MONASH UNIVERSITY

AUSTRALIA

ESTIMATING LONG RUN RELATIONSHIPS IN ECONOMICS: A COMPARISON OF DIFFERENT APPROACHES

Brett Inder

Working Paper No. 18/91

December 1991

DEPARTMENT OF ECONOMETRICS

ISSN 1032-3813

ISBN 0 86746 967 6

ESTIMATING LONG RUN RELATIONSHIPS IN ECONOMICS: A COMPARISON OF DIFFERENT APPROACHES

Brett Inder

Working Paper No. 18/91

December 1991

DEPARTMENT OF ECONOMETRICS, FACULTY OF ECONOMICS COMMERCE & MANAGEMENT

MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

ESTIMATING LONG RUN RELATIONSHIPS IN ECONOMICS:

A COMPARISON OF DIFFERENT APPROACHES

by Brett Inder

Department of Econometrics
Monash University
Clayton
Victoria 3168
AUSTRALIA

Abstract

One of the benefits of the Engle and Granger (1987) two-step procedure for modelling the relationship between cointegrated variables is that the "long run equilibrium" relationship can be estimated consistently by a straighforward OLS regression involving the levels of the variables. Test statistics with appropriate asymptotic distributions can also be computed fairly easily by applying the modifications of Park and Phillips (1988). However, the omission of dynamics may well be detrimental to the performance of the estimator in finite samples.

In this paper we use a Monte Carlo study to compare various estimators of the long run parameters. It is found that estimates which include the dynamics are much more reliable, even if the dynamic structure is overspecified. Furthermore, even though t statistics based on Park and Phillips' fully modified estimator are asymptotically valid, they do not have good finite sample properties. In contrast, the sizes of t tests based on an estimator which does make use of dynamics are very reliable.

*The author would like to thank Peter Burridge, Rob Engle, Clive Granger, Bruce Hansen, Keith McLaren, Adrian Pagan and anonymous referees for helpful discussions.

1. INTRODUCTION

The Engle and Granger (1987) two-step procedure for modelling the relationship between cointegrated variables has received a great deal of attention in recent years. One of its benefits is that the long run equilibrium relationship can be modelled by a straightforward regression involving the levels of the variables. All dynamics can be ignored and endogeneity of any of the variables has no effect asymptotically. In fact, the Ordinary Least Squares (OLS) estimator is "super-consistent"; it converges to the true value at a rate faster than in normal asymptotics. The attractiveness of these results can be seen easily by considering the vast number of applications of the procedure in recent literature.

There have, however, been concerns expressed at the approach taken in the two-step procedure. Some authors (for example, Banerjee *et al.*, 1986) stress that although the dynamics are asymptotically irrelevant in the first step, ignoring lagged terms may lead to substantial bias in finite samples. The small Monte Carlo study reported in Banerjee *et al* lends support to this allegation. Others (in particular, Park and Phillips, 1988) are more concerned with the fact that the OLS estimator in the first step has an asymptotic distribution which is non-normal, and depends on nuisance parameters. This makes inference difficult, and in particular the standard t statistics that are produced by most regression packages will not even be valid asymptotically.

Because these two groups of critics emphasise different aspects of the problem, naturally they recommend different solutions. Banerjee *et al* (1986) and many others, advocate estimating long run parameters in an unrestricted Error Correction Model (ECM) form, incorporating all the dynamics. Stock (1987) also recommends this, describing the estimator as nonlinear least squares (NLS). On the other hand, Phillips and Hansen (1990) (based on results in Park and Phillips, 1988) advocate using semi-parametric corrections to the OLS estimator to eliminate dependency on the nuisance parameters, and to give an estimator which follows a normal distribution asymptotically. They call this the Fully Modified OLS estimator.

Recent papers by Phillips and others have put a strong case for Modified OLS in preference to what we will describe as the Unrestricted ECM estimator. Phillips (1988) shows that the latter approach is not asymptotically optimal, as it takes no account of the possible endogeneity of the explanatory variables. A simple Monte Carlo study reported in Phillips and Hansen (1990) showed the ECM estimator to perform fairly well compared with Modified OLS, but t-statistics on the long run parameters can be quite misleading in the former case.

In this paper we make the following contributions to the debate:

1. We show that Phillips and Hansen's (1990) Monte Carlo design is 'biased' in favour of Modified OLS, and when a more realistic Monte Carlo is undertaken, the Unrestricted ECM estimator performs far better than OLS or Modified OLS.
2. We demonstrate that the semi-parametric corrections applied to OLS can also be applied to the ECM estimator, giving a Fully Modified Unrestricted ECM estimator which <u>is</u> asymptotically optimal.
3. We find that the effects of endogeneity on the bias and distribution of the ECM estimator are minimal.

The model and estimators are described in section 2. A Monte Carlo comparison is presented in section 3, with conclusions and recommendations in the final section.

2. THE MODEL AND ESTIMATORS

2.1 The Model

In this paper we focus on the modelling of one variable, y_t, although many of the issues discussed will apply to a system of equations. We consider the autoregressive model

$$\alpha(L)y_t = \mu + \beta'(L)x_t + u_t, \qquad (t=1, \ldots, T) \tag{1}$$

where y_t is a scalar, x_t is a kx1 vector of explanatory variables, u_t is a stationary error term, and $\alpha(L)$ and $\beta(L)$ are lag polynomials. Specifically,

$$\alpha(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \ldots - \alpha_p L^p, \tag{2}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_p$ are unknown scalar parameters, and

$$\beta(L) = \beta_0 + \beta_1 L + \beta_2 L^2 + \ldots + \beta_q L^q, \tag{3}$$

where $\beta_0, \beta_1, \ldots, \beta_q$ are kx1 vectors of unknown parameters. We assume that x_t is a vector of random variables and in particular that

$$x_t = x_{t-1} + v_t, \tag{4}$$

where v_t is stationary. By (4) we are implying that each of the regressors are integrated of order 1, or $I(1)$, and we further assume that y_t and x_t are cointegrated (see Engle and Granger, 1987), so y_t is $I(1)$.

The specification of the x_t vector as a random walk may seem rather restrictive, but the asymptotic results allow for v_t to follow a general ARMA process, so in a sense (4) could possibly be seen as a set of 'reduced form' equations for x_t . We allow for $Cov(u_t , v_t) \neq 0$, in which case x_t is endogenous.

Equation (1) can be rewritten as

$$\alpha(1)y_t = \mu + [\beta'(1) + \beta'(L) - \beta'(1)]x_t + [\alpha(1) - \alpha(L)]y_t + u_t$$

$$\Leftrightarrow \quad y_t = \frac{\mu}{\alpha(1)} + \frac{\beta'(1)}{\alpha(1)} x_t + \frac{[\beta(L) - \beta(1)]'(1-L)}{\alpha(1)(1-L)} x_t + \frac{[\alpha(1) - \alpha(L)](1-L)}{\alpha(1)(1-L)} y_t + \frac{u_t}{\alpha(1)}$$

$$= \lambda_1 + \lambda_2' x_t + \gamma_2'(L)\Delta x_t + \gamma_1(L)\Delta y_t + \varepsilon_t , \tag{5}$$

where $\lambda_1 = \mu/\alpha(1)$, $\lambda_2 = \beta(1)/\alpha(1)$, $\gamma_2(L) = \dfrac{\beta(L) - \beta(1)}{\alpha(1)(1-L)}$

and $\gamma_1(L) = \dfrac{\alpha(1) - \alpha(L)}{\alpha(1)(1-L)}$.

Alternatively the model can be written in ECM form:

$$\left[\alpha(L) \frac{(1-L)}{(1-L)} - \alpha(1) \frac{(1-L)}{(1-L)} L^p\right] y_t = \mu + \left[\beta(L) \frac{(1-L)}{(1-L)} - \beta(1) \frac{(1-L)}{(1-L)} L^p\right]' x_t - \alpha(1)L^p y_t + \beta(1)L^p x_t + u_t$$

$$\Leftrightarrow \quad \delta_1(L)\Delta y_t = \mu + \delta_2'(L)\Delta x_t - \alpha(1)[y_{t-p} - \lambda_2' x_{t-p}] + u_t, \tag{6}$$

where $\delta_1(L) = \dfrac{\alpha(L) - \alpha(1)L^p}{(1-L)}$, and $\delta_2(L) = \dfrac{\beta(L) - \beta(1)L^p}{1-L}$.

The parameter λ_2 measures the long run impact of x on y, and hence if primary interest is in the long run equilibrium relationship between x and y, λ_2 is a key parameter of interest.

2.2 OLS Estimator

Considering the version of the model given in (5), Engle and Granger (1987) point out that since y_t and x_t are I(1), and thus Δy_t and Δx_t are I(0), if we are interested only in λ_2, this can be estimated consistently by OLS, even if we omit the I(0) variables. In fact, Stock (1987) and others have shown that the OLS estimator of λ_2 converges to the true value at a rate of $O\left(\frac{1}{T}\right)$ instead of $O\left(\frac{1}{\sqrt{T}}\right)$ in traditional asymptotics applied to I(0) variables. It has thus become a very popular practice in applied research where interest is focused only on long run behaviour to estimate equations with y_t regressed only on a constant and x_t. This is also the recommended approach for the first stage of the Engle and Granger two-step procedure which has been widely used. In this case (5) can be written

$$y_t = \lambda_1 + \lambda_2' x_t + w_t , \tag{7}$$

where w_t is an I(0) disturbance.

Phillips and Durlauf (1986) derive the asymptotic distribution of the OLS estimator of λ in (7), where $\lambda' = (\lambda_1, \lambda_2)$. They show that $\hat{\lambda}$ has a distribution which is a ratio of integrals of functions of Brownian motion, and that this distribution is highly dependent on nuisance parameters. Similarly, the standard t and F statistics based on λ have equally complicated asymptotic distributions. So while it is relatively simple to obtain consistent estimates of λ by OLS, it is not possible to make inference by the use of standard test statistics.

2.3 Fully Modified OLS Estimator

In order to enable hypothesis tests to be carried out on λ, Phillips and Hansen (1990) (see also Park and Phillips (1988)) have proposed a Fully Modified OLS estimator. This estimator removes some second order bias in OLS, and also gives t statistics which follow a standard normal distribution asymptotically. Following Phillips and Hansen (1990), we assume the disturbance vector $e_t = (w_t, v_t)'$ (w_t is from equation (7) and v_t from equation (4)) is strictly stationary with zero mean, finite covariance matrix Σ and long run covariance matrix Ω. We decompose Ω as follows:

$$\Omega = \Sigma + \Lambda + \Lambda' ,$$

where $\Sigma = E(e_0 e_0')$ and $\Lambda = \sum_{k=1}^{\infty} E(e_0 e_k')$, and we define $\Delta = \Sigma + \Lambda$. We also let $X = [1, x_t]_{t=1,\dots,T}$. The Fully Modified OLS estimator is given by

$$\hat{\lambda}^{+} = (X'X)^{-1}[X'\hat{y}^{+} - DT(\hat{\Delta}_{21} - \hat{\Delta}_{22}' \hat{\Omega}_{22}^{-1} \hat{\Omega}_{21})], \qquad (8)$$

where

$$\hat{y}_t^{+} = y_t - \hat{\Omega}_{12} \hat{\Omega}_{22}^{-1} \hat{v}_t ,$$

$$D' = [0 \ I_k],$$

and $\hat{\Delta}_{21}$, $\hat{\Delta}_{22}$, $\hat{\Omega}_{12}$, $\hat{\Omega}_{22}$ are consistent estimates of the variances and covariances, based on OLS residuals $\hat{w}_t$ and $\hat{v}_t$.

Hansen and Phillips (1990) give the form of the t statistic for an element of λ, say λ_i, based on the modified estimator:

$$t(\lambda_i) = \frac{\hat{\lambda}_i^{+} - \lambda_i^{0}}{\left[(X'X)_{i,i}^{-1} \hat{\omega}_{11.2}\right]^{1/2}} , \qquad (9)$$

where $\hat{\omega}_{11.2} = \hat{\omega}_{11} - \hat{\Omega}_{12}\,\hat{\Omega}_{22}^{-1}\,\hat{\Omega}_{21}$. It can be shown that $t \Rightarrow N(0,1)$ as $T \to \infty$.

The use of fully modified OLS estimates and their associated t statistics thus provides a workable means of both estimating and performing hypothesis tests on the long run parameters of a model. The estimator should have even smaller bias than the already consistent OLS, and t statistics should have valid size asymptotically.

The main potential difficulty with the OLS and fully modified estimators is that they ignore the I(0) terms in (5). This will most likely have some effect on the precision of the estimators, and it will certainly have implications for the performance of t statistics. The disturbance w_t will clearly display substantial serial correlation, which is only dealt with in a non-parametric way in the developments of t statistics like (9). It is well known that the presence of undetected autocorrelated disturbances can lead to serious biases in estimates of standard errors and t statistics. Thus although the t statistic in (9) has a standard normal distribution asymptotically it may well be far from normal in finite samples. The Monte Carlo evidence of the next section confirms this fear.

2.4 Unrestricted ECM Estimator

It was shown above that if one's purpose is to estimate the long run parameter λ_2, it is asymptotically valid to omit the I(0) terms in (5), and estimate (7). However, there seems to be no reason why the full model cannot be estimated - one might expect this to lead to an improvement in efficiency in finite samples. The most obvious way to estimate the long run parameters would seem to be to estimate (1) or (6) by OLS, and then solve for λ_2. A nonlinear least squares algorithm could be used to obtain standard errors for the estimates. Either formulation would give the same inference on λ_2.

If nonlinear estimation is seen as an obstacle, identical results can be obtained by estimating (5) directly in a manner proposed by Bewley (1979). Bewley's estimator involves estimating (5) by instrumental variables (IV), with instruments for Δx_t and Δy_t being x_{t-1} and y_{t-1}. This estimator gives the same values for λ_2 and their standard errors as if (1) or (6) had been estimated by OLS [see Bewley (1986, p.69-73), and Wickens and Breusch, (1988)]. The only difference is the possible computational convenience of obtaining estimates and t statistics directly.

Phillips (1988) has shown that when x_t is exogenous, the above procedure is asymptotically equivalent to his fully modified estimator. In fact, the inclusion of the lagged terms in the estimated equation is an alternative to estimating Ω as a means of adjusting the OLS estimator. However, when x_t is endogenous, Phillips shows that the unrestricted ECM estimator does not adequately adjust OLS - it should yield some improvement, but the asymptotic distribution of the estimator is not free of nuisance parameters. Consequently, the t statistics do not have a standard normal distribution asymptotically.

2.5 Fully Modified Unrestricted ECM Estimator

Recall that the unrestricted ECM estimator is equivalent to estimating (5) by IV. The form of (5) suggests that the semi-parametric corrections advocated by Phillips and Hansen (1990) could be applied to the estimate of λ_2, the coefficient of the I(1) variable in (5). This would give an estimator which has the efficiency of using the full model, as well as the benefits of asymptotic optimality and an asymptotic distribution free of nuisance parameters.

To obtain this estimator we would proceed in two stages (see Gregory, Pagan and Smith, 1990).

1. Obtain unrestricted ECM estimates of λ and the coefficients of the lag polynomials $\gamma_1(L)$ and $\gamma_2(L)$, called $\tilde{\lambda}$, $\hat{\gamma}_1(L)$, $\hat{\gamma}_2(L)$.

2. Define $y_t^* = y_t - \hat{\gamma}_2(L)\Delta x_t - \hat{\gamma}_1(L)\Delta y_t$, and find the Fully Modified OLS estimator of λ in a regression of y^* on X.

Tests of hypotheses about λ can be based on the appropriate t statistics that come from the Fully Modified estimates in the second stage (see equation (9)).

This estimator does not suffer from the criticisms levelled at OLS and Modified OLS, namely that they omit possibly important dynamics. It also has the advantage over the standard Unrestricted ECM estimator of being asymptotically optimal, even in the presence of endogenous regressors. The Monte Carlo study to follow will explore just how much improvement it gives.

3. MONTE CARLO COMPARISON

3.1 Estimator precision

In their comparison of the OLS, Modified OLS and ECM estimators (among others), Phillips and Hansen's (1990) and Phillips and Loretan's (1991) Monte Carlo design allow for serial correlation in and dependence between u_t and v_t, but restricts the model by having $\beta_1 = \alpha_1 = 0$. In other words, the true data generating process has no dynamics (except via the error), so naturally it favours the OLS and Modified OLS estimators. The Unrestricted ECM estimator is based on an overspecified model, particularly as they allow for up to three lags of x_t and y_t in the ECM estimation. Despite this handicap, the ECM approach does reasonably well in most cases. It would seem only fair, though, to allow for some dynamics in the experiments, to see how Modified OLS performs in non-ideal situations.

In the light of these comments, we specify the following model in our Monte Carlo study of the estimators:

$$y_t = \mu + \beta_0 x_t + \beta_1 x_{t-1} + \alpha_1 y_{t-1} + u_t \qquad (t=1, \ldots, T), \qquad (10)$$

$$x_t = x_{t-1} + v_t \qquad (t=1, \ldots, T), \qquad (11)$$

where u_t and v_t are generated by the processes:

$$u_t = \rho_{11}\eta_{1t} \qquad (12)$$

$$v_t = \rho_{21}\eta_{1t} + \rho_{22}\eta_{2t} + \rho_{23}\eta_{1t-1}, \qquad (13)$$

with η_{1t} and η_{2t} being independently and identically distributed standard normal variables.

We will focus attention on the long run parameter $\lambda_2 = (\beta_0 + \beta_1)/(1 - \alpha_1)$, comparing the precision of the four estimators described in section 2. Bias, Root Mean Squared Error and Probability of Concentration are used to assess the estimators.

The following sets of parameter values are used in our Monte Carlo study:

$T = 50,\ 200;\ k = 1;\ x_0 = 1;\ y_0 = 1;\ \mu = 0.$

$$(\beta_0, \beta_1, \alpha_1) = \{(1, 0, 0), (.6, 0, .4), (.2, 0, .8),$$
$$(.6, .4, 0), (.4, .2, .4), (.1, .1, .8)\}.$$

$\rho_{11} = .2;\ (\rho_{21}, \rho_{22}, \rho_{23}) = \{(0, 1, 0), (.5, .866, 0), (.5, .707, .5)\}$

The various combinations of the ρ_{2i} allow for correlation between u_t and v_t, and hence the endogeneity of x_t, and for some serial correlation in v_t.

A few brief comments need to be made about how the estimators are computed. The estimates of Ω and Δ used in the modified estimators are based on the residuals $\hat{u}_t$, $\hat{v}_t$ and $\hat{w}_t$ from (10), (11) and (7) respectively. The lag truncation number used in calculating Ω and Δ for fully modified OLS depends on the values of β_1, α_1 and on the autocorrelation in v_t. Values chosen range from zero when β_1, α_1 and ρ_{23} are zero through to 11 when $\beta_1 = .1$, $\alpha_1 = .8$ and $\rho_{23} = .5$. The Modified ECM estimator uses a lag truncation of zero whenever $\rho_{23} = 0$, and one when $\rho_{23} = .5$. In both cases this lag truncation represents as close as possible to the "correct" truncation if all parameter values were known, so if anything the results should favour the modified estimators.[1]

Tables 1 and 2 present results on the precision of the estimators. We observe the following:

(i) In most cases the modified OLS estimator yields no improvement on OLS. When there is an improvement, the reduction in bias is fairly minimal, and clearly not adequate, in cases where $\alpha_1 \neq 0$. It seems the semi-parametric correction is insufficient to remove the autocorrelation in the error when the data generating process includes a lagged dependent variable.

(ii) The Unrestricted ECM estimator appears to perform very well. There is a huge improvement in precision over OLS and modified OLS in cases where α_1 or β_1 are non-zero. In a large number of cases it is the closest to the true value. This confirms the claim that the I(0) terms play a vital role in the precision of estimators in finite samples.

When $\alpha_1 = \beta_1 = 0$, the ECM estimator is based on an overspecified model, while OLS and modified OLS utilise the "correct" model. Despite this, the ECM estimator still performs very well in these cases. Its bias, RMSE and Pr(Concentration) is similar to the better of OLS and modified OLS in each situation. These results suggest the possible broad conclusion that it is better to <u>overspecify</u> the dynamics of the model than to underspecify.

(iii) The other striking observation from the tables is that the precision of the ECM and modified ECM estimators is quite similar. The difference in bias, RMSE and Pr(Concentration) is negligible in almost every case. Recall that the modifications to the estimator are intended to deal with autocorrelation in and cross-correlation between u_t and v_t. We would thus expect little difference between the estimators when $\rho_{23} = 0$, where u_t and v_t are independent of each other and over time. However, in the last block of the tables u_t and v_t have a covariance of 0.5, so we would expect the ECM estimator of the long run parameter to display simultaneity bias, which is then corrected by the modified ECM. However, it is obvious from the tables that the bias is not substantial in these experiments.

This point obviously requires further investigation, but it does suggest that the effects of simultaneity bias on long run parameters are much smaller than on short run parameters. In particular, the present evidence suggests that it may be unnecessary to use the modified ECM estimator: the standard unrestricted ECM estimator performs as well, if not better.

3.2 Hypothesis Testing

To examine the performance of t tests based on each estimator, we add a second explanatory variable to the model and perform tests on its coefficient. The model is

$$y_t = \mu + \beta_0 x_t + \beta_1 x_{t-1} + \gamma_0 z_t + \alpha_1 y_{t-1} + u_t , \qquad (14)$$

with x_t, u_t and v_t generated by (11), (12), and (13), and

$$z_t = z_{t-1} + w_t , \qquad (15)$$

where

$$w_t = \rho_{21}\eta_{1t} + \rho_{22}\eta_{3t} + \rho_{23}\eta_{1t-1} , \qquad (16)$$

and η_{3t} is another standard normal variate, independent of η_{1t} and η_{2t}.[2]

We will consider tests on the long run parameter $\lambda_3 = \gamma_0/(1-\alpha_1)$. We test $H_0 : \lambda_3 = 0$ against the two-sided alternative $H_1 : \lambda_3 \neq 0$ using critical values from the standard normal distribution. Nominal sizes of 5% and 1% were used, although results are given for the 5% values only. Under H_0, rejection probabilities are evaluated with $\gamma_0 = \lambda_3 = 0$, and under H_1, $\lambda_3 = .04$ for $T = 50$ and $.008$ for $T = 200$.[3] These values were chosen to give a realistic range of powers.

Tables 3 and 4 give the results on the sizes and powers of the various tests.[4] We first discuss their performance under H_0. The following general observations can be made.

(i) The rejection probabilities for the OLS based test are unacceptable. In only two situations is the probability close to the nominal value - the cases where $\beta_1 = \alpha_1 = 0$. Other values of the parameters give sizes up to 66% for a nominal size of 5%.

Modified OLS gives t-statistics whose sizes are generally no better than the OLS results. Neither test procedure shows any improvement for the larger sample size. This is not unexpected for OLS, since the t statistics are not based on the estimator's asymptotic distribution. The poor performance of t-statistics based on modified OLS suggests that in this case a very large sample is required for the asymptotics to take effect.

(ii) Rejection probabilities for the Unrestricted ECM estimator represent a vast improvement on OLS and modified OLS. With a nominal size of 5%, actual probabilities range from 1.9% to 7.4%, much more reasonable than the alternatives. It seems that the sizes of ECM-based tests are less vulnerable to changes in the values of the parameters of the data generating process: fluctuations in size are minimal, and seem not to follow any pattern. Significantly, the ECM estimator has reasonable sizes when $\beta_1 = \alpha_1 = 0$; that is, when there is no dynamics and estimation is based on an overspecified model.

(iii) Sizes of tests using the modified ECM estimator are inferior to those based on ECM. In some cases, in particular, where the errors are autocorrelated - the modified ECM has rejection probabilities of up to

27%, and in other cases the size is an small as zero! As with modified OLS, it seems that a very large sample is necessary for the test's asymptotic superiority to be apparent.

Power results are difficult to compare because of the widely disparate sizes, but we can observe the following.

(i) Power of the tests deteriorated dramatically as α_1 increases and hence γ_0 decreases. For example, in the first column of Table 3, sizes for the ECM test are relatively stable, but its power drops from 74.9% to 11.3% as α_1 increases, even though λ_3 is the same in each case. This suggests that tests of long run parameters are more difficult to perform when adjustment to the long run is slow.

(ii) Tests based on the ECM estimator seem to be slightly less powerful than OLS when there is no dynamics in the model, and hence ECM has been over-specified. For example, OLS has a power superiority of 2% in the first column of Table 3 with no dynamics. It is clear, though, that this overspecification has not led to a substantial loss of power.

(iii) In many other cases where reasonable sizes allow for valid power comparisons, it is clear that the ECM estimator gives tests with a substantial power superiority. For example, in the second block of Table 4, with $(\beta_0, \beta_1, \alpha_1) = (.6, .4, 0)$, the ECM-based test has a size of 5.3%, well below OLS-based size of 16.7%, and yet its power is 76.1% compared to 46.4%.

(iv) The semi-parametric corrections used in the modified OLS and modified ECM estimators appear to lead to some loss of power. For example, consider the last block of Table 4 with no dynamics: OLS has a power almost double that of its modified version, when both tests have similar size.

4. Concluding Remarks

In modelling long run relationships between I(1) variables, the researcher requires an estimator of the parameters which has good precision, and an hypothesis testing procedure which is reliable and powerful. The results contained in this paper suggest that if there is any possibility that the true relationship includes lagged values of variables, then it is unwise to use the OLS regression of y_t on x_t to estimate and perform tests on the parameters. The estimates can contain substantial bias, and test statistics are hopelessly unreliable.

The Monte Carlo results of section 3 also suggest that the semi-parametric approach used in Phillips and Hansen's (1990) fully modified OLS does not solve all the problems encountered by OLS. The bias is smaller, but still substantial, and t tests based on modified OLS can be just as misleading as those based on OLS.

The alternative advocated in this paper involves using the Unrestricted ECM estimator; in other words, including the dynamics in the estimation of long run parameters. The evidence suggests that this approach gives precise estimates and valid t statistics, even in the presence of endogenous explanatory variables. The long run parameters and their standard errors can be estimated directly using the instrumental variables method advocated by Bewley (1979), or by using a standard nonlinear least squares algorithm to estimate the model as given in (1) or (6).

If there is concern about residual autocorrelation or endogenous regressors, the modified ECM estimator and associated test statistics could be used. This estimator has a 'valid' asymptotic distribution, although its finite sample properties are not entirely convincing.

<u>Footnotes</u>

1. The Newey and West (1987) weights are used to ensure that $\hat{\Omega}$ is positive definite in each case.

2. All parameter values for this set of experiments are the same as given in section 3.1.

3. Obviously as α_1 varies, γ_0 also varies so as to keep λ_3 fixed at these values.

4. Results are based on 10,000 replications, so an estimate of the standard error of size estimates is .002 when the true size is .05. Standard errors are generally larger than this for power estimates.

References

Banerjee, A., J.J. Dolado, D.F. Hendry and G.W. Smith 1986, "Exploring equilibrium relationships in econometrics through static models: some Monte Carlo evidence", Oxford Bulletin of Economics and Statistics, 48, 253-277.

Bewley, R.A. 1979, "The direct estimation of the equilibrium response in a linear dynamic model", Economics Letters, 3, 357-361.

Bewley, Ronald, 1986, Allocation models: specification, estimation, and applications, (Ballinger, Cambridge, Massachusetts).

Engle, R.F. and C.W.J. Granger 1987, "Co-integration and error correction: representation, estimation, and testing", Econometrica, 55, 251-276.

Gregory, A.W., A.R. Pagan and G.W. Smith 1990, "Estimating linear quadratic models with integrated processes," Rochester Centre for Economic Research, Working Paper No. 247, University of Rochester.

Hansen, B.E. and P.C.B. Phillips 1989, "Estimation and inference in models of cointegration: A simulation study," Advances in Econometrics, forthcoming.

Park, J.Y. and P.C.B. Phillips 1988, "Statistical inference in regressions with integrated processes: Part 1", Econometric Theory, 4, 468-497.

Phillips, P.C.B. 1988, "Reflections on econometric methodology," the Economic Record, 64, 344-359.

Phillips, P.C.B. and S.N. Durlauf 1986, "Multiple time series regression with integrated processes", Review of Economic Studies, 53, 473-495.

Phillips, P.C.B. and B.E. Hansen 1990, "Statistical inference in instrumental variables regression with I(1) processes", Review of Economic Studies, 57, 99-125.

Phillips, P.C.B. and M. Loretan 1991, "Estimating long run equilibria," Review of Economic Studies, 58, 407-436.

Stock, J.H. 1987, "Asymptotic properties of least squares estimators of cointegrating vectors," Econometrica, 55, 1035-1056.

Wickens, M.R. and T.S. Breusch 1988, "Dynamic specification, the long-run and the estimation of transformed regression models," The Economic Journal, 98 (Conference 1988), 189-205.

Table 1

Precision of Estimators of Long Run Parameter λ_2: T = 50

		$(\rho_{21}, \rho_{22}, \rho_{23})=(0, 1, 0)$			$(\rho_{21}, \rho_{22}, \rho_{23})=(.5,.866, 0)$			$(\rho_{21}, \rho_{22}, \rho_{23})=(.5,.707,.5)$		
$(\beta_0,\beta_1,\alpha_1)$	ESTIMATOR	BIAS	RMSE	Pr(Conc)[1]	BIAS	RMSE	Pr(Conc)[1]	BIAS	RMSE	Pr(Conc)[1]
	OLS	.0000	.0129	.998	.0098	.0170	.983	-.0002	.0082	1.000
(1, 0, 0)	Modified OLS	.0002	.0140	.996	.0111	.0191	.973	.0060	.0017	.996
	ECM	.0002	.0136	.997	-.0010	.0117	.999	-.0037	.0081	.999
	Modified ECM	.0005	.0146	.996	-.0000	.0123	.996	-.0002	.0070	1.000
	OLS	-.0634	.0801	.459	-.0483	.0608	.595	-.0422	.0542	.653
(.6, 0,.4)	Modified OLS	-.0694	.0899	.441	-.0519	.0683	.566	-.0381	.0511	.702
	ECM	.0004	.0243	.952	-.0009	.0204	.969	-.0129	.0206	.968
	Modified ECM	.0000	.0258	.943	-.0008	.0218	.960	-.0040	.0149	.990
	OLS	-.2886	.3323	.061	-.2517	.2912	.059	-.2281	.2671	.089
(.2, 0,.8)	Modified OLS	-.3040	.3543	.059	-.2655	.3110	.057	-.2248	.2675	.093
	ECM	-.0072	.0899	.522	-.0058	.0782	.599	-.0369	.0703	.664
	Modified ECM	-.0062	.0946	.505	-.0054	.0824	.590	-.0101	.0584	.734
	OLS	-.0406	.0526	.681	-.0304	.0403	.803	-.0272	.0360	.854
(.6,.4, 0)	Modified OLS	-.0454	.0608	.646	-.0340	.0463	.767	.0240	.0337	.881
	ECM	.0002	.0140	.994	-.0002	.0120	.996	-.0073	.0117	.997
	Modified ECM	.0005	.0150	.992	-.0002	.0128	.995	-.0019	.0083	1.000
	OLS	-.0952	.1185	.295	-.0787	.0987	.358	-.0672	.0860	.423
(.4,.2,.4)	Modified OLS	-.1043	.1330	.275	-.0863	.1109	.346	-.0639	.0846	.457
	ECM	-.0000	.0245	.950	-.0006	.0209	.969	-.0112	.0193	.974
	Modified ECM	.0004	.0260	.943	-.0004	.0223	.959	-.0021	.0145	.988
	OLS	-.3247	.3734	.048	-.2876	.3321	.050	-.2611	.3057	.084
(.1,.1,.8)	Modified OLS	-.3421	.3980	.047	-.3033	.3546	.049	-.2581	.3068	.080
	ECM	-.0066	.0906	.525	-.0049	.0794	.600	-.0352	.0698	.670
	Modified ECM	-.0056	.0952	.504	-.0045	.0835	.591	-.0082	.0586	.727

1 Pr(Conc) is the proportion of times the estimator is within .05 of the true value.

Table 2

Precision of Estimators of Long Run Parameter λ_2: T = 200

		$(\rho_{21}, \rho_{22}, \rho_{23})=(0, 1, 0)$			$(\rho_{21}, \rho_{22}, \rho_{23})=(.5, .866, 0)$			$(\rho_{21}, \rho_{22}, \rho_{23})=(.5, .707, .5)$		
$(\beta_0, \beta_1, \alpha_1)$	ESTIMATOR	BIAS	RMSE	Pr(Conc)[1]	BIAS	RMSE	Pr(Conc)[1]	BIAS	RMSE	Pr(Conc)[1]
(1, 0, 0)	OLS	-.0000	.0032	1.000	.0025	.0042	1.000	-.0000	.0019	1.000
	Modified OLS	.0000	.0032	1.000	.0026	.0044	1.000	.0015	.0027	1.000
	ECM	-.0000	.0032	1.000	-.0000	.0027	1.000	-.0009	.0019	1.000
	Modified ECM	.0000	.0033	1.000	-.0000	.0028	1.000	.0001	.0016	1.000
(.6, 0, .4)	OLS	-.0165	.0219	.965	-.0125	.0167	.991	-.0114	.0146	1.000
	Modified OLS	-.0169	.0228	.959	-.0128	.0174	.986	-.0096	.0127	1.000
	ECM	-.0001	.0055	1.000	-.0001	.0047	1.000	-.0030	.0046	1.000
	Modified ECM	-.0000	.0056	1.000	-.0001	.0048	1.000	-.0005	.0029	1.000
(.2, 0, .8)	OLS	-.0913	.1153	.316	-.0804	.1010	.347	-.0754	.0931	.370
	Modified OLS	-.0929	.1182	.315	-.0818	.1033	.343	-.0723	.0901	.392
	ECM	-.0006	.0181	.981	-.0004	.0149	.992	-.0074	.0128	.999
	Modified ECM	-.0004	.0183	.980	-.0004	.0152	.993	-.0004	.0092	1.000
(.6, .4, 0)	OLS	-.0101	.0135	.997	-.0076	.0103	1.000	-.0069	.0090	1.000
	Modified OLS	-.0104	.0141	.995	-.0079	.0108	1.000	-.0056	.0075	1.000
	ECM	-.0000	.0032	1.000	-.0000	.0028	1.000	-.0018	.0027	1.000
	Modified ECM	.0000	.0033	1.000	-.0000	.0029	1.000	-.0002	.0017	1.000
(.4, .2, .4)	OLS	-.0247	.0326	.892	-.0207	.0272	.928	-.0182	.0232	.962
	Modified OLS	-.0254	.0338	.887	-.0213	.0282	.918	-.0162	.0211	.977
	ECM	-.0001	.0055	1.000	-.0001	.0047	1.000	-.0026	.0043	1.000
	Modified ECM	.0000	.0056	1.000	-.0001	.0048	1.000	-.0000	.0028	1.000
(.1, .1, .8)	OLS	-.1027	.1296	.272	-.0918	.1152	.295	-.0864	.1066	.321
	Modified OLS	-.1045	.1328	.275	-.0935	.1178	.294	-.0830	.1033	.340
	ECM	-.0005	.0181	.981	-.0003	.0149	.992	-.0071	.0126	.999
	Modified ECM	.0003	.0184	.981	-.0003	.0152	.993	.0007	.0093	1.000

1 Pr(Conc) is the proportion of times the estimator is within .05 of the true value.

Table 3

Performance of t tests of Long Run Parameter λ_3: T = 50

H_0: $\lambda_3 = 0$. Nominal size = .05. Powers calculated for $\lambda_3 = .04$

		$(\rho_{21}, \rho_{22}, \rho_{23})=(0, 1, 0)$		$(\rho_{21}, \rho_{22}, \rho_{23})=(.5, .866, 0)$		$(\rho_{21}, \rho_{22}, \rho_{23})=(.5, .707, .5)$	
$(\beta_0, \beta_1, \alpha_1)$	ESTIMATOR	SIZE	POWER	SIZE	POWER	SIZE	POWER
(1, 0, 0)	OLS	.058	.771	.138	.897	.056	.708
	Modified OLS	.069	.793	.254	.949	.236	.821
	ECM	.072	.749	.068	.853	.073	.942
	Modified ECM	.109	.804	.108	.892	.146	.963
(.6, 0, .4)	OLS	.334	.421	.353	.568	.376	.556
	Modified OLS	.526	.577	.550	.691	.604	.700
	ECM	.070	.434	.065	.567	.036	.352
	Modified ECM	.118	.528	.117	.649	.212	.532
(.2, 0, .8)	OLS	.608	.605	.606	.616	.627	.630
	Modified OLS	.623	.628	.637	.644	.658	.662
	ECM	.074	.113	.070	.140	.035	.035
	Modified ECM	.153	.213	.154	.250	.267	.225
(.6, .4, 0)	OLS	.144	.366	.156	.563	.171	.531
	Modified OLS	.455	.638	.465	.790	.537	.756
	ECM	.063	.730	.060	.843	.028	.693
	Modified ECM	.102	.791	.103	.887	.192	.802
(.4, .2, .4)	OLS	.354	.393	.362	.459	.390	.469
	Modified OLS	.579	.599	.599	.646	.635	.674
	ECM	.064	.423	.061	.550	.027	.306
	Modified ECM	.112	.520	.112	.640	.210	.497
(.1, .1, .8)	OLS	.609	.607	.606	.617	.627	.630
	Modified OLS	.640	.642	.649	.653	.672	.674
	ECM	.071	.110	.067	.136	.034	.032
	Modified ECM	.150	.210	.151	.246	.266	.222

Table 4

Performance of t tests of Long Run Parameter λ_3: T = 200

H_0: λ_3 = 0. Nominal size = .05. Powers calculated for λ_3 = .008

		$(\rho_{21}, \rho_{22}, \rho_{23})=(0, 1, 0)$		$(\rho_{21}, \rho_{22}, \rho_{23})=(.5, .866, 0)$		$(\rho_{21}, \rho_{22}, \rho_{23})=(.5, .707, .5)$	
$(\beta_0, \beta_1, \alpha_1)$	ESTIMATOR	SIZE	POWER	SIZE	POWER	SIZE	POWER
(1, 0, 0)	OLS	.052	.643	.133	.819	.052	.581
	Modified OLS	.000	.160	.015	.524	.066	.307
	ECM	.058	.639	.055	.763	.067	.908
	Modified ECM	.000	.174	.000	.306	.003	.602
(.6, 0, .4)	OLS	.362	.412	.369	.524	.388	.512
	Modified OLS	.484	.334	.520	.576	.606	.644
	ECM	.054	.491	.055	.453	.024	.245
	Modified ECM	.000	.033	.000	.072	.070	.101
(.2, 0, .8)	OLS	.655	.655	.658	.659	.660	.662
	Modified OLS	.618	.618	.629	.629	.619	.622
	ECM	.055	.091	.058	.103	.020	.016
	Modified ECM	.000	.002	.000	.003	.071	.044
(.6, .4, 0)	OLS	.146	.304	.167	.464	.183	.442
	Modified OLS	.205	.353	.214	.504	.403	.521
	ECM	.053	.633	.053	.761	.022	.579
	Modified ECM	.000	.167	.000	.300	.069	.256
(.4, .2, .4)	OLS	.379	.400	.375	.446	.405	.452
	Modified OLS	.586	.584	.615	.629	.651	.669
	ECM	.054	.334	.054	.449	.019	.214
	Modified ECM	.000	.032	.000	.070	.068	.070
(.1, .1, .8)	OLS	.657	.655	.659	.658	.659	.660
	Modified OLS	.632	.632	.641	.641	.628	.630
	ECM	.054	.090	.056	.101	.020	.016
	Modified ECM	.000	.002	.000	.003	.070	.042