than the nominal size of 0.05. This is not the case for the LM2 test when n = 24. For ρ_2 = 0.05 and also for ρ_2 = 0.1 in the case of X2, almost all estimated LM2 powers are below 0.05.

Of the two LM tests, as expected the one-sided test (LM1) is always more powerful than its two-sided counterpart. Power differences range from 0.025 to 0.223 when n = 24 and from 0.001 to 0.159 for larger sample sizes. In some extreme cases (X2 with n = 24), the percentage increase in power from using the one-sided test instead of its two-sided counterpart is greater than 200%.

The powers of the two POI tests are very similar suggesting a degree of insensitivity to the choice of $(\rho_{11},\rho_{21})'$ values in these tests. Power differences range from zero to 0.035 for n = 24 and zero to 0.003 for large sample sizes. Overall, the $s_{0.25}$ test does appear to have a slight power advantage which declines as n increases.

Typically both POI tests are more powerful than the LM1 test. Of the 252 LM1 powers calculated, only 18 are higher than those of one or both of the POI tests. Of these 18 cases, 7 lie on the boundary in the sense that $\rho_1 = 0$, and the rest almost always involve small ρ_1 values. LM1 powers in Tables 3 - 5 with stars next to them indicate powers that are significantly different from the lowest power of the corresponding POI tests. Overall, the POI tests do appear to have a clear power advantage over the LM1 test particularly for larger values of ρ_1 and ρ_2.

5. Concluding Remarks

Increasingly, survey data is being used in regression analysis. If disturbance correlation arising from block and/or subblock random effects is ignored, it can lead to inefficient regression estimates and predictions as well as misleading inferences. This paper addresses the problem of testing for subblock effects in the presence of block effects in a three-stage sampling regression model.

An obvious test would seem to be the LM test, particularly since a one-sided version is available for this one-sided testing problem. Unfortunately our Monte Carlo results indicate that its true size is about half of its nominal size. It is interesting to note the existence of almost identical findings for the small-sample size of the LM test for heteroscedasticity; see for example Breusch and Pagan (1979), Godfrey (1978), Honda (1988) and Lee and King (1993). It is tempting to suggest that the one-sided LM test be applied at twice the desired significance level. Sadly, we only have our limited simulation results to support this suggestion. The two-sided LM test seems to have better true sizes, however its use in place of the one-sided LM test can result in a large loss of power particularly for smaller sample sizes.

Our main finding is that the POI tests have extremely desirable small-sample properties. At least for the data sets used in our study, their true sizes correspond to the nominal size for all values of the nuisance parameter ρ_1 under H_0 and they almost always are more powerful than both LM tests when $\rho_1 > 0$. Which POI test to use is not an issue because both seem to have almost identical powers, particularly for larger sample sizes. It seems that the extra computational cost of applying a POI test is well rewarded.

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Table 1: Estimated sizes of the LM1 and LM2 tests $_0$ using asymptotic critical values at the 5% nominal level

Data Matrix	n	Test	ρ_1 0.0	0.05	0.1	0.2	0.3	0.4	0.5
X1	24	LM1	.015	.018	.016	.016	.016	.014	.013
		LM2	.059	.055	.050	.047	.038	.031	.026
	48	LM1	.019	.020	.021	.021	.022	.023	.023
		LM2	.033	.029	.031	.029	.028	.028	.027
X2	24	LM1	.013	.012	.013	.012	.012	.012	.010
		LM2	.048	.043	.040	.032	.030	.028	.024
	64	LM1	.019	.018	.018	.016	.016	.016	.017
		LM2	.047	.038	.032	.028	.025	.023	.022
X3	24	LM1	.021	.023	.023	.022	.019	.019	.018
		LM2	.059	.050	.045	.038	.033	.032	.031
	72	LM1	.025	.026	.025	.028	.025	.025	.023
		LM2	.042	.034	.030	.033	.030	.028	.024

Table 2: Calculated values of ρ^*_{10} and c for the $s_{0.1}$ and $s_{0.25}$ tests and estimated critical values for the LM1 and LM2 tests at the 5% significance level.

Data Matrix	n	$s_{0.1}$ test		$s_{0.25}$ test		LM1	LM2
		ρ^*_{10}	c	ρ^*_{10}	c	c	c
X1	24	.1252	.9591	.3298	.9924	1.2966	3.9919
	48	.1279	.9816	.3559	1.0786	1.4947	3.5528
X2	24	.1208	.9691	.3232	1.0112	1.0655	3.8244
	64	.1222	.9964	.3441	1.1103	1.2743	3.7645
X3	24	.1192	.9579	.3175	.9950	1.4068	4.0323
	72	.1313	.9987	.3693	1.1139	1.5421	3.6574

Table 3: Selected calculated sizes and powers of the POI and LM tests of $H_0 : \rho_2 = 0, \rho_1 \geq 0$ for X1 at the 5% level.

		n = 24 m = 2 s = 3 T = 4				n = 48 m = 2 s = 3 T = 8			
Tests	ρ_2	ρ_1 = 0.0	0.1	0.3	0.5	0.0	0.1	0.3	0.5
$s_{0.1}$	0.0	.050	.050	.050	.050	.050	.050	.050	.050
$s_{0.25}$		.050	.050	.050	.050	.050	.050	.050	.050
LM1		.043	.050	.047	.041*	.042	.048	.050	.045
LM2		.050	.041	.033	.024	.050	.037	.033	.031
$s_{0.1}$	0.05	.081	.084	.095	.115	.135	.146	.180	.243
$s_{0.25}$		.081	.084	.095	.117	.135	.146	.180	.244
LM1		.074	.080	.082*	.095*	.126	.139	.171	.216*
LM2		.049	.047	.044	.038	.093	.103	.116	.160
$s_{0.1}$	0.1	.119	.128	.155	.208	.245	.270	.342	.465
$s_{0.25}$		.118	.127	.156	.213	.244	.270	.343	.468
LM1		.109	.112*	.142*	.180*	.245	.280	.330	.417*
LM2		.056	.056	.061	.078	.180	.204	.260	.341
$s_{0.1}$	0.2	.217	.241	.311	.442	.469	.513	.626	.778
$s_{0.25}$		.216	.240	.315	.459	.468	.513	.627	.781
LM1		.198*	.217*	.277*	.370*	.484	.521	.601*	.715*
LM2		.100	.110	.147	.217	.405	.439	.518	.650
$s_{0.1}$	0.3	.335	.375	.490	.683	.651	.699	.809	.926
$s_{0.25}$		.335	.377	.501	.714	.650	.699	.811	.929
LM1		.308*	.337*	.440*	.593*	.661	.701	.784*	.886*
LM2		.182	.204	.277	.420	.593	.632	.728	.846
$s_{0.1}$	0.4	.464	.519	.668	.876	.781	.825	.913	.985
$s_{0.25}$		.465	.524	.685	.911	.782	.826	.915	.987
LM1		.434*	.484*	.607*	.788*	.790	.823	.889*	.955*
LM2		.270	.315	.432	.646	.742	.777	.857	.940
$s_{0.1}$	0.5	.594	.659	.821	.976	.870	.906	.968	1.000
$s_{0.25}$		.598	.668	.843	.998	.871	.906	.969	1.000
LM1		.565*	.619*	.751*	.932*	.873	.892*	.945*	.993*
LM2		.391	.448	.615	.853	.843	.870	.928	.990

* Difference in power between POI and LM1 test is significant, whereas the values without * indicates that the difference in power between POI and LM1 test is not significant.

Table 4: Selected calculated sizes and powers of the POI and LM tests of $H_0 : \rho_2 = 0, \rho_1 \geq 0$ for X2 at the 5% level.

		n = 24 m = 2 s = 3 T = 4				n = 64 m = 2 s = 4 T = 8			
Tests	ρ_2	ρ_1 = 0.0	0.1	0.3	0.5	0.0	0.1	0.3	0.5
$s_{0.1}$	0.0	.050	.050	.050	.050	.050	.050	.050	.050
$s_{0.25}$		.050	.050	.050	.050	.050	.050	.050	.050
LM1		.044	.049	.050	.049	.045	.049	.045	.046
LM2		.050	.041	.030	.024	.050	.035	.028	.024
$s_{0.1}$	0.05	.079	.083	.094	.115	.150	.164	.205	.283
$s_{0.25}$		.079	.082	.093	.114	.149	.163	.204	.282
LM1		.074	.084	.090	.102	.144	.154	.187	.250*
LM2		.040	.034	.027	.031	.083	.080	.088	.127
$s_{0.1}$	0.1	.115	.124	.151	.205	.284	.315	.402	.548
$s_{0.25}$		.114	.123	.150	.206	.282	.313	.401	.548
LM1		.107	.123	.148	.185*	.271	.304	.375*	.492*
LM2		.050	.046	.043	.060	.159	.179	.224	.333
$s_{0.1}$	0.2	.206	.229	.299	.435	.548	.598	.719	.864
$s_{0.25}$		.208	.229	.301	.442	.548	.598	.721	.866
LM1		.204	.228	.281*	.381*	.553	.579	.673*	.804*
LM2		.087	.093	.124	.187	.402	.443	.558	.707
$s_{0.1}$	0.3	.316	.355	.471	.681	.742	.790	.888	.972
$s_{0.25}$		.318	.358	.479	.698	.744	.792	.891	.973
LM1		.304	.339	.436*	.599*	.739	.767*	.854*	.947*
LM2		.150	.169	.235	.378	.619	.658	.762	.900
$s_{0.1}$	0.4	.436	.492	.648	.893	.864	.900	.963	.997
$s_{0.25}$		.443	.500	.663	.907	.866	.903	.965	.998
LM1		.430	.479	.602*	.723*	.849	.878*	.946*	.988*
LM2		.234	.261	.379	.602	.771	.815	.900	.977
$s_{0.1}$	0.5	.561	.630	.812	1.000	.935	.958	.991	1.000
$s_{0.25}$		.572	.643	.827	1.000	.936	.960	.992	1.000
LM1		.555	.606*	.748*	.947*	.920*	.944*	.981*	1.000
LM2		.342	.397	.544	.851	.874	.902	.967	.998

* Difference in power between POI and LM1 test is significant, whereas the values without * indicates that the difference in power between POI and LM1 test is not significant.

Table 5: Selected calculated sizes and powers of the POI and LM tests of $H_0 : \rho_2 = 0, \rho_1 \geq 0$ for X3 at the 5% level.

		n = 24 m = 2 s = 3 T = 4				n = 72 m = 2 s = 3 T = 12			
Tests	ρ_2	ρ_1 = 0.0	0.1	0.3	0.5	0.0	0.1	0.3	0.5
$s_{0.1}$	0.0	.050	.050	.050	.050	.050	.050	.050	.050
$s_{0.25}$		.050	.050	.050	.050	.050	.050	.050	.050
LM1		.044	.050	.047	.046	.040	.043	.040	.044
LM2		.050	.039	.029	.025	.050	.037	.033	.028
$s_{0.1}$	0.05	.081	.085	.097	.119	.190	.208	.261	.354
$s_{0.25}$		.081	.085	.097	.120	.190	.208	.261	.354
LM1		.077	.081	.088	.104*	.190	.198	.235	.310*
LM2		.045	.042	.040	.040	.142	.152	.174	.233
$s_{0.1}$	0.1	.122	.131	.161	.219	.354	.389	.482	.620
$s_{0.25}$		.121	.131	.161	.223	.354	.389	.482	.620
LM1		.115	.121	.154	.199*	.357	.381	.448*	.548*
LM2		.067	.061	.072	.099	.293	.312	.373	.480
$s_{0.1}$	0.2	.225	.250	.327	.472	.620	.664	.765	.878
$s_{0.25}$		.224	.250	.330	.484	.620	.664	.765	.878
LM1		.219	.250	.314	.419*	.619	.659	.743*	.825*
LM2		.120	.139	.184	.273	.562	.600	.678	.782
$s_{0.1}$	0.3	.350	.393	.518	.725	.783	.821	.897	.966
$s_{0.25}$		.349	.394	.525	.744	.783	.821	.897	.966
LM1		.334	.377	.484*	.652*	.780	.808	.862*	.924*
LM2		.215	.250	.335	.499	.736	.766	.830	.905
$s_{0.1}$	0.4	.485	.544	.701	.912	.878	.907	.958	.994
$s_{0.25}$		.486	.547	.712	.930	.878	.907	.959	.994
LM1		.476	.531	.657*	.850*	.870	.886*	.927*	.977*
LM2		.332	.375	.518	.744	.837	.862	.908	.970
$s_{0.1}$	0.5	.619	.688	.853	.994	.934	.954	.986	1.000
$s_{0.25}$		.622	.693	.864	1.000	.934	.954	.986	1.000
LM1		.595	.656*	.807*	.969*	.918	.928*	.970*	.999
LM2		.464	.528	.701	.931	.899	.914	.960	.998

* Difference in power between POI and LM1 test is significant, whereas the values without * indicates that the difference in power between POI and LM1 test is not significant.