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A SYSTEM OF DEMAND EQUATIONS SATISFYING EFFECTIVELY GLOBAL REGULARITY CONDITIONS *

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ABSTRACT

A parametric specification of an indirect utility function in terms of expenditure and two unit cost functions is proposed. Specification of these unit cost functions in terms of regular functions leads to the notion of an "effectively globally regular" system of demand equations; that is, a system of demand equations that is regular over a cone in expenditure-price space, and for which the regular region includes all points in any given sample, and all values of nominal expenditure and prices generating higher values of real expenditure than the sample minimum. This general model nests a number of popular demand systems, such as the Linear Expenditure System, as special cases. An empirical application demonstrates the value of the generalization.

J.E.L. Classification: D11, D12.

1. INTRODUCTION

Microeconomic theory provides a firm foundation for the estimation of systems of demand equations. In its most transparent form, this theory states that such demand equations should be consistent with the maximization of a utility function subject to a budget constraint, generating systems of equations satisfying homogeneity, monotonicity, symmetry and curvature restrictions.

Three approaches to the translation of these restrictions into empirical application may be identified. In the primal approach, the demand equations are derived literally by specifying a direct utility function and solving the constrained maximization problem. While this approach leads to demand systems which satisfy the above regularity conditions by construction, the need to derive analytical solutions to the first order conditions restricts its application to utility functions of the origin-translated C.E.S. form, such as the Klein-Rubin. A second approach is the Rotterdam methodology, which attempts to impose the regularity restrictions on log-differential approximations to the demand equations.

This paper is in the spirit of the third approach, which exploits the theory of duality among direct utility functions, indirect utility functions, and cost functions, and the regularity conditions on these functions which make them equivalent representations of the underlying preferences. Duality theory allows systems of demand equations to be derived from these dual representations via simple differentiation, according to Roy's Identity or Shephard's Lemma. This approach was popularized by Diewert (1974, 1982), and led to the use of flexible functional forms such as the Generalized Leontief of Diewert (1971) and the Translog of Christensen, Jorgenson and Lau (1978). While such flexible functional forms lead to demand equations which can attain arbitrary elasticities at a point in price-expenditure space, such systems generally satisfy globally only homogeneity with respect to prices and expenditure, and often violate monotonicity and, in particular, curvature restrictions, either within the sample, or at points close to the sample. Lau (1986) discusses the characterisation of regularity of such systems, and finds that the domain of regularity is rather limited.

Much recent work has been devoted to deriving demand systems that satisfy regularity over a wider domain. Many of these methods are based on series expansions - see Barnett (1983, 1985), Barnett, Lee and Wolfe (1985, 1987), Barnett and Yue (1988a, b), Gallant (1981, 1984) and Gallant and Golub (1984).

This paper generalizes a parametric representation of the indirect utility function in terms of expenditure and price indexes that was introduced in Cooper and McLaren (1988, 1992) in order to generate demand systems in the spirit of the Almost Ideal Demand System of Deaton and Muelbauer (1980), but with improved regularity properties. This is achieved by the use of regular functional forms for unit cost functions which are components of the indirect utility function. The generalization includes as nested cases a number of known separable demand systems, such as the linear expenditure system, and hence provides a consistent framework for the testing of the restrictions of additivity.

The parametric representation of the indirect utility function in terms of unit cost functions is introduced in Section 2, where conditions for regularity are specified. Section 3 considers possible specifications for the unit cost functions, and Section 4 provides an empirical application using Australian data. The final result is a demand system satisfying what is denoted "effectively global regularity"; that is, the domain of regularity includes the entire sample and all other possible values of nominal expenditure and prices generating real expenditure greater than the minimum value observed in the sample.

2. THE REPRESENTATION OF PREFERENCES

Let $x \in \Omega^n$ represent an n-vector of commodities, $p \in \Omega^n_+$ represent the corresponding vector of prices, and let $c>0$ represent total expenditure (cost), where Ω^n $\left(\Omega^n_+\right)$ is the non-negative (positive) orthant. We will assume that preferences can be represented by the (direct) utility function $u = U(x)$ where U satisfies regularity conditions RU:

RU1 : U is continuous,

RU2 : U is non-decreasing,

RU3 : U is strictly quasi-concave.

Dual to U(x) is the indirect or "Marshallian" utility function defined by

$$(2.1) \qquad u = U^M(c,p) = \max_x \left\{U(x) \ : \ p'x \le c \ , \ x \in \Omega^n\right\} = U\left(X^M(c,p)\right)$$

where the adjective "Marshallian" (and hence superscript M) refers to the arguments of the corresponding functions (the solutions $X^M(c,p)$ are usually referred to as Marshallian demand equations). As a notational convention, upper case letters will represent functional forms for the corresponding values represented by lower case letters - for example, $u = U(x)$; $x^M = X^M(c,p)$; etc. The indirect utility function $U^M(c,p)$ corresponding to a direct utility function $U(x)$ satisfying RU1 - RU3 will satisfy the regularity conditions RIU:

RIU1 : U^M is continuous on Ω_+^{n+1},

RIU2 : U^M is homogeneous of degree zero (HDO) in (c,p),

RIU3 : U^M is non-increasing in p,

RIU4 : U^M is non-decreasing in c,

RIU5 : U^M is quasi-convex in p.

By Roy's Identity,

$$(2.2) \qquad X^M(c,p) = -\frac{\partial U^M/\partial p}{\partial U^M/\partial c} .$$

Also dual to $U(x)$ is the (Hicksian) expenditure or cost function defined by

$$(2.3) \qquad c = C(u,p) = \min_x \left\{p'x : U(x) \geq u,\ x \in \Omega^n\right\} = p'X^H(u,p) .$$

Here the adjective "Hicksian" (and hence superscript H) refers to the arguments (u,p) of the corresponding functions. (The solutions $X^H(u,p)$ are usually referred to as Hicksian (or utility-compensated) demand equations. Since we have only one function C( ), the H is deleted as superfluous.)

The cost function $C(u,p)$ corresponding to a direct utility function $U(x)$ satisfying regularity conditions RU will satisfy the regularity conditions RC:

RC1 : C is continuous on $\mathcal{R} \times \Omega^n_+$,

RC2 : C(u,p) is homogeneous of degree one in p,

RC3 : C is non-decreasing in p,

RC4 : C is non-decreasing in u,

RC5 : C is concave in p.

$U^M(c,p)$ and C(u,p) are related by

$$U^M(C(u,p),p) \equiv u \ ; \quad C(U^M(c,p),p) \equiv c \ ;$$

and by Shephard's Lemma:

$$(2.4) \qquad X^H(u,p) \ = \ \partial C/\partial p \ .$$

The primal approach to demand system specification begins by specifying a functional form for U(x) directly, and deriving $X^M(c,p)$, and hence $U^M(c,p)$, by explicit solution of (2.1), or by deriving $X^H(u,p)$, and hence C(u,p), by explicit solution of (2.3). If the functional form U(x) satisfies the regularity conditions RU over Ω^n, then $U^M(c,p)$ will satisfy regularity conditions RIU over Ω^{n+1}_+, C(u,p) will satisfy regularity conditions RC over $\mathcal{R} \times \Omega^n_+$, and the functions $X^M(c,p)$, $U^M(c,p)$, $X^H(u,p)$ and C(u,p) will be said to be globally regular.

The best-known example of this approach is the Cobb-Douglas specification of U, which we write for convenience in logarithmic form:

$$(2.5a) \qquad U \ = \ \sum \beta_i \ \ell n \ x_i \ , \qquad \beta_i > 0$$

where the β_i may be normalized to sum to unity, $\sum \beta_i = 1$.

The derived globally regular equations are

$$(2.5b) \qquad X_i^M(c,p) = \beta_i c/p_i$$

$$(2.5c) \qquad U^M(c,p) = \sum \beta_i \ \ell n \ \beta_i + \ell n \ (c/P)$$

$$(2.5d) \qquad X_i^H(u,p) = \frac{\beta_i}{p_i} C(u,p)$$

$$(2.5e) \qquad \ell n \ C(u,p) = - \sum \beta_i \ \ell n \ \beta_i + u + \ell n \ P,$$

where the price index P(p) is defined by $\ell n \ P = \sum \beta_i \ \ell n \ p_i$. A generalization that preserves global regularity (for a suitable choice of parameters) is the Constant Elasticity of Substitution (CES) form.

By duality, it is convenient to modify the indirect utility function (2.5c) by generalizing the price index P from a Cobb-Douglas to a CES form:

$$(2.6a) \qquad P(p) = \left[\sum \beta_i \ p_i^{\rho}\right]^{1/\rho}$$

and ignoring the redundant constants.

Provided $\rho < 1$, the corresponding globally regular functions are:

$$(2.6b) \qquad U^M = \ell n \ (c/P)$$

$$(2.6c) \qquad X_i^M = \left(\beta_i \ p_i^{\rho-1}\right) c \ / \sum\left(\beta_j \ p_j^{\rho}\right)$$

$$(2.6d) \qquad \ell n \ C = u + \ell n \ P$$

$$(2.6e) \qquad X_i^H = \beta_i \ p_i^{\rho-1} \ P^{(1-\rho)} \ e^u \ .$$

Two well-known generalizations that do not preserve global regularity are considered next.

Firstly, consider the Translog function. The Translog direct utility function generalizes (2.5a) by adding quadratic terms in $\ell n\ x_i$:

$$(2.7a) \qquad U = \sum \beta_i \ \ell n\ x_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ \ell n\ x_i \ \ell n\ x_j \ .$$

The implied U( ) is not globally regular unless $\gamma_{ij} = 0 \quad \forall i,j$, and the functions X_i^M, U^M, X_i^H and C cannot be derived in closed form (and are not globally regular). By analogy, both U^M and C could also be represented by Translog functional forms. The advantage of the dual approach is that the implied demand equations may be derived by Roy's Identity or Shephard's Lemma, respectively, but again the functions U^M (or C) cannot satisfy the regularity conditions RIU (or RC) globally, except in the trivial cases. Indeed, the implied regions of regularity of these functions are quite limited.

A second generalization is accomplished by origin translation. Hence consider

$$(2.8a) \qquad U(x) = \sum \beta_i \ \ell n\ (x_i - \gamma_i) \qquad \gamma_i \geq 0$$

which, for $\beta_i > 0$ and $\sum \beta_i = 1$, is regular over the cone $x \in \Omega^n + \{\gamma\}$, i.e. $x_i > \gamma_i$, $i = 1,\dots,n$. Solution for X_i^M then gives the Linear Expenditure System (LES):

$$(2.8b) \qquad X_i^M(c,p) = \gamma_i + \frac{\beta_i}{p_i} (c - \kappa P1)$$

with corresponding functions

$$(2.8c) \qquad U^M(c,p) = \sum \beta_i \ \ell n\ \beta_i + \ell n\ (c - \kappa P1) - \ell n\ P2$$

$$(2.8d) \qquad X_i^H(u,p) = \gamma_i + \frac{\beta_i}{p_i} (C(u,p) - \kappa P1)$$

(2.8e) $C(u,p) = P2 \exp(u - \sum \beta_i \ell n\, \beta_i) + \kappa P1$

where the price indexes P1(p) and P2(p) are defined by $P1(p) = (\sum \gamma_i p_i) / \kappa$, $\kappa = \sum \gamma_i$, and $\ell n\, P2 = \sum \beta_i \ell n\, p_i$, $\sum \beta_i = 1$. The functions (2.8a,b,c) are not globally regular unless $\gamma_i = 0$ for all i = i,...,n, whereas $\gamma_i > 0$ are necessary for the usual interpretation as "subsistence" parameters, and empirical models often lead to estimates of $\gamma_i > 0$. Models with γ_i strictly positive are, however, regular over an unbounded region, the cone $x_i > \gamma_i$ for U(x), and the region $c > \kappa P1$ for U^M and hence X^M.

Since little progress has been made in the search for globally regular demand equations, the regularity properties of the LES are appealing; regularity is assured over regions of increasing c for given p. Since this is analogous to increasing real income, which characterises much time series data, the potential exists to define demand systems whose regular regions automatically include post-sample data. Such demand systems are as regular from an empirical viewpoint as globally regular systems.

We note that the indirect utility function of the Linear Expenditure System may be represented in terms of two price indexes P1 and P2 and that the regularity properties of the Linear Expenditure System depend upon the properties of these price indexes. Define P(p) to be a price index if it satisfies the properties of a unit cost function, i.e. the regularity properties RP:

RP1 : P is continuous in p,

RP2 : $P(p) > 0$ for $p \in \Omega_+^n$,

RP3 : P is homogeneous of degree 1 (HD1), RP4 : P is non-decreasing,

RP5 : P is concave,

RP6 : P(1) = 1.

In the next section we develop this approach by specifying the functional form of an indirect utility function whose regularity properties derive from the regularity properties of two general price indexes which satisfy properties RP1 to RP6.

3. <u>THE GENERAL EXPONENTIAL FORM</u>

If the indirect utility function is to be defined in terms of two price indexes, it is natural that these price indexes should act to deflate nominal expenditure c, i.e. to enter in the form c/Pk, k = 1,2.

Hence consider the indirect utility function in the general form:

$$(3.1) \qquad U^M(c,p) = \frac{\left(\frac{c}{\kappa P1}\right)^{\mu} - 1}{\mu} \cdot \left(\frac{c}{P2}\right)^{\eta}$$

where Pk(p), k = 1,2, are two price indexes satisfying regularity properties RP, and parameters μ,η,κ satisfy $0 \leq \eta \leq 1$, $\mu \geq -1$, and $\kappa > 0$. In an appendix it is shown that provided these conditions are satisfied, U^M satisfies conditions RIU over the region $\left\{(c,p) : c > \kappa P1(p)\right\}$ and so U^M, and hence the set of Marshallian demand equations, are regular over this region. Since, for any given units of measurement of expenditure and prices, κ may be chosen such that the regular region includes any desired base value of real expenditure and hence all larger values, we define such demand systems to be "effectively globally regular".

Demand equations are most easily represented in share form. From the logarithmic form of Roy's Identity:

$$(3.2) \qquad W_i^M(c,p) = \frac{p_i X_i^M}{c} = - \frac{\partial U^M / \partial \ln p_i}{\partial U^M / \partial \ln c} = EP1_i(1-Z) + EP2_i Z$$

where $EPk_i(p) = \frac{\partial \ln Pk}{\partial \ln p_i}$, k = 1,2 ,

$$Z = \frac{\eta \frac{R^\mu - 1}{\mu}}{R^\mu + \eta \frac{R^\mu - 1}{\mu}} ,$$

and $R = \frac{c}{\kappa P1}$. Thus Z is a mapping of real expenditure (R) into the [0,1] interval. If expressed in terms of c, P1(p) and P2(p), it can be seen that (3.2) is an example of Lewbel's fractional demand systems, (Case (vi), EXP demands), in Lewbel (1987).

If $\mu \le 0$, then W_i^M varies between $EP1_i$ for low real income ($c = \kappa P1$ and $Z = 0$) and $EP2_i$ for high real income ($Z = 1$). However, if $\mu > 0$, then as $c \to \infty$, $Z \to \frac{\eta}{\mu+\eta}$ and W_i^M converges to a fixed weighted average of $EP1_i$ and $EP2_i$. For this general model the Slutsky matrix has typical term

$$S_{ij} = \left(c / p_i p_j\right) \left[T_{1ij}(1-Z) + T_{2ij} Z\right]$$

where $T_{1ij} = EP1_{ij} + W_i^M W_j^M - W_i^M \delta_{ij} - \mu\left(W_i^M - EP1_i\right) \left(W_j^M - EP1_j\right)$

$$T_{2ij} = EP2_{ij} + W_i^M W_j^M - W_i^M \delta_{ij} + \eta\left(W_i^M - EP2_i\right) \left(W_j^M - EP2_j\right)$$

$$EPk_{ij} = \partial EPk_i / \partial \ln p_j , \qquad k = 1,2$$

and δ_{ij} is the Kronecker delta.

The responsiveness of budget shares to real expenditure is given by

$$\partial W_i^M / \partial c = \left(EP2_i - W_i^M\right) \left[(-\mu)Z + \eta(1-Z)\right] .$$

Also of interest is the behaviour of marginal budget shares $MBS_i = \partial \left(p_i X_i^M\right) / \partial c$. For the general model

$$MBS_i = \left(EP2_i - EP1_i\right) \left(c\, \partial Z / \partial c + Z\right) + EP1_i .$$

Marginal budget shares will generally depend upon both expenditure and prices except in certain special cases. Independence with respect to expenditure would apply if either of the following conditions were satisfied: (i) P1 = P2 or (ii) $c\partial Z / \partial c + Z$ does not depend upon Z. The condition P1 = P2 is degenerate and implies that budget shares W_i^M are invariant with respect to expenditure. Since $c\partial Z / \partial c + Z = \eta + (1-\eta) Z - (\mu+\eta) (1-Z)$, the only way that condition (ii) can be met for arbitrary Z is if $\mu = -1$ and $\eta = 1$ (see Case 1 below). Even under either of these two conditions marginal budget shares will be price-dependent unless the price indexes are of the Cobb-Douglas form. Generally, the ability of model (3.1) to represent flexible MBS characteristics with respect to both expenditure and prices is an important feature of this model.

Expenditure elasticities E_i for the general model exhibit similarly flexible characteristics. Since $E_i = MBS_i / W_i^M$, we observe

$$E_i = \frac{EP1_i + (EP2_i - EP1_i)\ [\eta + (1-\eta)Z - (\mu+\eta)Z(1-Z)]}{EP1_i + (EP2_i - EP1_i)Z}$$

so that, for the poor ($Z \to 0$), $E_i \to 1 + (EP2_i / EP1_i - 1)\eta$, while for the rich ($Z \to 1$), $E_i \to 1$. For Z = 0, luxuries ($E_i > 1$) may therefore be identified by $EP2_i > EP1_i$ and necessities by $EP2_i < EP1_i$. However, for

Z in the [0,1] range, it is possible for E_i to move through unity and hence for budget shares to exhibit regions of non-monotonic response to expenditure. In all cases expenditure elasticities asymptote to unity.

Two nested special cases are of particular interest:

Case 1 : $\mu = -1$, $\eta = 1$. In this case, U^M is of the form

$$\left(1 - \frac{\kappa P1}{c}\right)\left(\frac{c}{P2}\right)$$

$$= \left(\frac{c - \kappa P1}{c}\right)\frac{c}{P2}$$

$$= \frac{c - \kappa P1}{P2}$$

which is the Gorman Polar Form, a generalization of the LES (2.8c).

Case 2 : $\mu = 0$. In this case U^M is of the MPIGLOG form, a generalization of the PIGLOG preferences of Muellbauer which allows enhanced regularity over PIGLOG, introduced by Cooper and McLaren (1988, 1992).

The advantage of a parametric form such as (3.1) is that the specification of an indirect utility function satisfying conditions RIU has been reduced to the problem of specifying two unit cost functions satisfying conditions RP. At this stage, many specifications of P1 and P2 would be possible, but since our interest is in the effective global regularity of U^M, we will concentrate on examples where P1 and P2 satisfy conditions RP over Ω^n_+. An obvious choice would be to employ CES unit cost functions.

The resulting general specification nests a number of known functional forms. For example, $\mu = -1$, $\eta = 1$, P1 linear and P2 Cobb-Douglas gives the Linear Expenditure System, while $\mu = 0$ generates a class of functional forms which is AIDS-like but with enhanced regularity properties (see Cooper and McLaren (1992)). Hence this opens up the possibility of testing a number of existing models against more general but regular alternatives.

4. EMPIRICAL SPECIFICATION

The models of the previous section relate to individuals or households. In Cooper and McLaren (1992) the issue of aggregation across individuals in the context of MPIGLOG preferences is addressed in detail, and it is shown that an appropriate estimating form of (3.2) is

$$(4.1) \qquad w_i = EP1_i(1-Z) + EP2_iZ + \theta_i'd + u_i$$

where d is a vector of explanatory variables acting as a proxy for the change in distribution of real expenditure over the sample period, and the θ_i are parameters satisfying $\Sigma\theta_i = 0$. Given CES specifications for the price indexes

$$Pk(p) = \left[\sum \beta k_i \, p_i^{\rho k}\right]^{1/\rho k}, \quad \sum_i \beta k_i = 1, \quad k = 1,2 ,$$

the elasticity terms in (4.1) take the form

$$EPk_i = \beta k_i \, p_i^{\rho k} \Big/ \left[\sum \beta k_j \, p_j^{\rho k}\right] .$$

With this specification, sufficient conditions for effective global regularity of the systematic part of (4.1) over the region $c > \kappa P1$ are

$$\beta k_i \geq 0 , \qquad i = 1,..,n , \quad k = 1,2 ,$$

$$\rho k \leq 1 , \qquad k = 1,2 ,$$

$$0 \leq \eta \leq 1$$

$$\mu \geq -1 .$$

For purposes of estimation, an error term u_i has been appended additively in (4.1). For time series data, the specific error terms u_{it} are assumed to be distributed multivariate normal, with

$$E(u_{it}) = 0$$

$$E(u_{it}u_{js}) = \omega_{ij} \quad \text{for} \quad s = t,$$

$$= 0 \quad \text{for} \quad s \neq t .$$

In addition, the budget constraint implies that $\Sigma_i u_{it} = 0$, and hence $\Omega = [\omega_{ij}]$ is singular. For purposes of estimation, one equation is deleted, and as usual the parameter estimates are invariant to the deleted equation. (See McLaren (1990)).

The model (4.1) and a number of nested special cases were estimated using annual Australian data covering the period 1954/55 to 1990/91. The data used are based on Adams, Chung and Powell (1988) who constructed a data base ending in 1985/86, and are updated to 1990/91 using a similar methodology. For this period, the available categories are: Food (F), Tobacco and Alcohol (T), Clothing (C), Rent (R), Durables (D) and Other (O). The rent component poses a problem with Australian data, because of its high imputed component, and would be unlikely to be explained by a static allocation model. Similarly, it is unlikely that durables would be well suited to such a model, and hence these two categories are excluded in the empirical work. The variables used to proxy the effect of changing distribution of real expenditure over the sample period were: the rate of inflation (I), the rate of unemployment (U), and the participation rate (P). Estimation was carried out using the LSQ option of TSP, which is well-suited to the estimation of systems with complex cross-equation constraints.

5. RESULTS

Estimation results for the general specification (4.1) are reported in Table 1.

Table 1: General Model Results

Functional Form Parameters

μ	η	$\rho1$	$\rho2$	κ
-0.268	0.639	1.056	-7.871	0.617
(-0.7)*	(2.5)	(10.0)	(-2.9)	(4.6)

		Food	Tobacco	Clothing	Other
Price Index Share Parameters					
P1:	$\beta1_i$	0.417	0.145	0.191	0.247
		(5.9)	(6.1)	(5.8)	(2.0)
P2:	$\beta2_i$	0.001	0.014	0.002	0.983
		(0.6)	(1.5)	(0.7)	(72.7)
Aggregation Parameters					
Inflation:	θ_{1i}	0.007	0.063	0.059	-0.129
		(0.42)	(3.0)	(3.2)	(-3.1)
Unemployment:	θ_{2i}	0.127	-0.301	-0.366	0.540
		(2.4)	(-9.5)	(-10.4)	(5.9)
Participation:	θ_{3i}	-0.154	-0.254	-0.268	0.676
		(-2.4)	(-4.6)	(-3.9)	(5.1)
Summary Statistics					
	R^2	0.994	0.953	0.983	0.993
	D.W.	0.84	1.09	1.40	1.04

System Log-likelihood 514.602

* Asymptotic t values in parentheses.

The most important point to highlight from the results in Table 1 is that the parameter estimates satisfy the sufficient conditions for effective global regularity without the need to impose constraints, with the minor exception of $\rho 1$ which at 1.056 is only marginally above its limiting value of 1. This very minor violation of sufficient conditions for regularity is present in all the estimated submodels. It is unlikely to lead to actual regularity violation in practice. However, after selecting a preferred model we re-estimate imposing the regularity restriction.

The summary statistics indicate that the general model fits the data extremely well, even though estimation is in share form. Although the Durbin-Watson statistics may be suggestive of residual autocorrelation, it seems probable that this is a consequence of splicing techniques in the data series. To obtain an improvement here it would be preferable to revise the data rather than make technical model corrections.

At base period prices (p_j = 1 for all j) the price index share parameters βk_i (k = 1,2) become the price elasticities EPk_i. Thus the $\beta 1_i$ ($\beta 2_i$) may be interpreted as the low (high) income budget shares evaluated at base period prices. The budget share parameter estimates given in Table 1 therefore imply that "Other" is a luxury while "Food", "Tobacco and Alcohol" and "Clothing" are all necessities. While the asymptotic budget shares of the three necessities are insignificantly different from zero on the basis of t-tests, a likelihood ratio test rejects the joint hypothesis.

A final point of interest to note from the parameter estimates is that the freely estimated scale parameter κ, at approximately 0.6, provides an extension of the assured regular region to values of real expenditure well below the minimum value in the sample.

There are a variety of models nested within the general specification which are of interest. Table 2 provides a summary.

Table 2: Summary of Specific Model Results

Specific Model	Functional Form Parameter Restrictions			Log-likelihood
	μ	η	κ	
1	0	1	1	499.034
2	0	1	free	514.187
3	-1	1	1	500.987
4	-1	1	free	501.612
5	0	free	1	512.472
6	0	free	free	514.478
7	-1	free	1	512.458
8	-1	free	free	513.851
9	free	1	1	502.927
10	free	1	free	514.281
11	free	free	1	512.765
12	free	free	free	514.602

In Table 2, Model 12 represents the general model. Note that in all the above models, a CES specification is maintained for both price indexes, and hence $\rho 1$ and $\rho 2$ are freely estimated. Models 1, 2, 5 and 6 are "MAIDS-like" (Cooper and McLaren (1992)) in the sense that $\mu = 0$ implies a logarithmic form for the first real expenditure term in the indirect utility function. On the other hand, models 3 and 4 are "LES-like" in the sense that the Gorman Polar Form (GPF) (see Case 1, Section 3) estimates P1 as effectively linear (although P2 remains CES, not Cobb-Douglas). In fact, $\rho 1$ is approximately unity in all models.

There are several alternative sequences of nesting which are worth discussing. These generally involve successive one-parameter restrictions for which the critical value of χ^2 is 3.8. On a likelihood ratio test, the LES-like models 3 and 4 are dominated by models 7 and 8, suggesting a rejection of the GPF. On the other hand, subsequently freeing up μ (models 11 and 12) does not lead to a significant improvement once η has been freed up. That is, models 7 and 8 compare favourably with models 11 and 12. The freeing up of the scaling parameter κ is also of little statistical value once η has been freed. However, free estimation of κ is desirable on economic grounds since the estimate, which in all models is significantly less than unity, implies that the regular region extends well below the minimum value of real expenditure in the sample (as well as, necessarily, above, as holds when κ is constrained to unity - see Cooper and McLaren (1992)). It may also be noted that in the restricted MAIDS-like models 1 and 2, the freeing up of the scaling parameter κ achieves much the same effect as freeing up η. In fact, model 2 is not statistically inferior to any of the models in which it is nested (including the general model 12). This is not true for the LES-like models 3 and 4, which are clearly dominated by models 7 and 8 respectively.

It is interesting to note that of the two alternative non-nested branches (μ = 0 and μ = -1), the simplest model on the μ = 0 branch which cannot be rejected relative to the general model is the restricted

MAIDS-like model 2, while on the other hand on the $\mu = -1$ branch the simplest model which cannot be rejected relative to the general model is not LES-like, but takes the more complex form ($\eta \neq 1$) of model 8. Models 2 and 8 are not nested, but strong grounds for preferring model 2 lie in its more parsimonious parameterisation and the simplicity of its functional structure.

Our preferred model is therefore based on model 2. As discussed previously, when freely estimated, $\rho 1$ has a tendency to slightly exceed unity ($\rho 1 = 1.023$ in the case of model 2). The detailed parameter estimates for a restricted version of model 2 in which $\rho 1$ is constrained to unity are reported in Table 3.

Table 3: Results for the Preferred Model

Functional Form Restrictions: $\mu = 0$, $\eta = 1$, $\rho 1 = 1$

Functional Form Parameters

$\rho 2$	κ
-8.697 (-3.2)	0.558 (18.8)

		Food	Tobacco	Clothing	Other
Price Index Share Parameters					
P1:	$\beta 1_i$	0.497 (35.9)	0.174 (23.9)	0.229 (25.4)	0.100 (3.5)
P2:	$\beta 2_i$	0.0003 (0.6)	0.009 (1.8)	0.0008 (0.8)	0.989 (146.2)
Aggregation Parameters					
Inflation:	θ_{1i}	0.004 (0.3)	0.058 (2.8)	0.055 (3.1)	-0.117 (-3.1)
Unemployment:	θ_{2i}	0.110 (3.5)	-0.303 (-12.5)	-0.360 (-11.3)	0.554 (9.6)
Participation:	θ_{3i}	-0.158 (-3.1)	-0.237 (-5.8)	-0.254 (-4.0)	0.649 (6.0)
Summary Statistics					
	R^2	0.995	0.954	0.983	0.993
	D.W.	0.77	1.16	1.42	1.11

System Log-likelihood 514.161

6. CONCLUSION

The General Exponential Form (GEF) indirect utility function, introduced in section 3, is particularly designed for empirical application. While not necessarily globally regular, the function may be easily constrained to be regular over an unbounded region which includes all points in any given sample, and all values of nominal expenditure and prices generating higher values of real expenditure. Included as nested special cases are a number of well-known demand systems, including the Linear Expenditure System. The LES is a parsimonious demand system that has been found to fit extremely well in a number of applied studies, but has been criticized for the additive preference structure. The model of this paper allows a simple generalization away from additivity, and an empirical example demonstrates the value of this generalization.

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Appendix 1: Regularity of $U^M(c,p)$

In the paper it is claimed that if $U^M(p,c)$ has the form (3.1) then if P1, P2 satisfy RP1 - RP6 and $0 \le \eta \le 1$, then $U^M(p,c)$ satisfies RIU1 - RIU5 over $P = \{c,p;\ c > 0,\ p > 0,\ (c/P1) \ge 1\}$.

Consider these properties in turn,

RIU1: Follows from RP1 and the continuity of (3.1).

RIU2: Given P1 and P2 satisfy RP3, RIU2 is satisfied over the entire positive orthant Ω_+^{n+1} .

RIU3: $$U^M_{p_i} = -\frac{1}{P1}\left(\frac{c}{\kappa P1}\right)^{\mu}\frac{\partial P1}{\partial p_i}\left(\frac{c}{P2}\right)^{\eta} - \left[\frac{\left(\frac{c}{\kappa P1}\right)^{\mu}-1}{\mu}\right]\eta\left(\frac{c}{P2}\right)^{\eta}\frac{1}{P2}\frac{\partial P2}{\partial p_i}$$

Hence, given P1 and P2 satisfy RP4 and RP2, if $\eta > 0$ then RIU3 will be satisfied at least wherever $(c/P1) \ge 1$.

RIU4: By RIU2, $cU^M_c = -\Sigma p_i U^M_{p_i}$, and hence RIU4 will be satisfied over the same domain as RIU3.

RIU5: Define normalized prices $s_i = p_i/c$.

Let $U^M(p,c) = U^M(s,1) = V(s) = g(s)/f(s)$,

where $g(s) = -\dfrac{\left(\kappa P1(s)\right)^{-\mu}-1}{-\mu}$ and $f(s) = (P2(s))^{\eta}$. Now $\dfrac{x^{-\mu}-1}{-\mu}$ is increasing provided $x > 0$, and concave provided $\mu \ge -1$. Since an increasing concave function of a concave function is concave, the negative sign makes g(s) convex. Provided $\kappa P1(s) < 1$ (i.e. $c > \kappa P1(p)$) then $g(s) \ge 0$. f(s) is concave provided $0 \le \eta \le 1$. From Greenberg and Pierskalla (1971, p.155), g(s)/f(s) is quasi-convex provided g(s) is convex, $g(s) \ge 0$, f(s), ≥ 0, f(s) is concave.