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Working Paper No. 8/91

September 1991

DEPARTMENT OF ECONOMETRICS

ISSN 1032-3813

ISBN 0 86746 981 1

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(Revised Version)

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ABSTRACT

While the Almost Ideal Demand System (AIDS) has received increasing attention in empirical studies of consumer demand, the fact that the underlying PIGLOG cost function is not globally regular has often led to violation of negative semi-definiteness in the estimated Slutsky matrix. This paper suggests a modification to the PIGLOG class of preferences, termed MPIGLOG, which preserves regularity in a wider region of expenditure-price space. In an empirical section the AIDS special case of PIGLOG is compared with the analogous special case of MPIGLOG, which we term MAIDS. The comparison demonstrates the improved regularity features of MAIDS.

I. INTRODUCTION

It could be argued that the popularization of duality theory by authors such as Diewert (1974, 1982) has revolutionized the estimation of consumer demand systems. Previously, the derivation and estimation of consumer demand systems from globally regular direct utility functions which satisfy everywhere the neoclassical monotonicity and curvature conditions were based on minor variants of the Cobb-Douglas and CES functions. The only alternative was to estimate local approximations to the demand systems themselves, using representations such as the double log model, which were in fact nonintegrable.

With the advent of duality theory the potential, at least, was introduced for extending the range of empirically tractable integrable demand systems, through the specification of globally regular indirect utility functions or cost functions, and application of Roy's Identity, or Shephard's Lemma, respectively. However, with the possible exception of Houthakker's Indirect Addilog Model (Houthakker (1960)), this potential was not immediately realized. Instead, attention turned to the specification of locally flexible functional forms for indirect utility functions and cost functions, such as the Translog of Christensen, Jorgenson and Lau (1973) and the Generalized Leontief of Diewert (1971). While such specifications allow systems of demand equations which can attain arbitrary elasticities at a point in price-expenditure space, the generating functions are far from globally regular. Indeed, such estimated demand equations usually satisfy only homogeneity with respect to prices and expenditure, and violate both monotonicity and curvature conditions. Methods to characterize the domain of regularity of such functions were introduced by Caves and Christensen (1980), and are further expounded in Lau (1986). In general, it has been found that the domain of regularity of such functions is rather limited.

Both the Translog and Generalized Leontief are examples of a series expansion approach. Further developments within this approach aimed at improving regularity are: Barnett's minflex Laurent (see Barnett (1983, 1985), Barnett and Lee (1985) and Barnett, Lee and Wolfe (1985, 1987)); Gallant's Fourier Flexible Form (see Gallant (1981, 1984), Gallant and Golub (1984)); Barnett's Asymptotically Ideal Model (AIM) (see Barnett and Yue (1988), Barnett, Geweke and Yue (1988)); and the Generalized Barnett and Generalized McFadden models of Diewert and Wales (1987).

An alternative approach to the specification of demand systems based on dual representation of preferences is typified by the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980), which is based on a particular specification of the PIGLOG cost function, itself a special case of the price-independent generalized linear (PIGL) form of Muellbauer (1975). In the AIDS special case, Deaton and Muellbauer specify one of the aggregator functions to be Translog, to ensure local flexibility. In view of the known inability of the Translog to achieve global regularity it is to be expected that AIDS may violate regularity properties. However, it is demonstrated in Section II below that the class of PIGLOG cost functions is inherently non-regular - outside of an arbitrarily restrictive regularity region implied shares fall outside of the (0,1) interval, and the Slutsky matrix fails to be negative semi-definite. Even if the Translog price aggregator were replaced by a globally regular function, AIDS-like demand equations would be guaranteed to violate regularity as real expenditure grows.

In this paper we introduce a class of preference representations in the spirit of PIGLOG, which can be seen to be a generalization of the (implied) PIGLOG indirect utility function, but which potentially allows the imposition of regularity conditions over a more extensive region of price-expenditure space. For simplicity, we will refer to this preference representation as MPIGLOG (for modified PIGLOG). Since the most well-known application of PIGLOG is the AIDS demand system, our empirical example will compare AIDS with the empirical specification that results if MPIGLOG is parameterized in a similar way. This empirical model will be referred to as MAIDS which, while inherently more regular than AIDS, will nonetheless be potentially non-regular because of the Translog specification. However, this example is still able to demonstrate the improved regularity properties of the new representation.

Section II elaborates on the regularity issue as it affects PIGLOG, and Section III presents MPIGLOG as a regular modification to PIGLOG. Section IV discusses the aggregation problem in the context of MPIGLOG. A particular empirical specification which allows for a comparison between PIGLOG and MPIGLOG, i.e. between AIDS and MAIDS, is presented in Section V, and an empirical example is provided in Section VI, where it is found that MAIDS is indeed more regular than AIDS.

II. REGULARITY ISSUES

Consider the PIGLOG specification of a cost function,

$$\ell n\ C(u,p) = (1-u)\ \ell n\ a(p) + u\ \ell n\ b(p) \tag{1}$$

where a(p) and b(p) are positive and homogeneous of degree one (HD1) functions in a vector of prices p, and u is a given level of utility.

In order to discuss regularity, we note firstly the standard properties of a regular cost function:

C1 C is non-negative

C2 C is HD1 in p

C3 C is non-decreasing in u

C4 C is non-decreasing in p

C5 C is concave in p.

The only regularity conditions which (1) obviously satisfies globally are C1 and C2. It is a trivial matter, however, to design a and b to satisfy C3, by defining $\ell n\, b = \ell n\, a$ + (a positive function). More problematic are C4 and C5. For example, as Deaton and Muellbauer point out, a set of sufficient conditions for (1) to be concave in p would be that a and b be concave and $0 \leq u \leq 1$. However, on the one hand these conditions are not necessary, and on the other they do not represent a very realistic set of sufficient conditions, since the condition $0 \leq u \leq 1$ is in no way implied for the evaluation of the indirect utility function dual to (1). Consequently, even if concavity of the cost function were imposed in estimation, it could not be guaranteed in simulation work in the context, for example, of a computable general equilibrium model employing the PIGLOG specification.

To discuss C4 and C5 in more detail, it is useful to write (1) in terms of two reparameterized price aggregator functions P_1 and P_2, where $P_1 = a$ and $P_2 = \ell n\, (b/a)$. Hence:

$$\ell n\, C(u,p) = \ell n\, P_1 + u\, P_2 \qquad (2)$$

where: P_1 is HD1 in p,

P_2 is HD0 in p.

It is important to note that it is the aggregator function P_2 which "does the work" (in the sense of providing Working-Leser type share equations) but that this function must be HD0 to satisfy C2. For Deaton and Muellbauer's AIDS specification, P_1 is Translog and P_2 is Cobb-Douglas. Generalizations of P_1, such as Chalfant's (1982), do not affect the regularity problems, which arise primarily from P_2. It is true that the Translog specification for P_1 presents an additional source of irregularity in the case of the specific AIDS form, but that source of irregularity is not our chief concern here. Even if P_1 were specified along more regular lines following Diewert and Wales, the functional form (2) would exhibit regularity problems. The problem is that P_2 cannot be simultaneously HD0, non-decreasing and concave in p. Euler's theorem rules out the HD0, non-decreasing combination, and the HD0, concave combination, for any non-trivial functions. These conditions on P_2, together with $u \geq 0$ and P_1 HD1 non-decreasing and concave, are merely a set of sufficient conditions. If any of the three incompatible conditions on P_2 were not specified, regularity of (2) could only be imposed at the expense of further arbitrary restriction of the range of u.

In order to motivate an inherently more regular alternative to (2), it is useful to outline the implications of PIGLOG irregularity for the share form of the underlying demand system. Let $q_i = Q_i(c,p)$ denote an arbitrary Marshallian demand function, where c represents the level of total expenditure. Let $E_i = \partial \ln Q_i / \partial \ln c$ and $M_{ij} = \partial \ln Q_i / \partial \ln p_j$ denote the (Marshallian) expenditure and price elasticities respectively ($j = 1, \ldots, n$). Let $w_i = p_i q_i / c$ denote the i^{th} expenditure share. A standard consumer utility maximization problem subject to a linear budget constraint yields aggregation, homogeneity, symmetry and concavity restrictions which we summarise:

Aggregation	Engel:	$\Sigma_i\, w_i\, E_i = 1$	
	Cournot:	$w_j + \Sigma_i\, w_i\, M_{ij} = 0$	
Homogeneity	$E_i + \Sigma_j\, M_{ij} = 0$		
Symmetry	$w_i\, (M_{ij} + w_j\, E_i)$	symmetric	
Concavity	$[w_i\, (M_{ij} + w_j\, E_i)]$	a negative semi-definite matrix.	

Define $R = \ell n(c/P_1)$, and for the PIGLOG specification (2), let

$\varepsilon_{ki} = \partial\, \ell n\, P_k \,/\, \partial\, \ell n\, p_i$ and

$\varepsilon_{kij} = \partial^2\, \ell n\, P_k \,/\, \partial\, \ell n\, p_i\, \partial\, \ell n\, p_j$, $(k = 1,2)$, $(i,j = 1, \ldots, n)$.

Note that, since P_1 is HD1 but P_2 is HD0, $\Sigma_i\, \varepsilon_{1i} = 1$ while $\Sigma_i\, \varepsilon_{1ij}$, $\Sigma_i\, \varepsilon_{2i}$ and $\Sigma_i\, \varepsilon_{2ij}$ all = 0. Shephard's Lemma applied to (2) gives:

$$w_i = \varepsilon_{1i} + \varepsilon_{2i}\, R, \tag{3}$$

and it follows that:

$$E_i = 1 + \varepsilon_{2i}/w_i\ , \tag{4}$$

and:

$$M_{ij} = (\varepsilon_{1ij} + \varepsilon_{2ij}\, R - \varepsilon_{2i}\, \varepsilon_{1j})/w_i - \delta_{ij} \tag{5}$$

where δ_{ij} is the Kronecker delta.

It is clear that specification of P_1 HD1 and P_2 HD0 has been sufficient to maintain Engel and Cournot aggregation, homogeneity and symmetry for the PIGLOG expenditure share system (3). However, it is equally apparent from (3) that for real expenditure sufficiently large,

w_i will violate the (0,1) interval.

Moreover, maintenance of concavity is even more fragile. In terms of the previous notation, a typical term in the Slutsky matrix is:

$$S_{ij} = (c/p_i p_j)(w_i M_{ij} + w_i w_j E_i) \tag{6}$$

and some manipulation leads to the revealing formulation for the PIGLOG specification:

$$S_{ij} = (c/p_i p_j)(\zeta_{1ij} + \zeta_{2ij} R) \tag{7}$$

where:

$$\zeta_{1ij} = \varepsilon_{1ij} + \varepsilon_{1i} (w_j - \delta_{ij}), \text{ and}$$

$$\zeta_{2ij} = \varepsilon_{2ij} + \varepsilon_{2i} (w_j - \delta_{ij}) + \varepsilon_{2i} \varepsilon_{2j}.$$

The nature of the concavity violation problem for PIGLOG is now apparent. As R increases, the ζ_{2ij} terms dominate in S_{ij}. But, on the diagonal, $\varepsilon_{2i}{}^2$ is necessarily positive. For the specific Cobb-Douglas formulation of P_2 in AIDS, $\varepsilon_{2ii} = 0$. Hence $\zeta_{2ii} = \varepsilon_{2i}(w_i - 1) + \varepsilon_{2i}^2$. For $\varepsilon_{2i} > 0$, the positive ε_{2i}^2 term tends to dominate as w_i tends to unity. For $\varepsilon_{2i} < 0$, the entire ζ_{2ii} term is necessarily positive for w_i in the (0,1) interval. Thus it is clear that there is a tendency for the required non-positivity of S_{ii} to be violated as R increases, and this may occur well before w_i violates the (0,1) interval.

Both (3) and (7) indicate the one source of irregularity - the term in R. Yet this term is the essential component of the Working-Leser type specification (3). However, another way of looking at the problem is the intransigence of the elasticity ε_{2i} - no matter how complex this is as a function of prices it does not allow the response of the i^{th}

share to growth in real expenditure to be modified for higher incomes. In the next section we suggest a modification to the PIGLOG cost function (2) which allows an amelioration of the effect of growth in real expenditure on shares, avoiding violation of the (0,1) interval and maintaining concavity under the modest restriction $c \geq P_1$.

III. A MODIFIED PIGLOG SPECIFICATION OF PREFERENCES (MPIGLOG)

As an alternative to (2), consider the modification

$$\ell n\ C(u,p) = \ell n\ P_1 + u\ P_2 \ /\ [C(u,p)]^{\eta} \tag{8}$$

where now P_1 is HD1 in p, and P_2 is HDη in p.

Since (8) does not have an explicit analytical representation as a cost function, we continue the discussion in terms of the dual indirect utility function. For notational clarity in moving from functions to their inverses we employ upper case letters for functional forms such as C( ) and U( ) and lower case letters for the values of such variables appearing as arguments in the respective inverse functions. For purposes of comparison we note firstly that the indirect utility function dual to the PIGLOG cost function (2) would be:

$$U(c,p) = [\ell n\ (c/P_1)]\ /\ P_2 \tag{9}$$

while for the implicit MPIGLOG cost function (8) the dual would be:

$$U(c,p) = [\ell n\ (c/P_1)]\ (c^{\eta}/P_2). \tag{10}$$

Note that (10) nests (9) when $\eta = 0$. To discuss the regularity of MPIGLOG consider the standard properties of a regular indirect utility function:

U1 U is HD0 in (c,p)

U2 U is non-decreasing in c

U3 U is non-increasing in p

U4 U is quasiconvex in p.

By specifying P_1 and P_2 to be positive functions, HD1 and HDη respectively, (10) clearly satisfies U1 and U2 provided $\eta \geq 0$. If P_1 and P_2 are further specified to be non-decreasing in prices then U3 will be satisfied over the region $c \geq P_1$. If P_1 and P_2 are both concave then, over the region $c \geq P_1$, U will be quasiconvex in p (for relevant results on quasiconvex functions see Greenberg and Pierskalla (1971)). Concavity of P_2 will require the further restriction $0 \leq \eta \leq 1$. If these regularity conditions were imposed, then $\eta = 0$ would imply $P_2 \equiv 1$, the degenerate regular form of PIGLOG.

Applying Roy's Identity to (10) we obtain the MPIGLOG share equations:

$$w_i = \frac{\varepsilon_{1i} + \varepsilon_{2i} R}{1 + \eta R} \tag{11}$$

where the ε_{1i} and ε_{2i} are defined, as before, as the i^{th} price elasticities of P_1 and P_2 respectively, but where now $\Sigma_i\ \varepsilon_{1i} = 1$ and $\Sigma_i\ \varepsilon_{2i} = \eta$, and where $R = \ell n(c/P_1)$ as before. Observe that, in the region $c \geq P_1$, we have $R \geq 0$ and so the restrictions $\varepsilon_{1i} \geq 0$ and $\varepsilon_{2i} \geq 0$ are sufficient to ensure $0 \leq w_i \leq 1$, and these restrictions are natural ones to impose on the two price aggregator functions. In general the ε_{1i} and ε_{2i} will be functions of prices unless both P_1 and P_2 are Cobb-Douglas, but even that case provides an interesting illustration of the model in general. To facilitate comparison with PIGLOG note that for MPIGLOG:

$$\partial w_i / \partial \ln c = \frac{\varepsilon_{2i} - \eta w_i}{1 + \eta R} \tag{12}$$

whereas for PIGLOG $\partial w_i / \partial \ln c = \varepsilon_{2i}$. Equation (12) shows how in MPIGLOG the response of shares to growth in real expenditure is modified both by the expenditure level itself and by the pre-existing value of the share.

To provide further interpretation, consider only the non-degenerate case $\eta > 0$. (The degenerate case $\eta = 0$ follows by a limiting argument.) Then equation (11) indicates that, for given prices, the share w_i moves monotonically from ε_{1i} for the "poor" (if we interpret the lower extreme of the regular region for real expenditure as "subsistence", following Deaton and Muellbauer), asymptoting to (ε_{2i}/η) for the "rich". A particularly useful form for interpretation is provided if we define:

$$Z = \eta R/(1 + \eta R) \tag{13}$$

so that:

$$w_i = \varepsilon_{1i} (1 - Z) + (\varepsilon_{2i}/\eta)Z, \tag{14}$$

giving a transparent representation of the MPIGLOG shares as weighted averages of the shares of the rich (ε_{2i}/η) and poor (ε_{1i}).

We note from equation (14) that the potentially unbounded variable, real income, only enters through the bounded variable, Z. Furthermore, the potentially unbounded price terms enter through the homogeneous of degree zero price terms, ε_{1i} and ε_{2i}. Thus, in the spirit of "cointegration accounting", the right hand side of (14) is appropriate for explaining the bounded budget shares, w_i. In contrast, from equation (3), the PIGLOG share equations do not satisfy cointegration accounting unless the homogeneity restrictions on the ε_{1i} and the ε_{2i} are relaxed, a possibility which may explain the frequent rejection of homogeneity restrictions in non-regular models such as AIDS. Of course, this simply reflects the fact that cointegration accounting is the statistical counterpart of the theoretical regularity of economic models.

Further properties of MPIGLOG are readily derived. The expenditure elasticities satisfy:

$$E_i = 1 + (\varepsilon_{2i}/w_i - \eta)(1 - Z), \tag{15}$$

while the price elasticities satisfy:

$$M_{ij} = [\varepsilon_{1ij}(1-Z) + (\varepsilon_{2ij}/\eta)Z - \varepsilon_{2i}\varepsilon_{1j}(1-Z)]/w_i - \delta_{ij} + \eta\varepsilon_{1j}(1-Z) . \tag{16}$$

These relationships demonstrate that the expenditure elasticities range from $1 + \varepsilon_{2i}/\varepsilon_{1i} - \eta$ for the poor towards unity for the rich. As real spending power rises, E_i tends toward unity, rising from below in the case of a necessity or falling from above in the case of a luxury.

Although sufficient conditions exist for MPIGLOG to be globally regular, in many cases it may be useful (for enhanced flexibility) to employ a locally regular flexible functional form, such as the Translog, for P_1. Such a specification would allow on the one hand, a test of regularity under a more general maintained hypothesis than that of PIGLOG. On the other hand, where an extended region of regularity is desirable <u>á priori</u>, the MPIGLOG specification would appear to be <u>prima facie</u> superior to PIGLOG. The evidence for this may be seen on application of (6) to the MPIGLOG specification. This demonstrates that a typical term of the MPIGLOG Slutsky matrix is:

$$S_{ij} = (c/p_i p_j)\ [\xi_{1ij}\ (1 - Z) + \xi_{2ij}\ Z] \tag{17}$$

where:

$$\xi_{1ij} = \varepsilon_{1ij} + \varepsilon_{1i}\ (w_j - \delta_{ij}), \text{ and}$$

$$\xi_{2ij} = \varepsilon_{2ij}/\eta + (\varepsilon_{2i}/\eta)\ (w_j - \delta_{ij})$$

$$+ \eta(\varepsilon_{2i}/\eta - w_i)(\varepsilon_{2j}/\eta - w_j)\ .$$

As R increases Z rises from zero to unity and the Slutsky matrix terms asymptote to $(c/p_i p_j)\xi_{2ij}$. For the diagonal terms, $\xi_{2ii} = \varepsilon_{2ii}/\eta + (\varepsilon_{2i}/\eta)(w_i - 1) + \eta(\varepsilon_{2i}/\eta - w_i)^2$. While the squared term is clearly positive, it asymptotes to zero as R increases, since w_i asymptotes to ε_{2i}/η. If P_2 is chosen Cobb-Douglas as for AIDS, the only possible source of violation of non-positivity of S_{ii} comes from the term ξ_{1ii}. But this term has less weight as R increases. Thus, for P_1 Translog and P_2 Cobb-Douglas, MPIGLOG (under this specification, MAIDS) would appear on prima facie grounds to be inherently more regular than PIGLOG under this same specification (i.e. AIDS). The object of our empirical section is to test this conjecture.

The demand equations generated by the MPIGLOG indirect utility function are in fact a special case of Lewbel's (1987) LOG2 demands, case five of his characterization of the seven possible cases of fractional demand systems. However, while Lewbel speculated on the possible improved global regularity of fractional demand systems, his analysis is restricted to the application of Roy's Identity to an indirect utility function satisfying differentiability and homogeneity conditions only, and does not consider the more demanding conditions of monotonicity and curvature.

IV. AGGREGATION

Before turning to the empirics we provide a link between the microeconomic specification of MPIGLOG and a form to which aggregate time series data may be applied. Empirically, one advantage of MPIGLOG over PIGLOG is that MPIGLOG does not satisfy exact aggregation, except in the degenerate PIGLOG case. On the one hand, this would allow a test of exact aggregation under the MPIGLOG maintained hypothesis. On the other hand, where lack of exact aggregation is to be desired on á priori grounds, the MPIGLOG specification provides a rationale for the introduction of specific macro effects when the MPIGLOG share equations are estimated using macro data. Obviously, the macro estimating form will not allow all the underlying microeconomic parameters to be fully identified, except under the assumption of identical preferences. Nevertheless, we make this assumption for expository purposes and ease of interpretation of the empirical results. However, we note that this assumption is not crucial to the approach we adopt. We could, if we wish, interpret our parameter estimates as expectations of the distribution of micro parameters along the same lines as developed by Barnett (1979) for the Rotterdam model.

In the aggregation we find that a set of additional terms naturally arise. These terms are shown to depend upon the distribution of household expenditure. This creates an avenue for business cycle influences on the macro shares. Our empirical results demonstrate their importance.

Let the superscript h denote an individual household. The MPIGLOG specification (14) under identical preferences is:

$$w_i^h = \varepsilon_{1i}(1 - Z^h) + (\varepsilon_{2i}/\eta)Z^h \tag{18}$$

where

$$Z^h = \eta\ell n(c^h/P_1)/[1 + \eta\ell n(c^h/P_1)]. \tag{19}$$

Suppose now that time series data are only available at the aggregate level. Define observable macro average shares as:

$$w_i = \Sigma_h c^h w_i^h / \Sigma_h c^h . \tag{20}$$

A "macro" form of (18) becomes:

$$w_i = \varepsilon_{1i}(1 - Z^*) + (\varepsilon_{2i}/\eta)Z^* \tag{21}$$

where:

$$Z^* = \Sigma_h c^h Z^h / \Sigma_h c^h . \tag{22}$$

Of course, Z^* cannot be constructed from macro data on c ($\equiv$ average c^h) alone. Nevertheless, if we define macro Z as:

$$Z = \eta\ell n(c/P_1)/[1 + \eta\ell n(c/P_1)], \tag{23}$$

we may write (21) in the form:

$$w_i = \varepsilon_{1i}(1 - Z) + (\varepsilon_{2i}/\eta)Z + (\varepsilon_{1i} - \varepsilon_{2i}/\eta)(Z - Z^*) . \tag{24}$$

V. EMPIRICAL SPECIFICATION

To operationalise (24) some suitable parameterization must be employed for the term in $Z - Z^*$. In view of the obvious sensitivity of $Z - Z^*$ to the distribution of real spending power, we parameterize as follows:

$$(\varepsilon_{1i} - \varepsilon_{2i}/\eta)(Z - Z^*) = \eta\mu_i'x \tag{25}$$

where x is a vector of explanators sensitive to the distribution of real spending power and μ_i is a vector of parameters satisfying $\Sigma_i\mu_i = 0$.

Note that, since $\eta = 0$ for PIGLOG, the term (25) does not enter in this case. The specification (25) leads to the estimating form as:

$$w_i = \varepsilon_{1i}(1 - Z) + (\varepsilon_{2i}/\eta)Z + \eta\mu_i' x \qquad (26)$$

where the ε_{1i} and ε_{2i} have an interpretation as the elasticities of P_1 and P_2. Turning therefore to these price aggregators, we note that AIDS corresponds to the specification of P_1 Translog and P_2 Cobb-Douglas, i.e.

$$\ell n\, P_1 \;=\; \kappa + \sum_i \alpha_i\, \ell n\, p_i + 1/2 \sum_i \sum_j \gamma_{ij}\, \ell n\, p_i\, \ell n\, p_j \qquad (27)$$

$$\ell n\, P_2 \;=\; \sum_i \beta_i\, \ell n\, p_i \qquad (28)$$

where $\sum_i \alpha_i = 1$, $\gamma_{ij} = \gamma_{ji}$, $\sum_j \gamma_{ij} = 0$ and $\sum_i \beta_i = 0$, leading to the AIDS share equations (from (3)):

$$w_i \;=\; \varepsilon_{1i} + \beta_i R \qquad (29)$$

where $\varepsilon_{1i} = \alpha_i + \sum_j \gamma_{ij}\, \ell n\, p_j$ and $R = \ell n(c/P_1)$.

While it is well-known that a Translog function cannot be globally regular, and that the imposition of local (sample) regularity is a non-trivial task (see Diewert and Wales (1987)), recall from Section II that it is in fact the specification of P_2 that is at the heart of the regularity problem of AIDS. Thus one interesting possibility for the specification of an empirical version of MPIGLOG would be to choose P_1 and P_2 as in (27) and (28), except that $\Sigma\beta_i = \eta$. Regularity for P_2 would merely require $\beta_i \geq 0$. (Of course this constraint is inconsistent with η=0 except in the trivial case in which AIDS is known to be regular.) This specification leads to the share equations

$$w_i = \frac{\varepsilon_{1i} + \beta_i R}{1 + \eta R} + \eta \mu_i' x , \qquad (30)$$

(with ε_{1i} as in AIDS), which nests (29) when $\eta = 0$.

For estimation with time series data, (30) is written as

$$w_{it} = \frac{\varepsilon_{1it} + \beta_i R_t}{1 + \eta R_t} + \eta\mu_i' x_t + u_{it} \qquad (31)$$

where the error term u_{it} satisfies

$$\Sigma_i u_{it} = 0 ,$$

$$E(u_{it}) = 0 ,$$

$$E(u_{it}u_{js}) = \omega_{ij} \text{ for } s = t ,$$

$$= 0 \text{ for } s \neq t .$$

Clearly the characteristics of u_{it} will be sensitive to the adequacy of the use of the x_t variables as proxies for the weighted average real spending power deviation. For purposes of estimation, one equation is deleted, and as usual the parameter estimates are invariant to the deleted equation.

VI. AN EMPIRICAL COMPARISON OF AIDS WITH MAIDS

Variants of (31) were estimated using Australian annual data for the period 1954 to 1989. The data used is based on that of Chung and Powell (1987) for 1954 to 1986, updated to 1989. Their work indicated a particular problem with the rent component[1], and we have excluded this category from our data. Similarly, it can be argued that durables are unlikely to be well explained by a static allocation model, and this category has also been excluded. This leaves four categories of expenditure: Food (F), Tobacco and Alcohol (T), Clothing (C) and Other (O).

Such a sample of data was considered well-suited to an initial comparison of MAIDS with AIDS, with the disadvantage of a small number of categories more than outweighed by the advantages of a relatively long data period using annual data, and simplicity of presentation. All estimation was carried out using the LSQ option of TSP (Version 4.2).

Following the estimation methodology introduced by Deaton and Muellbauer, it is possible to consider numerous specifications of the AIDS and MAIDS models by, for example, specifying a Stone Price Index for P_1 in its role as a deflator, to simplify estimation and provide starting values for system estimation. It is then possible to test for zero, homogeneous and/or symmetric price effects. Such results are reported in an earlier version of the paper available on request, and are not included here since our main interest is regularity[2]. In general, initial estimates of both models using only expenditure and prices provided a good fit to the data, as judged by R^2, but quite low Durbin-Watson statistics. This is in contrast to the findings of Deaton and Muellbauer (1980) for British data, but consistent with the findings of Chung and Powell (1987) in the fitting of alternative models to the data being used. This finding is also consistent with the expectation that the exclusion of the term due to macro-aggregation will lead to serial correlation if income distribution has been changing systematically over the sample. Since specific data on income distribution is not available over the sample period, we introduce a number of proxy variables. Those variables for which data were freely available, and which we would expect to be significant determinants of changes in income distribution are: the rate of inflation (I), the rate of unemployment (U) and the participation rate (P). While the theory of aggregation outlined in Sections IV and V would suggest that these variables should not appear in AIDS (since η=0), these variables were also included in AIDS in order to simplify the comparison, and to allow concentration solely on the effects of considering a more regular formulation. Introduction of these variables has the desired "purging" effect, and raises the D.W. values to more acceptable (but still low) levels. These variables were henceforth included in estimation, in the form of deviations about their sample means, in order to preserve the interpretation of the ε_{ki}.[3]

Also in common with Deaton and Muellbauer, it was found that the scale parameter, κ, was not well defined by the data. Since κ corresponds to a normalization of P_1, in all estimation κ was set to zero and prices normalized to equal c at the beginning of the sample. This normalization has little effect on AIDS, and ensures that MAIDS is regular over the region of expenditure-price space defined by $\{(c, p): c \geq c_1$, p in the regular region of $P_1\}$ where c_1 is the initial (minimum) sample value of c.

Estimation results for the AIDS specification are reported in Table 1, and results for the MAIDS specification are reported in Table 2. Asymptotic t-values are reported in parentheses, and L represents the system log-likelihood value. The parameters of the specification of P_1, equation (27), were constrained to satisfy symmetry $(\gamma_{ij} = \gamma_{ji})$ and homogeneity $(\sum_j \gamma_{ij} = 0)$. A * by a parameter estimate denotes that this value has been derived by restriction (either explicitly, as in the case of homogeneity, or implicitly, as in the case of adding up restrictions), while a . indicates that the corresponding value was constrained by symmetry. While both models fit the data well, MAIDS performs significantly better on the basis of a comparison of likelihood values, and with an asymptotic t value of 5.01, η is significantly greater than zero. A chi-square test of the restriction implied by the AIDS specification results in a calculated χ^2 statistic of 27.12, to be compared with a critical value of χ_1^2 (.01) = 6.63. The significantly non-zero value of η may also be interpreted as evidence against the exact aggregation properties of the AIDS specification.

Table 1: MACRO AIDS

	1:F	2:T	3:C	4:O
α_i	0.317 (71.67)	0.125 (29.22)	0.155 (34.08)	0.403* (55.24)
β_i	-0.099 (-6.90)	-0.067 (-4.77)	-0.066 (-4.48)	0.232* (9.06)
γ_{iF}	0.155 (9.77)	-0.027 (-2.29)	0.019 (1.82)	-0.147* (-7.22)
γ_{iT}	•	-0.009 (-0.55)	-0.003 (-0.31)	0.039* (1.88)
γ_{iC}	•	•	0.052 (3.64)	-0.068* (-3.55)
γ_{iO}	•	•	•	0.176* (4.33)
μ_{Ii}	0.004 (0.17)	0.044 (2.15)	0.049 (2.38)	-0.096* (-2.85)
μ_{Ui}	0.254 (4.65)	-0.168 (-3.14)	-0.042 (-0.77)	-0.043* (-0.48)
μ_{Pi}	-0.118 (-1.18)	-0.012 (-0.12)	0.010 (0.10)	0.120* (0.72)
R^2	.993	.952	.979	.995
D.W.	1.17	1.04	0.71	0.78

L (system) = 476.731.

Table 2: MACRO MAIDS

	1:F	2:T	3:C	4:O
α_i	0.315 (79.70)	0.122 (28.51)	0.156 (40.08)	0.407* (63.38)
β_i	0.116 (2.51)	0.025 (1.01)	0.019 (0.78)	0.791* (6.89)
γ_{iF}	0.182 (9.79)	-0.048 (-3.12)	-0.006 (-0.41)	-0.128* (-5.51)
γ_{iT}	•	0.0003 (0.01)	-0.008 (-0.59)	0.056* (1.90)
γ_{iC}	•	•	0.057 (3.41)	-0.043* (-1.89)
γ_{iO}	•	•	•	0.116* (2.27)
μ_{Ii}	-0.018 (-0.94)	0.047 (2.30)	0.036 (2.03)	-0.066* (-2.14)
μ_{Ui}	0.066 (1.35)	-0.239 (-5.37)	-0.140 (-3.29)	0.312* (3.74)
μ_{Pi}	-0.169 (-1.84)	-0.067 (-0.67)	0.023 (0.26)	0.213* (1.43)
R^2	.994	.949	.984	.996
D.W.	0.93	1.02	1.04	0.98

L (system) = 490.290 η = 0.951
(5.01)

While the tabulated Durbin-Watson (D.W.) critical values are not appropriate in a situation such as this, as descriptive statistics the low D.W. values are indicative of some serial correlation in the (quite small) residuals. This may be due to the fact that the three proxy variables do not adequately capture the distribution effects over the sample, or alternatively it may be due to some other form of misspecification. A popular alternative specification would allow for dynamics. Nonetheless, a well specified dynamic model typically rests upon an appropriate equilibrium relationship, and results regarding the economic properties of static models provide information for such a specification.[4]

Of possibly greater importance than these statistical results are the economic properties of the models. First, we note that over the entire sample, the MAIDS estimates satisfy one of the sufficient conditions for regularity, that ε_{1i} and ε_{2i} be positive. Second, we check curvature conditions, by evaluating the eigenvalues of the Slutsky matrix over the sample period. For AIDS, negative semi-definiteness is violated within the sample period, with one of the three non-zero eigenvalues becoming consistently positive from 1966 onwards. Interestingly, P_1 itself is not concave over the sample period, so AIDS suffers from two possible sources of irregularity. For MAIDS, on the other hand, the Slutsky matrix is negative semi-definite over the <u>entire</u> sample period.

To investigate this regularity further, we analyze the properties of P_1 as estimated for MAIDS. Note that a sufficient condition for P_1 to be concave would be that the α_i be positive (as they are) and that the matrix $[\gamma_{ij}]$ be negative semi-definite. In fact, however, $[\gamma_{ij}]$ turns out to have two of the three non-zero eigenvalues positive. (This also occurred for AIDS, and perhaps illustrates the difficulties suggested by Diewert and Wales (1987) in using constraints on $[\gamma_{ij}]$ to impose regularity on a Translog). The α_i are sufficiently positive, however, to outweigh this effect, giving a P_1 that is concave for the prices occurring over the sub-sample 1954 to 1974. For the prices over the remainder of the sample, P_1 becomes non-concave. However, concavity is merely a sufficient condition for the quasiconvexity of the indirect utility function, along with the condition that $0 \leq \eta \leq 1$, which is satisfied. As noted above, P_1 is not sufficiently non-concave to violate the quasiconvexity of the indirect utility function over the entire sample. For relative prices "like" those observed over the sample period, the estimated demand system will be regular for all values of expenditure greater in real terms than the minimum observed in the sample. However, it would be possible to construct points in price space where both monotonicity and curvature would be violated for the estimated parameter values. Regularity could only be assured by imposing global regularity on P_1. While this is not possible for a Translog specification of P_1 it would be possible to impose regularity on P_1 using the methodology of Diewert and Wales. The advantage of the MPIGLOG parametric specification is that the regularity conditions of an indirect utility function can be expressed in terms of the regularity of unit cost functions. In contrast, for the PIGLOG specification, concavity of P_1 is not a sufficient condition to ensure quasiconvexity of the indirect utility function, and even for sets of relative prices within the concave region of P_1, there will exist values of expenditure at which the indirect utility function is neither quasiconvex nor monotonic in prices.

Another interesting comparison of the economic characteristics of the two models is to compare the behaviour of their expenditure elasticities over the sample. Since these elasticities are essentially monotonic over the sample, Table 3 reports their values at the beginning (1954), middle (1972) and end (1989) values. Note how for AIDS the necessities (Food, Tobacco and Alcohol, Clothing) exhibit declining income elasticities as real expenditure rises, while for MAIDS the income elasticities rise. Real expenditure increases by more than 60% over the sample period. At the midpoint of the sample, the elasticities are very similar, but the behaviour of elasticities as expenditure differs from the midpoint value is radically different for the two models.

Table 3

Income Elasticities

	F	T	C	O
		AIDS		
1954	0.68	0.49	0.54	1.59
1972	0.57	0.41	0.32	1.44
1989	0.51	0.33	0.19	1.38
		MAIDS		
1954	0.42	0.26	0.17	1.99
1972	0.67	0.48	0.44	1.34
1989	0.74	0.56	0.52	1.24

The two models were also estimated without the presence of the variables proxying the effects of changing income distribution. In general, the results for the parameter estimates are qualitatively similar, but the D.W. statistics are somewhat lower (for example, for MAIDS the average D.W. statistic falls from 0.99 to 0.61, while for AIDS it falls from 0.92 to 0.48) and the system likelihoods fall to 462.7 for

MAIDS and 442.8 for AIDS. While the calculated χ^2 statistic for a test of MAIDS vs. AIDS is 39.8 in this case, its higher value must be tempered by the poorer serial correlation properties of the errors.

VII. CONCLUSION

In this paper we have presented a modification of the PIGLOG class of functional forms, MPIGLOG. In a theoretical section we have provided strong prima facie evidence in favour of improved regularity properties of MPIGLOG relative to PIGLOG. We have also outlined an approach to dealing with the aggregation problem for MPIGLOG. In a similar way, it is possible to modify and regularise PIGL.

In an empirical illustration we have compared the most widely applied member of PIGLOG, AIDS, with a corresponding parameterization of MPIGLOG which we have termed MAIDS. An empirical application has demonstrated that the tendency for improved regularity in MAIDS is borne out in practice for our data set.

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FOOTNOTES

The authors are grateful to Alan Powell for his continued encouragement, and for providing access to his data, and to Priya Parameswaran for providing research assistance. We thank two anonymous referees for their helpful comments.

1. In Australia, the majority of the rent component is imputed rent on owner-occupied dwellings.

2. The results of these tests were that homogeneity was rejected for both models, but conditional on homogeneity, symmetry restrictions were not rejected for either model. See Cooper and McLaren (1988).

3. As pointed out by a referee, the aggregation correction for MAIDS corresponds to a specification error in the case of AIDS. An alternative approach would be to preserve comparability by excluding these macro terms from both models. Results for this case are summarised at the end of this section.

4. A referee has brought to our attention that the types of dynamics often included in similar models have recently been found to be unnecessary if distributional effects following aggregation are modelled carefully.