



**AgEcon** SEARCH  
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search

<http://ageconsearch.umn.edu>

[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

MONASH

WP no. 8/92

**M O N A S H**  
**U N I V E R S I T Y**



GIANNINI FOUNDATION OF  
AGRICULTURAL ECONOMICS  
LIBRARY  
SEP 08 1992

**THE FORM OF TIME VARIATION OF SYSTEMATIC RISK:  
SOME AUSTRALIAN EVIDENCE**

**Robert D. Brooks, Robert W. Faff and John H.H. Lee**

**Working Paper No. 8/92**

**July 1992**

**DEPARTMENT OF ECONOMETRICS**

ISSN 1032-3813

ISBN 0 7326 0358 7

THE FORM OF TIME VARIATION OF SYSTEMATIC RISK:

SOME AUSTRALIAN EVIDENCE

Robert D. Brooks, Robert W. Faff and John H.H. Lee

Working Paper No. 8/92

July 1992

DEPARTMENT OF ECONOMETRICS, FACULTY OF ECONOMICS COMMERCE & MANAGEMENT

MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

# THE FORM OF TIME VARIATION OF SYSTEMATIC RISK: SOME AUSTRALIAN EVIDENCE

Robert D. Brooks\*, Robert W. Faff\*\* and John H.H. Lee\*

\* Department of Economics and Finance, Royal Melbourne Institute of Technology, GPO Box 2476V, Melbourne 3001, and Department of Econometrics, Monash University.

\*\* Department of Accounting and Finance, Monash University, Clayton 3168.

## ABSTRACT

Many studies have investigated the issue of time stationarity of an asset's systematic risk. While there is considerable evidence to suggest that an asset's systematic risk is best described by some stochastic parameter model, little work has been conducted in determining the most appropriate stochastic parameter model. This paper addresses this issue. We extend the study conducted by Faff, Lee and Fry (1992) to investigate which varying coefficient model best describes the systematic risk of assets in the Australian equity market, for those assets for which a constant coefficient model is found to be inadequate. Our testing strategy is point optimal (see King (1987a)) given that this approach to testing is designed to have good small sample properties. Our results suggest that generally in cases where a stochastic parameter is appropriate a Hildreth-Houck (1968) random coefficient model is the preferred model.

## 1. INTRODUCTION

The notion of risk and its measurement is of fundamental importance to modern finance theory. For many years the dominant paradigm of the equilibrium risk-return trade-off has been the capital asset pricing model (CAPM). This model states that the relevant risk measure is the systematic risk or beta, because all other risk measures can be diversified away through holding the market portfolio. Empirically this systematic risk is often estimated by applying ordinary least squares (OLS) to the market model.

In recent times the literature on the market model has shown a great deal of concern with the issue of whether beta is time varying. Such variation in beta may arise through the influence of microeconomic or macroeconomic factors. The evidence from this literature suggests that beta is not constant but is best described by some type of stochastic parameter model<sup>1</sup>. For instance, Fabozzi and Francis (1978), Francis and Fabozzi (1980), Fabozzi, Francis and Lee (1982) and Alexander and Benson (1982) suggest the Hildreth-Houck (1968) random coefficient model as an appropriate model of time variation in systematic risk. Alternatively Sunder (1980), Garbade and Rentzler (1981), Alexander, Benson and Eger (1982) and Simonds, La Motte and McWhorter (1986) propose a random walk coefficient model as most appropriate. Another possibility is to consider a more general alternative. Such an alternative is Rosenberg's (1973) AR(1) or return to normalcy coefficient model. This alternative has been suggested by Bos and Newbold (1984) and Faff, Lee and Fry (1992)<sup>2</sup>.

Given the existence of a number of alternatives the obvious question is which alternative is best. This issue has received scant attention in the literature, although it was considered by Bos and Newbold. Using classical large sample tests to compare the Hildreth-Houck model and the Rosenberg model, they found in favour of the Hildreth-Houck model. However this may merely be a reflection of the low power of their tests.

The aim of this paper is to extend the study conducted by Faff, Lee and Fry (1992), by further investigating the issue of whether the constant coefficient model is an adequate simplification of reality using a more powerful testing strategy. Moreover we go on to

determine which varying coefficient model is most appropriate if the constant coefficient model is found to be inadequate<sup>3</sup>.

Faff, Lee and Fry (1992) used a sample of monthly data from the Australian equities market. They employed a locally best invariant test for the AR(1) coefficient model (see King (1987b)) for individual and portfolio betas over two five year time periods: 1978 to 1982 and 1983 to 1987. It was found that across all variations of their analysis a non-trivial degree of beta non-stationarity was evident. Using continuous returns and a value weighted market index they found evidence, (at the 5 % significance level) of non-stationarity for 11.3 % and 12.9 % of the individual firms in each sample period, respectively. Generally, when assets were formed into portfolios and as the size of the portfolios was increased they displayed a greater degree of non-stationarity. For example, in the 1983 to 1987 period for portfolios (sorted by firm size) of 5, 10 and 20 assets, the rejection rates observed were 24.2 %, 45.2 % and 53.3 %, respectively. Finally, constancy rejection rates were slightly higher for randomly formed portfolios of 10 assets compared to their size based counterparts. For example, in the 1983 to 1987 period the rejection rate was 47 %.

The testing strategy used in our study is point optimal (see King (1987a)). We first consider Brooks' (1992a) point optimal test for constant coefficients against the alternative of a Rosenberg (1973) coefficient. For those cases where we reject coefficient constancy we then apply Brooks and King's (1992) approximately point optimal invariant test of the Hildreth-Houck (1968) model against the Rosenberg (1973) alternative.

The plan of our paper is therefore as follows. In section two we provide some motivation for time variation in systematic risk and then outline the alternative parametric forms of time variation. Section three explains our testing strategy. Section four then reports the results from extending the Faff, Lee and Fry (1992) study. Section five contains some concluding remarks.

## 2. MODELS OF TIME VARIATION

While it would be appealing to derive the time variation in systematic risk directly along with the derivation of the market model, this is typically not the approach taken in the literature. Such literature has typically attempted to overcome supposed deficiencies in the derivation of the market model.

A number of authors have argued that time variation may be due to microeconomic factors at the level of the firm. Fabozzi and Francis (1978) suggest the reasons of alterations to the product mix or changes in leverage or dividend policy as giving rise to time variation in systematic risk. Bos and Newbold (1984) claim that changes in the operational structure of the firm may be the cause of time variation in systematic risk. Dielman and Nantell (1982) argue that the key operational change is likely to be merger activity. Turnbull (1977) identified maturity and growth of the firm as important determinants of systematic risk. Therefore, as the firm matures and its growth rate fluctuates through time, then so too may its beta risk change. Time variation in systematic risk due to microeconomic factors is also consistent with some of the arguments provided by Blume (1975). For example, he suggested that when firms engage in any project which is risky, the risk of the project may tend to be less extreme over time.

Alternatively macroeconomic factors may lead to time variation in the systematic risk. Both Fabozzi and Francis (1978) and Bos and Newbold (1984) claim that business cycle factors such as inflation and unemployment may account for the time variation in systematic risk. Another possibility is to attribute time variation in systematic risk to the behaviour of portfolio managers as is done in Alexander, Benson and Eger (1982).

Despite the desirability of actually modelling the factors that lead to time variation of systematic risk, their unobservability prevents one from doing so. Accordingly we now consider some simple parametric models for time variation of systematic risk.

First consider the market model:

$$R_{it} = \alpha_i + \beta_i R_{mt} + u_{it}$$

where  $R_{it}$  is the return on asset  $i$ ,  $R_{mt}$  is the return on the market portfolio and  $u_{it}$  is assumed to be distributed  $IN(0, \sigma^2)$  and  $\alpha_i$  and  $\beta_i$  are unknown firm specific parameters.

This paper considers the possibility that the systematic risk  $\beta$  varies over time. The two possible alternative varying coefficient models for  $\beta$  which we consider are Hildreth and Houck's (1968) random coefficient model and Rosenberg's (1973)  $AR(1)$  coefficient model. Accordingly one has three possible models for  $\beta$ :

$$(a) \quad \beta_t = \bar{\beta}, \quad \text{for all } t,$$

$$(b) \quad \beta_t = \bar{\beta} + v_t,$$

$$(c) \quad \beta_t = \phi_1 \beta_{t-1} + (1 - \phi_1) \bar{\beta} + w_t,$$

where  $v_t$  is assumed to be distributed as  $IN(0, \lambda_0 \sigma^2)$  and  $w_t$  is assumed to be distributed as  $IN(0, \lambda_1 \sigma^2)$ .

Model (a) is the constant coefficient model, (b) is the random coefficient model and (c) is the  $AR(1)$  coefficient model. All of the models are desirable because they possess mean reversion properties for  $\beta$ . In the constant coefficient model  $\beta$  is always fixed at its mean value. Alternatively in the random coefficient model any deviation of  $\beta$  away from its mean value is confined to the period in which that deviation occurred. The  $AR(1)$  coefficient model has more interesting mean reversion properties as in this case the effect of



any deviation from the mean value persists over time. However, in the long run  $\beta$  reverts to its mean value. As this mean value is of some economic significance, such mean reversion properties are important.

Further to this these possible models have a strong justification in the previous empirical literature. Fabozzi and Francis (1980) and Fabozzi, Francis and Lee (1982) using New York Stock Exchange data find evidence in favour of the random coefficient model. Francis and Fabozzi (1980) also find similar evidence for data on mutual funds. Bos and Newbold (1984) using New York Stock Exchange data find limited evidence in favour of the  $AR(1)$  coefficient model. Faff, Lee and Fry (1992) using Australian data also find evidence in favour of the  $AR(1)$  coefficient model.

### 3. TESTING PROCEDURE

The first test that one requires is one to discriminate the constant coefficient model from either of the varying coefficient models. To carry out such a test one requires a choice of alternative between the two possible varying coefficient models. The most appropriate choice would be one which also provides good power against the other possible alternative. Brooks (1992b) considers the problem of determining which of the two alternative varying coefficient models is the most appropriate alternative to test against. The analysis of this problem suggests that the more general alternative is to be preferred. Therefore this paper chooses the  $AR(1)$  coefficient model as the alternative, as it is the most general and encompasses the random coefficient model as a special case.

This is therefore a problem of testing whether  $\lambda$  is significantly different from zero. The interesting feature to this problem given our choice of the more general alternative is the presence of the nuisance parameter  $\phi_1$ . The problem of testing the constant coefficient model against the alternative of an  $AR(1)$  coefficient model has been well researched. For this problem, Watson and Engle (1985) suggested an approximate version of Davies' (1977) test, King (1987b) suggested a locally best invariant (*LBI*) test, Shively (1988) suggested a

point optimal test for this problem and, by way of an empirical power comparison shows its superiority to Watson and Engle's (1985) and King's (1987b) tests. Brooks (1992a) showed that improvements can be made to Shively's (1988) test. Recently King and Shively (1992) suggested the use of a locally most mean powerful test for this problem. On the basis of power considerations this study uses the version of the point optimal test suggested by Brooks (1992a).

The point optimal invariant test for this problem is of the form:

$$s(\lambda_1, \phi_1) = \tilde{u}'(I + \lambda_1\Omega(\phi_1))^{-1}\tilde{u}/\hat{u}'\hat{u}$$

where  $\tilde{u}$  is the generalised least squares (*GLS*) residual vector assuming covariance matrix  $(I + \lambda_1\Omega(\phi_1))$ ,  $\hat{u}$  is the *OLS* residual vector and  $\Omega(\phi_1)$  is a matrix with element  $(s, t)$  equal to  $(R_{ms}R_{mt}\phi_1^{|s-t|}/(1 - \phi_1^2))$ , where  $R_{mt}$  is the return on the market portfolio and the regressor with the varying coefficient. The test rejects the null hypothesis of a constant coefficient model for small values of the test statistic.

The test is made operational by a choice of  $\lambda_1$  and  $\phi_1$  values. Following Brooks (1992a),  $\phi_1$  is chosen to maximise the average power of the test over the grid of points  $\phi = (0.1, 0.5, 0.9)$ , while  $\lambda_1$  is chosen to make this maximised average power equal to 0.5. This test is then applied. If  $H_0$  is rejected a varying coefficient model is considered more appropriate. For those cases where rejection occurs a test is then carried out to discriminate between the two possible varying coefficient alternatives.

For those cases where one rejects the constant coefficient model, one then has to determine which of the varying coefficient alternatives is best. This is therefore a problem of testing the *AR*(1) parameter  $\phi_1$  in the presence of a significant  $\lambda$  parameter. As such the problem of testing the random coefficient model against the *AR*(1) coefficient model has not been extensively researched. Bos and Newbold (1984) consider this problem and solve it by using asymptotically valid *Wald* and *Likelihood Ratio* tests. Even with their comparatively large sample, they believe such tests lack power. Accordingly this paper

uses the approximately point optimal invariant (APOI) test suggested by Brooks and King (1992), which is designed to have good power properties in finite samples.

The approximately point optimal invariant test for this problem is of the form:

$$s(\lambda_0, \lambda_1, \phi_1) = \tilde{u}'(I + \lambda_1\Omega(\phi_1))^{-1}\tilde{u}/\hat{u}'(I + \lambda_0\Omega_0)\hat{u}$$

where  $\hat{u}$  is the *GLS* residual vector assuming covariance matrix  $(I + \lambda_0\Omega_0)$ , where  $\Omega_0$  is a diagonal matrix with typical element  $(R_{mt}^2)$ . The test rejects the null hypothesis of the random coefficient model for small values of the test statistic.

The test is made operational by a choice of  $\lambda_0$ ,  $\lambda_1$  and  $\phi_1$ . For choices of the  $\lambda$  values one requires both a lower and upper bound. The lower bound on  $\lambda$  is obvious and is set to 0, the upper bound is more difficult as there is no theoretical restriction on the value it can take. Therefore this paper follows Brooks and King's compromise solution of choosing the upper bound to be  $(10/\max R_{mt}^2)$ . Given this range  $\lambda_0$  is chosen in conjunction with the critical value to ensure that the size of the test equals its desired level at the endpoints and is below its desired level between the endpoints. The values for  $\lambda_1$  and  $\phi_1$  are chosen arbitrarily,  $\lambda_1$  to be half of the upper bound on  $\lambda$  and  $\phi_1$  to be 0.5. The evidence in Brooks and King (1992) suggests that these choices do not adversely effect the power of the test.

Having explained the tests at a theoretical level we are at the stage of their empirical implementation. Because all of the unknown parameters required for *GLS* estimation are fixed when testing, it is possible to transform the data, and then estimate the model by *OLS*. Therefore given the appropriate transformations it is possible to calculate all of the test statistics in terms of *OLS* residuals. The details of this are provided in the appendix.

#### 4. EMPIRICAL RESULTS

The basic data used in this paper is the same as that analysed by Faff, Lee and Fry (1992). Their data is on monthly returns on ordinary Australian equities obtained from the Price Relatives File of the Centre for Research in Finance at the Australian Graduate School of Management. We analyse both of their two sub-periods, being from 1978:1 to 1982:12 and 1983:1 to 1987:9. In the first sub-period this gives a sample of 159 assets, while in the second sub-period the sample is of 310 assets. There are 94 assets which are common to both sub-periods. All returns data is on a continuously compounded basis. The returns are analysed individually for the each asset in the respective sub-period and then for certain portfolios of assets. The return on the market portfolio is based on a value weighted index. We do not consider the alternative case in Faff, Lee and Fry (1992) where the return on the market portfolio is based on an equally weighted index.

First let us consider the results for testing  $\beta$  stationarity for individual assets. For our initial problem of testing constant coefficients against AR(1) coefficients the POI test requires the choice of parameter values for  $\lambda_1$  and  $\phi_1$ . Based on a five percent level of significance the parameter values chosen for the first sub-period are  $\lambda_1 = 0.0071954$  and  $\phi_1 = 0.561$ , while for the second sub-period the choice of parameter values is  $\lambda_1 = 0.0061606$  and  $\phi_1 = 0.544$ . These parameters are fixed over all the assets and portfolios we consider as their choice depends only on the explanatory variable, the return on the market portfolio.

The application of the test to the data gives the results in Table 1. These results enable comparison of Faff, Lee and Fry's (1992) use of a Burr (1942) approximation to the distribution of King's (1987) LBI test and the POI test. The first point to note is the remarkable similarity in the overall number of assets for which  $\beta$  stationarity is rejected. In each case one only finds a small number of additional assets are rejected by the POI test. Further one could also note the degree of similarity in the proportions of assets rejected across the two sub-periods.

Comparing at the level of which individual assets are rejected one also finds a large overlap between the rejections of the two tests. Table 2 shows at the different significance

levels the number of assets rejected by both tests (common) and those for which only one test rejects.

Accordingly we now take the 34 assets from the first sub-period and the 76 assets for the second sub-period which either test has rejected at the 10 % significance level and analyse which of the two possible varying coefficient models is most appropriate. To do this we use the APOI test of random against AR(1) coefficients. Given a 5 percent significance level the choice of parameter values for the first sub-period is  $\lambda_0 = 0.03339818$ ,  $\lambda_1 = 0.025329$ ,  $\phi_1 = 0.5$ , and for the second sub-period the choice of parameter values is  $\lambda_0 = 0.03547816$ ,  $\lambda_1 = 0.025953$  and  $\phi_1 = 0.5$ . These parameter values are again fixed over all the different assets and portfolios considered as their choice only depends on values of the explanatory variable, the return on the market portfolio.

Using the APOI test gives the results in Table 3. These results show that the large majority of cases of individual assets with  $\beta$  non-stationarity appear to be better modelled by a random coefficient model rather than an AR(1) model for both of the sub-periods considered. This result is consistent with that of Bos and Newbold (1984) and more significantly has been found with a test that is designed to have good power properties.

It is of interest to consider whether there is any relationship between the form of beta non-stationarity and certain firm characteristics. The link between beta non-stationarity and firm characteristics was investigated by Faff, Lee and Fry (1992). The characteristics they considered were riskiness (measured by the OLS point estimate of beta), firm size (measured by market capitalisation) and industry sector membership.

We therefore examine the link between our three alternative models for beta and the above characteristics. The results for the different sub-periods are presented in Table 4. This table shows the proportions of firms with particular characteristics that are best modelled by a particular beta process. In this table the higher risk is represented by an increase in beta, and S1 represents those firms with the lowest market capitalisation, while S5 represents those firms with the highest market capitalisation.

The most interesting feature of the results is the difference across the two sub-periods. In the second sub-period the degree of both types of non-stationarity increases greatly for high risk firms, while in the first sub-period this result occurs for medium risk firms. With respect to firm size in the first sub-period the proportion of random betas falls markedly for medium size firms, while for the second sub-period the link is not so apparent. In terms of industry classification resources stocks display a greater degree of non-stationarity of both types in the second sub-period.

Let us now consider the results with respect to portfolios of assets. In this study we consider only a subset of the portfolios analysed in Faff, Lee and Fry (1992). We consider size based portfolios of five assets, ten assets and twenty assets based on market capitalisation. For the five asset portfolios in the first sub-period we consider 31 of these portfolios, while for the second sub-period we consider 62 such portfolios. For the ten asset portfolios in the first sub-period there are 15 such portfolios while in the second sub-period there are 31 such portfolios. For the twenty asset portfolios there are 7 such portfolios in the first sub-period and 15 such portfolios in the second sub-period. Analysis of this data yields the results in Table 5.

If we again compare the two tests on the basis of the number of rejections, we again find the numbers to be very close. Comparing percentages of rejections we find that the proportion of size based portfolios for which  $\beta$  stationarity can be rejected differs from those proportions found in the case of individual assets. In the first sub-period the rejection probabilities are lower, while in the second they are higher. Further the proportion of rejections increases as the portfolio size increases.

This result is somewhat counter-intuitive. For example, Ferson and Harvey (1991, p.52) state, "Of course, portfolio betas are more stable than individual common stock betas." The rationale for this statement is that as more assets are combined into portfolios the extent of instability in individual firm betas will tend to offset one another and hence be diversified away. However this ignores the complications of background noise. While increasing portfolio size is likely to reduce actual beta instability, it also reduces background noise making beta instability easier to detect. To the extent that this reduction in back-

ground noise more than offsets the diversification effect, then one would expect greater beta instability to be detected in portfolios as opposed to individual stocks. This background noise argument was proposed by Collins, Ledolter and Rayburn (1987) in explaining a similar result to ours which they found with American data.

Comparing at the level of individual portfolios of size five, for the first sub-period one finds few common rejections of beta stationarity. The only common rejection is for that portfolio which both tests are able to reject at the 1 % significance level. For the second sub-period the results are similar to those of the individual assets with a large number of common rejections. For the larger portfolios of size ten and twenty there is again a large degree of overlap between the tests as is shown in Table 6.

One can now consider the appropriate form of beta non-stationarity for those cases where beta stationarity has been rejected at the 10 % significance level by either test. For the portfolios of size five, in the first sub-period this gives 7 portfolios, while for the second sub-period it gives 20 portfolios. For the portfolios of size ten, this gives 5 portfolios in the first sub-period, and 15 portfolios in the second sub-period. For the portfolios of size twenty this gives 5 portfolios in the first sub-period, and 8 portfolios in the second sub-period. The results of testing for a Hildreth-Houck (1968) beta against a Rosenberg (1973) beta are given in Table 7.

We again find the result that the preferred model of beta non-stationarity is the simpler Hildreth-Houck (1968) model in both sub-periods and for all portfolio sizes, a result consistent with that found for the analysis of individual assets.

Finally we consider some randomly chosen portfolios of 10 assets and analyse the  $\beta$  stationarity for these portfolios. In each of our sub-periods we consider 100 such portfolios. In Table 8 we consider the testing of  $\beta$  stationarity for such portfolios. We again note the similarity in the number of rejections by each of the tests. Further the proportion of rejections in the first sub-period is quantitatively similar to the proportion in the other cases. A difference does however appear when one considers the second sub-period, where one finds the proportion of rejections to be larger than in the case of individual assets

or size based portfolios. When comparing at the level of individual portfolios one finds a number of rejections common to both tests, details of which are provided in Table 9.

Again one needs to consider the appropriate form of beta non-stationarity for those cases where beta stationarity is rejected. This involves 29 portfolios in the first sub-period and 54 portfolios in the second sub-period. The results are given in Table 10.

Examining the appropriate form of beta non-stationarity we again find the bulk of evidence in favour of the simpler Hildreth-Houck (1968) model in both sub-periods.

## 5. CONCLUSIONS

This study has extended the study conducted by Faff, Lee and Fry (1992) in considering the analysis of time stationarity of systematic risk with Australian data. We first considered the issue of whether or not, beta was time stationary using a more sophisticated test than that used in Faff, Lee and Fry (1992). Despite this our results largely concur with the earlier results.

Secondly, we considered the determination of the appropriate form of the time variation for the systematic risk between the competing Hildreth-Houck (1968) and Rosenberg (1973) models. The bulk of our evidence is consistent with that of the simpler Hildreth-Houck (1968) model. This result accords with Bos and Newbold (1984) who found a similar result with American data.



## ENDNOTES

1. Faff, Lee and Fry (1992) reviews the results of this previous research.
2. While there are other possibilities such as, the ARMA(1,1) coefficient model of Ohlson and Rosenberg (1982) and Collins, Ledolter and Rayburn (1987) the random coefficient, random walk coefficient and AR(1) coefficient models are the most popular.
3. However, one should note that for the purposes of our study we do not consider the random walk coefficient alternative. This is because we do not consider such an alternative theoretically appealing given its absence of a mean reversion property. Further we omit from our study consideration of the more complex ARMA(1,1) coefficient model.

## ACKNOWLEDGEMENTS

The authors wish to thank Max King and Kim Sawyer for helpful comments on earlier versions of this paper.

## REFERENCES

- Alexander, G.J. and P.G. Benson (1982), 'More on Beta as a Random Coefficient', *Journal of Financial and Quantitative Analysis* 17, 27-36.
- Alexander, G.J., Benson, P.G. and C.E. Eger (1982), 'Timing Decisions and the Behaviour of Mutual Fund Systematic Risk', *Journal of Financial and Quantitative Analysis* 17, 579-602.
- Blume, M.E. (1975), 'Betas and their Regression Tendencies', *Journal of Finance* 10, 785-795.
- Bos, T. and P. Newbold (1984), 'An Empirical Investigation of the Possibility of Systematic Stochastic Risk in the Market Model', *Journal of Business* 57, 35-41.
- Brooks, R.D. (1992a), 'Alternative Point Optimal Tests for Regression Coefficient Stability', *Journal of Econometrics*, forthcoming.
- Brooks, R.D. (1992b), 'The Robustness of Point Optimal Testing for Rosenberg Random Regression Coefficients', mimeo, Monash University.
- Brooks, R.D. and M.L. King (1992), 'Testing Hildreth-Houck Against Return to Normalcy Random Regression Coefficients', mimeo, Monash University.
- Burr, I.W. (1942), 'Cumulative Frequency Functions', *Annals of Mathematical Statistics* 13, 215-232.
- Collins, D.W., Ledolter, J. and J. Rayburn (1987), 'Some Further Evidence on the Stochastic Properties of Systematic Risk', *Journal of Business* 60, 425-428.
- Dielman, T.E. and T.J. Nantell (1982), 'Tender Offer Mergers and Shareholders Wealth: A Random Coefficient Regression Approach', *Proceedings of the Business and Economics Statistics Section of the American Statistical Association*, 334-339.

Fabozzi, F.J. and J.C. Francis (1978), 'Beta as a Random Coefficient', *Journal of Financial and Quantitative Analysis* 13, 101-115.

Fabozzi, F.J., Francis, J.C. and C.F. Lee (1982), 'Specification Error, Random Coefficient and the Risk-Return Relationship', *Quarterly Review of Business and Economics* 2, 23-30.

Faff, R.W., Lee, J.H.H. and T.R.L. Fry (1992), 'Time Stationarity of Systematic Risk: Some Australian Evidence', *Journal of Business Finance and Accounting* 19, 253-270.

Ferson, W.E. and C.R. Harvey (1991), 'Sources of Predictability in Portfolio Returns', *Financial Analysts Journal* May-June, 49-56.

Francis, J.C. and F.J. Fabozzi (1980), 'Stability of Mutual Fund Systematic Risk Statistics', *Journal of Business Research* 8, 263-275.

Garbade, K. and J. Rentzler (1981), 'Testing the Hypothesis of Beta Stationarity', *International Economic Review* 22, 577-586.

Hildreth, C. and J.P. Houck (1968), 'Some Estimators for a Linear Model with Random Coefficients', *Journal of the American Statistical Association* 63, 584-595.

King, M.L. (1987a), 'Towards a Theory of Point Optimal Testing', *Econometric Reviews* 6, 169-218.

King, M.L. (1987b), 'An Alternative Test for Regression Coefficient Stability', *Review of Economics and Statistics* 69, 379-381.

King, M.L. and T.S. Shively (1992), 'Locally Optimal Testing When A Nuisance Parameter is Present Only Under the Alternative', *Review of Economics and Statistics*, forthcoming.

Ohlson, J. and B. Rosenberg (1982), 'Systematic Risk of the CRSP Equal-Weighted Common Stock Index: A History Estimated by Stochastic Parameter Regression', *Journal of Business* 55, 121-145.

Rosenberg, B. (1973), 'The Analysis of a Cross-Section of Time Series by Stochastically Convergent Parameter Regression', *Annals of Economic and Social Measurement* 2, 399-428.

Shively, T.S. (1988), 'An Analysis of Tests for Regression Coefficient Stability', *Journal of Econometrics* 39, 367-386.

Simmonds, R., La Motte, L. and A. McWhorter (1986), 'Testing for Nonstationarity of Market Risk: An Exact Test and Power Considerations', *Journal of Financial and Quantitative Analysis* 21, 209-220.

Sunder, S. (1980), 'Stationarity of Market Risk: Random Coefficient Tests for Individual Stocks', *Journal of Finance* 35, 883-896.

Turnbull, S.M. (1977), 'Market Value and Systematic Risk', *Journal of Finance* 32, 1125-1142.

Watson, M.W. and R.F. Engle (1985), 'Testing for Regression Coefficient Stability with a Stationary AR(1) Alternative', *Review of Economics and Statistics* 67, 341-346.

TABLE 1 - TESTS FOR BETA STATIONARITY					
INDIVIDUAL ASSETS					
Rejections at different significance levels					
		0.01	0.05	0.10	
1978:1982					
	LBI	8	18	28	
		5.0 %	11.3 %	17.6 %	
	POI	9	22	30	
		5.7 %	13.8 %	18.9 %	
1983:1987					
	LBI	17	40	68	
		5.5 %	12.9 %	21.9 %	
	POI	19	43	69	
		6.1 %	13.9 %	22.3 %	

TABLE 2 - PERFORMANCE OF DIFFERENT TESTS			
INDIVIDUAL ASSETS			
• Rejections at different significance levels			
	0.01	0.05	0.10
1978:1982			
Common	7	16	24
LBI Only	1	2	4
POI Only	2	6	6
1983:1987			
Common	14	35	61
LBI Only	3	5	7
POI Only	5	8	8

TABLE 3 - FORM OF BETA NON-STATIONARITY					
INDIVIDUAL ASSETS					
Rejections at different significance levels					
		0.01	0.05	0.10	
1978:1982					
	H-H	34	30	28	
		21.4 %	18.9 %	17.6 %	
	AR(1)	0	4	6	
		0.0 %	2.5 %	3.8 %	
1983:1987					
	H-H	71	59	54	
		22.9 %	19.0 %	17.4 %	
	AR(1)	5	17	22	
		1.6 %	5.5 %	7.1 %	



TABLE 4 - BETA TYPE AND FIRM CHARACTERISTICS				
	Constant	H-H	AR(1)	Total
1978:1982				
Beta				
$\hat{\beta} \leq 0.8$	78.8 %	18.2 %	3.0 %	99
$0.8 \leq \hat{\beta} \leq 1.2$	69.6 %	21.7 %	8.7 %	23
$\hat{\beta} \geq 1.2$	83.7 %	13.5 %	2.7 %	37
Size				
S1	74.2 %	25.8 %	0.0 %	31
S2	90.6 %	6.3 %	3.1 %	32
S3	87.4 %	6.3 %	6.3 %	32
S4	71.9 %	25.0 %	3.1 %	32
S5	68.7 %	25.0 %	6.3 %	32
Industry				
Resources	78.6 %	19.0 %	2.4 %	42
Industrials	78.6 %	17.1 %	4.3 %	117
1983:1987				
Beta				
$\hat{\beta} \leq 0.8$	84.7 %	11.5 %	3.8 %	131
$0.8 \leq \hat{\beta} \leq 1.2$	80.6 %	13.9 %	5.5 %	72
$\hat{\beta} \geq 1.2$	60.7 %	27.1 %	12.2 %	107
Size				
S1	64.5 %	22.6 %	12.9 %	62
S2	67.7 %	21.0 %	11.3 %	62
S3	79.0 %	17.8 %	3.2 %	62
S4	83.8 %	8.1 %	8.1 %	62
S5	82.2 %	17.8 %	0.0 %	62
Industry				
Resources	67.4 %	23.7 %	8.9 %	135
Industrials	81.7 %	12.6 %	5.7 %	175

TABLE 5 - TESTS FOR BETA STATIONARITY				
SIZE BASED PORTFOLIOS				
Rejections at different significance levels				
		0.01	0.05	0.10
PORTFOLIO SIZE = 5				
1978:1982	LBI	1 3.2 %	2 6.5 %	4 12.9 %
	POI	1 3.2 %	3 9.7 %	4 12.9 %
1983:1987	LBI	8 12.9 %	15 24.2 %	19 30.6 %
	POI	9 14.5 %	15 24.2 %	17 27.4 %
PORTFOLIO SIZE = 10				
1978:1982	LBI	1 6.7 %	2 13.3 %	5 33.3 %
	POI	1 6.7 %	2 13.3 %	4 26.7 %
1983:1987	LBI	6 19.4 %	14 45.2 %	15 48.4 %
	POI	6 19.4 %	12 38.7 %	14 45.2 %
PORTFOLIO SIZE = 20				
1978:1982	LBI	1 14.3 %	2 28.6 %	4 57.1 %
	POI	0 0.0 %	1 14.3 %	5 71.4 %
1983:1987	LBI	5 33.3 %	8 53.3 %	8 53.3 %
	POI	6 40 %	7 46.7 %	8 53.3 %

TABLE 6 - PERFORMANCE OF DIFFERENT TESTS			
SIZE BASED PORTFOLIOS			
Rejections at different significance levels			
	0.01	0.05	0.10
PORTFOLIO SIZE = 5			
1978:1982			
Common	1	1	1
LBI Only	0	1	3
POI Only	0	2	3
1983:1987			
Common	8	15	16
LBI Only	0	0	3
POI Only	1	0	1
PORTFOLIO SIZE = 10			
1978:1982			
Common	1	2	4
LBI Only	0	0	1
POI Only	0	1	0
1983:1987			
Common	6	12	14
LBI Only	0	2	1
POI Only	0	0	0
PORTFOLIO SIZE = 20			
1978:1982			
Common	0	1	4
LBI Only	1	1	0
POI Only	0	0	1
1983:1987			
Common	5	7	8
LBI Only	0	1	0
POI Only	1	0	0

TABLE 7 - FORM OF BETA NON-STATIONARITY				
SIZE BASED PORTFOLIOS				
Rejections at different significance levels				
		0.01	0.05	0.10
PORTFOLIO SIZE = 5				
1978:1982	H-H	7 22.6 %	6 19.4 %	4 12.9 %
	AR(1)	0 0 %	1 3.2 %	3 9.7 %
1983:1987	H-H	17 27.4 %	15 24.2 %	14 22.6 %
	AR(1)	3 4.8 %	5 8.1 %	6 9.7 %
PORTFOLIO SIZE = 10				
1978:1982	H-H	5 33.3 %	4 26.7 %	3 20.0 %
	AR(1)	0 0.0 %	1 6.7 %	2 13.3 %
1983:1987	H-H	13 41.9 %	12 38.7 %	10 32.3 %
	AR(1)	1 3.2 %	2 6.5 %	4 12.9 %
PORTFOLIO SIZE = 20				
1978:1982	H-H	4 57.1 %	4 57.1 %	4 57.1 %
	AR(1)	1 14.3 %	1 14.3 %	1 14.3 %
1983:1987	H-H	6 40.0 %	6 40.0 %	4 26.7 %
	AR(1)	2 13.3 %	2 13.3 %	4 26.7 %

TABLE 8 - TESTS FOR BETA STATIONARITY				
RANDOM PORTFOLIOS				
Rejections at different significance levels				
		0.01	0.05	0.10
1978:1982				
	LBI	6	19	25
	POI	5	15	22
1983:1987				
	LBI	30	47	51
	POI	33	47	54

TABLE 9 - PERFORMANCE OF DIFFERENT TESTS			
RANDOM PORTFOLIOS			
Rejections at different significance levels			
	0.01	0.05	0.10
1978:1982			
Common	5	14	18
LBI Only	1	5	7
POI Only	0	1	4
1983:1987			
Common	30	46	51
LBI Only	0	1	0
POI Only	3	1	3

TABLE 10 - FORM OF BETA NON-STATIONARITY				
RANDOM PORTFOLIOS				
Rejections at different significance levels				
		0.01	0.05	0.10
1978:1982				
	H-H	28	25	21
	AR(1)	1	4	8
1983:1987				
	H-H	53	42	41
	AR(1)	1	12	13

## APPENDIX

For the random coefficient model the appropriate transformations are:

$$\hat{R}_{it} = R_{it}/(1 + \lambda_0 R_{mt}^2)^{1/2}, \quad \hat{C}_t = 1/(1 + \lambda_0 R_{mt}^2)^{1/2}, \quad \hat{R}_{mt} = R_{mt}/(1 + \lambda_0 R_{mt}^2)^{1/2}.$$

The denominator of Brooks and King's (1992) APOI test can then be found as the sum of squared residuals from an *OLS* regression of  $\hat{R}_{it}$  on  $\hat{C}_t$  and  $\hat{R}_{mt}$ .

For the *AR*(1) coefficient model the appropriate transformations are:

$$\bar{R}_{it} = R_{it}/R_{mt}, \quad \bar{C}_t = 1/R_{mt}, \quad \bar{R}_{mt} = R_{mt}/R_{mt},$$

for the first observation, and for the remaining observations in the sample,

$$\bar{R}_{it} = R_{it}/R_{mt} - \phi_1 R_{it-1}/R_{mt-1},$$

$$\bar{C}_t = 1/R_{mt} - \phi_1 1/R_{mt-1},$$

$$\bar{R}_{mt} = R_{mt}/R_{mt} - \phi_1 R_{mt-1}/R_{mt-1}.$$

Then another transformation is required. For the first observation this is,

$$\tilde{R}_{it} = \bar{R}_{it}/L_{1,1}, \quad \tilde{C}_t = \bar{C}_t/L_{1,1}, \quad \tilde{R}_{mt} = \bar{R}_{mt}/L_{1,1}.$$

and for the remaining observations in the sample,

$$\tilde{R}_{it} = (\bar{R}_{it} - L_{t,t-1} \tilde{R}_{it-1})/L_{t,t},$$



$$\tilde{C}_t = (\bar{C}_t - L_{t,t-1}\tilde{C}_{t-1})/L_{t,t},$$

$$\tilde{R}_{mt} = (\bar{R}_{mt} - L_{t,t-1}\tilde{R}_{mt-1})/L_{t,t}.$$

where the  $L_{i,j}$  are elements of a tri-diagonal matrix of the form:

$$L_{1,1} = (1/R_{m1}^2 + \lambda_1/(1 - \phi_1^2))^{1/2},$$

$$L_{t,t} = (\phi_1^2/R_{mt-1}^2 + 1/R_{mt}^2 + \lambda_1 - L_{t,t-1}^2)^{1/2},$$

$$L_{t,t-1} = -\phi_1/(R_{mt-1}^2 * L_{t-1,t-1}).$$

and the numerator of both Brooks' (1992a) test and Brooks and King's (1992) test can be found as the sum of squared residuals from an *OLS* regression of  $\tilde{R}_{it}$  on  $\tilde{C}_t$  and  $\tilde{R}_{mt}$ .

