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DYNAMIC OPTIMAL CONTROL OF PLANT PRODUCTION: AN APPLICATION TO DETERMINE OPTIMAL GREENHOUSE TEMPERATURE STRATEGIES[1]

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UITTREKSEL: DINAMIESE OPTIMALE BEHEER VAN PLANTPRODUKSIE: 'n TOEPASSING OM OPTIMALE GLAHUIS-TEMPERATURE TE BEPAAL

In die artikel word die beginsels betrokke by die optimale beheer van dinamiese plantgroeimodelle ontleed. Dit begin met 'n uiteensetting van die struktuur van besluitnemingsprobleme en die voortvloeiende vereistes van modelle wat ontwerp is om hierdie besluite te ondersteun. Drie basiese raamwerke word beskou, insluitende Pontrjagin se beginsel, dinamiese programmering en die gebruik van numeriese optimeringsmetodes. Die artikel verduidelik die onderliggende beginsels van hierdie benaderings en verwys na hul sterk- en swakpunte. Daar word afgesluit met 'n toepassing van numeriese optimeringsbeginsels op glashuistemperatuur strategieë. Die resultate toon die werking van die algoritme en verwys na die toepassing daarvan vir navorsings- en praktiese besluitneming.

ABSTRACT

This article delineates the principles for obtaining optimal control policies based on dynamic plant growth models. It starts with a brief outline of the structure of decision problems and the resulting requirements for models designed to support these decisions. The concepts of open-loop and closed-loop control are discussed next. Emphasis is placed on the methods for solving the control problem. Three basic frameworks are considered including Pontrjagin's Maximum Principle, Dynamic Programming and the use of Numerical Optimization methods. The article outlines the principle structure of these approaches and addresses their strengths and weaknesses. It concludes with an application of Numerical Optimization methods to greenhouse temperature strategies. The results demonstrate how the algorithms work and lead to conclusions with respect to their applicability for research purposes and practical decision-making.

1. Introduction: Decision problems and model requirements

As usually conceived, decision problems involve controllable and uncontrollable or exogenous variables. They also involve a system structure that relates the exogenous and controllable variables to outputs in terms of performance measures that enter an objective function. Figure 1 illustrates this setting.

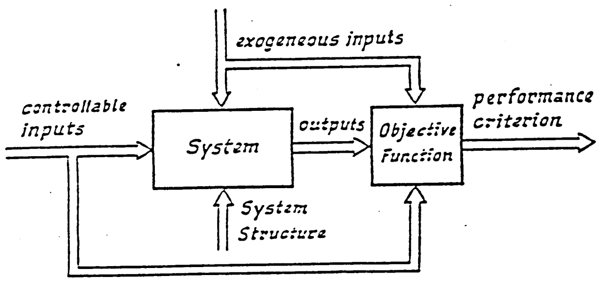

Figure 1: Components of decision problems

Decision-making in general includes both determining the system structure and setting the controllable inputs. In terms of modeling, however, it is often useful to distinguish between two broad categories of decision problems according to which is emphasized: the design of a system structure or the setting of controllable inputs. The latter can be referred to as regulatory or tactical decision problems. Such problems typically concentrate on decisions that must be frequently made under a given structural design. Examples are temperature control, fertilizer and pest management or the marketing of goods. In contrast, decision problems that focus on establishing a new system or changing the structure of an existing one are called design problems. They are normally broader in scope and thus affect larger parts of the overall system. Decisions of that sort have to be made infrequently and are long-run in nature. In a management context such problems are often referred to as strategic decision problems (Harsh *et al*, 1988). This article is mainly concerned with regulatory or tactical decisions.

Horticultural operations can be viewed as dynamic systems. Tactical decision-making then involves the adjustment of the controllable variables at any point in time where such adjustments are possible. Since the effects of these adjustments normally extend beyond the time at which the subsequent adjustment can be made, the decision problems are dynamic in nature. In essence, this means that today's decisions influence the future set of possible actions. Solving problems of this nature requires the use of dynamic models in order to establish dynamic control schemes.

2. Optimal control of dynamic systems

Decision problems of the above nature are dynamic control problems. This means that in order to solve the problem, a control law or optimal policy must be found which maximize (or minimize, respectively) an objective function of the basic form[2]

$$Z = \int_{u}^{T} g\{x(t), u_c(t), u_e(t), t\}\, dt + h\{x(T), T\} \qquad (1)$$

where t indicates the time variable and T marks the end of the planning horizon; $x(t)$ is the vector of state variables of the system to be controlled and $u_c(t)$ and $u_e(t)$ represent the vectors of controllable and exogenous inputs respectively; g is a function that relates the values of $x(t)$, $x_c(t)$ and $u_e(t)$ to the performance measure. Thus the first term (Z) in the above equation could for example represent the present value of the accumulated profit over the time horizon T. The term $h\{x(T), T\}$ represents the terminal value of the state vector and is included to provide continuity with future periods beyond T, if the duration of the system is assumed to be greater than T. If the model is a continuous time representation of the system, the state variables are related to the controllable and exogenous inputs via a set of first order differential equations that are usually called equations of motion (*cf.* Jacob, 1982).

$$x(t) = a\{x(t-1), u_c(t), u_e(t), t\} \qquad (2)$$

According to the particular form in which the optimal policy is formulated, two control loop structures can be distinguished, namely open loop control and closed loop control. The concepts of open loop and closed loop control are illustrated in Figure 2. The controller is a model representation of the system to be controlled, which is used to derive the optimal policy. In the case of open loop control the solution of the optimization problem yields the time paths of all controllable variables. That is, the optimal control is determined as a function of time for a specified initial state vector.

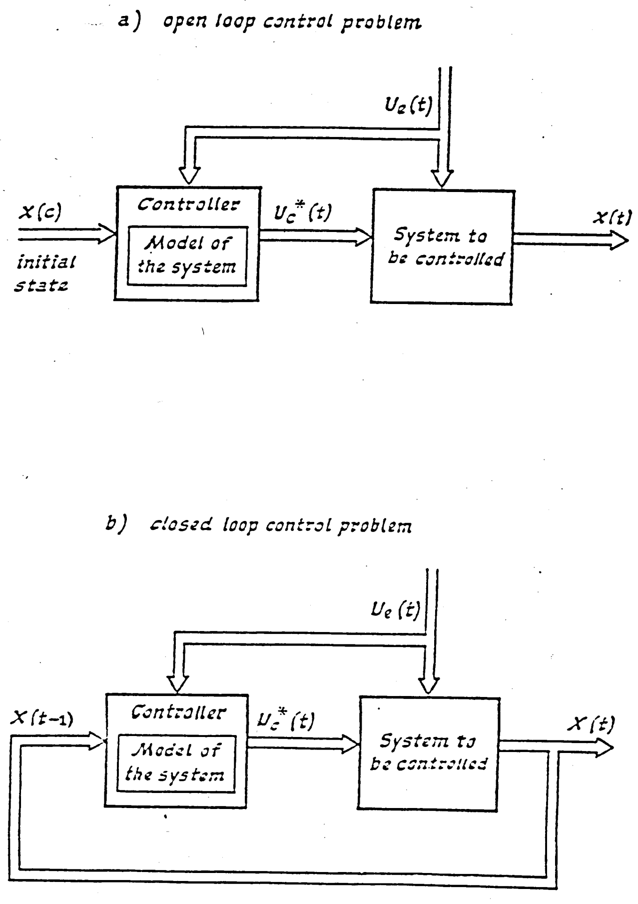

Figure 2: Open loop and closed loop control

It is obvious that determining the time paths of all controllable variables over the rest of the planning horizon in advance is a valid control formulation only under particular circumstances, namely,

- o if the problem is deterministic, i.e. the system itself as well as the future time paths of all exogenous variables are perfectly known, or
- o adjustments of the controllable variables at later points in time are precluded by the nature of the system, or
- o the exogenous inputs and the state variables of the system are not observable.

If these conditions are not given, then it is possible to revise the initial policy after new information on the true state of the system and the values of the exogenous inputs becomes available. Such adjustments are explicitly considered in a closed loop of feedback control formulations (part b of Figure 3). The optimal policy in this case is determined as a function of the observed values of the state and the exogenous input sector. The closed loop policy thus describes a rule for computing the values of the controllable inputs after certain observations have been made rather than determining them in advance.

While for most management problems the conditions are such that the preferable control formulation is that of the closed loop, deriving optimal policies of this form is a rather complex task. This is particularly true if a complex system like a whole firm must be considered as is the case in medium term or annual planning of the production program. Therefore in these cases optimal controls are often formulated in an open loop form assuming that the stochastic variables of the system take on their expected values. In the short term management of single crops the expected time paths of the relevant state variables associated with these controls then serve as reference or target inputs, so the control problem now becomes that of finding a closed loop policy which minimizes the costs associated with deviations from this target set. Figure 3 illustrates this setting.

The controller I yields an open loop policy for and the expected states associated with it for the overall system ($x^*(t)$). A subset of $x^*(t)$ then serves as target set for the short term control of a subsystem. This control is formulated in a closed loop form. From time to time the target set will be updated by using the current as the new initial state in the open loop problem.

The general configuration of Figure 3 largely reflects common management practices. In this respect the open loop control schemes refers to planning activities that are usually carried out this way, yielding an initial policy and target values of state variables and performance measures. In day-to-day decision-making the initial policy is then revised in order to make the system approach the target states as closely as possible. The latter refers to the closed loop subproblem. However, instead of using formal models to derive the optimal policy, this is commonly done intuitively relying on rules of thumb and past management experience. With the advent of on-farm microcomputers there are numerous possibilities to improve this type of decision-making, and thus increase the performance of the overall system.

3. Approaches to solving the control problem

Optimal control problems can be solved according three basic approaches (Adby and Dempster, 1974):

- o Analytical solution using Pontrjagin's Maximum Principle.

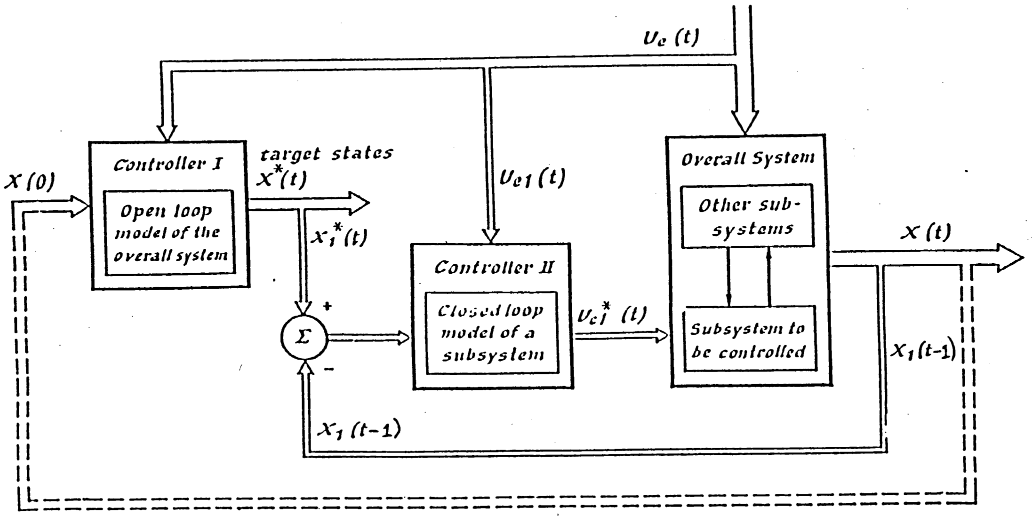

Figure 3: Combination of open loop and closed loop control

- o Numerical solution according to Bellman' principle of Dynamic Programming.
- o Numerical solution using Numerical Optimization Procedures.

Pontrjagin's Maximum Principle is an extension of the classical Calculus of Variation. The general idea of the approach is to decompose the problem of optimizing over the total planning horizon into single stage or static decision problems that can be solved analytically for any point in time.

This is done by establishing the Hamiltonian function:

$$H\{x,u_c,u_e,p,t\} = g\{x,u_c,u_e,t\} + p\{t\}\, f\{x,u_c,u_e,t\} \qquad (3)$$

which then is to be maximized with respect to u_c. The Maximum Principle gives the necessary conditions for maximizing the Hamiltonian and establishing the adjoint function p{t}, and thus for solving the original control problem. Applying Pontrjagin's Maximum Principle to quantitatively solve optimal control problems requires the analytical solution of the resulting equations. This however is possible only for certain classes of models, e.g. linear or linear-quadratic ones. Thus, Pontrjagin's approach in many cases either imposes serious restrictions on the model structure or, in turn, only general properties of the resulting optimal control can be given, while numerical results are not obtainable (For applications *cf* Bazlen, 1985; Buchwald, 1987).

Bellman's Dynamic programming framework basically follows the same idea as the Maximum Principle: A multi-period or multistage decision problem is decomposed into a number of single-stage subproblems. Bellman's principle then states that an overall control policy is an optimal control if, at any point in time, the remaining decisions are optimal with respect to the states that result from earlier decisions. Consequently, if the possible states of the system can be determined in advance, a numerical solution of the control problem can be obtained recursively moving backward from the last stage toward the initial one.

Typically, the dynamic programming framework is applied to problems where time and state variables take on discrete values. When this is the case, the method of solution does not impose any restrictions of the structure of the models that govern the transition from one state to another. However, if the state variables are continuous by nature, which is the predominant case for plant growth models, discretizing them at a reasonable level of accuracy is likely to result in problems of dimensionality. This means that the number of different states to be considered becomes large enough to cause computational problems with respect to computer storage capacity and/or computing time. Therefore dynamic programming applications are largely limited to problems where the state space is either discrete by nature or can by subdivided into a relatively small number of discrete values. Examples are replacement problems, inventory management, and marketing (*cf*: Lentz, 1985; 1987a; 1987b; Håkansson, 1987).

In recent years a large number of numerical optimization procedures for general nonlinear problems have been developed. Applying those jointly with a simulation model that computes the development of a crop for a given set of controls is the third way of approaching the optimal control problem. Figure 4 delineates the basic concept: For a given set of control variables the simulation models yield an objective function value at the end of the planning horizon. From this the optimization procedure derives an improved set of controls and starts a new simulation run. This feedback loop is repeated until the objective function value converges on the optimum.

Since the simulation model runs independently from the optimization algorithm there are no restrictions with respect to its structure. This property makes it both feasible and desirable to use numerical optimization techniques to obtain optimal temperature strategies for greenhouse crops. In the remainder of the article the principles of this approach are outlined in more detail.

4. Application of numerical optimization procedures to determine optimal greenhouse temparature strategies

The general idea of this approach is based on the closed loop concept. It incorporates three components: the simulation model, the performance criterion or objective function and the optimization method. To start the optimization the user has to choose the initial value of the controllable variables. Then the numerical optimization algorithm adjusts the policy variables iteratively in order to respectively maximize or minimize the performance criterion.

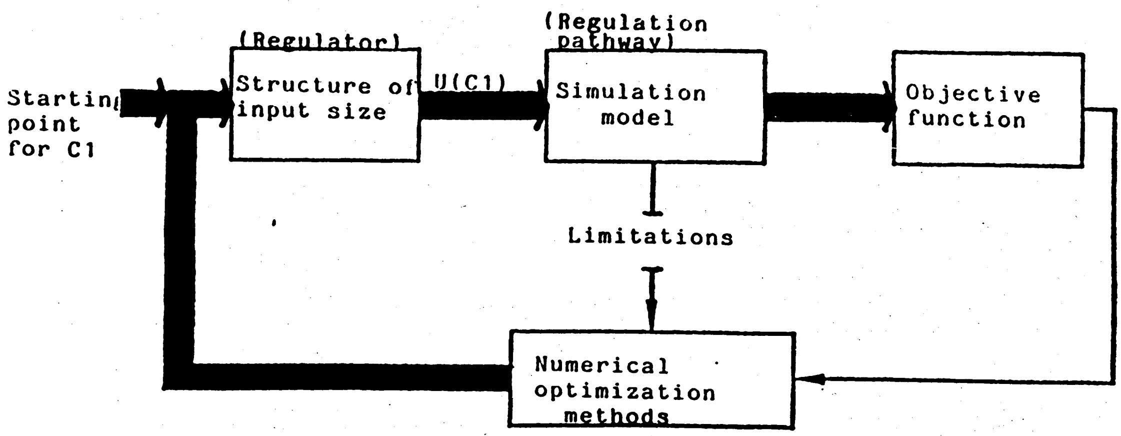

Figure 4: Direct optimization of simulation models (Jacob, 1982)

In many practical optimization problems there are constraints on the values of some of the policy or state variables which restrict the feasible region. The numerical optimization approach offers two possibilities to handle this. Either penalty functions are involved in the simulation model or the search algorithms are forced to adjust the values of the policy variables until all variables are in the feasible region.

4.1 Characteristics of numerical optimization procedures

In recent years the use of computers has led to a widespread development of numerical optimization methods. In order to determine search methods which are generally suitable for optimizing bio-economic models a classification scheme was developed. First the methods are classified by the mathematical structure of the simulation model which can be optimized.

Three main classes are distinguished, namely linear, quadratic or non-linear systems. Bio-economic models are usually non-linear systems. Second, the optimization methods are classified by the requirement of gradient information about the objective function. For this the algorithms are divided into three groups. The first group of optimization methods requires first and second order partial derivatives, the second group uses only first order partial derivatives and the last group requires no external gradient information. When complex non-linear bio-economic models are optimized it may often be difficult or even impossible to compute gradient information about the objective function. Therefore it seems to be advisable to concentrate on search methods without gradient information. The third step divides search algorithms into simultaneous and sequential methods. The first group of methods starts evaluating the objective function at several points distributed over the whole search region. The point representing the best value is then selected as the optimum. When using these methods all values of the objective function can theoretically be computed simultaneously. In contrast to this, sequential methods move from one point to the next.

The last classification criterion points out two different search techniques which are available in both groups: either the values of the policy parameters are varied randomly or they are adjusted systematically in order to determine the optimum. Figure 5 shows a contour map of the two dimensional function and illustrates the principal search techniques. The top left example in Figure 5 belongs to simultaneous- systematical methods. In this case the objective function is evaluated one each point of the grating and the point representing the best value is selected as the optimum. The top right example represents a simultaneous-random method. Contrary to the first example this procedure evaluates the objective function at several randomly distributed points. The bottom left example traces the path of the Gauss-Seidel-Strategy which belongs to the category of the sequential-systematical procedures. Usually these methods first determine a direction of search and then carries out a single line search that yields a local minimum.

The search goes on iteratively until a stop criterion is fulfilled. The remaining example depicts the evolution strategy "EVOL" of Rechenberg (Schwefel, 1977). The algorithm belongs to the sequential-random methods and represents a simplified imitation of the principles of biological evolution. In the first step, the mutation, the coordinate values of the actual point will be changed randomly to create a second point. In the next step, the selection, the point representing the better value of the objective function will be selected for further mutation.

Practical search algorithms often combine several of these basic search techniques.

4.2 The bio-economic simulation model

Figure 6 illustrates the structure of the bio-economic simulation model used to calculate the performance criterion of different temperature strategies. The simulation model incorporates five submodels: (1) outdoor weather (2) heating energy (3) greenhouse climate (4) biological growth and (5) economic evaluation. The model is influenced by several environmental (exogenous) variables. The temperature set points and the start of cultivation belong to the controllable (policy) inputs. The uncontrollable inputs are formed by the parameters of the greenhouse structure, the climate, data, the product prices and the opportunity costs.

The average daily outdoors temperature, solar radiation and the daily courses of these variables form the input of the submodel "outdoor weather" which computes the temperature and radiation of each hour. The output of this model is the input

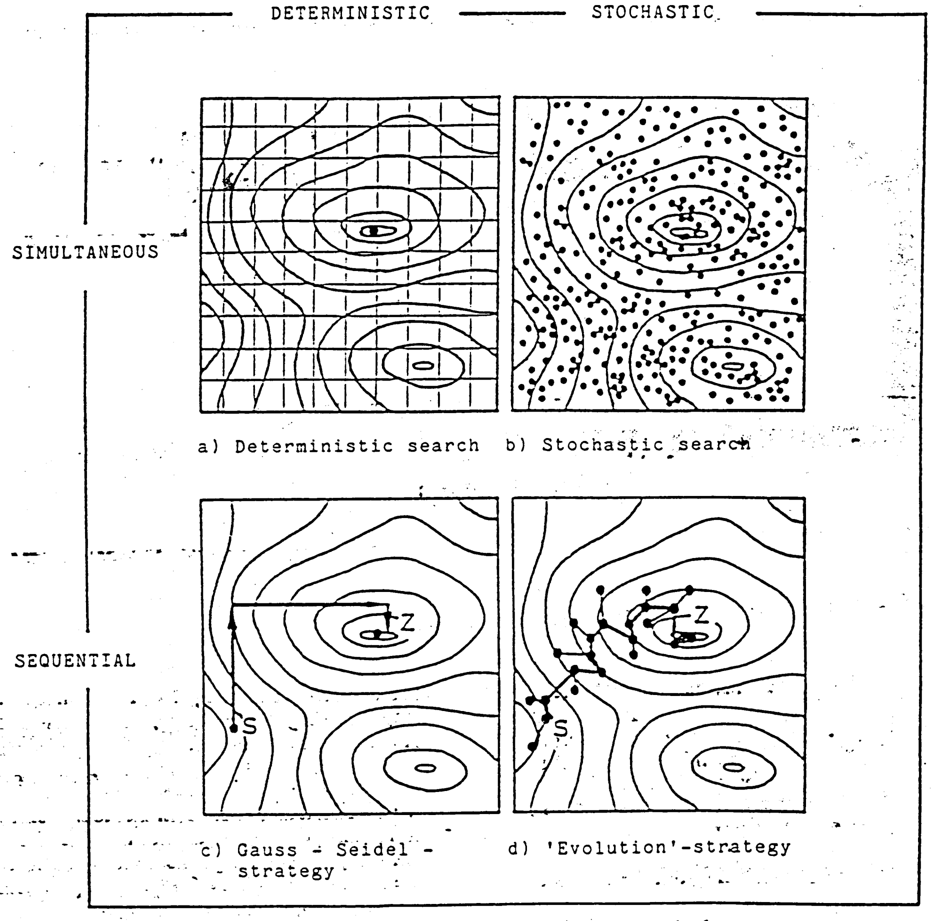

Figure 5: Different numerical optimization methods

of the heating energy model and the greenhouse climate model. both models are based on the concept of the k-'model (Schockert & Von Zabeltitz, 1979). The biological growth models consider solar radiation and temperature as the main factors influencing the growth rate. These were developed by Krug (1985), Liebig (1984) and Bazlen (1985) for radish, cucumbers and lettuce, respectively. The economic model summarizes the output of the different submodels and exogenous variables to determine the performance criterion for different policies.

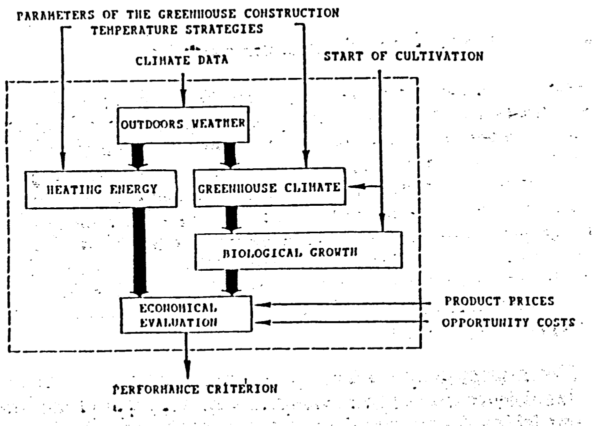

Figure 6: The bio-ecological simulation model

4.3 Some results of the optimization

An example illustrates the results obtained from combining the bio-economic simulation model and numerical optimization.

A starting point of 10°C has been chosen for the greenhouse cultivation of lettuce in autumn in the vicinity of Hannover, West Germany. The goal was to maximize the revenue minus energy costs. Assuming that the period of cultivation will not be extended the search algorithm 'EVOL' selects a temperature strategy that results in energy savings of about 15% (Figure 7). The calculated optimal temperature settings appear to be reasonable. The temperature set points decrease during the cultivation period due to the diminishing solar radiation which is the limiting growth factor.

Table 1 shows another example for the greenhouse cultivation of radish in autumn in the same area. Again the return minus energy costs was maximized. The wide range of the obtained optimization results are remarkable. The reason for this might be a response surface with a lot of local optima, sharp ridges and narrow valleys. This points to the legitimacy of some of the models. It may be that under certain given conditions all these models may be acceptable, but not under some other conditions. When all are applicable, the most efficient should be used. However, the question concerning which of these solutions is most likely to be correct is still not solved.

As the results in Table 1 already point out, the optimum cannot be determined with certainty. The relative advantages and disadvantages of the different search methods depend on the response surface of the simulation model and the localization

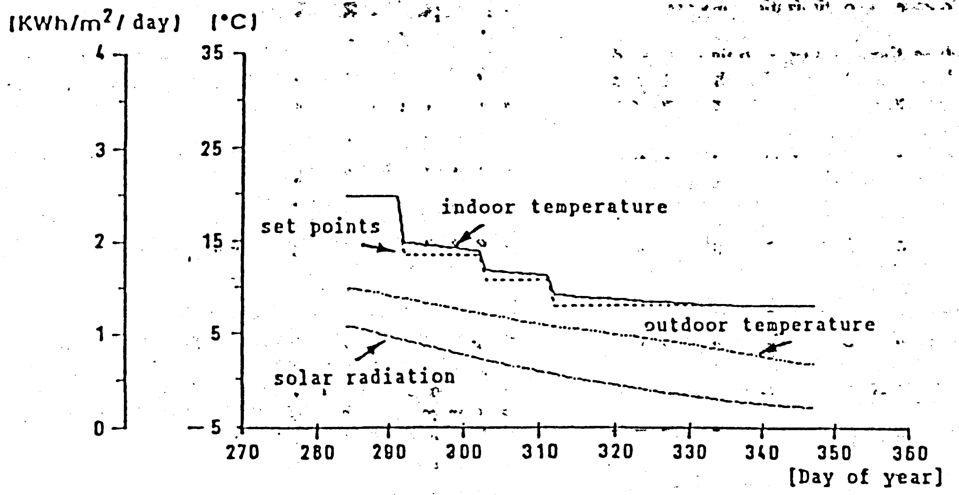

Figure 7: Optimal temparature strategies for lettuce

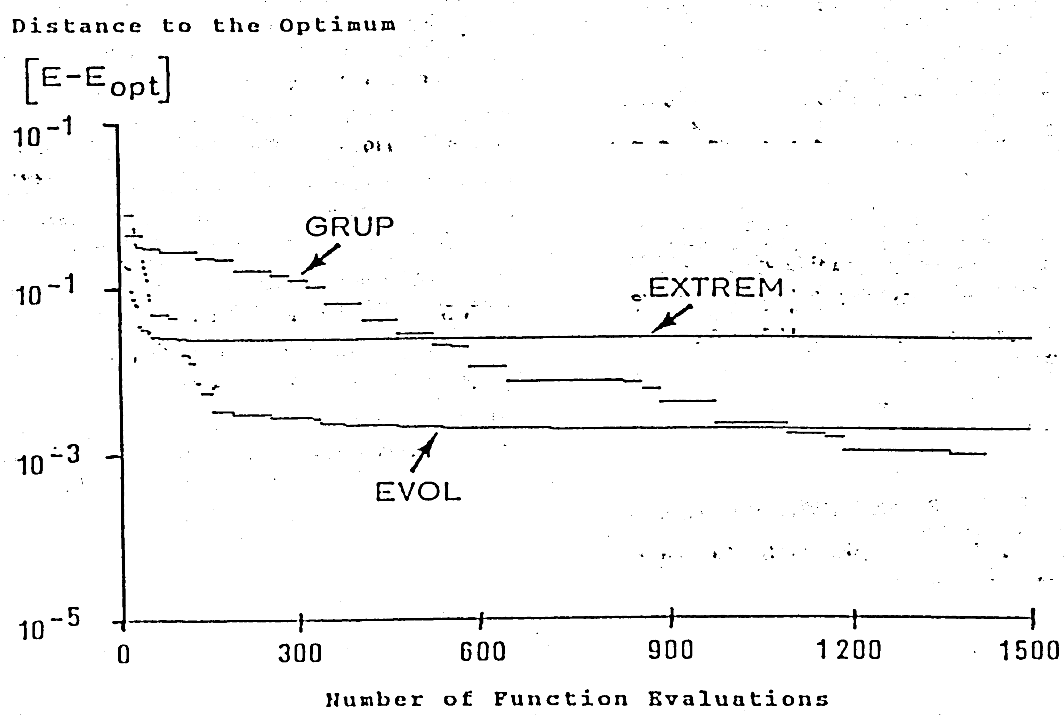

Figure 8: Speed of convergence of three different methods

of the initial point. If the search starts close to the optimum, the systematical methods gain the best values of the objective function. However, if nothing is known about the response surface of the simulation model it will be better to use search strategies with a random component because these scatter several state points over a wide range within the feasible region.

The test of different algorithms on well known test problems often shows that the systematic methods have a higher speed of convergence (number of function evaluations). Nevertheless Figure 8 depicts that for a complex simulation model the evolution exhibit a higher performance.

5. Conclusions

Numerical optimization techniques do open a number of opportunities to derive optimal controls from complex simulation models. The performance of different methods depends on the nature of the problem. In general, if little is known about the surface of the solution space, random strategies in general perform better than directed search methods.

Although there is no explicit restriction with respect to the structure of the simulation model, one should bear in mind that smooth surfaces facilitate the optimization, while sharp ridges and narrow valleys decrease the performance or may cause a complete failure of the algorithm. Therefore, if deriving an optimal control law is the final objective, simulation models should be constructed as to ease the optimization process.

Applying numerical search methods is not a straight forward matter, since the algorithms cannot determine the optimum with certainty. Thus their successful application requires experience and knowledge about the simulation model. Although these methods are valuable tools in research, their use in practical decision-making is however limited.

Table 1: Optimal temperature strategies for radish in autumn using different methods of optimization (start of cultivation: 14.10)

Search method	Temperature set points [°C] Until emergence	Growth phase 1	2	3	4	Revenue minus Heating costs [DM/m²]
Initial values	8,0	8,0	8,0	8,0	8,0	1,82
Deterministic						
EXTREM	8,8	8,4	8,0	8,2	6,8	2,09
GLOBEX	3,7	7,5	11,8	11,7	3,2	2,41
Stochastic						
EVOL	16,5	4,7	10,7	6,4	3,5	2,78
GRUP	14,8	3,3	9,1	7,9	4,4	2,84

Notes

1. The basic research for this article was done while the authors were visiting professors at the Michigan State University in 1988. The authors wish to thank Steve Harsh and Jan Groenewald for valuable comments.

2. See for example Adby and Dempster (1974) and Schwefel (1977)

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