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Working Paper No. 16/91

December 1991

DEPARTMENT OF ECONOMETRICS

ISSN 1032-3813

ISBN 0 86746 951 X

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MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

Cointegration and Aggregation

by

Gábor Kőrösi* and László Mátyás**

** La Trobe University, Department of Econometrics, Bundoora, Australia*

*** Monash University, Department of Econometrics, Clayton, Australia*

Introduction

Macroeconometric models frequently assume that the behaviour of economic agents is uniform, and thus the behaviour of a single agent characterizes the aggregate behaviour of the agents. This assumption of the existence of a *representative agent* renders modelling simpler, and frequently feasible. It was always recognized that there may be some outliers, some agents who, for any particular reason, may behave differently, but the maintained hypothesis is that the effects of these perturbations cancel out each other, and hence they can be ignored.

Recently the concept of cointegration has gained considerable popularity among macromodellers, *e.g.*, when studying consumer behaviour, or money demand. There is an abundance of papers on the cointegrated nature of the behaviour of representative agents.

In this paper we demonstrate that the above mentioned two concepts, the assumption of representative agents and of cointegration may frequently be incompatible. The problem is all too significant as the majority of empirical studies of cointegration explicitly or implicitly assumed the existence of representative agents.

Theory

Definition: Two time series x_t and y_t are cointegrated: $x_t,\, y_t \sim \mathrm{CI}(d,b)$) if

— both variables are integrated to the same order d: $x_t,\, y_t \sim \mathrm{I}(d)$,

— there is a $z_t = y_t - \beta x_t$ linear combination which is integrated to a lower order: $z_t \sim \mathrm{I}(d-b)$, $b > 0$. The $(1, -\beta)$ vector is called the cointegrating vector.

There are n agents in an economy measured over T time periodes. Their behaviour is represented by two time series, x_{it} and y_{it} $(i = 1, \ldots, n;\, t = 1, \ldots, T)$. Their relation is characterized by the $y_{it} = \beta_i x_{it} + u_{it}$ regression. x_t and y_t $(t = 1, \ldots, T)$ denote the aggregate time series: $x_t = \sum x_{it}$ and $y_t = \sum y_{it}$.

Property 1: If time series x_{1t} is integrated to order 1, but all other time series x_{it} $(i = 2, \ldots, n)$ are stationary, the aggregate time series x_t is integrated to order one. More generally, if $x_{it} \sim \mathrm{I}(d_i)$, and $d = \max d_i$, then $x_t \sim \mathrm{I}(d)$.

This means that an aggregated time series "inherits" the integration of the single individual with the integration of the highest order.

Property 2: If time series x_{1t} and y_{1t} are cointegrated to order 1, *i.e.*, $x_{1t},\, y_{1t} \sim \mathrm{CI}(1,1)$ with cointegrating vector $(1, \beta_1)$, and all other x_{it} and y_{it} $(i = 2, \ldots, n)$ are stationary, the x_t and y_t aggregate time series are cointegrated to order 1, and their cointegrating vector is $(1, -\beta_1)$ as well, regardless of all other β_i coefficients. More generally, if $x_{1t},\, y_{1t} \sim \mathrm{CI}(d,b)$, and for all other agents $x_{it},\, y_{it} \sim \mathrm{I}(d_i)$ where $d_i \leq (d-b)$, then the two aggregated time series $x_t,\, y_t \sim \mathrm{CI}(d,b)$.

This means that an aggregate time series may be cointegrated even if the variables are cointegrated for only one individual agent, but for all the other agents they are not.

Property 3: If x_{1t}, $x_{2t} \sim$ CI(2,1) and all other x_{it} $(i = 3, \ldots, n)$ are stationary, the aggregate time series x_t is integrated to order one. More generally, if x_{1t}, $x_{2t} \sim$ CI(d, b) and all other $x_{it} \sim$ I(d_i) $(i = 3, \ldots, n)$ where $d_i \leq (d - b)$, then $x_t \sim$ I$(d - b)$.

This means that an aggregate time series may be integrated to an order different from the order of the integration of any of its components.

Property 4: If x_{it}, $y_{it} \sim$ CI(1,1) with cointegrating vector $(1, \beta_i)$ $(i = 1, \ldots, n)$, and $\beta_i \neq \beta_j$ for any i and j, then time series x_t and y_t are not necessarily cointegrated. (As a matter of fact they need not be integrated, *c.f.*, property 3.)

This means that aggregate time series may not be cointegrated even if the variables are cointegrated for all individual agents.

Conclusion:

If a macroeconometric model is based on microeconomic theory and the data generating process is a dynamic non–stationary one, even one single outlier may change the characteristics of the aggregate model. The assumed microeconomic behaviour may not be observed in the aggregate time series, and the dynamic characteristics of the aggregate time series may reflect the behaviour of one single extreme agent. If there are several agents with extreme behaviour the characteristics of the aggregate series may not correspond to any of its components. In such cases any inference from the aggregate time series may be severely biased, and thus useless. Therefore results of (co)integration analysis of aggregate time series should be interpreted with very strong reservation and caution.

In practice these strange characteristics may not always be apparent because of the limited length of the time series: the behaviour of the extreme agent(s) may not dominate the characteristics of a relatively short observation period. However, by obtaining more observations, so increasing the reliability of measurement, the model may actually become worse rather than better: the formerly unrecognized behaviour may become dominant.

The only real solution of the problem seems to be the use of micro data (*e.g.*, panel data) which facilitates the identification of outliers, instead of aggregation.