Learning in repeated multiple unit combinatorial auctions: An experimental study

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Abstract

The motivation of this paper is to understand trader behaviour and learning in a complex setting where finding a best strategy might not be intuitive. The assertion made is that feedback information can help in updating strategies through repeated bidding processes. The paper explores this assertion through the results of a series of repeated multiple unit combinatorial auction laboratory experiments where item and package traders interact under three information treatments: 1) basic information feedback on market prices and status of their own bids; 2) basic information feedback and all winning bids; and 3) market prices and the status of all bids. We compare bidding behavior with a local optimal package selection model. We then estimate an experience weighted attraction learning (EWA) model of bidding behavior. We observe that package traders follow price feedback information more closely than item traders, especially in the basic treatment information. With additional information package traders substantially deviate from best response bidding strategy resulting in a loss of efficiency. Finally, item traders tend to remember their past experiences more than package traders in low information environments. In high information environments the trend is reversed. The implications of this study could be significant for market design. The standard assumption that more information in combinatorial market design is better for traders may not hold in all cases.

Key words: Experience Weighted Attraction Learning; best response bidding strategy; multiple unit combinatorial auctions; package selection

JEL Code: D03, D44

Introduction

Combinatorial auctions enable traders to submit bids on bundles of items. Many studies have shown that combinatorial auctions achieve high allocative efficiency with traders having economies of scope (Bichler et al., 2005). Combinatorial auctions have been successfully applied in a number of cases, such as, procurement of goods and services (Hohner et al., 2003, Elmaghraby and Keskinocak, 2004), transportation services, spectrum auction (Koboldt et al., 2003); school meals (Epstein et al., 2004) and fisheries site allocation (Department of Primary Industries, 2007). It has also been tested in wide range of markets such as airport slot allocation (Rassenti et al., 1982), emission trading schemes (Porter et al., 2009), environmental payment services (Iftekhar et al., 2011, Iftekhar et al., 2013) and fisheries quota allocation (Iftekhar and Tisdell, 2012).

Increasing implementation of combinatorial markets in different sectors has led to greater interest in understanding how traders' behavior influences performances of auctions. Compared to a single item auction, the combinatorial auction equilibrium bidding strategy is not always clear to the traders due to complementary valuations of the items. For example, consider three items are on sell A, B and C and there are three traders 1, 2 and 3. Assume Trader 1 submitted a bundle bid {AB, $10}; Trader 2 an item bid {C, $5}, and Trader 3 did not submit an offer in the first auction (round). The winners in this auction are traders 1 and 2. While Trader 3 did not submit any bid they are still interested in bundle BC. However, with the current set of information it is not clear how much Trader 3 should bid on BC in the following auction (Bichler et al., 2005). This problem is even more complex in multiple unit combinatorial auctions as traders can not only combine different items but also demand multiple units of individual items. For example, if a trader is interested in two items and they can vary the levels by five levels then in total they can chose from 25 different combinations (including an option for null bid). In repeated setting this problem is ameliorated by providing market information.

Theoretical analysis of bidding behavior in multiple unit combinatorial auctions is still in its infancy and difficult to implement (Armstrong, 2000). As a result, testing of designs though "wind tunnel" laboratory experiments have gained in popularity (Bichler et al., 2005, Brewer, 1999, Cox et al., 2002, Goeree and Holt, 2010, Ledyard et al., 2009, Lunander and Nilsson, 2004, Takeuchi et al., 2010). Results from the experiments are used to formulate bidding behavior (Palfrey, 1983). Many studies concentrated on fully rational models like Bayesian Nash Equilibrium model (Cason, 1995, Chen and Takeuchi, 2010, Neugebauer and Perote, 2008) and constant relative risk aversion model (Holt Jr, 1980, Riley and Samuelson, 1981), and some on adaptive models like quantal response equilibrium bidding model (Goeree et al., 2002). Very few studies have estimated parameter values for learning models based on auction data (Bazzan et al., 2010, Erev and Roth, 1998). We are not aware of any study which has estimated parameter values of the experience weighted attraction learning algorithm(Camerer, 2003) for multiple unit repeated combinatorial auction games.

This study makes a contribution in understanding the role of market information on traders' behavior in multiple unit combinatorial auctions. Using standard learning theory we formally test the change in bidding behavior with the provision of different market information. In the experiments 4 humans compete against 4 robots using the Experience Weighted Attraction (EWA) learning algorithm of Camerer (2003). This allows the robots to learn from the market to select their appropriate bidding strategies whereas in other auction studies a fixed strategy, such as random, sincere or Nash equilibrium bidding has been used (Cason, 1995, Chen and Takeuchi, 2010). Traders could be either item traders (have strong preference for a single type) or a package trader (prefer both types).

We pose two research questions: (1) Do traders follow price signals more closely when market information is limited? and (2) Do traders use the same learning model irrespective of the amount of market information?

Founded on our research questions we identify the following predictions.

Prediction 1: Traders will follow the price signals more closely with basic feedback information. In the basic information feedback treatment traders only receive feedback about their own bids and feedback prices computed from the submitted bids. In other information treatments they receive additional market information which might influence them to deviate from following feedback price signals. Within this question we will also speculate that:

Prediction 1a: Package traders will follow the item feedback price signals more closely than item traders. Package traders use the price signals to determine bid prices to maximize their expected profits. For item traders, feedback prices provide guidance in the bid formulation to maximize their expected profits by forming winning combinations with complementary bids. Therefore, item traders might find it difficult to follow price signals (Bikhchandani and Mamer, 1997).

Prediction 2: All types of traders will use the same learning model in all information treatments

To answer these questions and explore these predictions we analyzed bidding strategies of participants (traders) in a series of economic experiments under three information treatments. In the basic feedback treatment the traders received information on market prices and status of their own bids. In the second treatment, they received market prices and status of their own bids and all winning bids. With the third treatment they could observe all bids and their respective status from previous round. Based on current literature, we select two bidding models: best response bidding strategy model (Parkes, 2006) and experience weighted attraction learning model (Camerer, 2003, Camerer and Hua Ho, 1999). The first model records how closely traders follow the price feedbacks and assumes adaptive behavior of traders. The second model assumes learning through experience and market information feedback. Using the threshold accepting algorithm we find the set of EWA parameter values that minimize the distance between simulated and observed paths. We then compared the results from the models in different information treatments with different trader types to discern the effect of market information on bidding strategies.

Experimental design

A series of experiments were conducted to explore the impact of information on the performance of multiple unit combinatorial auction games. The experimental market consists of eight traders competing for resource extraction in two regions A and B[1]. We describe our experimental design, environment and procedures below.

We implemented a 3x3 design. Based on available literature, we designed three information treatments. In the first treatment, the traders could only see the status of their own bids and market information processed in terms of item price feedbacks. No other feedback was provided under this treatment. In the second treatment, information on winning bids (bid choice and status) was revealed along with item price feedbacks. Finally, in the third treatment, bids submitted by all traders and the status of their bids were revealed along with the information already provided under Treatments 1 and 2.

The treatments were blocked by three sets of human (H) and robot (R) traders[2]. Each treatment/experimental set combination included three types of traders. Traders had the choice to submit up to three bids; one of each of the two on individual regions (hereafter item A and item B) and one on the package (hereafter package AB)[3]. In other words, they could demand quotas for individual regions as well as for both regions up to their maximum capacity. Trader type A has a preference for region A, trader type B has a preference for region B and trader type AB with equal preference for both regions. In the first experimental set all human traders were the AB trader type. The proportion of AB trader type gradually reduced to 50% and 0% in experiment sets 2 and 3 respectively. Table 1 summarizes the three combinations of trader types.

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[1] The experiment has been modelled in terms of a fisheries quota auction as part of a larger fishery project. Contextualising the problem in a hypothetical fishery met the needs of the larger project and assisted in explaining the connectivity of the packages to the traders. There was no evidence that the context impacted on the generalities of the results.

[2] Each auction round consisted of 4 human and 4 robot traders. Having an even distribution of humans and robots allowed us to have a symmetric distribution of valuation types. However, the human traders were not informed that they were playing against robots. Not informing the human traders that robots also traded allows for the control findings for a larger study into the impact of knowledge of and future role of robot traders.

[3] Allowing the traders to only submit 2 item and one package offer promotes efficiency and potentially leads to quicker convergence.

Table 1: Distribution of trader type in different experimental set

Trader Type	Experimental Set		
	1	2	3
A	2R	1H and 1R	2H
B	2R	1H and 1R	2H
AB	4H	2H and 2R	4R

Four experimental sessions were conducted for each treatment/experimental set combination. Each session consisted of 20 rounds of combinatorial auctions with constant trader valuations. Given constant valuations, traders were able to learn and respond to the outcomes of prior auctions, and use the information from previous auctions (rounds) to revise their bids[4]. This research explored how they revised their bids and used different amounts of information from previous auctions.

The auction model

We applied a first price selling auction format where the aim of the auctioneer was to sell multiple units of a set of items, where u_k represents the number of units of item *k* available for sale. *N* traders { $i = 1,2,\ldots,n$ } participate in the auction, each submitting a set of bids *M* $\{j = 1,2,\ldots,m\}$. $\lambda_{ij}^{k} \geq 0$ and p_{ij} is the number of units of item *k* and respective bid price asked in bid *j* from trader *i*. The auctioneer's objective is to maximize revenue (*Z*) by selling available units for target items. Formally:

$$Z = \max \sum_{i=1}^{N} \sum_{j=1}^{M} p_{ij} x_{ij} ;\ s.t. \sum_{i,j} \lambda_{ij}^{k} x_{ij} \leq u_k , \sum_{i} x_{ij} \leq 1 \quad and\ x_{ij} \in \{0,1\} \qquad \ldots (1)$$

Here, x_{ij} is a binary variable, indicating winning ($x_{ij} = 1$) and losing ($x_{ij} = 0$) condition of respective bid. The first constraint ensures that the sell is not greater than the capacity, whereas the second constraint makes sure that each trader could win a maximum of one package (Xia *et al*., 2004). At the end of an auction, the auctioneer uses the results from the revenue maximization problem to process

[4] In traditional iterative bidding allocations are made at the end of series of iterations. In our experiments allocations are made in each round in order to comply with induced value theory principle that traders earn trader income based on their performance in each auction (round).

market feedback information. There are a number of alternative ways feedback information could be processed. In this paper, we have used the data envelopment analysis based procedure developed by Aparicio *et al.* (2008) to process feedback prices. Their procedure calculates prices for individual items based on the most expensive units available in the market. Readers are referred to that paper for details of the procedure.

The economic environment

In each auction there were a total of 8 quotas for region A and 8 quotas for region B available from the central authority. Each trader could purchase a maximum of 4 quotas for a single region. We have used the specifications used by Iftekhar and Tisdell (2012) to generate individual traders valuation:

$$v_i^{ab} = v_i^a \cdot \left(q_i^a\right)^{\alpha_i} \cdot \left(1 + q_i^b\right)^{\beta_i} + v_i^b \cdot \left(q_i^b\right)^{\alpha_i} \cdot \left(1 + q_i^a\right)^{\beta_i} \ldots (2)$$

In the model, the individual trader's valuations for different combinations are expressed in terms of four parameters: v_i^a, v_i^b, α_i and β_i. The first and second parameters represent the value for an individual quota for region a and b respectively for trader i. The next two terms determine quota value superadditivities. The parameter α_i is used to model trader i's economies of scale in valuation from acquiring multiple quotas for a given region. Parameter β_i determines trader i's economies of scope in valuation from winning quotas for different regions together. q_i^a and q_i^b indicate the number of quotas under consideration for region A and B respectively by trader i. The parameter values for the trader types are shown in Table 2.

Table 2: Parameter values for different trader types

Trader Type	v_i^a	v_i^b	α_i	β_i
A	$7.6	$2	0.4	0
B	$2	$7.6	0.4	0
AB	$4	$4	0.4	0.4

Experimental procedures

The experimental sessions were conducted in the Experimental Economics Laboratory at the University and so traders were recruited from the University student population. On arrival traders were provided with a set of instructions and quiz to test understanding of the tasks to be undertaken during the experiment (an example set of instructions and associated quiz are provided in Appendix A and B respectively). Overall, 36 independent computerized sessions were conducted (3 treatments x 3 blocks x 4 sessions). Each session lasted approximately one and half hours. In addition to their auction earnings, subjects received a show-up fee of A$10 (~U. S. $11). The average earnings (including the show up fee) was A$23 (~U. S. $25).

We used TESS© (the Experimental Software System) and GAMS© (the General Algebraic Modelling System) to program our experiments. The experimental software TESS© collected all bids from each group, computed the final allocation and payoff[5] for each trader and sent information back to the trader's screen. Each round, once all the human bids were lodged, the experimental software TESS© called a GAMS© optimization model to (a) determine the robot best response strategies using a parameterized version of the Experience Weighted Learning (EWA) of Iftekhar and Tisdell (2012) and (b) determine the set of successful bids. The best response strategy was based on the expected surplus which is calculated as the difference between the maximum valuation and the current computed value of the package.

[5] Pay-off to traders was a function of the amount of money they could earn (i.e., value - bid) from their wining package.

Result and discussion

In this section we analyze and discuss our experimental data and findings[6]. We begin by looking at the trends and patterns in locally optimal bid selection strategy. Then we discuss the parameter estimation of the Experience Weighted Attraction (EWA) learning model.

Myopic Best-Response Bidding Behavior

In repeated combinatorial auctions, feedback about the current market is often provided in the form of prices for individual items. It has been observed that traders adopt a myopic best response bidding strategy, where in each round traders select new bundles to submit to maximize their utility given the current ask prices for bundles or goods (Wurman et al., 2001, Parkes, 2006). To analyze the bundle selection behavior, we calculated the bundle prices for different combination of packages based on the current market price. We then identified the package with highest expected pay-off. Finally, we estimated the ratio of the valuations of the local optimal package and the valuation of the submitted package. We refer to this ratio as locally optimal bid proximity (LOBP). A value of LOBP equals to 1 indicates a locally optimal package selection by traders.

Table 3 summarizes the panel regression models of local optimal package selection with information treatments and trader types. Overall, across all information treatments average value of local optimal selection is 0.75 (± 0.39). Yet, with more information average value of LOBP significantly declined from 0.76 (± 0.39) in Treatment 1 to 0.74 (± 0.39) in Treatment 3. As expected, traders followed price information more closely in Treatment 1 where were traders were only provided with the status of their own bids and item prices (Figure 1).

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[6] Since we have compared the aggregate outcomes under different information treatment somewhere else IFTEKHAR, M. S. & TISDELL, J. G. 2015. Bidding and performance in multiple unit combinatorial fishery quota auctions: Role of information feedbacks. *Marine Policy,* 62**,** 233-243. we do not present them here. It was found that there was no significant difference in average revenue earned in one information treatment from another in terms of average revenue earned. However, allocative efficiency was significantly lower in Treatments1 and 2 compared to Treatment 3, although no significant difference was found between Treatments1 and 2. In our analysis we concentrate only on human traders.

On the other hand, the average values of LOBP indicate that different types of traders followed price feedback signals differently. Package traders (AB) followed price signals significantly more closely than item traders. No significant difference between two item trader types (A and B) was found. Our results conform to the predictions from theory proposed by Bikhchandani and Mamer (1997). They suggested in the theory that unless the feedback prices are personalized (non-anonymous) and discriminatory for individual packages, it is difficult for feedback prices to guide small traders or traders with interests on a single or few items to form potential winning combinations and take full advantages of the price feedbacks (Bikhchandani and Mamer, 1997).

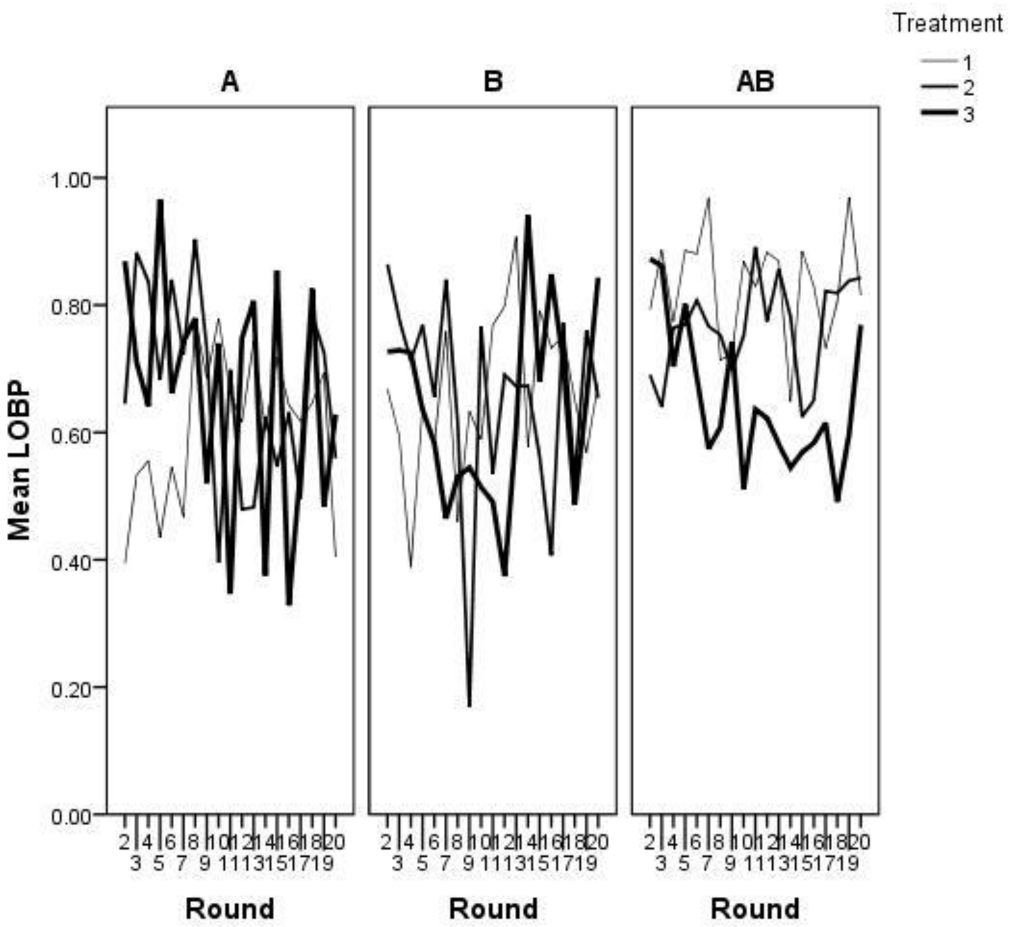

Figure 1: Locally optimal bid proximity (LOBP) values in different information treatments for different trader types

For item traders (item A and item B), the proportion of best response bidding (i.e., LOBP = 1) is similar in all information treatments. For instance, the proportion of best response bidding for trader Type A for Treatment 1, 2 and 3 were 28%, 36% and 30% respectively. Similar estimates for trader type B were

23%, 23% and 32%. Nonetheless, in package bids (AB) with the provision of more information the proportion of bids with best-response strategy gradually declined (from 71.1% to 48.5%). This is much higher than observations made by Pikovsky (2008) in his laboratory experiments. He observed that only 7%, 33% and 13% of bids were best-response bids in their experiments with ALPS, Clock Combinatorial and iBundle combinatorial auction designs respectively.

Moreover, it has been observed that traders who have won in the previous round have followed price signals (0.79 ± 0.32) significantly more closely than losing traders (0.63 ± 0.40). However, the winning traders gradually deviate from this strategy with more information. The value of LOBP declined from 0.84 (± 0.29) in Treatment 1 to 0.69 (± 0.37) in Treatment 3. Losing traders also showed a declining trend but at a much lower rate.

Table 3: Panel regression model with AR (1) disturbances. Local optimal package selection with information treatments and trader types

Equation	1			2			3		
	Coef.	P	Std. Err.	Coef.	P	Std. Err.	Coef.	P	Std. Err.
Constant	0.58	**	(0.04)	0.72	**	(0.03)	0.65	**	(0.04)
Win condition (lag)	0.09	**	(0.01)	0.09	**	(0.01)	0.09	**	(0.01)
Information Treatment 1	0.08	*	(0.04)	0.02		(0.04)			
Information Treatment 2	0.06	^	(0.04)				-0.02		(0.04)
Information Treatment 3				-0.06	^	(0.04)	-0.08	*	(0.04)
Trader Type A	-0.01		(0.04)	-0.09	*	(0.04)			
Trader Type B				-0.08	*	(0.04)	0.01		(0.04)
Trader Type AB	0.08	*	(0.04)				0.09	**	(0.04)
Wald statistics	53.88	**		53.88	**		53.88	**	

Note: significance *** p<0.01; ** p<0.05, ^ p<0.10.

There could be a number of reasons why traders do not systematically follow a myopic best response bidding strategy. It might be cognitively difficult for the traders to evaluate all the potential combinations of bids. For example, in our experiments a trader has potential 25 packages to choose in a single round. Although we have substantially reduced the difficulty by automating the calculation of expected payoff for different combinations, it might still be difficult for them to select an optimal package in every round.

There is also the possibility that item price feedback mechanisms may not always support the Walrasian allocation (Bikhchandani and Mamer, 1997). In other words, there might be instances when a losing trader might have higher valuation for a sub-set of package compared to the standing values computed by using the item price feedback. Moreover, the DEA based prices are conditioned on the most expensive units available in the market (Iftekhar et al., 2011). As a result, computed values for some provisional winning packages might be higher than their winning bids. Therefore it might not be beneficial for the traders to only consider the price feedbacks, especially when other types of information about the market are available.

It has been observed that traders are influenced by their own experience, as indicated by the significant value of lagged winning condition. They follow price signals more closely when they are winning and participating with only their own offer and market price information (information Treatment 1) than in more complex environments. In other information treatments they gradually move away from price feedbacks, as indicated by the significant effects of information treatments.

Finally, traders may not always be motivated by rational expectations. It may take time for them to understand their strategic position in the market. The repeated procedure of the auction has often been used as a learning mechanism for the traders to elicit and learn about their true valuations and standing in the market. Many learning models have been developed to understand trader's adoption of a strategy in a given environment.

The Experience Weighted Attraction (EWA) learning model

There are many learning algorithms proposed in the auction literature. In this paper, we have implemented the Experience Weighted Attraction (EWA) learning model. Depending on the parameter value choice the model can take different forms of learning, which allow us to understand the shift in learning model in different information environments. In the EWA it is assumed that an agent *i* has a set of strategies $g \in G$ for pricing a package *j*. Each strategy g has an attraction, $q_{ij}^{g}(t)$, attached to it. Attractions determine the probabilities of choosing different strategies through a logistic response function. In the parametric form of EWA, there are four major parameters:

- A parameter ϕ_i captures the decay or depreciation of past attractions.

- Another parameter δ_i depicts the weight each trader attaches to foregone payoffs relative to realized payoffs.
- The attraction sensitivity parameter λ_i determines trader's sensitivity to the values of attractions.
- Finally, a parameter k_i controls the rate at which attractions grow (Ho et al. 2008).

Depending on parameter values, the EWA model could take different forms. For example, as the value of δ_i moves towards 0 and the value of k_i moves towards 1, the EWA takes the form of cumulative choice reinforcement learning where the agent only respond to pay-off from chosen strategy. On the other hand, with a value of δ_i = 1 and k_i = 0 traders treat pay-offs and attractions attached to each strategies equally, resembling a belief model (Camerer 2003).

In order to estimate the parameter values from the experiment data we have followed the simulated model fit approach where, given a set of parameter choice values, a simulated path is compared with the observed data (McAllister, 1991). We are interested in identifying the set of parameter choice values that would minimize the sum of squared distance (SSD_i) between the observed strategy choice and simulated strategy choice sets of the trader i in an experimental session using the following set of equations:

$$SSD_i = \sum_j \sum_g \sum_t \left[I\left(s_{ij}^g(t)\right) - p_{ij}^g(t) \right]^2$$

$$p_{ij}^g(t+1) = \frac{e^{\lambda_i \cdot q_{ij}^g(t)}}{\sum_H e^{\lambda_i \cdot q_{ij}^h(t)}}$$

$$q_{ij}^g(t) = \frac{\phi_i \cdot N_i(t-1) \cdot q_{ij}^g(t-1) + \left[\delta_i + (1-\delta_i) \cdot I\left(s_{ij}^g(t)\right)\right] \cdot R_{ij}^g}{N_i(t)}$$

$$\begin{aligned}
&where \\
&R_{ij}^g = \begin{cases} v\left(s_{ij}^g\right) - v\left(s_{ij}^1\right) & if \;\; CV_{ij}(t) \geq v\left(s_{ij}^g\right) \\ 0 & otherwise \end{cases} \qquad \ldots (3) \\
&I\left(s_{ij}^g(t)\right) = \begin{cases} 1 & if \;\; s_{ij}^g(t) = s_{ij}^*(t-1) \\ 0 & otherwise \end{cases} \\
&N_i(t) = \phi_i \cdot (1-k_i) \cdot N_i(t-1) + 1
\end{aligned}$$

In the EWA algorithm, there are two main parameters updated after every round, the attraction attached to an individual strategy ($q_{ij}^g(t)$) and the experience weight $N_i(t)$. The updating of $N_i(t)$ is a function of previous experience weight $N_i(t-1)$, memory retention parameter (ϕ_i) and attraction growth rate (k_i). Attraction weights are revised in terms of lagged attractions $q_{ij}^g(t-1)$ and a lagged experience weight $N_i(t-1)$ multiplied by ϕ_i and adding the expected payoff R_{ij}^g from a strategy. In order to be competitive, traders calculate expected payoffs only for the sub-set of strategies with valuations ($v\left(s_{ij}^g\right)$) lower than the current market value ($CV_{ij}(t)$). The entire value of expected payoff is added if the strategy was met in the previous cycle. Otherwise, the expected payoff is discounted by the factor δ_i. The attractions were then normalized with respect to the updated experience weight, $N_i(t)$ to get the final values for $q_{ij}^g(t)$. Updated attractions are used to probabilistically choose a strategy, $p_{ij}^g(t+1)$.

In order to estimate the parameter values we used the threshold acceptance (TA) algorithm of Dueck and Scheuer (1990). It is a local search heuristic which accepts solutions not worse than a certain threshold to avoid local minima (Gilli and Këllezi, 2002). The TA algorithm has an easy parameterization and has been implemented in many situations, such as portfolio optimization and down-side risk constraints (Liu, 2011, Schumann, 2011, Winker, 1995). The primary steps of the algorithm are given in Figure 2. At the beginning, the modeller has to define the search configuration by fixing the number of iterations ($n_{iteration}$) the algorithm should search and the number of times the search should be repeated ($n_{restarts}$). The threshold sequence τ_r traces the amount of slack allowed in the objective value in the current iteration, which decreases to zero during the course of iterations. Initially the algorithm randomly generates a set of parameter values and computes the value of the objective function ($x^c = SSD_i$) defined in equation 3. In the following iteration, a new candidate solution is generated in the neighbourhood of the current parameter values set and the objective function is recalculated. And then, the objective function values are compared. The new values are accepted if the difference between the current and the previous objective function values is less than the current value of τ_r. This process continues until the termination rule is satisfied. At the end of the iterations the set of parameter values with minimized objective function values is selected as the solution (Winker and Maringer, 2007).

Initialize $n_{restarts}$ and n_{rounds}
Initialize threshold sequence, τ_r
for $k = 1{:}\, n_{restarts}$ do
    Randomly generate current solution $x^c \in X$
    for $r = 1{:}\, n_{rounds}$ do
        Generate $x^n \in N(x^c)$ and compute $\Delta = f(x^{\mathrm{n}}) - f(x^{\mathrm{c}})$
        if $\Delta < \tau_r$ then $x^{\mathrm{n}} = x^{\mathrm{c}}$
    end for
    $\vartheta_k = f(x^c)$, $x^{sol}_{(k)} = \mathrm{x^c}$
end for
$\mathrm{x^{sol}} = x^{sol}_{(k)}, k | \vartheta_k = \min\{\vartheta_1, \dots, \vartheta_{nRestarts}\}$

Figure 2: Pseudo – code for threshold accepting algorithm (Winker and Maringer 2007)

For our search we generated 25 random seeds (i.e., $n_{restarts}$= 25) and iterated for 1000 iterations (i.e., $n_{iteration}$= 1000). The values of τ_r started with 0.1 and are reduced to 0.05 and 0 after 200 and 400 iteration respectively. Neighbors of parameter value were searched within 0.01 standard deviation of the previous value. These search specifications showed promise after some initial experimentation. We followed the work of Camerer and Ho (Camerer and Ho, 1998, Camerer and Hua Ho, 1999) in fixing the parameter search space between 0 to 1. We have discretized bids into twenty strategies with valuations ranging from 5% to 100%.

Results from the threshold accepting algorithm search indicate that traders have used a hybrid model (Table 4). The mean values of the parameters are ϕ_i (0.34 ± 0.27), k_i (0.55 ± 0.31), δ_i (0.26 ± 0.33) and λ_i (0.34 ± 0.27). A value of ϕ_i closer to zero indicates that traders are adapting to the changing market condition quickly and discounting previous experiences. The value of δ_i determines the amount of discount traders apply to expected payoff from previously unselected strategy. A value of 0.26 indicates that traders are heavily motivated by the pay-offs related to their chosen strategy in previous round, although they have considered un-chosen strategies as well. λ_i measures how sensitive traders are to strength of expected pay-off attached to each strategy. A value of λ_i closer to one indicates traders will always select strategies with maximum expected pay-offs and vice versa. However, the estimated value of 0.34 indicates that traders frequently chose strategies with inferior expected payoffs. As observed in the previous section, traders did not always follow the best response strategy. k_i is the speed of selecting a strategy. A value of 0.55 suggests that traders explore the market before settling down and lock in a strategy.

Table 4: Mean (standard deviation) values of the EWA parameters and goodness of fit for information treatments and trader types

Trader Type	Treatment	ϕ_i	k_i	δ_i	λ_i	SSD_i
A	1	0.37 ± 0.19	0.40 ±0.35	0.17 ±0.25	0.37 ±0.19	33.95 ±6.19
	2	0.32 ±0.19	0.50 ±0.31	0.07 ±0.10	0.33 ±0.19	31.73 ±6.77
	3	0.34 ±0.27	0.63 ±0.23	0.13 ±0.17	0.33 ±0.27	32.20 ±7.33
	Total	0.34 ±0.22	0.50 ±0.32	0.12 ±0.19	0.34 ±0.22	32.68 ±6.80
B	1	0.42 ±0.31	0.51 ±0.31	0.31 ±0.36	0.40 ±0.31	35.22 ±4.74
	2	0.28 ±0.28	0.61 ±0.35	0.40 ±0.36	0.28 ±0.27	34.28 ±5.92
	3	0.39 ±0.28	0.62 ±0.25	0.43 ±0.37	0.41 ±0.28	34.12 ±6.93
	Total	0.37 ±0.30	0.58 ±0.31	0.38 ±0.36	0.37 ±0.29	34.53 ±5.95
AB	1	0.22 ±0.28	0.65 ±0.27	0.48 ±0.36	0.22 ±0.29	39.53 ±2.10
	2	0.22 ±0.28	0.54 ±0.26	0.30 ±0.34	0.23 ±0.28	37.70 ±3.08
	3	0.48 ±0.27	0.56 ±0.31	0.09 ±0.24	0.47 ±0.27	37.99 ±2.78
	Total	0.31 ±0.30	0.58 ±0.28	0.29 ±0.35	0.31 ±0.30	38.36 ±2.81
Total	1	0.35 ±0.27	0.50 ±0.33	0.30 ±0.34	0.35 ±0.27	35.78 ±5.38
	2	0.28 ±0.25	0.55 ±0.31	0.25 ±0.32	0.28 ±0.25	34.35 ±6.08
	3	0.40 ±0.28	0.61 ±0.26	0.24 ±0.32	0.40 ±0.28	34.52 ±6.63
	Total	0.34 ±0.27	0.55 ±0.31	0.26 ±0.33	0.34 ±0.27	34.89 ±6.08

Estimated parameter values are significantly different for different trader types (Table 5). On average, item traders have higher values than package traders for ϕ_i (AB - A= -0.03, and AB - B = -0.06*) and λ_i (AB - A= -0.03, and AB - B = - 0.06*) and lower values for k_i (AB - A= 0.08*, and AB - B = 0.00). These values indicate that item traders discount their previous experience at a lower rate than package traders. They also explore more and have higher propensity to select the package with the highest expected payoff more frequently than package traders. Value of δ_i was lower for item trader A (AB - A= 0.17*), and higher for item trader B than package trader (AB – B = - 0.09*) indicates the variation in item traders responsiveness to expected pay-offs of unselected strategies.

Table 5: Univariate ANOVA. Differences in EWA parameter values for different trader types

	ϕ_i			k_i			δ_i			λ_i		
Source	Type III Sum of Squares	Mean Square	Sig.	Type III Sum of Squares	Mean Square	Sig.	Type III Sum of Squares	Mean Square	Sig.	Type III Sum of Squares	Mean Square	Sig.
Corrected Model	0.38	0.19	^	1.09	0.54	**	9.48	4.74	**	0.42	2.89	^
Intercept	88.10	88.10	**	233.82	233.82	**	52.63	52.63	**	88.03	1207.08	**
Trader Type	0.38	0.19	^	1.09	0.54	**	9.47	4.74	**	0.42	2.89	^
Error	56.90	0.07		71.83	0.09		74.43	0.10		56.59	0.07	
Total	149.02			310.09			137.39			148.77		
Corrected Total	57.28			72.92			83.91			57.02		
Adjusted R square	0.04			0.12			0.11			0.01		

Note: significance *** p<0.01; ** p<0.05, ^ p<0.10.

Table 6 presents the analysis of variance comparing information treatments for different trader types. The significant effects of additional market information on most parameters (except for values of ϕ_i and λ_i for trader type A) indicate changes in traders' learning models in different environment.

This is most prominent for package traders (Table 6). For example, in full information treatment (Treatment 3) package traders have significantly lower values for δ_i and higher values for ϕ_i and λ_i compared to in other information treatments. This suggests that in full information treatment package traders discount their past experiences less and respond more moderately to recent experiences. They are likely to repeat their past choices and more sensitive to expected payoff attached to individual strategies. On the other hand, the value of k_i showed a non-linear trend with the propensity of locking a strategy in more quickly in basic information treatment (Treatment 1). This shows that with less market information package traders are driven to lock into a strategy early in the auction. This was likewise mentioned in the previous section that in basic information treatment (Treatment 1) package traders have followed myopic best response strategy at a higher proportion than item traders.

On the other hand, for item traders the effect of information treatment is most significant on the parameter k_i . The alterations in the values indicate that with additional market information item traders are likely to take a strategy early in the auction, which is opposite to the trend observed for package traders. Values for other parameters do not show clear trends for item traders, supporting our previous observations that additional market information has no significant effect on their bidding behaviour in terms of adoption of the best Response bidding strategies.

Table 6: Univariate ANOVA. Differences in EWA parameter values in different information treatments for different trader types

	ϕ_i			k_i			δ_i			λ_i		
Source	Type III Sum of Squares	Mean Square	Sig.	Type III Sum of Squares	Mean Square	Sig.	Type III Sum of Squares	Mean Square	Sig.	Type III Sum of Squares	Mean Square	Sig.
Trader Type A												
Corrected Model	0.09	0.05		2.50	1.25	**	0.46	0.23	**	0.13	0.07	
Intercept	33.02	33.02	**	71.96	71.96	**	4.17	4.17	**	32.82	32.82	**
Treatment	0.09	0.05		2.50	1.25	**	0.46	0.23	**	0.13	0.07	
Error	12.90	0.05		25.59	0.09		10.08	0.04		13.00	0.05	
Total	46.29			98.92			14.82			46.28		
Corrected Total	13.00			28.09			10.53			13.13		
Adjusted R square	0.00			0.08			0.04			0.00		
Trader Type B												
Corrected Model	1.01	0.51	**	0.78	0.39	*	0.79	0.40	*	0.98	0.49	**
Intercept	38.40	38.40	**	96.87	96.87	**	41.43	41.43	**	38.70	38.70	**
Treatment	1.01	0.51	**	0.78	0.39	*	0.79	0.40	*	0.98	0.49	**
Error	24.27	0.08		26.36	0.09		37.35	0.13		24.05	0.08	
Total	64.12			124.32			79.80			64.23		
Corrected Total	25.28			27.14			38.15			25.03		
Adjusted R square	0.03			0.02			0.02			0.03		
Trader Type AB												
Corrected Model	3.06	1.53	**	0.48	0.24	*	5.13	2.56	**	2.76	1.38	**
Intercept	19.66	19.66	**	70.68	70.68	**	17.72	17.72	**	19.48	19.48	**
Treatment	3.06	1.53	**	0.48	0.24	*	5.13	2.56	**	2.76	1.38	**
Error	15.56	0.08		16.13	0.08		20.63	0.10		15.68	0.08	
Total	38.62			86.85			42.76			38.26		
Corrected Total	18.62			16.60			25.75			18.44		
Adjusted R square	0.16			0.02			0.19			0.14		

Note: significance *** p<0.01; ** p<0.05, ^ p<0.10.

In summary, we observe that package traders follow price feedback information more closely than item traders especially when they are in low information treatment (Treatment 1) when only status of their own bid and market feedback price information are released. However, with additional information package traders substantially deviate from best response bidding strategy. Finally, parameter estimations of EWA algorithm indicates that item traders tend to repeat their past selection more frequently than package traders, especially in low information environment. In high information environment the trend is reversed.

Conclusion

We questioned whether traders in combinatorial markets follow the equilibrium bidding strategy given different amounts of market information, whether they follow price signals more closely when market information is limited and whether they use the same learning model irrespective of the amount of market information. The research found that on average across all information treatments package traders followed price feedback information more closely than item traders. With additional information package traders substantially deviated from best response bidding strategy. They also deviate from their equilibrium bidding strategy when they are bidding on individual points. Overall, traders did not systematically follow a myopic best response bidding strategy. Potential reasons for this could include the cognitive complexity of evaluating the complete lot of bid combinations that the item price feedback mechanism may not always support the Walrasian allocation, and traders may not be consistently motivated by rational expectations. The results of the Experience Weighted Attraction learning modelling suggest that traders use a hybrid model and package traders change their learning model substantially in full information treatment.

References

APARICIO, J., LANDETE, M., MONGE, J. & SIRVENT, I. 2008. A new pricing scheme based on DEA for iterative multi-unit combinatorial auctions. *Top,* 16**,** 319-344.

ARMSTRONG, M. 2000. Optimal multi-object auctions. *The Review of Economic Studies,* 67**,** 455.

BAZZAN, A. L. C., DE OLIVEIRA, D. & DA SILVA, B. C. 2010. Learning in groups of traffic signals. *Engineering Applications of Artificial Intelligence,* 23**,** 560-568.

BICHLER, M., PIKOVSKY, A. & SETZER, T. 2005. An analysis of design problems in combinatorial procurement auctions. *Wirtschaftsinformatik,* 47**,** 126-134.

BIKHCHANDANI, S. & MAMER, J. W. 1997. Competitive Equilibrium in an Exchange Economy with Indivisibilities. *Journal of Economic Theory,* 74**,** 385-413.

BREWER, P. J. 1999. Decentralized computation procurement and computational robustness in a smart market. *Economic Theory,* 13**,** 41-92.

CAMERER, C. 2003. *Behavioral game theory: Experiments in strategic interaction*, Princeton University Press Princeton, NJ.

CAMERER, C. & HO, T. H. 1998. Experience-weighted attraction learning in coordination games: Probability rules, heterogeneity, and time-variation. *Journal of Mathematical Psychology,* 42**,** 305-326.

CAMERER, C. & HUA HO, T. 1999. Experience-weighted Attraction Learning in Normal Form Games. *Econometrica,* 67**,** 827-874.

CASON, T. N. 1995. An experimental investigation of the seller incentives in the EPA's emission trading auction. *The American Economic Review,* 85**,** 905-922.

CHEN, Y. & TAKEUCHI, K. 2010. Multi-object auctions with package bidding: An experimental comparison of vickrey and ibea. *Games and Economic Behavior,* 68**,** 557-579.

COX, J. C., OFFERMAN, T., OLSON, M. A. & SCHRAM, A. 2002. Competition for versus on the rails: A laboratory experiment. *International Economic Review,* 43**,** 709-736.

DEPARTMENT OF PRIMARY INDUSTRIES 2007. Auction of marine aquaculture sites in Victoria: a review of concepts and practice. *Evaluation Report 10.* Melbourne: Department of Primary Industries.

DUECK, G. & SCHEUER, T. 1990. Threshold accepting: a general purpose optimization algorithm appearing superior to simulated annealing. *Journal of computational physics,* 90**,** 161-175.

ELMAGHRABY, W. & KESKINOCAK, P. 2004. Combinatorial Auctions in Procurement. *The Practice of Supply Chain Management: Where Theory and Application Converge.*

EPSTEIN, R., HENRIQUEZ, L., CATALAN, J., WEINTRAUB, G. Y., MARTINEZ, C. & ESPEJO, F. 2004. A combinatorial auction improves school meals in Chile: a case of OR in developing countries. *International Transactions in Operational Research,* 11**,** 593-612.

EREV, I. & ROTH, A. E. 1998. Predicting how people play games: Reinforcement learning in experimental games with unique, mixed strategy equilibria. *The American Economic Review,* 88**,** 848-881.

GILLI, M. & KËLLEZI, E. 2002. The threshold accepting heuristic for index tracking. *Applied Optimization,* 70**,** 1-18.

GOEREE, J. K. & HOLT, C. A. 2010. Hierarchical package bidding: A paper & pencil combinatorial auction. *Games and Economic Behavior,* 70**,** 146-169.

GOEREE, J. K., HOLT, C. A. & PALFREY, T. R. 2002. Quantal response equilibrium and overbidding in private-value auctions. *Journal of Economic Theory,* 104**,** 247-272.

HOHNER, G., RICH, J., NG, E., REID, G., DAVENPORT, A. J., KALAGNANAM, J. R., LEE, H. S. & AN, C. 2003. Combinatorial and quantity-discount procurement auctions benefit Mars, incorporated and its suppliers. *Interfaces,* 33**,** 23-35.

HOLT JR, C. A. 1980. Competitive bidding for contracts under alternative auction procedures. *The Journal of Political Economy***,** 433-445.

IFTEKHAR, M. S., HAILU, A. & LINDNER, R. K. 2011. Item price information feedback in multiple unit combinatorial auctions: design issues. *IMA Journal of Management Mathematics,* 22**,** 271-289.

IFTEKHAR, M. S., HAILU, A. & LINDNER, R. K. 2013. Choice of item pricing feedback schemes for multiple unit reverse combinatorial auctions. *Journal of the Operational Research Society,* 64**,** 1571 - 1582.

IFTEKHAR, M. S. & TISDELL, J. G. 2012. Comparison of simultaneous and combinatorial auction designs in fisheries quota market. *Marine Policy,* 36**,** 446-453.

IFTEKHAR, M. S. & TISDELL, J. G. 2015. Bidding and performance in multiple unit combinatorial fishery quota auctions: Role of information feedbacks. *Marine Policy,* 62**,** 233-243.

KOBOLDT, C., MALDOOM, D. & MARSDEN, R. 2003. The first combinatorial spectrum auction. *DotEcon DP*.

LEDYARD, J., HANSON, R. & ISHIKIDA, T. 2009. An experimental test of combinatorial information markets. *Journal of Economic Behavior & Organization,* 69**,** 182-189.

LIU, Y. H. 2011. Incorporating scatter search and threshold accepting in finding maximum likelihood estimates for the multinomial probit model. *European Journal of Operational Research,* 211**,** 130-138.

LUNANDER, A. & NILSSON, J.-E. 2004. Taking the Lab to the Field: Experimental Tests of Alternative Mechanisms to Procure Multiple Contracts. *Journal of Regulatory Economics,* 25**,** 39-58.

MCALLISTER, P. H. 1991. Adaptive approaches to stochastic programming. *Annals of Operations Research,* 30**,** 45-62.

NEUGEBAUER, T. & PEROTE, J. 2008. Bidding 'as if'risk neutral in experimental first price auctions without information feedback. *Experimental Economics,* 11**,** 190-202.

PALFREY, T. R. 1983. Bundling decisions by a multiproduct monopolist with incomplete information. *Econometrica: Journal of the Econometric Society***,** 463-483.

PARKES, D. C. 2006. Iterative combinatorial auctions. *In:* CRAMTON, P., SHOHAM, Y. & STEINBERG, R. (eds.) *Combinatorial Auctions.* Cambridge MA: The MIT Press.

PIKOVSKY, A. 2008. *Pricing and bidding strategies in iterative combinatorial auctions.* PhD, Munchen eingereicht und durch die Fakultat fur Informatik.

PORTER, D., RASSENTI, S., SHOBE, W., SMITH, V. & WINN, A. 2009. The design, testing and implementation of Virginia's NOx allowance auction. *Journal of Economic Behavior & Organization,* 69**,** 190-200.

RASSENTI, S. J., SMITH, V. L. & BULFIN, R. L. 1982. A combinatorial auction mechanism for airport time slot allocation. *Bell Journal of Economics,* 13**,** 402-417.

RILEY, J. G. & SAMUELSON, W. F. 1981. Optimal auctions. *The American Economic Review,* 71**,** 381-392.

SCHUMANN, E. 2011. Portfolio Optimisation with Threshold Accepting. *System,* 1**,** 0.3363246.

TAKEUCHI, K., LIN, J. C., CHEN, Y. & FINHOLT, T. A. 2010. Scheduling with package auctions. *Experimental Economics,* 13**,** 476-499.

WINKER, P. 1995. Identification of multivariate AR-models by threshold accepting. *Computational Statistics & Data Analysis,* 20**,** 295-307.

WINKER, P. & MARINGER, D. 2007. The threshold accepting optimisation algorithm in economics and statistics. *Optimisation, econometric and financial analysis***,** 107-125.

WURMAN, P. R., WELLMAN, M. P. & WALSH, W. E. 2001. A parametrization of the auction design space. *Games and Economic Behavior,* 35**,** 304-338.