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Economic Motivations for Limited Dependent and Qualitative Variable Models.

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Abstract: The greater availability of survey data, a succession of papers dealing with the statistical issues arising from the analysis of such data and the appearance of software packages, such as LIMDEP (Greene (1991)), have led to a remarkable increase in the application of limited dependent and qualitative variable models in economics. Economic analysis of the behaviour of individual decision makers often leads to models which are of a limited dependent or qualitative variable nature. This paper attempts to show how the use of limited dependent and qualitative variable models naturally arises from the more general framework of modifications to traditional economic optimisation problems.

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1. Introduction.

In recent years econometrics has embraced the use of limited dependent and qualitative variable models in applied work. This is largely due to the greater availability of survey data and an increasing awareness of aggregation bias in time series regional and economy-wide modelling. However, the statistical models used were developed in other fields, such as psychology and transportation studies, and have been transplanted into econometrics with little attention paid to their economic foundations. This seems less than satisfactory.

Surveys of the statistical properties of the limited dependent variable models in use in econometrics have been published at regular intervals (see, *inter alia*, Amemiya (1975, 1981, 1984, 1985), Gourieroux (1989), Hensher and Johnson (1981), McFadden (1976, 1982, 1985), and Maddala (1983)). In addition many recent econometrics textbooks (see *inter alia*, Chow (1983), Greene (1990), Judge, Griffiths, Hill, Lütkepohl and Lee (1985,1988) and Maddala (1988)) include chapters on these models and several computer programs (e.g. SHAZAM (White, Wong, Whistler and Haun (1990)), TSP (Hall, Schnake and Cummins (1987)) and LIMDEP (Greene (1991)) allow the estimation of many of the models. However, until the recent publication of Pudney's (1989) book, much less had been said in survey format about the economic situations that motivate such models.

This paper provides a brief overview of the economic foundations of such models so that economists can see how these models might relate to problems that they may be interested in. As such it does not dwell upon the statistical properties of the models, but indicates how modifications to the traditional optimisation problems of microeconomics generate particular limited dependent and qualitative variable models. An outline of the type of models which are classified as limited dependent and qualitative variable is given in section 2. In Section 3 it is shown how modifications to traditional optimisation problems give rise to such limited dependent and qualitative variable models. The final section of the paper contains some concluding remarks.

2. Limited Dependent and Qualitative Variable Models.

Limited dependent variable models should be used whenever the data on the variable of interest or its observed realisation is censored or truncated. Qualitative variable models should be used whenever the data of interest is discrete. In this section the various types of discrete variables and the concepts of censoring and truncation are defined. The main types of models for use with such data are also discussed.

Discrete variables may be either nominal (i.e. data classified into groups), ordinal (i.e. data classified into groups which have some ordering) or a count (i.e. can only be a non-negative integer). Such data usually comes from cross sectional surveys and relate to individual decision making units. The variable of interest may either have been coded as a discrete variable, or discreteness may have arisen due to the grouping of a continuous variable.

The Logit and Probit models are appropriate when one works with discrete data in binomial (two outcomes) or multinomial (more than two outcomes) situations such as modelling home ownership (Mingche (1977)) or choice of transportation mode (Domenich and McFadden (1975)). For a more comprehensive list see Gourieroux (1989) and Maddala (1983). The use of such *discrete* data renders the conventional application of ordinary least squares (OLS) regression techniques inappropriate as typically the disturbances in the linear model are heteroscedastic. Although the application of (feasible) generalised least squares resolves this issue the use of a linear regression model is still inappropriate. That is, it ignores the fact that with discrete data it is the determinants of the probabilities of each of the outcomes of the dependent variable which are to be modelled. The use of linear regression models does not constrain these probabilities to lie between zero and one, even for the observations in the sample.

Count data models are required when the dependent variable can only take on values of a non-negative integer and again concern modelling outcome probabilities. These models were developed in the biometric literature, and as yet have only had minimal applications in economics. One final point to note is that the discrete data themselves may be of primary interest or alternatively it may be the observed representation of a continuous latent variable which is of primary interest.

Logit models are widely used in applications of qualitative variable models because of the computational ease and ease of interpretation associated with their use. These models arise from the assumption that the random component of the model follows the Logistic distribution. This distribution is similar to the Normal distribution except that it has more area in the tails (i.e. has so called 'heavy tails') leading to a greater proportion of outliers.

A property of the Logit model in the multinomial situation is the independence of irrelevant alternatives (IIA). This implies that the odds ratio between two alternatives is independent of alterations to other alternatives in the choice set. The 'textbook' example of this property is the so called red bus/blue bus problem (see Debreu (1960)). That is, suppose that an individual when offered a choice between commuting to work by train or by bus has a probability of choosing the train equal to *2/3* . If an additional alternative is made available, say an additional bus just like the first bus in all respects except that it is of a different colour, then we would expect the individuals probability of choosing the train to remain at *2/3* and the probability of choosing the bus to be split according to colour preference. However, the IIA property implies that the probability of choosing the train must fall to *1/2* and those for either bus must equal *1/4* if the odds of selecting the train relative to the first bus is to remain at *2:1*.

Testing of the IIA property is important. A researcher cannot determine *a priori* whether the property is an appropriate representation of consumer preferences. Thus the researcher should test for IIA. Furthermore, if a model incorporating IIA provides an adequate representation of consumer preferences then considerable advantages are gained in model specification and estimation. The IIA property and testing for its existence are discussed in Section 3.

Probit models are derived from the random variable(s) in question having a Normal distribution. This assumption will allow all random variables in a model to be Normal as we typically only consider linear transformations of variables in our models and linear combinations of normally distributed variables are themselves normally distributed.

A Probit model allows for a more realistic modelling of choice behaviour since, unlike the Logit model, it allows for a general pattern of inter-relationships between alternatives in the choice set. Unfortunately, a major limitation to its use is its computational complexity which prevents its use in applied settings with choice sets of a realistic size. This computational restriction stems from the fact that in multinomial choice problems with M alternatives the choice probabilities in the Probit model are defined by the cumulative distribution function of an $(M - 1)$ dimensional Normal distribution. In practice this means that only small choice sets with *4* or less alternatives are amenable to modelling with a Probit model. This is in contrast to vast majority of econometric work where the assumption of normality ensures analytical and computational tractability of the model specification. However, recent work on the estimation by simulation technique (see McFadden (1989), Ruud (1991)) offers a solution to the computational problems and should lead to more empirical work using Probit models in the future.

Both the Logit and Probit models are invariant to the scaling of the data on the dependent variable. In the logit model the variance is known to be $\pi^2/3$, while in the probit model it is normalised to one in estimation. The interpretive consequence of this is that one is unable to say anything about variability in responses which could be of economic importance.

In situations where the variable of interest is a non-negative integer, which represents the number of occurrences of an economic event in a given time period (i.e. a count), then appropriate statistical models need to be used. When analysing such data it is natural to model the probability that the economic event will occur $j = 1, 2, ...$ times in the given time interval. The statistical models typically used to model count data in economics are either Poisson or compound Poisson (e.g. Negative Binomial) and a good statistical discussion of these models may be found in Cameron and Trivedi (1986).

Models of the Tobit type are required when the variables of interest are not observed over their full range. That is, when they have been subject to either censoring or truncation. There exist a large variety of models of the Tobit type, many of which are discussed in Amemiya (1984,1985). When a variable is censored, values in a certain range are all transformed to, or recorded as a single value. For example, in modelling demand for airline travel the variable to be used in modelling may be the number of tickets sold. However, if a plane is full then demand is larger than, or equal to, the number of seats sold. The demand for travel is censored when it is transformed to the number of tickets sold. In practice, most occurrences of censored variables will involve censoring at zero. Truncation is primarily a function of the nature of the sample taken. In other words, a sample is taken from a subset of the population of interest and a model (or inference) is desired for the whole population. For example, a sample of hours worked by females may be taken in a labour force survey (i.e. a sample of working females) and inferences desired about all females. Of the two, censoring is the more common in economics.

The use of censored or truncated data rule out the use of a conventional OLS regression approach since the usual consistency and unbiasedness properties of regression estimators are violated. Tobit type models have found widespread application in economics since Tobin's (1958) paper, which considered the modelling of household expenditure on durable goods (see Amemiya (1984, 1985) for a more comprehensive list of applications). It is possible to consider a wide variety of distributions in these models (Appendix 2 of Pudney (1989) gives results concerning a number of distributions). Typically, censoring or truncation is at zero, which usually gives a sample with only non-negative values. It is however possible to consider censoring or truncation at any constant (see Maddala (1983)).

Pudney (1989, pp.155-156) points out that a major difficulty with Tobit type models in a system framework is that one cannot ensure that they satisfy the budget constraint. Unlike the usual situation, we cannot ensure consistency with the budget constraint by simply deleting an equation as the parameter estimates are not invariant to this procedure. In a recent paper on modelling household meat consumption in Sydney and Melbourne (Bartley, Ball and Weeks (1988)) the authors are faced with just this problem. However, their proposed solution, which modifies the seemingly unrelated regression model, is not ideal since it discards information. An appropriate solution using a seemingly unrelated regression procedure may be found in Heien and Wessels (1990).

3. Economic Motivations

This section provides some economic motivations for the use of limited dependent and qualitative variable models. The vast majority of these models are derived from modifications to consumer or household choice problems as the literature has primarily concerned itself with these problems. However this should not imply that other economic stories would not produce such models. Silberman and Talley (1974) arrive at a Probit model in considering the public choice problem of government regulatory decision making. Duncan (1980) in considering the choice of plant location and product type for a neoclassical firm arrives at a Probit model for a discrete/continuous choice problem. McGillivray and Ozcowski (1991) model the allocation of Australian foreign aid using a Tobit type model.

The traditional optimisation problem can be characterised as the maximisation of a continuous deterministic objective function, with continuous deterministic arguments, subject to a continuous linear deterministic set of constraints (see Varian (1984)). Several modifications to this problem, in particular random utility maximisation, will be considered in this section. Depending on the particular modification(s) made, any of the models discussed in the earlier statistical section may be obtained.

Random Utility Maximisation.

The traditional approach is based on the maximisation of a deterministic utility function. This is unlikely to be practically relevant and it may be more appropriate to work in a random utility framework to generate models of discrete and/or discrete/continuous choice behaviour. Hanemann (1984, pp 543 - 544) justifies random utility models in the following way:

"A random utility model arises when one assumes that, although a consumer's utility function is deterministic for *him*, it contains some components which are unobservable to the econometric investigator and are treated by the observer as random variables. The unobservables could be characteristics of the consumer and/or attributes of the commodities. This concept, therefore, combines two ideas that have a long history in economics - the idea of a variation in tastes among individuals in a population and the idea of unobserved variables in econometric models".

Luce (1977) points out that the use of these models in economics can be based on two considerations. First, one could choose a model purely on the basis of statistical criteria. Alternatively, models could be derived from a reconstituted economic theory based upon probabilistic rather than algebraic assumptions. This second point has led to the development of the random utility maximisation model in economics (see *inter alia* McFadden (1973, 1978, 1981), Manski (1977)).

The random utility function is defined as:

$$U_{ij} = V_{ij}(Z_{ij}, X_i) + e_{ij} = Z_{ij}\alpha + X_i\beta_j + e_{ij} \quad (1)$$

where α and β_j are column vectors providing information on the marginal utilities with respect to the relevant characteristics. In this case U_{ij} is interpreted as an indirect utility function. The deterministic component V_{ij} can be thought of as the expected utility the individual can obtain and the random component e_{ij} represents unobservable factors, measurement errors, unobservable variations in preferences and/or random individual behaviour. The deterministic component of this model is further broken up into two parts one involving X_i which represents characteristics of the individual choice maker and one involving Z_{ij} representing characteristics of the alternatives in the choice set.

Manski (1977) points out that the majority of econometric work has considered the deterministic component to the neglect of the random component. An important paper in this field is McFadden (1981) where the random utility maximisation framework is used to evaluate the properties of the models proposed for modelling choice behaviour.

Within this model Ben-Akiva and Lerman (1985, pp.56-57) identify four main sources of randomness in the utility function. These are:

(i) Unobservable attributes of goods,

(ii) Unobservable variations in preferences,

(iii) Measurement errors in the data,

(iv) Use of proxy variables.

Quandt (1956) would add to this list the uncertainty facing the individual consumer. The utility gained from the consumption of a particular good is not known with certainty. In a world of non-zero information costs the consumer must also choose the optimal level of information. As such, apparent randomness may arise from a deterministic utility function, which is particularly relevant if one allows for a learning process in preference formation. It represents an example of bounded rationality. Bounded rationality which recognises limitations in the decision making processes of individuals has been used by Williamson (1981) in his work on transaction cost economics. The researcher is typically unable to distinguish between randomness arising from measurement error or from consumer decision making. While the theoretical difference may be important, their observational equivalence prevents it having any practical consequences.

However, it should be noted that a possible criticism of the random utility model lies in its structure of an additive error, which is likely to be inappropriate. Clearly from equation (1) randomness can arise from one of three sources in the unconstrained case. These three sources are: via randomness in the consumers' perception of the attributes of the available goods, via randomness in the observed preferences of consumers, or from randomness in the functional form of the direct utility function. It is not obvious that the existence of randomness in any of these components will give rise to statistical models with the same properties as those with an additive error term.

The random utility maximisation literature to date, in contrast to the usual treatment of utility maximisation which is *ex ante*, has proceeded from the *ex post* observation that utility has been maximised by the optimising unit. Viewing the problem from the *ex ante* perspective is likely to have greater economic significance and provide justification for the inclusion of randomness in the attributes of the arguments of the utility function.

An alternative to randomness in the utility function is randomness in the constraints. Such randomness could arise due to the existence of soft budget constraints (see Kornai (1980)). For example, taxation and social security laws which are not rigidly enforced produce *ex ante* uncertainty in the budget constraint. Accordingly, randomness in choice behaviour may still be generated by maximisation of a deterministic utility function subject to random constraints.

Random utility maximisation models can be complex unless the deterministic component of the utility function is kept simple. Having specified a utility function, individuals are assumed to make selections that maximise their utility. Thus, if individual i faces J alternatives, the probability that the first alternative is chosen is

$$P_{i1} = prob[U_{i1} > U_{i2} \quad and \quad U_{i1} > U_{i3} \ldots \quad and \quad U_{i1} > U_{iJ}],$$

which in turn is equivalent to,

$$P_{i1} = prob[e_{i2} - e_{i1} < V_{i1} - V_{i2} \quad and \quad \ldots e_{iJ} - e_{i1} < V_{i1} - V_{iJ}].$$

Depending on the distributional assumptions made about the random component we may obtain either Logit or Probit models. Under the usual assumption that disturbances are normally distributed we obtain a Probit model. On the other hand, if the disturbances are assumed to have independent Extreme Value distributions then a (mixed) Logit model results. Since the Extreme Value distribution is the limiting distribution of the maximum of n independent and identically distributed random variables as n tends to infinity there are situations, e.g. utility maximisation, in which the assumption of extreme value distributed errors may be justified (see Börsch-Supan (1985)).

For the Logit model we need to distinguish two cases. If in the random utility model $\alpha = 0$ then the model is Multinomial Logit (MNLGT), which can be referred to as a "characteristics of the chooser" model. Conversely if all the elements in β_j are zero then the model is conditional Logit (CLGT), which is a "characteristics of the alternative" model. Further this model implies that individuals have homothetic preferences. Although the MNLGT and CLGT models are different statistically, they are derived from the same random utility maximisation approach.

The choice of one Logit model over the other is mainly a matter of data availability, not of behavioural assumptions. In practice, it is very likely that a particular survey would collect data on Z_{ij} but not X_i (in which case the CLGT model would be used) or collect data on X_i but not Z_{ij} (in which case the MNLGT model would be used). Hence, either model is frequently estimated on the basis of data availability. In such a situation, both models would be mis-specified if the mixed Logit is the true model because both models would have omitted relevant explanatory variables. It is particularly important therefore to carry out diagnostic tests on Logit models once estimated. Pagan and Vella (1989) and McFadden (1987) suggest some tests which can be used for Logit models and these tests are easily implemented in LIMDEP (Greene (1991)).

A confusion between "MNLGT" and "CLGT" has arisen in the literature as many authors do not distinguish the two models and the words "MNLGT" and "CLGT" are used interchangeably (e.g. Ben-Akiva and Lerman (1985), Pudney (1989)). Note that the mixed Logit model is most appropriate in general and should be used if the data are available apart from the extremely rare cases where the theory suggests otherwise.

Other distributional assumptions have been used in an attempt to produce choice models which allow a general pattern of inter-relationships between alternatives and are computationally tractable (see *inter alia*, McFadden (1981), Madan (1986)). Typically these models are based upon the assumption of a hierarchical structure in the choice set. Dominant amongst these hierarchical models are the Generalised Extreme Value (GEV) model and a special case of GEV, the Nested Multinomial (NMNL) model (see McFadden (1978, 1981)). Such models are consistent with the random utility maximisation model (Börsch-Supan (1990), McFadden (1981)) and may be estimated. For example, the estimation of the NMNL is discussed in Hensher (1986, 1991) and is available in LIMDEP (Greene (1991)). Practical implementation of these models does however involve the specification of the hierarchical structure for the choice set. Often this may not be apparent to the researcher.

It is also possible that the choice set is both discrete and continuous. This situation is considered in Dubin and McFadden (1984) and Hanemann (1984). Dubin and McFadden are concerned with modelling the choice of residential appliances and the consumption of electricity. Hanemann's example of such a situation is the discrete choice of a particular brand of product and then the continuous choice of the quantity of that brand to consume. A recent Australian example from the field of transport economics is Hensher, Milthorpe and Smith (1990) who use 1981/82 data for Sydney to estimate discrete/continuous models. Again models for such behaviour are typically developed within the random utility maximisation framework.

Independence of Irrelevant Alternatives

As this is a particularly important property of the widely used Logit models it is treated separately from the topic of random utility maximisation although it is related to the latter. The IIA property arises out of the use of Logit models within the random utility maximisation model. It is, however, also a representation of the Luce (1959) choice axiom. A model for discrete choice behaviour may be derived from either the "economic" random utility approach or the "psychology" choice axiom approach. These two are related, indeed psychologists, for example Thurstone (1927), were involved in the early development of the random utility model. Thus, insights into the IIA property may be gained from both approaches.

Various attempts (see *inter alia* McFadden (1973), (1981) and Börsch-Supan (1985)) have been made to state the necessary and sufficient conditions for the IIA property to hold. From these one obtains the following list:

1) All the alternatives in the choice set are distinct. For alternatives to be distinct the consumer must not view them as identical (an example of such alternatives would be differentiated products such as petrol). In other words, when the choice set should not contain two or more alternatives which are close substitutes for each other.

2) The stochastic terms in the underlying model, typically a random utility maximisation model, are independent.

3) The odds of choosing alternative i over alternative $j \neq i$ are independent of all other alternatives and the number of alternatives in the choice set.

4) The elasticity of the choice probability of alternative i with respect to the attributes of any other alternative $j \neq i$ is constant, that is independent of i (see Hausman (1975) and Malhotra (1984)).

5) The social surplus function as defined in McFadden (1981) is separable.

6) Of the axioms of choice required to make use of a utility function, continuity and transitivity are of key importance. The continuity assumption allows the derivation of systems of demand equations by simple differentiation. For the preferences of the individual to be transitive requires that if A is preferred to B and B is preferred to C then A is preferred to C. Luce (1959) and Ben-Akiva and Lerman (1985) interpret the IIA property as being a probabilistic interpretation of transitivity. In fact, the IIA property is stronger than that of transitivity. Transitivity is a purely ordinal concept whereas IIA with its explicit use of choice probabilities is a cardinal concept. This arises as orders of magnitude matter in IIA to keep the odds ratio invariant to the number of alternatives in the choice set.

Essentially these conditions are the same and the list could be drastically reduced to include just points 2 and 3. However, in the light of the previous literature it would be misleading to make such a reduction. Note also that this property may apply to models other than Logit models. For example, it is possible to have an independent Probit model which exhibits IIA (see Currim (1982)). However, in the literature the existence of IIA is usually taken to be synonomous with the use of a Logit model. The Logit model has been extended by McFadden (1978) to the Generalised Extreme Value (GEV) model and

McFadden (1981) to the Nested Multinomial Logit (NMNL) model. These extensions are designed to tackle the problems associated with the IIA property inherent in Logit while retaining computational tractability and also to allow for the testing of the IIA property.

Hausman and McFadden (1984) propose two alternative tests for IIA. The first test is an application of Hausman's (1978) specification error test. The idea is simple: if the IIA property is valid, then the model structure and parameters are unchanged when choice is analysed conditional on a restricted subset of the full choice set. The second test proposed in Hausman and McFadden (1984) is a classical large sample (i.e. asymptotic) test (namely a Likelihood ratio, Wald or Lagrange multiplier test). While the use of the first specification test requires no specific alternative, the second test specifies an alternative model, the so-called NMNL model. The idea of an NMNL model is to make relative probabilities of choosing two alternatives in a choice set dependent on characteristics of not only these two alternatives, but also other alternatives in the choice set. The NMNL is not the only alternative model that we could specify. Tse (1987) sets up the DOGIT model (see Gaudry and Dagenais (1979), Hensher (1982)) as the alternative to the Multinomial Logit model. Further alternatives could be considered but it must be pointed out that the use of an alternative model specification does not represent a model selection exercise. The alternative used is merely the departure from the null in the direction in which high power of the test is desired.

Both tests for IIA proposed by Hausman and McFadden (1984) are available in the frequently-used statistical software LIMDEP (Greene, 1991). One problem with the first Hausman-McFadden test is that it requires the inverse of the estimated covariance matrix, which sometimes may not exist in small samples. Further, LIMDEP can only calculate a covariance matrix to a maximum order of 50 by 50 so computational problems arise if a model has more than 50 parameters. The problem with the second Hausman-McFadden test is that it involves specifying the structure of the NMNL model.

Small and Hsiao (1985) propose a test that avoids these problems and can be easily implemented. The Small-Hsiao test is based on the Likelihood ratio test of McFadden, Tye and Train (1977), which Small and Hsiao show is biased in large samples towards failing to reject IIA. Yet again, the basic idea is to see how different the maximum likelihood values are with the full choice set and with the restricted choice set. As yet the Small-Hsiao test has not been pre-programed in LIMDEP, but it is easy to compute within the package.

It should be pointed out that rejection of IIA can also be caused by other mis- specifications in the model. In particular, omission of relevant variables (e.g. estimating a MNLGT or CLGT model while the true model is a mixed Logit) can produce the rejection. Thus, the IIA tests cannot identify IIA versus true omission of relevant variables as a source of rejection. However, this lack of identification is no barrier to performing the test, which in any case detects a failure of the model under consideration. This consideration is of course true of any testing situation. Tests only ever reject the null model or fail to reject it. Therefore they never accept a particular model. Rejection of the null can always be for a variety of reasons other than the specified alternative. In this context it is perhaps best to view IIA testing as a general test for model mis-specification and, as a matter of course, IIA testing should go hand in hand with other diagnostic tests such as those discussed in McFadden (1987) and Pagan and Vella (1989).

Further there are problems in the economic interpretation of testing IIA. The current testing paradigm requires the restricted model to be of the CLGT form for IIA to be accepted. However the use of the CLGT form implies that only the characteristics of the alternatives matter. It is in this model that it is least plausible that additions to the choice set of new bundles of characteristics leaves the structure of the choice probabilities unchanged. This inappropriateness of the use of the CLGT model is further reinforced if the consumers' perception of the value of alternatives is conditional upon the known availaibility of other alternatives.

Kinks etc.

The traditional consumer optimisation problem maximises utility subject to a linear budget constraint. This framework implies an institutional setting of efficient markets with negligible transaction costs (see Deaton and Muellbauer (1980, p.5)). In certain situations we can find a non-linear budget constraint. The existence of price discrimination of the first and second kind (see Pigou (1962, p.279)) will generate non-linear budget constraints. For example:

(i) The pricing schemes of public authorities will frequently be non-linear. This will apply to the provision of services such as gas, water and electricity.

(ii) A large number of products are sold with discounts for bulk buying (e.g. foodstuffs). One will typically find the unit price to be lower the larger is the quantity of the commodity bought.
(iii) The interest rate paid on bank deposits will usually be related to the amount deposited. Usually the rate of interest will be greater the greater is the amount deposited.
However the applied researcher must be careful to distinguish between price discrimination and joint supply (see Demsetz (1973)).

Hausman (1985) makes the case for durable goods on the basis of household production theory. The household will purchase the durable as an input into its production process. Therefore the purchase decision is joint with the utilisation decision. In this sense the problem of a non-linear budget constaint is closely linked to the problem of discrete/continuous modelling and thus similar models could be utilised.

The most popular form of non-linear budget constraint that is modelled is in the area of labour supply. Interaction of the tax and social security systems of Western countries produces a non-linear budget constraint. However, the actual constraint produced is quite complex, which usually restricts researchers to modelling the observed discrete choice between non-participation, part-time and full-time work and may lead to the use of ordered (Logit or Probit) models. Models with non-linear budget constraints are surveyed in Moffit (1990) and the estimation of the resultant models has been considered by Moffit (1986) and Megdal (1987).

Modelling with non-linear budget constraints can be of importance in policy making. Symons and Walker (1990) using this framework have considered the work disincentive effects of certain forms of welfare payments. Zabalza, Pissarides and Barton (1980) also use this framework to model the retirement decision. Applications of this type of model are clearly of interest in an Australian context (e.g. analysis of government policy initiatives in the superannuation/retirement saving field).

The simple version of the traditional approach to consumer theory rules out corner solutions and restricts attention to interior solutions. Those at an interior solution will already be consuming the product and will make the required marginal substitutions in response to a price change. However, it is possible that some individuals not consuming a good (i.e.

at a corner solution) can be induced to do so by a price fall. In theory in response to a price fall continuous substitution from the corner solution will occur. However, if there are non-negligible transactions costs in consumption small changes will not be observed and discrete switches will. That is, such individuals will make a discrete switch rather than the smooth adjustment made by those already consuming the good since consumption of a good must by definition be non-negative. This is a problem of asymmetric information which is only present in the case of a price fall.

Pudney (1989, pp. 158-159) identifies three main reasons for the existence of corner solutions:

(i) Shortness of observation period,

(ii) Involuntary behaviour,

(iii) Utility maximisation.

The shortness of observation period is quite likely in applied work. This is particularly so with durables. Although individuals may not purchase a durable during the observation period they are likely to obtain consumption services from a purchase in a prior period. Therefore models of purchases are not necessarily models of consumption.

Individuals may be at a corner solution because they have no choice (e.g the labour supply decision affected by involuntary unemployment). Individuals could also reach an optimum at a corner solution by way of an economically rational decision. For example, the modelling of the demand for alcohol may be a case of this decision making process. Atkinson, Gomulka and Stern (1990) find in their modelling of U.K. alcohol consumption from the family expenditure survey that approximately 20% of households record zero purchases.

The existence of such situations can be difficult to handle. Corner solutions give rise to censoring or truncation in the observed data. As has been noted earlier this rules out the application of conventional OLS regression techniques and takes us to Tobit type models. The standard Tobit model results from a normality assumption. However, this may not be the appropriate model to use in empirical work. More general Tobit models allowing for asymmetric behaviour are considered in Atkinson, Gomulka and Stern (1990) and Fry (1991).

Pudney (1989, pp.262-279) considers the effect of rationing, which is essentially a form of non-price allocation, on choice behaviour and finds that it gives rise to censoring and truncation in observed data. Both the empirical importance of and the need for appropriate modelling of rationing was noted by Tobin and Houthakker (1951). Rationing elements are present in the allocation of many commodities. Rationing is frequently used in the allocation of tickets to entertainment. Further to this in some countries it is used to allocate basic commodities.

It should also be noted that there exist forms of non-price allocation other than rationing. A recent example is the paper by Hartley and Trengrove (1990) which considers capacity constraints for airline travel. Another example is queueing (see Barzel (1974)) which is used in the allocation of certain commodities. Further, a rationing interpretation is placed on corner solutions by Pudney (1989, p.159). Therefore we should not be surprised that in both of these cases Tobit type models are obtained.

Other Motivations.

Many modelling situations may be motivated in more than one way. In particular, it is possible to use a random utility maximisation model to motivate many of the following situations. However, it is of interest to consider competing ways of arriving at appropriate limited dependent and/or qualitative variable models as they may yield insights into current research problems. The majority of the following situations are, in reality, interpretations of the latent variable modelling approach used by statisticians/econometricians to analyse the properties of these models (see *inter alia* Gourieroux (1989), Greene (1990)). Essentially this approach involves specifying a model structure for the underlying latent variable(s) and a known mapping from these variable(s) to their observable realisation. The combination of distributional assumptions made in the latent variable and the mapping to the observable variable(s) then defines the probability structure of the model for the observable variable(s). This structure is then typically used in maximum likelihood estimation of the model.

Pudney (1989, p.92) claims that many important realistic choices are discrete. This gives rise to a choice set consisting of a finite number of mutually exclusive alternatives. This breaches the continuity assumption of traditional theory and therefore rules out this approach. Economic examples of such discrete choices are easily found. One may deal with indivisible goods which give rise to discrete choices. The purchase of durables is a good example of this indivisibility. Deaton and Muellbauer (1980, p.345) point out that a key aspect of modelling the demand for durables is the distinction between purchases and consumption. Economic modelling of fertility is another example. While parenthood rights are not traded in markets one can still determine the shadow prices attached to children. Biology dictates that the fertility choice set be discrete. The choice of occupational status (see Robertson and Symons (1990)) is also a situation where individuals are faced with a set of discrete alternatives.

The majority of real world goods are heterogeneous and therefore of different quality (e.g. durable goods). These differences in quality are likely to be discrete. As Ben-Akiva and Lerman (1985, p.35) point out, these differences arise because individuals are interested in the attributes of goods and not the goods *per se*. The attributes of the goods are continuous and are of interest to the individual consumer in a household production framework. They are however unobservable and are therefore latent variables. What is observed is the choice of the individual amongst discrete alternatives.

If the economic decisions underlying an unordered multinomial variable are made sequentially over time (e.g. educational attainment, see Maddala (1983,pp.49-51), fines default, see Stagoll and Fry (1991)) then the estimation of the multinomial model can be reduced to the successive estimation of binary models. Such a reduction may occur in a wide variety of applications, particularly if we consider that economic agents often adopt lexicographic choice behaviour (see Earl (1990, p.729)). The use of sequential models is considerably easier than the use of the multinomial Logit or Probit models discussed earlier. There is, however, a disadvantage in that the random factors influencing decisions at the different stages must be assumed independent, which in many economic situations will be inappropriate. One area where sequential models may prove useful is in the analysis of sequential games.

When the multinomial variable represents an ordering or ranking then ordered variants of the Logit and Probit models are appropriate. Economic examples of such behaviour include voting outcomes in public choice (Zavonia and McElvey (1975)) and ratings of corporate bonds (Terza (1985)). Ordered models may also be useful in the case of "grouped" data where the data have been collected in ranges (e.g. earnings, see Stewart (1983)). To apply ordered models it is argued that the underlying economic variable is an unobservable, continuous variable and that the values that are observed are in a clearly ordered sequence.

If we have panel data available to us (e.g. N firms and T time periods) which records behaviour of interest as qualitative and/or limited (e.g. censored or truncated) then the fixed and random effects panel data models (see Chamberlain (1980), Hsiao (1986)) may be modified to produce panel qualitative and limited dependent variable models.

Panel data models allow the modelling of dynamic behavioural responses and so extend the range of economic situations which can be analysed. An example of a panel data study is Hausman and Taylor (1981) who were interested in earnings determination. An example of Chamberlain's panel data Logit model is Cecchetti (1986) who studied the newsstand prices of magazines. Another area in which such models may prove useful is the analysis of repeated games or any other economic situation where the outcomes of the same decision are observed several times.

As noted earlier when the dependent variable is a non-negative integer count data models are required. While there is scope to use these models in econometrics, their use to date has been limited. A key paper using these models is Hausman, Hall and Griliches (1984) who consider the modelling of research and development activity by firms. Unfortunately the variable of interest is latent, and for its observed realisation use is made of the number of patents applied for in a given period. By its very nature the dependent variable in their paper is a count requiring the consideration of the count data models.

Cameron and Trivedi (1986) also suggest a number of economic situations in which count data models may be relevant. In the context of consumption of medical services they suggest that the observed realisation may be the number of visits to a medical practitioner in a specified period. In the context of labour economics they suggest that important counts with an economic interpretation may be number of strikes per period, and number of spells of unemployment for an individual in a given period. A further example of the modelling of count data is the recent paper by Terza and Wilson (1990) which develops a mixed Multinomial-Poisson approach in modelling choices among types of trips and frequency of trips.

However there may be interpretive difficulties in the economic modelling of counts. The number of times an event occurs, while important, may be secondary to its intensity at each occurrence (e.g. in the area of strikes both their frequency and duration may be important). The fact that count data models are only concerned with the number of occurences may therefore limit their potential applicability in economics.

4. Conclusion.

Many excellent surveys of the statistical aspects of limited dependent and qualitative variable models exist, some of which were mentioned in the introduction to this paper. Although it must be said that most of these are mainly concerned with the specification and estimation of the models, less attention has been paid to model evaluation and, in particular, to diagnostic testing. In many of these models the distributional assumption is crucial to the statistical properties of the model and thus should be tested. However, until the recent paper by Pagan and Vella (1989) researchers were unable to find, in one place, details of the available diagnostic tests.

This paper, however, has considered the economic motivations for limited dependent and qualitative variable models, some of which have not been discussed in the recent book by Pudney (1989), and as such may be viewed as a complement to the book. The paper has shown how modifications to the traditional economic optimisation problem can be seen as relaxations of the requirement that the objective function and the constraints be continuous and deterministic, and that the constraints be linear, and how these modifications can give rise to limited dependent and qualitative variable models. In interpreting the results of applied work it is important that researchers are aware of these foundations. The use of these models is not a mere exercise in statistics but is of clear relevance to the economist. Furthermore by understanding that these models do have economic foundations, economists may be able to see their theories implemented in a more appropriate fashion.

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