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ABSTRACT

The MPIGLOG specification of an indirect utility function leads to a parametric representation in terms of expenditure and two unit cost functions. Specification of these unit cost functions in terms of regular, flexible functions leads to the notion of an "effectively globally regular" system of demand equations. Three examples demonstrate the success of such a specification in achieving regularity and flexibility.

1. INTRODUCTION

Microeconomic theory provides a firm foundation for the estimation of systems of demand equations. In its most transparent form, this theory states that such demand equations should be consistent with the maximization of a utility function subject to a budget constraint, generating systems of equations satisfying homogeneity, monotonicity, symmetry and curvature restrictions.

Three approaches to the translation of these restrictions into empirical application may be identified. In the primal approach, the demand equations are derived literally by specifying a direct utility function and solving the constrained maximization problem. While this approach leads to demand systems which satisfy the above regularity conditions by construction, the need to derive analytical solutions to the first order conditions restricts its application to utility functions of the Cobb-Douglas form, or its variants such as C.E.S. and Klein-Rubin. A second approach is the Rotterdam methodology, which attempts to impose the regularity restrictions on log-differential approximations to the demand equations.

This paper is in the spirit of the third approach, which exploits the theory of duality among direct utility functions, indirect utility functions, and cost functions, and the regularity conditions on these functions which make them equivalent representations of the underlying preferences. Duality theory allows systems of demand equations to be derived from these dual representations via simple differentiation, according to Roy's Identity or Shephard's Lemma. This approach was popularized by Diewert (1974, 1982), and led to the use of flexible functional forms such as the Generalized Leontief of Diewert (1971) and the Translog of Christensen, Jorgenson and Lau (1978). While such flexible functional forms lead to demand equations which can attain arbitrary elasticities at a point in price-expenditure space, such systems generally satisfy only homogeneity with respect to prices and expenditure globally, and often violate monotonicity and, in particular, curvature restrictions, either within the sample, or at points close to the sample. Lau (1986) discusses the characterisation of regularity of such systems, and finds that the domain of regularity is rather limited.

Much recent work has been devoted to deriving demand systems that satisfy regularity over a wider domain. Many of these methods are based on series expansions - see Barnett (1983, 1985), Barnett, Lee and Wolfe (1985, 1987), Barnett and Yue (1988a, b), Gallant (1981, 1984) and Gallant and Golub (1984).

This paper builds on a parametric representation of the indirect utility function in terms of expenditure and price indices that was introduced in Cooper and McLaren (1988, 1991) in order to generate more regular demand systems in the spirit of the Almost Ideal Demand System of Deaton and Muelbauer (1980). Regularity is achieved by limiting the use of flexible functional forms to unit cost functions which are components of the indirect utility function rather than the indirect utility function itself. This allows the use of recent developments in imposing regularity (specifically, global curvature) on cost functions in Diewert and Wales (1987).

The parametric representation of the indirect utility function in terms of unit cost functions is introduced in Section 2, where conditions for regularity are detailed. Section 3 details possible specifications for the unit cost functions, and Section 4 details an empirical application using Australian data. The final result is an empirical demand system satisfying what is denoted "effectively global curvative conditions"; that is the domain of regularity includes the entire sample and all other possible values of real expenditure greater than the minimum value observed in the sample.

2. THE MPIGLOG SPECIFICATION OF PREFERENCES

The MPIGLOG class of preferences, introduced by Cooper and McLaren (1988, 1991), is specified in terms of an indirect utility function as

$$(2.1) \qquad U(c, p) = [\ln(c/P_1)](c/P_2)^{\eta}$$

where p is an n-vector of commodity prices, c is total expenditure, η is a parameter satisfying $0 \le \eta \le 1$, and P_1 and P_2 are functions of p yet to be specified, having the interpretation of price indices or unit cost functions. (Note that in Cooper and McLaren (1988, 1991) the functional form of U(c, p) was specified slightly differently in order to facilitate comparison with PIGLOG. Since this is not our purpose here, the specification (2.1) is the more natural). The term MPIGLOG is used to denote that (2.1) (or more precisely its dual cost function) can be interpreted as a modication of PIGLOG, itself a special case of the PIGL (price independent generalized linear) cost function of Muellbauer (1975), a particular parameterization of which leads to the AIDS (Almost Ideal Demand System) of Deaton and Muellbauer (1980).

By duality theory, an indirect utility function U(c, p) is a valid representation of preferences if it satisfies the following regularity conditions:

- U1 U is homogeneous of degree zero (HDO) in (c, p),
- U2 U is non-increasing in p,
- U3 U is non-decreasing in c,
- U4 U is quasi-convex in p.

If a function U satisfies U1 - U4 over the entire positive orthant $\Omega_+^{n+1} = \{c, p : c > 0, p > 0\}$ U is said to be globally regular. If U satisfies U1 - U4 over a region $R \subset \Omega_+^{n+1}$, then U is said to be locally regular. Flexible functional forms such as the Translog and Generalized Leontief typically have regular regions which are quite restricted, and in particular those regular regions are often bounded from above in the direction of real income.

For the specification (2.1), P_1 and P_2 can be interpreted as price indices or unit cost functions. The properties that a function P(p) should satisfy to qualify as a price index (unit cost function) are:

P1 $P(p) > 0$ for $p \in \Omega_+^n$,

P2 P is homogeneous of degree 1 (HD1),

P3 P is non-decreasing,

P4 P is concave.

It is shown in Appendix 1 that, provided P_1 and P_2 satisfy P1 - P4, then U(c, p) defined by (2.1) will satisfy U1 - U4 over the region $\mathcal{P} = \{c,p : c > P_1(p)\}$. While $\mathcal{P}$ is a proper subset of Ω_+^{n+1}, so that global regularity cannot be assured, in practice this distinction is unimportant, since it is always possible to choose the base of the prices to ensure that regularity is satisfied in the base period. In addition, regularity is then assured for higher levels of real income, as measured by c/P_1, and since most time series databases are characterized by real income rising through time, this property of (2.1) is particularly useful for empirical work. We will refer to this type of regularity property as "effectively globally regular".

Application of Roy's Identity to (2.1) gives the MPIGLOG share equations as

$$(2.2) \qquad w_i = \frac{\varepsilon_{1i} + \eta\varepsilon_{2i}R}{1 + \eta R}$$

where $\qquad \varepsilon_{ki} = \dfrac{\partial \ln P_k}{\partial \ln p_i}, \qquad k = 1, 2,$

and $\qquad R = \ln(c/P_1)$. If we define

$$(2.3) \qquad Z = \eta R/(1 + \eta R)$$

then (2.2) can be written as

$$(2.4) \qquad w_i = \varepsilon_{1i}(1-Z) + \varepsilon_{2i}Z$$

giving the interpretation of the shares of the MPILOG specification as a weighted average of the shares of the "rich" (ε_{2i}) and the "poor" (ε_{1i}), where the rich are defined by $Z = 1$ i.e. $c = \infty$, and the poor are characterized by $Z = 0$ i.e. $c = P_1$.

For this system, expenditure elasticities are given by

$$(2.5) \qquad E_i = \partial \ln Q_i/\partial \ln c$$
$$= 1 + \eta(\varepsilon_{2i}/w_i - 1)(1 - Z)$$

where $Q_i(c, p)$ are the Marshallian demand equations, and a typical term of the Slutsky matrix is

$$(2.6) \qquad S_{ij} = (c/p_ip_j)[\xi_{1ij}(1-Z) + \xi_{2ij}Z]$$

where $\qquad \xi_{1ij} = \varepsilon_{1ij} + \varepsilon_{1i}(w_j - \delta_{ij})$

$$\xi_{2ij} = \varepsilon_{2ij} + \varepsilon_{2i}(w_j - \delta_{ij})$$
$$+ \eta(\varepsilon_{2i} - w_i)(\varepsilon_{2j} - w_j)$$

3. SPECIFICATION OF UNIT COST FUNCTIONS

The MPIGLOG specification can be made operational by specifying functional forms for the price indices P_1 and P_2 . Effective regularity will be assured if P_1 and P_2 can be chosen to satisfy properties P1 - P4. Of course, it must be realized that the specification of regular unit cost functions raises many of the same difficulties as the specification of a regular indirect utility function. One obvious exception, however, is that the testing for, or imposition of, concavity is usually more straightforward than the testing or imposition of quasi-convexity. A second exception is that scale effects have been explicitly parameterized.

In general, the choice of unit cost functions will be subject to the usual tradeoff between regularity and flexibility. At this point, it may be useful to recall a result of Lau: "The implication of this theorem is that there can be no linear-in-parameters and parsimonious functional form for a normalized unit cost function which can fit arbitrary but theoretically consistent values of a normalized unit cost function and its first and second derivatives at any preassigned value of the normalized price and be itself theoretically consistent for all non-negative normalized prices. One has to be prepared to give up one or more of the desirable properties of an algebraic functional form." (Lau (1986), p.1557).

The main model to be considered below will be based on a Generalized McFadden specification of a unit cost function. (See Diewert and Wales (1987).) For comparison, two other specifications will be considered. Note that since P_2 describes asymptotic behaviour, while P_1 describes behaviour at a particular point in the sample, it seems reasonable a priori to consider generalizations of P_1.

3.1 Model 1: Cobb-Douglas P_1 amd P_2

Let (3.1) $\ln P_1 = \Sigma\alpha_i \ln p_i \quad , \qquad \iota'\alpha = 1$

(3.2) $\ln P_2 = \Sigma\beta_i \ln p_i \quad , \qquad \iota'\beta = 1 \; .$

The non-negativity restrictions $\alpha_i > 0$, $\beta_i > 0$ are sufficient to ensure P_1 and P_2 are globally regular, i.e. satisfy P1 - P4 over Ω^n_+, and hence the share equations corresponding to (2.2),

$$(3.3) \qquad w_i = \frac{\alpha_i + \eta\beta_i R}{1 + \eta R}$$

with $R = \ln c - \Sigma\alpha_j \ln p_j$,

constitute an effectively globally regular demand system. This 2n - 1 parameter demand system rivals the linear expenditure system, which also has 2n - 1 parameters and is effectively globally regular, as a parsimonious well-behaved demand system.

3.2 Model 2: Translog P_1 , Cobb-Douglas P_2

Let (3.1) be generalized to the Translog specification

$$(3.4) \qquad \ln P_1 = \sum_i \alpha_i \ln p_i + 1/2 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j \; .$$

Identification of the γ_{ij} requires $\gamma_{ij} = \gamma_{ji}$, $i \neq j$, and homogeneity requires $\sum_j \gamma_{ij} = 0 \;\; \forall\, i$.

The share equations corresponding to (2.2) are

$$(3.5) \qquad w_i = \frac{\alpha_i + \sum_j \gamma_{ij} \ln p_j + \eta\beta_i R}{1 + \eta R}$$

with $R = \ln(c/P_1)$.

P_1 is not globally non-decreasing or concave unless $\gamma_{ij} = 0, \;\; \forall_{i,j}$, in which case P_1 loses its flexibility (and Model 2 collapses to Model 1). Global concavity could be imposed by restricting the matrix of γ_{ij}

to be negative semi-definite, but Diewert and Wales (1987) show that this restriction makes P_1 "too concave", and hence defeats flexibility. Nevertheless, regularity of P_1 is merely a sufficient condition for effective global regularity of (2.1), and system (3.5) is a useful demand system, which may well possess a relatively large region of regularity. In Cooper and McLaren (1988) it was shown that a system similar to (3.5) attained regularity over a much wider region than the popular AIDS system.

3.3 Model 3: Generalized McFadden P_1, Cobb-Douglas P_2

To overcome the problems of imposing curvature conditions on popular flexible functional forms such as the Translog and Generalized Leontief, Diewert and Wales (1987) introduced a number of flexible functional forms which are more amenable to the imposition and testing of curvature conditions. In this example we will use the Generalized McFadden, defined for a time invariant unit cost function by

$$(3.6) \qquad C^1(p) = g^1(p) + \Sigma b_i p_i$$

where

$$(3.7) \qquad g^1(p) = (1/2)p_1^{-1} \sum_{i=2}^{n} \sum_{j=2}^{n} c_{ij} p_i p_j$$

and $c_{ij} = c_{ji}$. If $\tilde{C}$ is defined to be the $(n-1) \times (n-1)$ matrix of c_{ij}'s and $\tilde{p}^T = [p_2, p_3, \ldots, p_n]$, the Hessian matrix of C^1 is

$$(3.8) \qquad \nabla^2_{pp} C^1(p) = \nabla^2_{pp} g^1(p) = \begin{bmatrix} p_1^{-3}\tilde{p}^T\tilde{C}\tilde{p}, & -p_1^{-2}\tilde{p}^T\tilde{C} \\ -p_1^{-2}\tilde{C}\tilde{p}, & p_1^{-1}\tilde{C} \end{bmatrix} .$$

This Hessian matrix will be negative semi-definite, and hence C^1 will be concave, for all $p \in \Omega^n_+$, if and only if $\tilde{C}$ is negative semi-definite. Thus the global concavity of C^1 is easily tested. Further, if in unconstrained estimation C^1 is not concave, it is relatively easy to impose concavity on C^1 using a technique introduced by Wiley, Schmidt and Bramble (1973): reparameterize the matrix $\tilde{C}$ as

$$(3.9) \qquad \tilde{C} = -AA^T$$

where $A = [a_{ij}]$, $a_{ij} = 0$ for $i < j$, $i, j=1, \ldots, n-1$.

The parameterization adopted for Model 3 results if the functional form for $C^1(p)$, is used for P_1, and P_2 is maintained to be Cobb-Douglas as in Model 2. This will generate an indirect utility function (2.1) which is potentially effectively globally quasi-convex i.e. the Slutsky matrix of the implied demand system can be constrained to be effectively globally negative semi-definite.

The four economic properties of a cost function P1 - P4, may be characterized as non-negativity, homogeneity, monotonicity and curvature. Parametric forms for a cost function, such as Cobb-Douglas or its variants (CES etc.) may be constrained to satisfy these four properties globally. Flexible functional forms such as Translog and generalized Leontief can only be constrained to satisfy homogeneity (generalized Leontief) or homogeneity and non-negativity (Translog) globally without imposing restrictions that destroy flexibility. Empirically the most frequent of these violations to occur is curvature, followed by monotonicity and then non-negativity. For this reason, developments in imposing restrictions on flexible functional forms have typically concentrated on curvature, and the generalized McFadden is an example. However, if is worth considering these other two properties.

Considering monotonicity first, note that for C^1 defined by (3.6), we have

$$\frac{\partial C^1}{\partial p_1} = b_1 - p_1^{-2} g^1(p)$$

(3.10)

$$\frac{\partial C^1}{\partial p_i} = b_i + p_1^{-1} \sum_{j=2}^{n} c_{ij} p_j \qquad i = 2, \ldots, n.$$

If C^1 is concave, then by construction $g^1(p)$ is ≤ 0, and hence $b_1 \geq 0$ is sufficient to guarantee that C^1 is increasing in p_1. This is an aspect of the asymmetry of the generalized McFadden that is not generally recognized, and in particular may be useful if one good is violating monotonicity over the sample period. Conversely, for $i = 2, \ldots, n$, since concavity requires $c_{ii} < 0$, it can be seen that concavity guarantees that it is possible to find points in price space where $\partial C^1/\partial p_i$ becomes negative, even if $b_i > 0$.

Non-negativity generates similar problems. Concavity of C^1 requires that $g^1 < 0$, and since g^1 is quadratic in prices other than p_1, whereas the other term in C^1 is linear, it will always be possible to find points in price space at which C^1 becomes negative (except for the degenerate case of $g^1 \equiv 0$).

Thus the potential to impose curvature conditions in the generalized McFadden comes at a cost, but of course the relative importance of these properties and their potential violation is an empirical question. This is also more serious for the use of the Generalized McFadden as a stand-alone function. With its use as a specification of P_1 in the MPIGLOG specification, at all but one point in the sample space its properties will be combined with those of P_2, which if itself globally regular may attenuate the problems with P_1

and lead to an indirect utility function with an acceptable region of regularity.

Specifying P_1 as the C^1 of (3.6), and P_2 as Cobb-Douglas as in (3.2), the corresponding share equations are

$$w_1 = \frac{(b_1 p_1 - g^1(p))/P_1 + \eta\beta_1 R}{1 + \eta R}$$

(3.11)

$$w_i = \frac{(b_i p_i + (p_i/p_1)\sum_{j=2}^{n} c_{ij} p_j)/P_1 + \eta\beta_i R}{1 + \eta R} .$$

The specification of P_1 requires normalization, and the normalization chosen is

$$\sum_{i=1}^{n} b_i + \frac{1}{2}\sum_{i=2}^{n}\sum_{j=2}^{n} c_{ij} = 1.$$

This ensures that, if all p_i are normalized to a constant K in some base period, then $P_1 = K$ in that period.

4. EMPIRICAL SPECIFICATION AND RESULTS

The models of the previous section relate to individuals or households. In Cooper and McLaren (1991) the issue of aggregation across individuals in the context of MPIGLOG preferences is addressed in detail, and it is shown that an appropriate model is of the form

$$(4.1) \qquad w_i = \frac{\varepsilon_{1i} + \eta\varepsilon_{2i}R}{1 + \eta R} + \mu_i' x$$

where x is a vector of explanatory variables acting as a proxy for the change in distribution of real expenditure over the sample period, and the μ_i are parameters satisfying $\Sigma\mu_i = 0$. For purposes of estimation, an error term u_i is appended additively to (4.1). For time series data, the specific error terms u_{it} are assumed to be distributed multivariate normal, with

$$E(u_{it}) = 0$$

$$E(u_{it}u_{js}) = \omega_{ij} \quad \text{for} \quad s = t,$$

$$= 0 \quad \text{for} \quad s \neq t .$$

In addition, the budget constraint implies that $\Sigma_i u_{it} = 0$, and hence $\Omega = [\omega_{ij}]$ is singular. For purposes of estimation, one equation is deleted, and as usual the parameter estimates are invariant to the deleted equation. (See McLaren (1990)).

The three models of Section 3 were estimated using annual Australian data covering the period 1954/55 to 1988/89. The data used is based on Chung and Powell (1987) who constructed a data base ending in 1985/86, and this is updated to 1988/89 using a similar methodology. For this period, the available categories are: Food (F), Tobacco and Alcohol (T), Clothing (C), Rent (R), Durables (D) and Other (O). The rent component poses a problem with Australian data, because of its high imputed component, and would be unlikely to be explained by a static allocation model. Similarly, it is unlikely that durables would be well suited to such a model, and hence these two categories are excluded in the empirical work. The variables used to proxy the effect of changing distribution of real expenditure over the sample period were: the rate of inflation (I), the rate of unemployment (U), and the participation rate (P). Estimation was carried out using the LSQ option of TSP, which is well-suited to the estimation of systems with complex cross-equation constraints.

Parameter estimates of the three models are presented in Tables 1 to 3. Model 1 corresponds to the case of Cobb-Douglas representations of P_1 and P_2, and results are given in Table 1. Note that even for this simple model the explanatory power is quite high (recalling that the dependent variable is a share). Although the Durbin-Watson statistic is not strictly appropriate for models such as these, it is reported here as a descriptive statistic, and indicates some positive serial correlation in the (small) residuals. Note that the α_i and β_i are strictly

Table 1: Model 1 (Cobb-Douglas P_1 and P_2)

	1:F	2:T	3:C	4:O
α_i	0.309	0.122	0.153	0.416
	(52.01)	(26.61)	(34.40)	(55.28)
β_i	0.120	0.088	0.038	0.760
	(3.87)	(7.53)	(1.97)	(14.10)
μ_{Ii}	-0.046	0.053	0.051	-0.058
	(-1.78)	(2.67)	(2.75)	(-1.75)
μ_{Ui}	-0.046	-0.270	-0.175	0.490
	(-0.66)	(-6.26)	(-3.70)	(4.70)
μ_{Pi}	-0.207	-0.113	-0.042	0.362
	(-1.53)	(-1.09)	(0.43)	(2.06)
R^2	0.985	0.933	0.978	0.994
D.W.	0.749	0.897	1.36	1.063

L (system) = 456.043 η = 1.649
(4.11)

positive without constraint, and since for this model $\varepsilon_{1i} = \alpha_i$, $\varepsilon_{2i} = \beta_i$, this demand system is hence effectively globally monotonic. These conditions are also necessary for effective global quasi-convexity, but the η value of 1.649 does violate one of the sufficient conditions. Nevertheless, these parameter estimates produce a Slutsky matrix which is negative semi-definite over the entire sample period and beyond.

Model 2 nests Model 1 by generalizing P_1 from a Cobb-Douglas to a Translog unit cost function, and parameter estimates are presented in Table 2. While the extra price terms available in Model 2 contribute significantly to explanatory power as evidenced by the higher values of R^2, and by the fact that a likelihood ratio test clearly rejects the restrictions implicit in moving from Model 2 to Model 1, it is well known that the second order terms in a Translog are a source of non-regularity. A sufficient condition for global concavity of P_1 is the non-negativity of the ε_{1i} plus the negative semi-definiteness of the $[\gamma_{ij}]$ matrix. The non-negativity of the ε_{1i} relates to the monotonicity of the P_1 function, which cannot hold globally in the case of a non-trivial Translog function. In addition, the $[\gamma_{ij}]$ matrix has two of its three non-zero eigenvalues positive. But these are sufficient conditions, and over the sample period, P_1 is concave from 1954/51 until 1974/75, but becomes non-concave from 1975/76 to the end of the sample. Again, concavity of P_1 is merely a sufficient condition for quasi-convexity of the indirect utility function along with the condition that $0 \leq \eta \leq 1$, (which is satisfied), and indeed the Slutsky matrix is easily negative semi-definite over the entire sample period. This is an example of a case where the MPIGLOG specification corrects for the poor regularity property with respect to curvature of the Translog functional form. For relative prices "like" those observed over the sample period, the estimated demand system will be regular for all values of expenditure greater in real terms than the minimum observed in the sample. However, it would be possible to construct

Table 2: Model 2 (Translog P_1, Cobb-Douglas P_2)

	1:F	2:T	3:C	4:0
α_i	0.315	0.122	0.156	0.407
	(79.70)	(28.51)	(40.08)	(63.38)
β_i	0.127	0.027	0.020	0.832
	(4.49)	(1.15)	(0.080)	(14.28)
γ_{iF}	0.182	-0.480	-0.006	-0.128
	(9.79)	(-3.11)	(-0.41)	(-5.51)
γ_{iT}	-0.480	0.0003	-0.008	0.056
	(-3.11)	(0.014)	(-0.59)	(1.90)
γ_{iC}	-0.006	-0.008	0.057	-0.043
	(-0.41)	(-0.59)	(3.41)	(-1.89)
γ_{iO}	-0.128	0.056	-0.043	0.116
	(-5.51)	(1.90)	(-1.89)	(2.27)
μ_{Ii}	-0.018	0.047	0.036	-0.066
	(-0.94)	(2.30)	(2.03)	(-2.14)
μ_{Ui}	0.066	-0.239	-0.140	0.312
	(1.35)	(-5.37)	(-3.28)	(3.74)
μ_{Pi}	-0.169	-0.067	0.023	0.213
	(-1.84)	(-0.67)	(0.256)	(1.43)
R^2	0.994	0.949	0.984	0.996
D.W.	0.932	1.02	1.04	0.977

L (system) = 490.290 $\quad\quad \eta = 0.951$
(5.01)

$2\left(L^{(2)} - L^{(1)}\right) = 34.25 \quad\quad \chi^2_6\,(.05) = 12.59 \quad \chi^2_6\,(.01) = 16.81$

points in price space where both monotonicity and curvature are violated for the estimated parameter values.

The purpose of Model 3 is to attempt to resolve these questions of effective global curvature conditions. The parameter estimates for Model 3 are presented in Table 3. On statistical grounds, there is

Table 3: Model 3 (Generalized McFadden P_1, Cobb-Douglas P_2)

	1:F	2:T	3:C	4:0
a_i	0.312	0.223	0.150	0.360
	(10.55)	(6.73)	(4.97)	(11.97)
b_i	0.112	0.027	0.032	0.829
	(3.53)	(1.05)	(1.46)	(12.96)
c_{iF}	-0.041	-0.005	0.047	-
	(-1.54)	(-0.27)	(2.36)	
c_{iT}	-0.005	-0.1133	0.018	-
	(-0.27)	(-4.63)	(1.11)	
c_{iC}	0.047	0.018	-0.059	-
	(2.36)	(1.11)	(-2.35)	
μ_{Ii}	-0.016	0.050	0.033	-0.067
	(-0.90)	(2.42)	(1.87)	(-2.18)
μ_{Ui}	0.067	-0.232	-0.147	0.312
	(1.32)	(-4.99)	(-3.27)	(3.56)
μ_{Pi}	-0.170	-0.056	0.026	0.200
	(-1.86)	(-0.540)	(0.28)	(1.30)
R^2	0.994	0.945	0.983	0.995
D.W.	0.884	0.973	0.884	0.881

L (system) = 487.78 η = 0.895
(4.29)

probably little to choose between Models 2 and 3 although, since each model has 22 parameters (13 "model" parameters and 9 "macro" parameters), Model 2 fits better on the basis of R^2 and value of the likelihood. The advantage of Model 3 is that the generalized McFadden specification of P_1 allows simple testing of the global concavity of P_1, and, if violated, its imposition. For the parameter estimates reported in Table 3, the estimated $[c_{ij}]$ matrix is negative definite when estimated without concavity constraints imposed. Hence the estimated P_1 function is globally concave, and together with the fact that the estimated η of 0.895 is within the (0, 1) range, the parameter estimates correspond to an indirect utility function that is effectively globally quasi-convex, i.e. the implicit Slutsky matrix will be negative semi-definite over all expenditure and price combinations which satisfy $c/P_1(p) \geq 1$. Again, as with the other functional forms where monotonicity cannot be assured, monotonicity will eventually fail at some extreme relative price values. But over the sample period monotonicity is easily satisfied.

5. CONCLUSION

The MPIGLOG representation of the indirect utility function is a parametric representation of utility in terms of expenditure and two unit cost functions. The choice of regular unit cost functions generates demand equations regular over a cone in expenditure - price space whose size increases as real expenditure increases. The use of a generalized McFadden functional form for one of these unit cost functions allows the creation of a regular, flexible system of demand equations. In general, such functional forms are more appropriate for the cost functions occurring in an indirect utility function than for the indirect utility function itself. An empirical section compares three such demand systems that arise in this way, and the estimated demand systems confirm the empirical results.

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Appendix 1: Regularity of U(c, p)

In the paper it is claimed that if U(c, p) has the form

$$U(c, p) = [\ln(c/P_1)](c/P_2)^{\eta}$$

then if P_1, P_2 satisfy P1 - P4 and $0 \leq \eta \leq 1$, then U(c, p) satisfies U1 - U4 over $P = \{ c, p; c > 0, p > 0, (c/P_1) > 1\}$.

Consider these properties in turn,

U1: Given P_1 and P_2 satisfy P2, it is clear that U1 is satisfied over the entire positive orthant Ω_+^{n+1}.

U2: $$U_{p_i} = -\frac{1}{P_1}\frac{\partial P_1}{\partial p_i}\left(\frac{c}{P_2}\right)^{\eta} - \left[\ln\left(\frac{c}{P_1}\right)\right] \eta \left(\frac{c}{P_2}\right)^{\eta} \frac{1}{P_2}\frac{\partial P_2}{\partial p_i}$$

Hence, given P_1 and P_2 satisfy P3 and P1, if $\eta > 0$ then U2 will be satisfied at least wherever $(c/P_1) > 1$.

U3: By U1, $cU_c = \Sigma p_i U_{p_i}$, and hence U3 will be satisfied over the same domain as U2.

U4: Define normalized prices $s_i = p_i/c$. Then by U1,

$$U(c, p) = \frac{-\ln[P_1(p/c)]}{[P_2(p/c)]^{\eta}} = \frac{-\ln P_1(s)}{[P_2(s)]^{\eta}} = V(s),$$ say, and V will be

quasi-convex in s. From Greenberg and Pierskalla (1971, p.155), $V(s) = g(s)/f(s)$, say, is quasi-convex provided g(s) is convex, $g(s) \geq 0$, $f(s) \geq 0$, f(s) is concave. Since a concave, non-decreasing function of a concave function is concave, $\ln[P_1(s)]$ is concave and $[P_2(s)]^{\eta}$ is concave provided $0 \leq \eta \leq 1$. Hence g(s) $= -\ln[P_1(s)]$ is convex. Further, $g(s) \geq 0$ when $\ln[P_1(s)] \leq 0$ i.e. $P_1\left(\frac{p}{c}\right) \leq 1$, i.e. $P_1(p)/c \leq 1$ i.e. $P_1(p) \leq c$.