MONASH

WP NO. 10/91

MONASH
UNIVERSITY

AUSTRALIA

ALTERNATIVE POINT OPTIMAL TESTS FOR REGRESSION COEFFICIENT STABILITY

Robert D. Brooks

Working Paper No. 10/91

October 1991

DEPARTMENT OF ECONOMETRICS

ISSN 1032-3813

ISBN 0 86746 950 1

ALTERNATIVE POINT OPTIMAL TESTS FOR REGRESSION COEFFICIENT STABILITY

Robert D. Brooks

Working Paper No. 10/91

October 1991

DEPARTMENT OF ECONOMETRICS, FACULTY OF ECONOMICS COMMERCE & MANAGEMENT

MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

ALTERNATIVE POINT OPTIMAL TESTS FOR

REGRESSION COEFFICIENT STABILITY

Robert D. Brooks*

Departments of Economics and Econometrics, Monash University, Clayton, Victoria 3168, Australia

Abstract

In the context of the linear regression model, Shively (1988) has constructed a point optimal test for constant coefficients against the alternative of return to normalcy coefficients. This paper considers alternative methods for the choice of values of the unknown parameters required to conduct the test. These alternatives are based on Cox and Hinkley's (1974, p.102) idea of maximising some weighted average of powers. The paper explores the use of some simple weighting schemes and demonstrates by an empirical power comparison the usefulness of maximising some weighted average of powers in solving this testing problem.

* An earlier version of this paper was presented at a Monash University Department of Econometrics seminar. The author wishes to thank Max King, Kees Jan van Garderen, Brett Inder, Merran Evans, two anonymous referees and an Associate Editor for their helpful comments on earlier versions of this paper. The financial support of an Australian Postgraduate Research Priority Award is also gratefully acknowledged. All omissions in the paper are the sole responsibility of the author.

1. Introduction

The complexity of economic processes often leads the researcher undertaking regression analysis to consider the use of a varying coefficient model. A popular varying coefficient model is the return to normalcy process introduced by Rosenberg (1973). For example, it has been found to be of particular relevance in the literature on systematic risk in the capital asset pricing model by Sunder (1980), Ohlson and Rosenberg (1982), Bos and Newbold (1984), Collins, Ledolter and Rayburn (1987) and Faff, Lee and Fry (1991) among others. A further strength of this alternative is its ability to incorporate both Hildreth-Houck (1968) random coefficients and random walk coefficients as special cases.

It is therefore important to be able to test for the presence of return to normalcy coefficients. Econometric investigation of this problem has resulted in tests proposed by Watson and Engle (1985), King (1987) and Shively (1988). On the basis of an empirical power comparison, Shively's point optimal test appears to have the best power properties for this problem.

An interesting feature of the problem is the presence of a nuisance parameter only under the alternative hypothesis. The application of the point optimal test requires the choice of values for both the parameter of interest and the nuisance parameter at which power is to be optimised. Shively (1988), while choosing the value of the parameter of interest according to some criteria, handles the choice of the nuisance parameter in an arbitrary manner by setting it equal to a typical value. This would appear to introduce unnecessary arbitrariness into the test. This paper explores alternative solutions to overcome this arbitrariness based on Cox and Hinkley's (1974,p.102) idea of maximising some weighted average of powers.

The plan of this paper as follows. The testing problem is set out in Section 2. Shively's solution is then reviewed in Section 3. Possible alternatives for overcoming the arbitrariness in Shively's solution are discussed in Section 4. An empirical power comparison between these alternatives and Shively's test is conducted in Section 5.

2. The Testing Problem

Consider the model which has a single varying coefficient α_t,

$$y_t = x_t\alpha_t + z_t'\beta + \epsilon_t,$$

where z_t is a (kx1) vector,

$$\epsilon_t \sim IN(0, \sigma^2), \quad t = 1, 2, ..., n,$$

If α_t follows Rosenberg's return to normalcy process, one can write,

$$\alpha_t = \phi\alpha_{t-1} + (1-\phi)\bar{\alpha} + a_t,$$

where,

$$a_t \sim IN(0, \lambda\sigma^2), \quad t = 1, 2, ..., n,$$

ϵ_t is independent of a_t and, $0 \leq \phi \leq 1$. By repeated substitution, the model can be written as,

$$y_t = x_t\bar{\alpha} + z_t'\beta + v_t,$$

where,

$$v_t = x_t(\alpha_t - \bar{\alpha}) + \epsilon_t.$$

When the varying coefficient follows a return to normalcy process, the disturbances v_t will be both autocorrelated and heteroscedastic. One will find,

$$Var(v_t) = \sigma^2(1 + x_t^2\lambda/(1-\phi^2))$$
$$Cov(v_t, v_s) = (\lambda\sigma^2 x_t x_s \phi^{|t-s|})/(1-\phi^2).$$

When $\lambda = 0$ these are greatly simplified and reduce to,

$$Var(v_t) = \sigma^2 \quad Cov(v_t, v_s) = 0.$$

This simplifies the process of estimation as it allows the valid use of OLS. Therefore the testing problem of interest is :

$$H_0 : \lambda = 0 \quad against \quad H_a : \lambda > 0.$$

The problem is one-sided because λ is a ratio of variances which by definition must be non-negative. The interesting feature of this testing problem is the presence of the nuisance parameter ϕ only under the alternative hypothesis.

3. Shively's Point Optimal Invariant Test

Shively (1988) constructed a point optimal invariant (POI) test for this problem and reported a power comparison between the different solutions to this testing problem. A POI test rejects H_0 for small values of

$$T(\lambda_1, \phi_1) = w'[P(I + \lambda_1 \Omega(\phi_1))P']^{-1} w / w'w,$$

or,

$$T(\lambda_1, \phi_1) = \tilde{u}'(I + \lambda_1 \Omega(\phi_1))^{-1} \tilde{u} / \hat{u}'\hat{u}$$

where P is a matrix whose rows form an orthonormal basis for the orthogonal complement of (X Z), w=Pv=Py, $\tilde{u}$ is the GLS residual vector assuming covariance matrix $(I+\lambda_1\Omega(\phi_1))$ and $\hat{u}$ is the OLS residual vector. $\Omega(\phi)$ is the covariance matrix for the return to normalcy process which has a typical element of

$$\Omega(\phi)_{st} = x_s x_t \phi^{|s-t|}/(1-\phi^2).$$

Invariance is with respect to transformations of the form

$$y \rightarrow cy - (XZ)\begin{pmatrix} a \\ b \end{pmatrix},$$

where c and a are scalars and b is a (kx1) vector, and only holds provided that the choice of λ_1 value is allowed to adjust to the scale of x_t.

The POI test is made operational by choosing values for the unknown parameters λ_1 and ϕ_1. This choice should be non-arbitrary and made according to some optimality criteria. The approach aims to choose values for λ_1 and ϕ_1 which provide good power over a wide range of the parameter space.

On the basis of an empirical power comparison Shively (1988) recommends that one set $\phi_1 = 0.7$. He proposes that λ_1 be chosen so that the power of the test is 0.5 against that λ_1. This choice of λ_1 is denoted as λ_1^*. This gives the point optimal test as $T(\lambda_1^*, 0.7)$. Shively's choice of λ_1 is based on optimality criteria but his choice of ϕ_1 is made in an arbitrary fashion.

Shively (1988) conducts a Monte Carlo power comparison of his test and those suggested by Watson and Engle (1985) and King (1987). Watson and Engle (1985) propose a test based on an approximation to the procedure in Davies (1977). King (1987) constructs a locally best invariant (LBI) test for this problem which is also LBI against the random walk coefficient alternative.

For the three different experiments, the $T(\lambda_1^*, 0.7)$ is always superior to King's LBI test. The test is also nearly always superior to the approximate Davies test proposed in Watson and Engle (1985). From this, one concludes that the $T(\lambda_1^*, 0.7)$ test is to be preferred when testing the constant coefficient model against the return to normalcy alternative.

4. Alternative Testing Solutions

Despite the good power performance of the $T(\lambda_1^*, 0.7)$ test, one may be uncomfortable with the arbitrary choice of ϕ_1=0.7. A possible solution for choosing a ϕ_1 value is given in King (1989). For any given λ value the power of the test will be a function of ϕ. King (1989) argues that this function will have a minimum at some ϕ value. The value for ϕ should therefore be chosen so that this minimum power is maximised at the chosen λ_1^* value.

Unfortunately this suggestion does not work for our testing problem. The minimum is typically found at a very low ϕ_1 value. By choosing this ϕ_1 value the power function is flattened out so that the power barely rises above the size.

An alternative suggestion is given in Cox and Hinkley (1974, p.102). They propose choosing a test which maximises some weighted average of powers. The difficulty is then in determining the appropriate weighted average. For simplicity considerations the first option considered is a simple average over a set of ϕ values. Despite its simplicity, this tackles the problem of arbitrariness inherent in the choice of a typical value. A matter of interest is how large a grid of ϕ values is required in maximising the average power.

The optimal ϕ_1 value is denoted as ϕ_1^*. It is that ϕ_1 value which maximises the average power over the following three different grids:

$$\phi = 0.1, 0.5, 0.9$$

$$\phi = 0.05, 0.35, 0.65, 0.95$$

$$\phi = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$$

when $\lambda = \lambda_1^*$. In keeping with Shively (1988), λ is chosen to make this maximised average 0.5. This λ value is denoted as λ_1^*. The respective test statistic are denoted as $s_3(\lambda_1^*, \phi_1^*)$, $s_4(\lambda_1^*, \phi_1^*)$ and $s_9(\lambda_1^*, \phi_1^*)$.

The steps involved in carrying out the new test are as follows:

(a) Guess likely values for λ_1^* and ϕ_1^*, namely λ_1 and ϕ_1.

(b) Calculate the average power over the suggested set of ϕ values at $\lambda = \lambda_1$.

(c) By altering ϕ_1, find the value which maximises the average power.

(d) If the maximised average is less (greater) than the desired value then increase (decrease) λ_1. Steps (b) and (c) are then repeated.

(e) The procedure is complete when the maximised average power is at the desired value.

These steps are carried out for each of the three different tests. The smaller the number of points in the grid the lower the computational intensity of the test. Inclusion of the three different tests therefore allows assessment of the gains from the greater computational load.

5. Empirical power comparison

An empirical power comparison between the T, s_3, s_4, s_9 tests was conducted. The empirical power comparison was conducted using the following design matrices :

X1: n = 31. A constant as the only regressor and therefore the regressor with the varying coefficient.

X2: n = 41. A constant and the income and price of spirits data from Durbin and Watson (1951). Income is the regressor with the varying coefficient. Data is annual commencing 1870.

X3: n = 31. A constant and a linear time trend. The linear time trend is the regressor with the varying coefficient.

X4: n = 41. A constant, current Australian income and Australian consumption lagged one period. Lagged consumption is the regressor with the varying coefficient. Data is quarterly commencing 1959(4).

X5: n = 20. A constant and the Australian consumer price index. The CPI is the regressor with the varying coefficient. Data is quarterly commencing 1948(3).

X6: n = 25. A constant and Chow's (1957) money and automobile stock per capita data. Money is the regressor with the varying coefficient. Data is annual commencing 1921.

X7: n = 60. A constant, current Australian income, Australian consumption lagged one period and a full complement of seasonal dummy variables. Lagged consumption is the regressor with the varying coefficient. Data is quarterly commencing 1959(4).

The matrices $X1$ and $X2$ are taken from Shively (1988). The other matrices $X3$ through to $X7$ are chosen to provide alternatives to his power comparison done by Shively (1988). It is expected that Shively's test will perform well for $X1$ and $X2$ and that any problems with the arbitrary choice of ϕ_1=0.7 will show up for the other X matrices.

All of the test statistics considered can be written as ratios of quadratic forms in normal variables. Therefore exact critical values and powers can be computed for each test statistic in an analogous manner to that for the Durbin-Watson statistic.

For each design matrix the choice of points and five percent critical values for each test are contained in table 1. It is worth noting that the number of points in the grid of ϕ values has minimal impact on the choice of point. In all cases, Shively's test produces a choice of λ value lower than the average power tests. In some cases, namely $X3$, $X4$ and $X7$, Shively's choice of $\phi = 0.7$ appears too high given the choice of ϕ values made by the other tests.

The power results for each of the design matrices are contained in tables 2-5. For the $X1$ case, in Table 3 the average powers for the s_3, s_4 and s_9 tests are only 0.4 as it cannot be raised to 0.5. Except for $X5$ the powers of the tests are always an increasing function of both λ and ϕ. In the $X5$ case, the power of the s_3, s_4 and s_9 tests falls as ϕ increases from 0.9 to 0.99. For all cases the powers of the s_3, s_4 and s_9 tests are similar. This is not unexpected given that all three tests produce such similar choices of values for the unknown parameters required to conduct the test.

For $X5$ the tests' power are not altered by changes in λ values. In this case all of the power action with respect to λ appears to have occurred before λ reaches a value of 0.1. However, the choice of value for the nuisance parameter ϕ is important. For low ϕ values (less than 0.7) Shively's test fares poorly. Overall the average power tests with the exception of s_9 fare well, and appear less sensitive.

For the other X matrices, the results are similar in each case. Shively's test works best for combinations of very low λ values and high ϕ values. The opposite result is obtained for moderate to large λ values and small to moderate ϕ values where the average power tests are superior.

The greatest differences in power performance appear for the typical economic data in the $X3$, $X4$ and $X7$ cases. From these cases the extreme effects on Shively's test appear greatest. This suggests one should use of one of the s_3, s_4 or s_9 tests which do not suffer as greatly. On the basis of computational cost one would therefore advocate the use of the s_3 test.

References

Bos, T. and P.Newbold, 1984, An empirical investigation of the possibility of stochastic systematic risk in the market model, Journal of Business 57, 35 - 41.

Chow, G.C., 1957, Demand for automobiles in the United States: A study in consumer durables, (North-Holland, Amsterdam).

Collins, D.W., J.Ledolter and J.Rayburn, 1987, Some further evidence on the stochastic properties of systematic risk, Journal of Business 60, 425 - 448.

Cox, D.R. and D.V.Hinkley, 1974, Theoretical statistics (Chapman and Hall, London).

Davies, R.B., 1977, Hypothesis testing when a nuisance parameter is present only under the alternative, Biometrika 64, 247 - 254.

Durbin, J. and G.S.Watson, 1951, Testing for serial correlation in least squares regression II, Biometrika 38, 159 - 178.

Faff, R.W., J.H.H. Lee, and T.R.L. Fry, 1991, Time stationarity of systematic risk: Some Australian evidence, Journal of Business Finance and Accounting, forthcoming.

Hildreth, C. and J.P.Houck, 1968, Some estimators for a linear model with random coefficients, Journal of the American Statistical Association 63, 584 - 595.

King, M.L., 1987, An alternative test for regression coefficient stability, Review of Economics and Statistics 69, 379 - 381.

King, M.L., 1989, Testing for fourth-order autocorrelation in regression disturbances when first-order sutocorrelation is present, Journal of Econometrics 41, 285 - 301.

Ohlson, J. and B.Rosenberg, 1982, Systematic risk of CRSP equal-weighted common stock index : A history estimated by stochastic-parameter regression, Journal of Business 55, 121 - 145.

Rosenberg , B., 1973, The analysis of a cross-section of time series by stochastically convergent parameter regression, Annals of Economic and Social Measurement 2, 399 - 428.

Shively, T.S., 1988, An analysis of tests for regression coefficient stability, Journal of Econometrics 39, 367 - 386.

Sunder, S., 1980, Stationarity of market risk : Random coefficient tests for individual stocks, Journal of Finance 35, 883 - 896.

Watson, M.W. and R.F.Engle, 1985, Testing for regression coefficient stability with a stationary AR(1) alternative, Review of Economics and Statistics 67, 341 - 346.

Table 1 - Points and critical values for the four POI tests; X1 to X7				
	T	s_3	s_4	s_9
$X1$				
λ	0.436157	0.541614	0.545414	0.644355
ϕ	0.700000	0.675000	0.669000	0.620000
c	0.589150	0.551303	0.550654	0.523076
$X2$				
λ	0.124305	0.311233	0.312369	0.309856
ϕ	0.700000	0.625000	0.622000	0.587000
c	0.612033	0.432316	0.431580	0.433647
$X3$				
λ	0.124151	0.192491	0.193398	0.198450
ϕ	0.700000	0.519000	0.522000	0.512000
c	0.644589	0.579679	0.578670	0.577750
$X4$				
λ	0.039064	0.082289	0.083043	0.084069
ϕ	0.700000	0.496000	0.500000	0.500000
c	0.645043	0.514936	0.512708	0.510049
$X5$				
λ	0.000005	0.000020	0.000021	0.000017
ϕ	0.700000	0.607000	0.627000	0.550000
c	0.364332	0.131322	0.128378	0.150440
$X6$				
λ	0.031085	0.137640	0.138854	0.126866
ϕ	0.700000	0.643000	0.647000	0.574000
c	0.462013	0.191624	0.190639	0.199872
$X7$				
λ	0.014211	0.022366	0.022517	0.023404
ϕ	0.700000	0.552000	0.547000	0.535000
c	0.728636	0.672100	0.671419	0.665672

æ

Table 2 - Power of the six POI tests for X1 and X2						
ϕ	0.01	0.2	0.5	0.7	0.9	0.99
			$X1$			
$\lambda = 0.2$						
T	0.051	0.071	0.148	0.291	0.583	0.722
s_3	0.051	0.071	0.148	0.289	0.577	0.716
s_4	0.051	0.071	0.148	0.289	0.577	0.716
s_9	0.051	0.072	0.148	0.286	0.569	0.709
$\lambda = 1.0$						
T	0.052	0.127	0.425	0.719	0.916	0.954
s_3	0.052	0.128	0.429	0.722	0.917	0.954
s_4	0.052	0.128	0.430	0.722	0.917	0.954
s_9	0.053	0.130	0.433	0.724	0.917	0.954
			$X2$			
$\lambda = 0.01$						
T	0.050	0.055	0.067	0.085	0.121	0.140
s_3	0.050	0.055	0.066	0.082	0.113	0.130
s_4	0.050	0.055	0.066	0.082	0.113	0.130
s_9	0.050	0.055	0.066	0.082	0.112	0.128
$\lambda = 0.5$						
T	0.056	0.177	0.586	0.841	0.953	0.968
s_3	0.057	0.186	0.606	0.850	0.955	0.970
s_4	0.057	0.186	0.606	0.850	0.955	0.970
s_9	0.057	0.187	0.607	0.850	0.954	0.969

Table 3 - Power of the six POI tests for X3 and X4						
ϕ	0.01	0.2	0.5	0.7	0.9	0.99
			$X3$			
$\lambda = 0.2$						
T	0.184	0.274	0.478	0.641	0.781	0.818
s_3	0.242	0.325	0.498	0.634	0.752	0.780
s_4	0.242	0.325	0.498	0.634	0.753	0.781
s_9	0.245	0.327	0.498	0.633	0.750	0.778
$\lambda = 1.0$						
T	0.414	0.598	0.833	0.922	0.964	0.971
s_3	0.590	0.729	0.881	0.937	0.962	0.962
s_4	0.589	0.728	0.881	0.937	0.962	0.963
s_9	0.597	0.733	0.882	0.937	0.961	0.962
			$X4$			
$\lambda = 0.01$						
T	0.056	0.068	0.108	0.178	0.337	0.402
s_3	0.058	0.071	0.107	0.166	0.298	0.354
s_4	0.058	0.071	0.107	0.166	0.298	0.355
s_9	0.058	0.071	0.107	0.166	0.298	0.354
$\lambda = 0.5$						
T	0.122	0.331	0.777	0.941	0.988	0.992
s_3	0.174	0.413	0.822	0.952	0.989	0.993
s_4	0.172	0.412	0.823	0.953	0.989	0.993
s_9	0.173	0.412	0.823	0.953	0.989	0.993

Table 4 - Power of the six POI tests for X5 and X6						
ϕ	0.01	0.2	0.5	0.7	0.9	0.99
			$X5$			
$\lambda = 0.01$						
T	0.102	0.253	0.604	0.788	0.877	0.884
s_3	0.118	0.285	0.636	0.806	0.880	0.877
s_4	0.117	0.281	0.635	0.807	0.884	0.884
s_9	0.129	0.296	0.639	0.801	0.866	0.854
$\lambda = 1.0$						
T	0.102	0.253	0.604	0.788	0.877	0.884
s_3	0.118	0.285	0.637	0.807	0.880	0.878
s_4	0.117	0.281	0.635	0.808	0.884	0.884
s_9	0.129	0.296	0.639	0.801	0.866	0.854
			$X6$			
$\lambda = 0.01$						
T	0.051	0.074	0.149	0.255	0.409	0.463
s_3	0.051	0.075	0.145	0.239	0.380	0.430
s_4	0.051	0.075	0.145	0.239	0.380	0.430
s_9	0.051	0.075	0.145	0.239	0.378	0.428
$\lambda = 0.5$						
T	0.054	0.187	0.591	0.814	0.922	0.941
s_3	0.055	0.196	0.615	0.825	0.931	0.947
s_4	0.055	0.196	0.615	0.825	0.931	0.947
s_9	0.055	0.197	0.615	0.825	0.930	0.946

Table 5 - Power of the six POI tests for X7

ϕ	0.01	0.2	0.5	0.7	0.9	0.99
$\lambda = 0.01$						
T	0.087	0.121	0.229	0.382	0.628	0.709
s_3	0.101	0.136	0.237	0.369	0.580	0.653
s_4	0.102	0.137	0.237	0.368	0.578	0.651
s_3	0.103	0.138	0.237	0.366	0.573	0.645
$\lambda = 0.2$						
T	0.368	0.625	0.920	0.985	0.998	0.999
s_3	0.512	0.736	0.945	0.988	0.998	0.999
s_4	0.517	0.738	0.945	0.988	0.998	0.999
s_9	0.529	0.746	0.946	0.988	0.998	0.999
$\lambda = 0.5$						
T	0.433	0.711	0.959	0.995	1.000	1.000
s_3	0.599	0.822	0.976	0.996	1.000	1.000
s_4	0.604	0.825	0.976	0.996	1.000	1.000
s_9	0.619	0.832	0.977	0.996	1.000	1.000
$\lambda = 1.0$						
T	0.459	0.743	0.969	0.997	1.000	1.000
s_3	0.633	0.851	0.984	0.998	1.000	1.000
s_4	0.638	0.854	0.984	0.998	1.000	1.000
s_9	0.653	0.861	0.985	0.998	1.000	1.000