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Abstract
A mean shift can cause tests for a unit root to erroneously fail to reject the null hypothesis of the existence of a unit root. Perron (1990) and Hendry and Neale (1991) provide simulation evidence of this for (augmented) Dickey-Fuller tests in models without a time trend. This paper extends these analyses by considering a wider range of test statistics (including statistics proposed by Bhargava (1986)) applied to models (possibly including a time trend) subject to a shift in mean. Our simulation results show that, at least for alternatives close to the unit root, either an appropriate Bhargava test statistic or the suitably normalised OLS estimator of the unit root has higher power than the Dickey-Fuller or augmented Dickey-Fuller *t*-tests. In particular, in models with a trend, an increase in the mean shift does not reduce the power of Bhargava's R_2 test as much as it reduces the power of the other tests. Estimated response surfaces summarise the likely power loss due to any particular mean shift.

KEYWORDS: Testing; unit roots; structural break; mean shift; simulation.

1. Introduction

A mean shift can cause tests for a unit root to erroneously fail to reject the null hypothesis of the existence of a unit root. Perron (1990) and Hendry and Neale (1991) provide simulation evidence of this for (augmented) Dickey-Fuller tests in models without a time trend. This paper extends these analyses by considering a wider range of test statistics (including statistics proposed by Bhargava (1986)) applied to models (possibly including a time trend) subject to a shift in mean.

In the next section we outline various tests for a unit root, in models with and without a linear trend, and considers how a simple mean shift may be incorporated into the models. Section 3 outlines a Monte Carlo simulation study of the size and power properties of various tests for a unit root when applied to models subject to a mean shift. In Section 4 we show how estimated response surfaces can summarise the likely power loss due to any particular mean shift. Section 5 concludes.

2. Unit Root Tests with Mean Shifts

A simple but common representation of a nonstationary time series is the random walk with drift model

$$y_t = \mu + y_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim IN(0,\sigma^2) \qquad t = 1,\ldots,T$$

We can write

$$y_t = y_0 + \mu t + \sum_{i=1}^{t} \varepsilon_i$$

which is clearly nonstationary since $E[y_t] = \mu t$ is time-dependent. The random walk with drift model is sometimes used to explain a variable that is believed to have a time trend. A common alternative to this is a model which is stationary about a deterministic trend, *i.e.*

$$y_t - (m + \mu t) = \beta(y_{t-1} - (m + \mu(t-1)) + \varepsilon_t \qquad |\beta| < 1$$

where m is the mean of the stationary series $(y_t - \mu t)$. The problem we investigate here is that of a step change in m, such that $m = m_1$ for $t < t_0$, and $m = m_1 + m_2$ for $t \geq t_0$. Both Perron (1990) and Hendry and Neale (1991) argue that mean shifts may characterise many economic time series.

This paper seeks to examine (i) the extent to which the mean shift (or structural change) causes unit root tests to erroneously fail to reject the null hypothesis that a unit root exists, and (ii) the relative performance of a selection of different unit root tests under the influence of the mean shift. Recently, Hendry and Neale (1991) considered the impact of structural change in models without time trends on two Dickey-Fuller unit root tests. This study extends their analysis by considering models both with and without time trends and by examining the effect of mean shifts on unit root tests suggested by Bhargava (1986), in addition to Dickey-Fuller tests.

The Dickey-Fuller tests considered here are those commonly labelled τ_μ (used by Hendry and Neale), τ_τ, ρ_μ, ρ_τ and appropriate versions of the augmented Dickey-Fuller test of order one, labelled ADF(1) here. Note that there are two different ADF(1) tests, one when the augmented regression excludes a linear trend (as in Hendry and Neale's (1991) analysis), and another when the augmented regression includes a linear trend.

The first of the two Bhargava (1986) unit root test statistics considered here is

$$R_1 = \frac{\sum_{t=2}^{T}(y_t - y_{t-1})^2}{\sum_{t=1}^{T}(y_t - \bar{y})^2} \qquad \text{where } \bar{y} = T^{-1}\sum_{t=1}^{T} y_t$$

The R_1 test is a uniformly most powerful invariant test of the null hypothesis that y_t follows a random walk, against the one-sided stationary alternative that

$$(y_t - m) = \beta(y_{t-1} - m) + \varepsilon_t \qquad \varepsilon_t \sim IN(0,\sigma^2) \qquad 0 \leq \beta < 1$$

This test statistic (which is clearly equivalent to the Durbin-Watson test statistic) is invariant to the values of m and σ^2. 1% and 5% critical values are given in the first rows of table 1 of Sargan and Bhargava (1983).

The second of the Bhargava test statistics is

$$R_2 = \frac{\sum_{t=2}^{T}(y_t - y_{t-1})^2 - (T-1)^{-1}(y_T - y_1)^2}{(T-1)^{-2}\sum_{t=1}^{T}(R_t^* - R_t^{**})^2}$$

where $\quad R_t^* = (T-1)y_t - (t-1)y_T - (T-t)y_1$

and $\quad R_t^{**} = (T-1)[\bar{y} - 0.5(y_1 + y_T)]$

This is a locally most powerful invariant test in the neighbourhood of $\beta = 1$ of the null hypothesis that y_t follows a random walk with non-zero constant drift against one-sided stationary alternatives of the form

$$(y_t - m - \mu t) = \beta(y_{t-1} - m - \mu(t-1)) + \varepsilon_t$$

$$\varepsilon_t \sim IN(0,\sigma^2) \qquad 0 \leq \beta < 1$$

The R2 test statistic is invariant to the values of μ, m and σ^2. 5% critical values are given in table 1 of Bhargava (1986). A 1% critical value for sample size T = 100 is estimated by Monte Carlo simulation; the estimated value based on 8101 replications is 0.47.

3. Monte Carlo Simulation

The experimental design of our Monte Carlo simulation analysis sets three parameters as fixed, *i.e.* T = 100, $\sigma^2 = 1$ and $m_1 = 1$. Other parameters vary as follows:

$$m_2 = \{\ 0 \quad 0.5 \quad 1 \quad 2 \quad 4\ \}$$
$$\mu = \{\ 0 \quad 0.001 \quad 0.01 \quad 0.05 \quad 0.1\ \}$$
$$t_0 = \{\ 20 \quad 40 \quad 60 \quad 80\ \}$$
$$\beta = \{\ 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1\ \}$$

The sample size is fixed at T = 100 for two reasons. First, critical values for both the Dickey-Fuller and Bhargava tests seem to be available for only two sample sizes, $T = 50$ and 100. Second, much of the extensive literature on the finite sample performance of unit root tests suggests that for alternative hypotheses with β close to 1 these test all have very low power for small to moderate sample sizes. For example, DeJong *et al.* (1992, p. 431) recommend against using unit root tests when $T < 100$. Given that we fix both $m_1 = 1$ and $\sigma^2 = 1$, our choice of values for m_2 has the effect of defining the mean shift relative to the (standardised) value of σ^2, as is done by Hendry and Neale (1991). This experimental design defines a total of 500 experiments for the test statistics, which we evaluate at both 1% and 5% critical values. We set the number of replications of each experiment to 9604, so as to give a maximum 95% confidence interval for the test rejection frequency of 0.01.

For the experiments where $|\beta| < 1$ the initial value y_0 of the y_t series is given as

$$y_0 = \sigma m_1 + (1-\beta)^{-1/2}\varepsilon_0 \qquad \text{where } \varepsilon_0 \sim IN(0,\sigma^2)$$

so that y_0 has the same distribution as the other values in the (detrended) y_t series for $t < t_0$. As all the test statistics can be shown to be invariant to the value of y_0 when $\beta = 1$, we set $y_0 = 0$ in these cases.

We begin the analysis of the simulation results by commenting on the cases where $\beta = 1$, that is, where the null hypothesis is true so that we are estimating the sizes of the tests (not that these are not reported here in any detail). Using 1% critical values, all the rejection frequencies for every test statistic fall within the approximate 95% confidence interval about 0.01. In fact, if we report the rejection frequencies to two decimal places we find only one case (for the R_1 test) where the rejection frequency is not equal to 0.01.

For 5% critical values, examining the results for $\beta = 1$ we find for the Dickey-Fuller, R_2 and ADF(1) tests all the rejection frequencies fall between

0.04 and 0.06. Even to two decimal places there are only a couple of cases where the rejection frequency is not observed to be 0.05. The story for the R_1 test is rather different. The R_1 test is appropriate only when $\mu = 0$. As μ increases, *ceteris paribus*, the value of the R_1 test statistic falls, and hence so do rejection frequencies for the R_1 test. As a result, in 27 out of 100 experiments (with $\beta = 1$) the rejection frequency is not equal to 0.05 to two decimal places (in fact these are all *less* than 0.05) and in 21 of these cases the rejection frequency is less than 0.04. These cases all occur for relatively large values of μ (0.05 or 0.1).

In general these results indicate that the existence of a mean shift (*i.e.* when $m_2 \neq 0$) of which the econometrician is unaware does not prevent finding a unit root if one is present. Except for the problems with R_1 noted above, we can use the tabulated critical values to estimate the powers of the tests.

We now consider the cases where $m_2 = 0$ (and $|\beta| < 1$), so that no mean shift occurs at all (see Tables 1 and 2). Rejection frequencies will therefore be estimates of the power of the tests for a unit root. As one would expect, for any given value of μ rejection frequencies for each test decline as β increases towards one. For $\mu = 0$, R_1 seems to be the most powerful of the tests considered, rejection frequencies only deviating significantly from 1 when $\beta = 0.8$. R_1 is followed by ρ_μ, τ_μ, ADF(1) and R_2 in decreasing order of power. The R_2 test performs relatively poorly because it is inappropriate in situations where the alternative model has no time trend.

For μ non-zero (again with $m_2 = 0$, $|\beta| < 1$) the situation is slightly different. The ρ_τ, τ_τ, ADF(1) and R_2 tests are all invariant to the value of μ in the model. This is clearly illustrated by examining the rejection frequencies for these test statistics in Tables 1 and 2 for different non-zero values of μ and any given value of β and noting how little they vary. The R_1 test, however, is not invariant to μ and rejection frequencies for this test fall as μ increases, making it the least powerful of the tests when $\mu \neq 0$. This is explained by the fact that the R_1 test is inappropriate if the alternative model has a time trend – in such a situation the econometrician should prefer the R_2 test (one might regard using the R_1 test in situations where $\mu \neq 0$ as equivalent to wrongly performing a τ_μ test instead of a τ_τ test). For the other tests it is clear that in terms of decreasing power we have the order ρ_τ, τ_τ, and ADF(1) for all values of β. The relative power of the R_2 test changes with the value of β. For $\beta < 0.8$ the power of R_2 is between that of τ_τ and ADF(1). However, when $\beta = 0.8$ the R_2 test is the most powerful of the tests considered. This is because R_2 is locally most powerful in the neighbourhood of $\beta = 1$, and so we would expect it to out-perform the other tests (for $\mu \neq 0$) as β approaches 1.

One general point to note is that the rejection frequencies do not differ very much from 1 until β reaches 0.8 (with the exception of the R_1 test for non-zero values of μ). For this reason, we concentrate on values of $\beta \geq 0.6$, and extend the analysis to include values of $\beta = 0.85, 0.9, 0.95$. Tables 3 to 6 show the remainder of the results. Since the R_2 test and appropriate versions of the τ, ρ and ADF(1) tests are invariant to the value of μ we only show the effects of $\mu = 0$ and $\mu = 0.05$ on these tests.

Consider the cases where $\mu = 0$ ($|\beta| < 1$) and tests are performed using 1% critical values (Table 3; note that the results at 5% in Table 5 follow very similar patterns). For a small mean shift ($m_2 = 0.5$ or 1) we find that the rejection frequencies are not very much smaller than for the situation where $m_2 = 0$. However as m_2 increases the rejection frequencies tend to fall. This is most noticable when $m_2 = 4$ and the rejection frequencies fall very close to zero for every test statistic when $\beta = 0.8$. In fact, when $\beta = 0.8$ a mean shift of $m_2 = 2$ is sufficient to approximately halve the power of all the tests (except R_2) from the level experienced for $m_2 = 0$.

Indeed, we may identify some cases where the tests are biased, in the sense that there seems to be a lower probability of rejecting the null hypothesis that $\beta = 1$ when it is false than when it is true (indicated in Table 3 by rejection frequencies of less than 0.01). For example, see the rejection frequencies for the τ_{μ} test when $m_2 = 4$, $\beta = 0.6$ and $t_0 = 60$, and for the ADF(1) test when $m_2 = 4$, $\beta = 0.6$ or 0.8, and $t_0 = 40$ or 60. Values less than 0.008 are significantly biased in 9604 replications using a 5% test.

The position of the mean shift (t_0) does seem to have some effect on the rejection frequencies. Examination of Table 3 indicates that we have similar results, in general, for $t_0 = 20$ and 80, and similar results for $t_0 = 40$ or 60, with higher rejection frequencies in the former case.

The comparative performance of the tests for $\mu = 0$ and m_2 non-zero is very similar to the $\mu = m_2 = 0$ case. Thus the order in terms of decreasing power seems to be R_1, ρ_{μ}, τ_{μ} and ADF(1). The R_2 test is inappropriate here (as $\mu = 0$), so we might expect it to be the least powerful of the tests in this case. This is true for $m_2 = 0.5$, but as m_2 increases the relative performance of the R_2 test improves (although its absolute power declines), so that when $m_2 = 4$ it is the most powerful of the tests considered. Thus the mean shift does not reduce the power of the R_2 test as much as it reduces the power of the other tests.

We now turn to the cases where $\mu = 0.05$ ($|\beta| < 1$) and tests are performed using 1% critical values (Table 4; again the results for 5% follow similar patterns: see Table 6). In general we observe that as either m_2 or β increase the rejection frequencies for each test fall. It is fair to say therefore that the results in Table 4 follow a similar pattern to those in Table 3, with the notable exception of the R_1 test. Not being invariant to the value of μ, the power of the R_1 test falls towards zero as μ increases, and so the R_1 test is the least powerful test for μ non-zero.

One important difference between the results of Tables 3 and 4 is the effect of the position of the mean shift (t_0). It is true that for some cases where $\mu = 0.05$ the effect of the value of t_0 on rejection frequencies is the same as for the $\mu = 0$ cases. However, this seems to be the exception rather than the rule, and the precise effect of the value of t_0 seems unclear when μ is non-zero.

Comparing the performance of the tests from Table 4 leads us to exactly the same conclusions as we reached from Table 1 with R_2 being the most powerful test for β close to 1, and ρ_τ being the most powerful elsewhere. Note that these results hold for all other non-zero values of μ not reported here. The R_1 test is not invariant to the value of μ. This causes the R_1 test to be biased for many large values of μ (for example $\mu = 0.1$). In fact, as the value of μ increases, the power of the R_1 test falls to zero for any given value of m_2 and β, since the calculated value of the R_1 test statistic will fall as μ increases.

To summarise, we find that the introduction of a simple mean shift reduces the power of all of the tests for a unit root considered here. This reduction is more pronounced when the trend parameter μ is non-zero. However, the power advantage of the ρ_μ and R_1 tests over the more commonly used τ_μ and ADF(1) tests (when $\mu = 0$), and of the ρ_τ and R_2 tests over the τ_τ and ADF(1) tests (when $\mu \neq 0$), is maintained when $m_2 \neq 0$. It also seems to be the case that the larger is the mean shift m_2, the greater is the advantage of the R_1 and R_2 tests (when $\mu = 0$ and $\mu \neq 0$ respectively) over the Dickey-Fuller and augmented Dickey-Fuller tests. As an illustration of this, Figure 1 indicates the power curves (generated for $\beta = 0.6\ (0.05)\ 1$) of the tests when $m_2 = 0$ (the top four curves) and for $m_2 = 4$ (the bottom four curves), in both cases when $\mu = 0.05$. Notice how the power of the R_2 test dominates that of the other tests for $0.6 \leq \beta \leq 1$ when $m_2 = 4$.

4. Response Surfaces

We fit a response surface for the rejection frequencies for each of the test statistics. Response surfaces summarise the relationship between rejection frequency and the parameters of the simulation (here, m_2, μ, β and t_0). These estimated response surfaces should give us some idea of the probable power loss due to any particular mean shift. Since we are concerned with power we can exclude all experiments where $\beta = 1$.

It has already been indicated that the R_1 test is inappropriate in situations where μ is non-zero, and so as we are interested in examining the effect of a mean-shift on unit root tests, it is sensible to fit the response surface for R_1 only for those experiments where $\mu = 0$. Similarly, the response surface for the R_2 test is estimated only for those experiments with non-zero values of μ.

In addition, since the τ_τ, ρ_τ, R_2 tests and appropriate versions of the ADF(1) test are invariant to the value of μ we need only consider one value of μ, and so we fit response surfaces for the above tests only for experiments where $\mu = 0.05$. We thus estimate eight response surfaces, each with a maximum of 80 experiments.

For the rejection frequency π of each test statistic we use the logistic transformation $\ln(\pi/(1-\pi))$; note that this means that experiments with $\pi = 0$ or 1 will be omitted from the response surface. All variables in the response surfaces are multiplied by $(N\pi(1-\pi))^{1/2}$ (where N is the number of replications of each experiment; here this is 9604) so that each transformed rejection frequency will have unit variance. This heteroscedasticity transformation is also used by Hendry and Neale (1991).

When $m_2 = 0$ we need to explain the power of each test by β alone. In fact, preliminary investigation showed that using β^3 provided a better fit in each response surface regression than did β. Since the simulation results indicated similar patterns for $t_0 = 20$ or 80, and for $t_0 = 40$ or 60 (as indicated by Perron (1990), we also use km_2 as well as m_2, where $k = \min[t_0/100, (1-t_0/100)]$. As an illustration, we report below response surfaces of the rejection frequencies of tests using 1% critical values.

$$\mathrm{HL}(\tau_\mu) = \underset{(7.66)}{4.5075\mathrm{H}} - \underset{(-7.42)}{7.3890\mathrm{H}\beta^3} - \underset{(-3.64)}{0.6706\mathrm{Hm}_{2\ 2}} - \underset{(-2.83)}{1.6268\mathrm{H}k\mathrm{m}_{2\ 2}}$$

$R^2 = 0.6019$ DW = 0.7912 53 observations
AR(1) = 22.96 Norm(2) = 5.05 Hetero(1) = 1.05

$$\mathrm{HL}(\tau_\tau) = \underset{(14.31)}{5.0193\mathrm{H}} - \underset{(-16.48)}{10.3565\mathrm{H}\beta^3} - \underset{(-7.66)}{0.9598\mathrm{Hm}_{2\ 2}} + \underset{(2.40)}{0.7921\mathrm{H}k\mathrm{m}_{2\ 2}}$$

$R^2 = 0.8494$ DW = 0.8370 52 observations
AR(1) = 17.54 Norm(2) = 10.27 Hetero(1) = 1.30

$$\mathrm{HL}(\rho_\mu) = \underset{(8.39)}{4.5473\mathrm{H}} - \underset{(-7.08)}{6.2906\mathrm{H}\beta^3} - \underset{(-3.57)}{0.5343\mathrm{Hm}_{2\ 2}} - \underset{(-4.31)}{1.8739\mathrm{H}k\mathrm{m}_{2\ 2}}$$

$R^2 = 0.6824$ DW = 0.5817 46 observations
AR(1) = 24.16 Norm(2) = 2.31 Hetero(1) = 0.94

$$\mathrm{HL}(\rho_\tau) = \underset{(13.13)}{4.9982\mathrm{H}} - \underset{(-14.82)}{9.9118\mathrm{H}\beta^3} - \underset{(-6.90)}{0.8623\mathrm{Hm}_{2\ 2}} + \underset{(2.23)}{0.7355\mathrm{H}k\mathrm{m}_{2\ 2}}$$

$R^2 = 0.8185$ DW = 0.8325 51 observations
AR(1) = 17.51 Norm(2) = 10.51 Hetero(1) = 0.11

$$\mathrm{HL}(\mathrm{R}_1) = \underset{(8.77)}{4.7176\mathrm{H}} - \underset{(-7.19)}{6.2900\mathrm{H}\beta^3} - \underset{(-3.65)}{0.5315\mathrm{Hm}_{2\ 2}} - \underset{(-4.54)}{1.9070\mathrm{H}k\mathrm{m}_{2\ 2}}$$

$R^2 = 0.7136$ DW = 0.6039 43 observations
AR(1) = 20.63 Norm(2) = 1.58 Hetero(1) = 1.13

$$\mathrm{HL}(\mathrm{R}_2) = \underset{(23.92)}{4.9170\mathrm{H}} - \underset{(-25.45)}{9.7021\mathrm{H}\beta^3} - \underset{(-7.81)}{0.6330\mathrm{Hm}_{2\ 2}} + \underset{(0.32)}{0.0719\mathrm{H}k\mathrm{m}_{2\ 2}}$$

$R^2 = 0.8849$ DW = 0.6345 70 observations
AR(1) = 32.49 Norm(2) = 15.15 Hetero(1) = 0.003

$$\underset{[\mu\ =\ 0]}{\mathrm{HL(ADF(1))}} = \underset{(12.98)}{3.8714\mathrm{H}} - \underset{(-12.69)}{6.7325\mathrm{H}\beta^3} - \underset{(-4.43)}{0.7642\mathrm{Hm}_{2\ 2}} - \underset{(-3.04)}{1.7074\mathrm{H}k\mathrm{m}_{2\ 2}}$$

$R^2 = 0.7481$ DW = 0.8737 65 observations
AR(1) = 21.88 Norm(2) = 8.49 Hetero(1) = 0.03

$$\underset{[\mu\ =\ 0.05]}{\mathrm{HL(ADF(1))}} = \underset{(14.56)}{3.6990\mathrm{H}} - \underset{(-14.45)}{7.7630\mathrm{H}\beta^3} - \underset{(-7.69)}{1.1693\mathrm{Hm}_{2\ 2}} + \underset{(1.51)}{0.6086\mathrm{H}k\mathrm{m}_{2\ 2}}$$

$R^2 = 0.8000$ DW = 0.3749 72 observations
AR(1) = 47.89 Norm(2) = 9.09 Hetero(1) = 0.86

Numbers in parentheses below estimated coefficients are asymptotic *t*-ratios;

the serial correlation (denoted AR), normality, and heteroscedasticity tests are all asymptotically distributed as χ^2, with degrees of freedom given in parentheses.

As might be expected, the regressors β^3 and m_2 have negative coefficients in every response surface, indicating that power falls as β approaches 1, or as m_2 increases. However, the R^2 for each response surface is not particularly high. This may be because this Monte Carlo study uses a wide range of values of β, whereas other similar studies (for example, Hendry and Neale (1991)) have considered a smaller range of values of β so that any power function may be approximately linear.

The heteroscedasticity transformation seems to perform well, as we are unable to reject the null hypothesis of homoscedastic errors for any of the response surfaces, even at a 10% significance level. However, there is clear evidence of serial correlation in the residuals of each response surface regression. As our regression data have no specific order, it may be possible to reorder the data in such a way so as to remove this serial correlation. However, taken together with the relatively low R^2 values this evidence of serial correlation suggests that the relationships are misspecified, so that it is likely that the response surfaces could be improved upon.

We may use the response surfaces to predict the power loss for each test due to a particular mean shift. If a test (labelled Z) has an estimated response surface given by

$$H\hat{L}(Z) = \hat{\alpha}_0 H + \hat{\alpha}_1 H\beta^3 + \hat{\alpha}_2 Hm_2 + \hat{\alpha}_3 Hkm_2 \quad ,$$

then the power of the test will be given by $\exp(\gamma)[1+\exp(\gamma)]^{-1}$, where

$$\gamma = \hat{\alpha}_0 + \hat{\alpha}_1\beta^3 + \hat{\alpha}_2 m_2 + \hat{\alpha}_3 km_2 \quad .$$

5. Conclusions

Our simulation results show that, at least for alternatives close to the unit root, either an appropriate Bhargava test statistic or the suitably normalised OLS estimator of the unit root (*i.e.* ρ_{μ} or ρ_{τ}) has higher power than the Dickey-Fuller or augmented Dickey-Fuller *t*-tests. In particular, in models with a trend, an increase in the mean shift does not reduce the power of Bhargava's R_2 test as much as it reduces the power of the other tests. Estimated response surfaces illustrate how we may summarise the likely power loss due to any particular mean shift.

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Table 1

Rejection Frequencies Without a Mean Shift ($m_2 = 0$)

1% critical values

μ	β	τ_μ/τ_τ	ρ_μ/ρ_τ	R_1	R_2	ADF(1)
0	0.2	1.000	1.000	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.998	1.000
	0.6	0.998	1.000	1.000	0.952	0.969
	0.8	0.517	0.683	0.719	0.379	0.414
	1	0.010	0.010	0.010	0.010	0.009
0.001	0.2	1.000	1.000	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.997	0.994
	0.6	0.975	0.989	1.000	0.952	0.857
	0.8	0.296	0.364	0.719	0.380	0.235
	1	0.011	0.010	0.009	0.012	0.011
0.01	0.2	1.000	1.000	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.996	0.994
	0.6	0.976	0.989	0.999	0.955	0.861
	0.8	0.301	0.375	0.682	0.375	0.237
	1	0.011	0.010	0.010	0.011	0.010
0.05	0.2	1.000	1.000	0.963	1.000	1.000
	0.4	1.000	1.000	0.762	0.996	0.995
	0.6	0.978	0.991	0.363	0.952	0.865
	0.8	0.300	0.371	0.125	0.383	0.230
	1	0.010	0.011	0.010	0.011	0.011
0.1	0.2	1.000	1.000	0.000	1.000	1.000
	0.4	1.000	1.000	0.000	0.996	0.995
	0.6	0.978	0.988	0.000	0.947	0.860
	0.8	0.295	0.367	0.000	0.382	0.227
	1	0.010	0.010	0.007	0.010	0.010

Table 2

Rejection Frequencies Without a Mean Shift ($m_2 = 0$)

5% critical values

μ	β	τ_μ/τ_τ	ρ_μ/ρ_τ	R_1	R_2	ADF(1)
0	0.2	1.000	1.000	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.993	0.999
	0.8	0.879	0.955	0.969	0.722	0.798
	1	0.052	0.050	0.053	0.048	0.052
0.001	0.2	1.000	1.000	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	1.000	1.000
	0.6	0.999	1.000	1.000	0.992	0.981
	0.8	0.662	0.739	0.969	0.719	0.568
	1	0.051	0.050	0.051	0.050	0.051
0.01	0.2	1.000	1.000	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	1.000	1.000
	0.6	0.999	1.000	1.000	0.992	0.984
	0.8	0.661	0.735	0.956	0.718	0.564
	1	0.051	0.050	0.053	0.049	0.052
0.05	0.2	1.000	1.000	1.000	1.000	1.000
	0.4	1.000	1.000	0.996	1.000	1.000
	0.6	0.999	1.000	0.891	0.991	0.982
	0.8	0.664	0.741	0.454	0.726	0.562
	1	0.053	0.051	0.048	0.051	0.054
0.1	0.2	1.000	1.000	0.008	1.000	1.000
	0.4	1.000	1.000	0.000	1.000	1.000
	0.6	0.999	1.000	0.000	0.992	0.981
	0.8	0.666	0.745	0.002	0.720	0.562
	1	0.050	0.049	0.035	0.048	0.051

Table 3

Rejection Frequencies at 1% critical values when $\mu = 0$

m_2	β	t_0	τ_μ	ρ_μ	R_1	R_2	ADF(1)
0	0.6		0.998	1.000	1.000	0.952	0.969
	0.8		0.517	0.683	0.719	0.379	0.414
	0.85		0.255	0.383	0.418	0.181	0.218
	0.9		0.095	0.146	0.160	0.069	0.090
	0.95		0.027	0.040	0.042	0.023	0.027
0.5	0.6	20	0.998	1.000	1.000	0.942	0.951
		40	0.994	0.999	0.999	0.943	0.941
		60	0.995	0.999	1.000	0.947	0.942
		80	0.996	0.999	1.000	0.947	0.950
	0.8	20	0.485	0.662	0.700	0.358	0.390
		40	0.486	0.658	0.696	0.372	0.391
		60	0.486	0.658	0.694	0.373	0.396
		80	0.491	0.666	0.701	0.373	0.397
	0.85	20	0.252	0.368	0.402	0.181	0.213
		40	0.249	0.369	0.406	0.177	0.203
		60	0.253	0.377	0.412	0.179	0.211
		80	0.242	0.376	0.410	0.176	0.205
	0.9	20	0.088	0.140	0.155	0.064	0.082
		40	0.085	0.144	0.159	0.065	0.082
		60	0.089	0.143	0.157	0.066	0.086
		80	0.088	0.142	0.156	0.067	0.081
	0.95	20	0.029	0.043	0.046	0.022	0.029
		40	0.026	0.038	0.042	0.021	0.025
		60	0.025	0.039	0.041	0.023	0.026
		80	0.028	0.040	0.043	0.026	0.026
1.0	0.6	20	0.991	0.999	0.999	0.936	0.906
		40	0.982	0.996	0.997	0.940	0.854
		60	0.980	0.996	0.998	0.935	0.848
		80	0.988	0.998	0.999	0.938	0.891
	0.8	20	0.451	0.608	0.644	0.353	0.352
		40	0.421	0.574	0.613	0.354	0.328
		60	0.420	0.588	0.630	0.354	0.332
		80	0.448	0.617	0.660	0.351	0.357
	0.85	20	0.226	0.343	0.372	0.169	0.193
		40	0.208	0.325	0.352	0.173	0.172
		60	0.211	0.324	0.358	0.179	0.178
		80	0.221	0.343	0.380	0.174	0.187
	0.9	20	0.087	0.138	0.153	0.068	0.082
		40	0.081	0.127	0.142	0.065	0.072
		60	0.079	0.130	0.145	0.065	0.075
		80	0.084	0.139	0.154	0.064	0.076
	0.95	20	0.026	0.040	0.043	0.022	0.026
		40	0.026	0.041	0.044	0.024	0.023
		60	0.023	0.036	0.040	0.019	0.024
		80	0.026	0.038	0.040	0.021	0.025

2.0	0.6	20	0.917	0.969	0.971	0.877	0.664
		40	0.751	0.880	0.899	0.872	0.400
		60	0.720	0.870	0.900	0.868	0.364
		80	0.849	0.955	0.972	0.878	0.551
	0.8	20	0.318	0.439	0.453	0.283	0.236
		40	0.226	0.337	0.365	0.302	0.163
		60	0.219	0.341	0.381	0.295	0.163
		80	0.296	0.448	0.495	0.286	0.220
	0.85	20	0.166	0.244	0.258	0.146	0.138
		40	0.130	0.199	0.218	0.147	0.106
		60	0.130	0.212	0.239	0.149	0.107
		80	0.167	0.266	0.299	0.147	0.135
	0.9	20	0.066	0.104	0.115	0.061	0.059
		40	0.061	0.100	0.110	0.062	0.058
		60	0.060	0.098	0.111	0.062	0.055
		80	0.074	0.119	0.135	0.064	0.068
	0.95	20	0.025	0.039	0.041	0.024	0.026
		40	0.023	0.035	0.038	0.024	0.023
		60	0.023	0.036	0.038	0.020	0.024
		80	0.023	0.034	0.036	0.022	0.022
4.0	0.6	20	0.225	0.290	0.246	0.431	0.063
		40	0.011	0.027	0.030	0.492	0.001
		60	0.005	0.021	0.032	0.487	0.001
		80	0.069	0.198	0.289	0.431	0.013
	0.8	20	0.066	0.085	0.075	0.118	0.044
		40	0.014	0.023	0.024	0.146	0.009
		60	0.012	0.026	0.033	0.140	0.008
		80	0.045	0.103	0.135	0.116	0.035
	0.85	20	0.049	0.063	0.060	0.075	0.037
		40	0.016	0.025	0.028	0.088	0.012
		60	0.017	0.026	0.031	0.085	0.012
		80	0.047	0.088	0.110	0.077	0.040
	0.9	20	0.031	0.041	0.042	0.044	0.028
		40	0.020	0.029	0.030	0.048	0.018
		60	0.017	0.030	0.035	0.042	0.016
		80	0.037	0.061	0.068	0.043	0.031
	0.95	20	0.017	0.024	0.025	0.020	0.018
		40	0.017	0.024	0.026	0.022	0.016
		60	0.016	0.026	0.028	0.021	0.015
		80	0.020	0.030	0.034	0.019	0.019

Table 4

Rejection Frequencies at 1% critical values when $\mu = 0.05$

m_2	β	t_0	τ_τ	ρ_τ	R_1	R_2	ADF(1)
0	0.6		0.978	0.991	0.363	0.952	0.865
	0.8		0.300	0.371	0.125	0.383	0.230
	0.85		0.131	0.171	0.085	0.180	0.112
	0.9		0.052	0.063	0.057	0.070	0.047
	0.95		0.018	0.020	0.030	0.022	0.017
0.5	0.6	20	0.971	0.985	0.199	0.951	0.846
		40	0.972	0.988	0.145	0.947	0.840
		60	0.973	0.988	0.148	0.949	0.848
		80	0.966	0.984	0.203	0.951	0.835
	0.8	20	0.287	0.357	0.071	0.368	0.221
		40	0.298	0.367	0.060	0.373	0.230
		60	0.291	0.357	0.061	0.370	0.219
		80	0.286	0.355	0.079	0.367	0.221
	0.85	20	0.127	0.163	0.056	0.179	0.110
		40	0.134	0.168	0.053	0.178	0.114
		60	0.127	0.161	0.053	0.177	0.108
		80	0.137	0.176	0.066	0.184	0.112
	0.9	20	0.050	0.062	0.043	0.070	0.046
		40	0.053	0.063	0.041	0.067	0.048
		60	0.048	0.060	0.042	0.062	0.043
		80	0.051	0.061	0.047	0.071	0.045
	0.95	20	0.018	0.020	0.023	0.025	0.017
		40	0.018	0.021	0.025	0.021	0.019
		60	0.020	0.022	0.027	0.022	0.017
		80	0.021	0.021	0.028	0.023	0.020
1.0	0.6	20	0.950	0.973	0.081	0.937	0.790
		40	0.958	0.978	0.031	0.931	0.806
		60	0.957	0.978	0.034	0.931	0.801
		80	0.953	0.976	0.091	0.939	0.786
	0.8	20	0.272	0.333	0.040	0.344	0.207
		40	0.278	0.342	0.025	0.354	0.213
		60	0.276	0.345	0.026	0.352	0.216
		80	0.274	0.334	0.048	0.355	0.215
	0.85	20	0.127	0.162	0.037	0.172	0.105
		40	0.124	0.159	0.028	0.175	0.105
		60	0.127	0.158	0.030	0.172	0.108
		80	0.126	0.160	0.049	0.167	0.102
	0.9	20	0.050	0.063	0.034	0.071	0.044
		40	0.047	0.056	0.027	0.061	0.044
		60	0.048	0.059	0.030	0.066	0.048
		80	0.050	0.061	0.039	0.068	0.045
	0.95	20	0.019	0.021	0.024	0.023	0.018
		40	0.018	0.021	0.025	0.023	0.017
		60	0.019	0.021	0.025	0.021	0.018
		80	0.020	0.024	0.026	0.022	0.020

2.0	0.6	20	0.849	0.902	0.006	0.879	0.576
		40	0.880	0.931	0.000	0.872	0.630
		60	0.888	0.931	0.000	0.874	0.649
		80	0.843	0.898	0.008	0.882	0.565
	0.8	20	0.213	0.265	0.009	0.295	0.162
		40	0.243	0.296	0.002	0.311	0.182
		60	0.232	0.285	0.003	0.299	0.175
		80	0.216	0.269	0.015	0.294	0.163
	0.85	20	0.105	0.136	0.012	0.145	0.086
		40	0.109	0.136	0.004	0.146	0.092
		60	0.109	0.137	0.006	0.144	0.095
		80	0.106	0.133	0.019	0.148	0.087
	0.9	20	0.043	0.053	0.016	0.060	0.042
		40	0.045	0.054	0.012	0.060	0.043
		60	0.044	0.054	0.014	0.059	0.042
		80	0.044	0.051	0.025	0.060	0.043
	0.95	20	0.019	0.020	0.015	0.022	0.021
		40	0.019	0.020	0.018	0.022	0.020
		60	0.018	0.020	0.017	0.024	0.018
		80	0.018	0.019	0.020	0.020	0.016
4.0	0.6	20	0.264	0.347	0.000	0.439	0.089
		40	0.397	0.501	0.000	0.484	0.170
		60	0.435	0.512	0.000	0.480	0.187
		80	0.261	0.343	0.000	0.441	0.084
	0.8	20	0.076	0.101	0.000	0.122	0.055
		40	0.107	0.136	0.000	0.142	0.077
		60	0.106	0.129	0.000	0.132	0.082
		80	0.080	0.106	0.000	0.120	0.059
	0.85	20	0.051	0.066	0.001	0.082	0.043
		40	0.062	0.081	0.000	0.086	0.053
		60	0.064	0.077	0.000	0.080	0.054
		80	0.052	0.066	0.002	0.080	0.043
	0.9	20	0.031	0.039	0.003	0.043	0.028
		40	0.034	0.042	0.001	0.046	0.030
		60	0.030	0.034	0.001	0.041	0.029
		80	0.031	0.038	0.008	0.042	0.029
	0.95	20	0.018	0.020	0.009	0.021	0.017
		40	0.017	0.018	0.009	0.018	0.017
		60	0.016	0.017	0.009	0.019	0.017
		80	0.015	0.016	0.015	0.019	0.015

Table 5

Rejection Frequencies at 5% critical values when $\mu = 0$

m_2	β	t_0	τ_μ	ρ_μ	R_1	R_2	ADF(1)
0	0.6		1.000	1.000	1.000	0.993	0.999
	0.8		0.879	0.955	0.969	0.722	0.798
	0.85		0.639	0.780	0.821	0.475	0.575
	0.9		0.337	0.460	0.502	0.240	0.314
	0.95		0.123	0.169	0.182	0.095	0.120
0.5	0.6	20	1.000	1.000	1.000	0.991	0.998
		40	1.000	1.000	1.000	0.991	0.998
		60	1.000	1.000	1.000	0.991	0.996
		80	1.000	1.000	1.000	0.992	0.997
	0.8	20	0.864	0.946	0.961	0.706	0.779
		40	0.868	0.951	0.962	0.717	0.775
		60	0.861	0.944	0.944	0.713	0.779
		80	0.868	0.949	0.965	0.720	0.783
	0.85	20	0.630	0.770	0.808	0.472	0.561
		40	0.626	0.765	0.804	0.464	0.563
		60	0.632	0.770	0.810	0.474	0.565
		80	0.630	0.769	0.814	0.467	0.562
	0.9	20	0.318	0.437	0.477	0.229	0.296
		40	0.321	0.450	0.488	0.232	0.300
		60	0.329	0.455	0.494	0.231	0.315
		80	0.329	0.456	0.493	0.232	0.303
	0.95	20	0.130	0.175	0.192	0.095	0.131
		40	0.125	0.172	0.185	0.096	0.117
		60	0.126	0.172	0.190	0.097	0.122
		80	0.128	0.174	0.187	0.098	0.128
1.0	0.6	20	1.000	1.000	1.000	0.989	0.993
		40	1.000	1.000	1.000	0.988	0.988
		60	1.000	1.000	1.000	0.989	0.984
		80	1.000	1.000	1.000	0.990	0.989
	0.8	20	0.841	0.930	0.945	0.694	0.752
		40	0.804	0.906	0.928	0.692	0.708
		60	0.809	0.912	0.933	0.708	0.717
		80	0.830	0.928	0.952	0.688	0.742
	0.85	20	0.600	0.740	0.775	0.454	0.528
		40	0.574	0.720	0.762	0.462	0.507
		60	0.569	0.710	0.755	0.461	0.501
		80	0.591	0.746	0.794	0.460	0.529
	0.9	20	0.313	0.433	0.470	0.235	0.297
		40	0.297	0.416	0.453	0.229	0.279
		60	0.309	0.426	0.466	0.228	0.285
		80	0.322	0.445	0.488	0.229	0.301
	0.95	20	0.123	0.171	0.183	0.094	0.122
		40	0.123	0.167	0.182	0.099	0.121
		60	0.118	0.164	0.180	0.090	0.119
		80	0.121	0.167	0.181	0.090	0.119

2.0	0.6	20	0.998	0.999	1.000	0.980	0.946
		40	0.978	0.995	0.997	0.975	0.811
		60	0.967	0.994	0.997	0.974	0.782
		80	0.987	0.999	1.000	0.983	0.883
	0.8	20	0.712	0.831	0.848	0.621	0.608
		40	0.592	0.738	0.774	0.632	0.485
		60	0.582	0.736	0.784	0.633	0.467
		80	0.668	0.830	0.883	0.623	0.569
	0.85	20	0.486	0.625	0.650	0.409	0.427
		40	0.412	0.544	0.582	0.412	0.355
		60	0.410	0.554	0.607	0.420	0.355
		80	0.475	0.639	0.705	0.409	0.419
	0.9	20	0.273	0.368	0.394	0.221	0.246
		40	0.245	0.339	0.365	0.217	0.228
		60	0.244	0.341	0.371	0.213	0.221
		80	0.271	0.388	0.435	0.218	0.253
	0.95	20	0.119	0.160	0.172	0.097	0.114
		40	0.117	0.153	0.164	0.092	0.110
		60	0.112	0.154	0.170	0.091	0.110
		80	0.116	0.160	0.176	0.093	0.112
4.0	0.6	20	0.773	0.865	0.832	0.839	0.421
		40	0.208	0.371	0.411	0.821	0.045
		60	0.144	0.321	0.424	0.823	0.025
		80	0.402	0.715	0.857	0.834	0.138
	0.8	20	0.314	0.382	0.356	0.365	0.233
		40	0.107	0.165	0.181	0.404	0.073
		60	0.092	0.168	0.206	0.393	0.061
		80	0.219	0.384	0.501	0.374	0.165
	0.85	20	0.224	0.273	0.263	0.252	0.186
		40	0.093	0.139	0.151	0.285	0.078
		60	0.086	0.144	0.175	0.275	0.068
		80	0.195	0.326	0.411	0.259	0.165
	0.9	20	0.147	0.181	0.180	0.161	0.135
		40	0.092	0.127	0.140	0.175	0.086
		60	0.087	0.142	0.165	0.163	0.082
		80	0.152	0.239	0.287	0.160	0.142
	0.95	20	0.085	0.107	0.113	0.085	0.084
		40	0.076	0.105	0.116	0.087	0.076
		60	0.082	0.110	0.122	0.082	0.080
		80	0.097	0.137	0.155	0.081	0.096

Table 6

Rejection Frequencies at 5% critical values when $\mu = 0.05$

m_2	β	t_0	τ_τ	ρ_τ	R_1	R_2	ADF(1)
0	0.6		0.999	1.000	0.891	0.991	0.982
	0.8		0.664	0.741	0.454	0.726	0.562
	0.85		0.403	0.474	0.333	0.474	0.348
	0.9		0.190	0.225	0.232	0.233	0.178
	0.95		0.087	0.095	0.130	0.092	0.085
0.5	0.6	20	0.999	1.000	0.778	0.993	0.979
		40	0.999	1.000	0.713	0.991	0.978
		60	0.999	1.000	0.718	0.991	0.980
		80	0.999	1.000	0.783	0.992	0.976
	0.8	20	0.646	0.724	0.336	0.716	0.548
		40	0.653	0.729	0.302	0.707	0.559
		60	0.656	0.734	0.311	0.707	0.555
		80	0.647	0.718	0.362	0.703	0.546
	0.85	20	0.392	0.459	0.259	0.473	0.342
		40	0.398	0.463	0.238	0.469	0.343
		60	0.387	0.455	0.232	0.462	0.338
		80	0.398	0.472	0.275	0.477	0.353
	0.9	20	0.196	0.233	0.190	0.233	0.180
		40	0.201	0.239	0.183	0.239	0.188
		60	0.186	0.220	0.182	0.229	0.177
		80	0.193	0.229	0.197	0.231	0.179
	0.95	20	0.082	0.091	0.116	0.094	0.080
		40	0.082	0.090	0.115	0.094	0.085
		60	0.087	0.094	0.120	0.091	0.084
		80	0.087	0.098	0.124	0.096	0.089
1.0	0.6	20	0.998	0.999	0.592	0.991	0.963
		40	0.998	1.000	0.428	0.988	0.969
		60	0.997	0.999	0.426	0.988	0.970
		80	0.997	0.999	0.612	0.989	0.962
	0.8	20	0.618	0.695	0.236	0.684	0.524
		40	0.633	0.716	0.185	0.700	0.533
		60	0.632	0.708	0.190	0.689	0.539
		80	0.628	0.704	0.273	0.693	0.534
	0.85	20	0.393	0.460	0.192	0.460	0.338
		40	0.386	0.453	0.152	0.459	0.342
		60	0.392	0.461	0.160	0.461	0.344
		80	0.384	0.446	0.225	0.460	0.334
	0.9	20	0.193	0.233	0.151	0.236	0.180
		40	0.188	0.221	0.133	0.229	0.175
		60	0.187	0.218	0.143	0.227	0.177
		80	0.189	0.218	0.170	0.231	0.176
	0.95	20	0.085	0.091	0.110	0.094	0.081
		40	0.084	0.093	0.105	0.094	0.081
		60	0.085	0.092	0.110	0.091	0.084
		80	0.086	0.096	0.126	0.098	0.085

2.0	0.6	20	0.983	0.994	0.208	0.982	0.880
		40	0.988	0.996	0.049	0.973	0.906
		60	0.990	0.995	0.051	0.975	0.912
		80	0.980	0.993	0.221	0.981	0.871
	0.8	20	0.543	0.616	0.085	0.636	0.447
		40	0.572	0.652	0.038	0.635	0.490
		60	0.565	0.638	0.046	0.628	0.470
		80	0.532	0.610	0.126	0.623	0.439
	0.85	20	0.334	0.397	0.081	0.412	0.300
		40	0.343	0.411	0.047	0.409	0.302
		60	0.346	0.407	0.053	0.409	0.306
		80	0.337	0.398	0.127	0.417	0.297
	0.9	20	0.175	0.208	0.083	0.211	0.165
		40	0.183	0.209	0.064	0.214	0.169
		60	0.178	0.207	0.075	0.213	0.165
		80	0.175	0.211	0.126	0.217	0.162
	0.95	20	0.084	0.092	0.078	0.094	0.085
		40	0.080	0.090	0.079	0.089	0.083
		60	0.085	0.091	0.079	0.091	0.082
		80	0.074	0.085	0.100	0.088	0.073
4.0	0.6	20	0.701	0.782	0.001	0.844	0.362
		40	0.796	0.873	0.000	0.820	0.498
		60	0.840	0.890	0.000	0.815	0.545
		80	0.674	0.772	0.002	0.842	0.340
	0.8	20	0.270	0.330	0.003	0.377	0.208
		40	0.330	0.399	0.001	0.393	0.269
		60	0.345	0.391	0.000	0.387	0.277
		80	0.272	0.328	0.009	0.375	0.211
	0.85	20	0.200	0.241	0.007	0.266	0.171
		40	0.239	0.288	0.002	0.285	0.210
		60	0.226	0.261	0.002	0.257	0.205
		80	0.197	0.238	0.021	0.261	0.169
	0.9	20	0.123	0.149	0.015	0.161	0.117
		40	0.147	0.172	0.008	0.169	0.137
		60	0.138	0.155	0.013	0.157	0.130
		80	0.125	0.148	0.049	0.156	0.117
	0.95	20	0.080	0.087	0.042	0.088	0.079
		40	0.083	0.087	0.039	0.086	0.081
		60	0.077	0.081	0.047	0.079	0.073
		80	0.070	0.078	0.077	0.083	0.072

Figure 1: Power when m_2 = 0 or 4