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ECONOMIC ANALYSIS OF MAIZE YIELD RESPONSES TO FERTILIZER AND RAINFALL VARIATIONS AT DUNDEE

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UITTREKSEL: EKONOMIESE ONTLEDING VAN MIELIE-OPBRENGSREAKSIE OP KUNSMIS- EN REËNVALVARIASIES TE DUNDEE

Data van 'n langtermyn eksperiment is gebruik om 'n vooruitskattingsvergelyking te ontwikkel wat 'n verband uitdruk tussen mieliegraanopbrengs en stikstof- (N-) en fosfaat- (P-) toedienings en totale neerslag (Ri) in verskillende groeistadia van die mielieplant. Hierdie stadia is 'n vooraanplantingstydperk en 'n reeks nie-oorvleuelende fisiologiese-groeifases vanaf saai tot wasdom. Die vergelyking is 'n gemengde kwadratiese (N-) en vierkantswortel- (P-) veelterm wat lineêre Ri-, Ri*N- en Ri*P-terme bevat het. Die funksie is gebruik om die reaksie van mieliegraanopbrengs op N en P te bepaal, en die kunsmisvereistes vir maksimale wins en minimum koste N- en P-kombinasies. Die effek van Lae, Mediane en Hoë reënvalvlakke is ook ondersoek. Daar is gevind dat opbrengsreaksie en winsgewendheid nadelig beïnvloed is deur verhoogde neerslagvlakke. Die resultate toon die belangrikheid van die inagneming van reënval by kunsmisaanbevelings.

ABSTRACT

Data from a long term field experiment were used to develop a predictive equation relating maize grain yield to nitrogen (N) and phosphate (P) applications, and total precipitation (Ri) in different growth stages of the maize plant. These stages were a pre-planting period, followed by a sequence of non-overlapping physiological growth phases from sowing to maturity. The equation was a mixed quadratic (N) and square-root (P) polynomial containing linear Ri, Ri*N and Ri*P terms. The function was used to determine the response of maize grain yields to N and P, the fertilizer requirements for maximum profits and least cost N and P combinations. The effects of Low, Median, and High levels of rainfall were also investigated. It was found that yield response and profitability were adversely affected by increased levels of precipitation. The results indicate the importance of considering rainfall in fertilizer recommendations.

1. Introduction

The maize producer is continually faced with decisions that affect his financial well-being, and a critical decision concerns fertilization levels. Fertilizer costs represented about 51% of total allocated costs of maize production in the Dundee area in 1986/87 (Directorate of Agricultural Production Economics, 1986/87). In 1987/88 the average expenditure on fertilizer, for the Dundee Study Group, amounted to 44% of total allocated costs per hectare, and 32% of average maize gross income per hectare (Directorate of Agricultural Production Economics, 1987/88). Thus, fertilizer usage is a prime determinant of profitability, and research and advice have concentrated on fertilizer recommendations. Furthermore, fertilizer costs are an aspect of production which can be manipulated by the producer (Farina *et al*, 1980). It is not economic to improve production by substantial applications of fertilizers, and a decision of profound consequence in maize cultivation centres on the determination of optimum fertilization rates.

In order to determine the fertilization rates that will maximize his profits the maize producer needs to equate the value of the marginal product of fertilizer (VMP) to the price of fertilizer. This requires the existence of a physical production function relating grain yields to fertilization levels. A description of the derivation of a model relating maize grain yields to applied N (N), applied P (P) and the amount of rainfall in different growth phases was given in a previous article (Berry *et al*, 1989). The purpose of this article is to examine the economic properties of this model, and the effects of rainfall on maize grain responses and profitability will be highlighted.

2. Estimated response function

The estimated response function that was derived (Berry *et al.* 1989) is as follows:

$$
\begin{aligned}
Y = {} & 5090{,}477 + 4{,}248*N - 0{,}080*N^2 - 83{,}757*P + 864{,}087*P^{0,5} \\
& + 1{,}620*N*P^{0,5} \\
& - 9{,}474*R1 + 0{,}078*P*R1 \\
& - 4{,}088*R2 - 0{,}276*N*R2 - 0{,}184*P*R2 \\
& - 13{,}433*R5 - 0{,}089*N*R5 \\
& - 32{,}355*R7 \\
& - 4{,}049*R9 + 0{,}096*N*R9 + 0{,}088*P*R9 \\
& + 2{,}551*R15 + 0{,}037*N*R15 \\
& - 36{,}254*R17
\end{aligned}
\qquad (1)
$$

Adjusted R^2 = 0,897 D.F. = 255 F-value = 126,788

where : Y = estimated maize grain yield (kg/hectare)
N = N applied (kg/hectare)
P = P applied (kg/hectare)
Ri = total rainfall (mm) in different growth stages as follows:
R1 = 1st September to planting
R2 = sowing to emergence

R5 = emergence to the end of the juvenile stage
R7 = the end of the juvenile stage to tassel initiation
R9 = tassel initiation to the beginning of grain filling
R15 = the beginning of grain filling to the end of grain filling
R17 = the end of grain filling to physiological maturity.

Pre-planting precipitation (P-P) is represented by R1. The rainfall for the remaining stages (from sowing to maturity) will be referred to as the growing-season rainfall (G-S).

This function was selected since it had the highest adjusted R^2 value, and the lowest predicted residual sum-of-squares (PRESS statistic) of all possible models (Berry *et al*, 1989).

It is clear from equation (1) that variations in the amount of fertilizer and rainfall will have an influence on grain yields. Thus levels of production which maximize profits and determine the least cost combination of nutrients will be affected by fertilizer and maize prices, and by rainfall. It was therefore decided to investigate the effect of rainfall on the response of maize to fertilization and on the profitability of maize production.

Fifty years of rainfall data from the Dundee Research Station were used to derive the Median rainfall for each growth period contained in the yield function. Two additional rainfall amounts were calculated for each growth period:

(a) Lowest (L): Average of the 10 years having the lowest precipitation levels for a particular growth period.
(b) Highest (H): Average of the 10 years having the highest precipitation levels for a particular growth period.

Low, Median and High levels of rainfall were also determined for the pre-planting period.

3. Maximum predicted maize grain yields

The maximum maize grain yield was determined by equating the first derivatives (the marginal physical products) of the yield equation to zero (Heady and Dillon, 1961). The derivatives with respect to N and P are given in Equations 2 and 3.

$$\frac{dY}{dN} = 4{,}248 - 0{,}161{*}N + 1{,}620{*}P^{0,5} - 0{,}276{*}R2 - 0{,}089{*}R5 + 0{,}096{*}R9 + 0{,}037{*}R15 = 0 \quad (2)$$

$$\frac{dY}{dP} = -83{,}757 + \frac{432{,}044}{P^{0,5}} + \frac{0{,}810{*}N}{P^{0,5}} + 0{,}078{*}R1 - 0{,}184{*}R2 + 0{,}088{*}R9 = 0 \quad (3)$$

The highest predicted yield of 9512 kg/hectare is associated with Low levels of pre-planting and growing-season rainfall. This yield and the associated application rates of N and P were taken as base values, and the percentage differences between these values and those associated with the other eight rainfall combinations are presented in Table 1.

It is apparent that the maximum predicted yield declines with increasing amounts of rainfall in either or both periods. This detrimental effect of rainfall is linked to the hydromorphic nature of the "experimental" soils at the Dundee Research Station. Soils of this type become water-logged in the presence of excessive moisture, and this leads to denitrification and stifling of the roots (Farina, 1970). Other studies (Berry, 1989) indicate that high levels of rainfall can result in yield reductions. For example, the amount of radiation and temperature may be reduced, and this will adversely affect maize grain yields.

Table 1: Percentage variations in maximum predicted grain yields, applied N and applied P for various rainfall levels

	GROWING-SEASON RAINFALL		
	LOW	MEDIAN	HIGH
A. YIELD			
PRE-PLANTING RAINFALL:			
LOW	0,0	-2,7	-47,8
MEDIAN	-2,2	-3,6	-48,3
HIGH	-3,1	-2,0	-45,5
B. APPLIED N			
PRE-PLANTING RAINFALL:			
LOW	0,0	12,0	3,2
MEDIAN	4,7	17,7	9,4
HIGH	13,1	28,1	21,2
C. APPLIED P			
PRE-PLANTING RAINFALL:			
LOW	0,0	22,4	29,8
MEDIAN	22,7	52,8	64,6
HIGH	69,4	117,7	141,1

Note: The maximum predicted maize grain yield for Low levels of pre-planting and growing season rainfall = 9512 kg/ha, and the associated N level = 204 kg/ha, and the associated P level = 78kg/ha.

4. Production curves and marginal physical products

Predicted grain yields are dependent on fertilization rates and the amount of rainfall. If one input is varied while others are held constant a single input-output curve is obtained (Heady *et al*, 1961). This technique was used to show the effects of fertilization and rainfall on maize grain yield.

4.1 Response curves for N: Applied P held constant

Five levels of P and Median levels of pre-planting and growing-season rainfall were used to derive the yield response curves illustrated in Figure 1. The response curves for N, when P is held constant, indicate that higher levels of P result in larger predicted grain yields, and this can be attributed to the positive interaction between the two nutrients.

Figure 1 highlights the effects of increasing quantities of N and P for a constant level of rainfall. Figures 2 and 3 show the yield response to varying amounts of N and rainfall when P is held constant at 60 kg/hectare.

In the case of Figure 2 varying amounts of growing-season rainfall are coupled with a Median pre-planting rainfall. Figure 3 was derived by varying the amount of pre-planting rainfall and

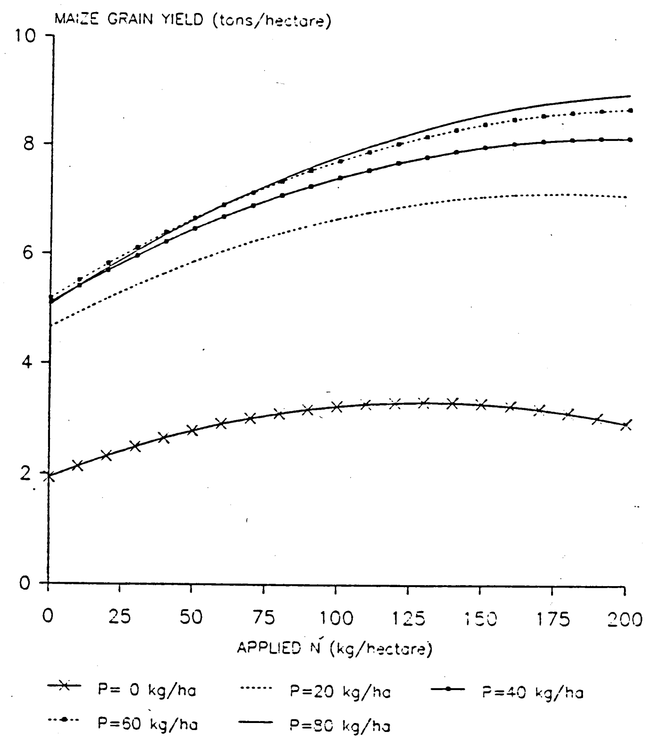

Figure 1: Predicted yield response to N for five levels of P and median levels of pre-planting and growing season rainfall

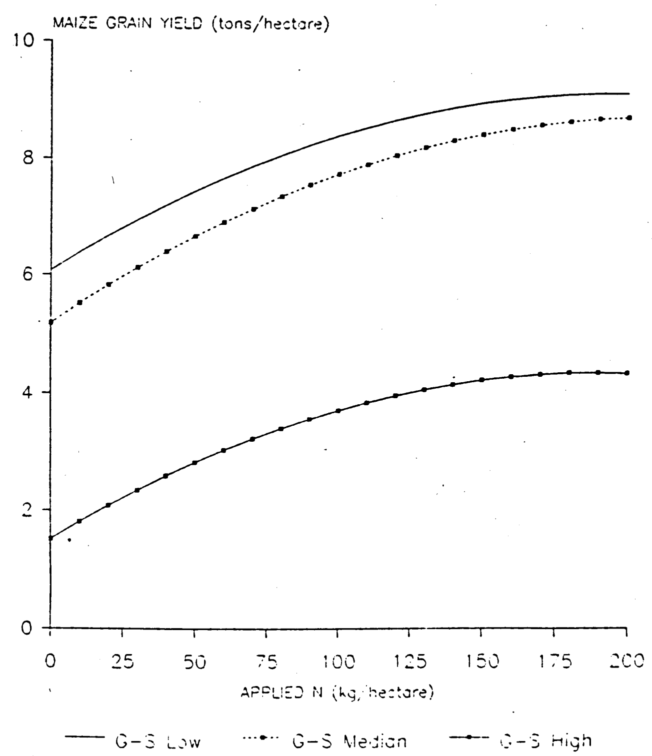

Figure 2: Predicted yield response to N and varying amounts of growing-season (G-S) rainfall (P = 60kg/ha and median pre-planting (P-P) rainfall

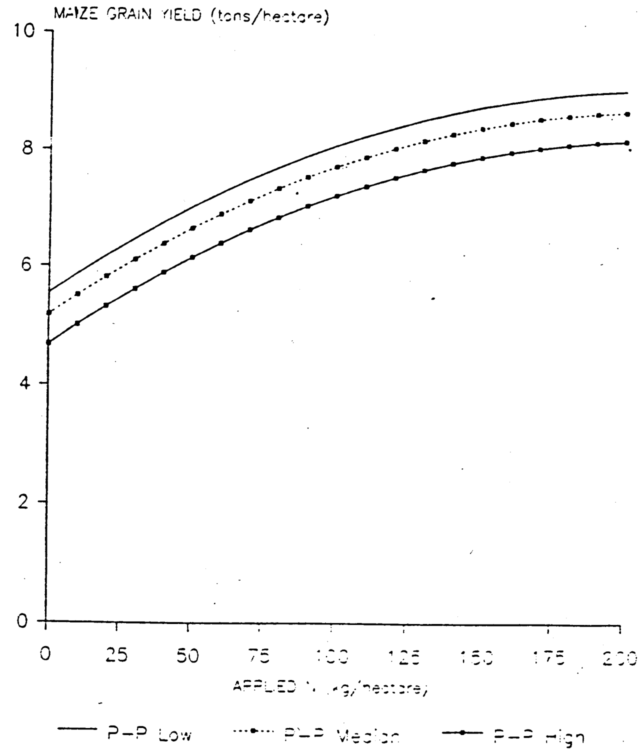

Figure 3: Predicted yield response to N and varying amounts of pre-planting (P-P) rainfall (P = 60 kg/ha and median growing-season (G-S) rainfall

keeping the growing-season rainfall at the Median level. Both sets of response curves confirm the negative influence of precipitation.

4.2 Marginal physical products of N

The partial derivative dY/dN (equation (2)) is not affected by changes in the amount of precipitation prior to sowing (R1). It will, however, alter with different levels of rainfall from sowing to the end of the juvenile stage (R2 and R5), and from tassel initiation to the end of grain filling (R9 and R15). These variations are shown in Table 2, which contains the marginal physical products of N for a P rate of 60 kg/hectare.

Marginal physical products decline with higher levels of N. In other words, each increment of N causes grain yield to increase by a smaller amount than that for the previous N increment, reflecting the Law of Diminishing Returns. For example, from Table 2, at around 100 kg N/hectare, if N is increased by one kg maize yield is estimated to increase by 15 kg under Low growing-season rainfall conditions. In the case of a quadratic polynomial, the marginal physical products decline by constant amounts, as is evident in Table 2. The negative marginal physical products in Table 2 indicate Stage III of the production function. In this zone increased applications of N result in a decline in grain yield.

The response of maize grain yields to N is also influenced by the amount of growing-season rainfall. For any particular level of N, say 150 kg/hectare, the largest increase in grain yield occurs with a Median level of rainfall during the growing-season. This is a result of the N*Ri interaction terms, where R2 and R5 have a negative effect on the marginal physical product, whilst R9 and R15 have a positive effect.

Table 2: Marginal physical products of N for various rainfall levels (P = 60 kg/ha)

RAINFALL LEVEL		MARGINAL PHYSICAL PRODUCTS OF N					
		N (KG/HECTARE)					
PRE-PLANTING	GROWING-SEASON	0	50	100	150	200	250
	LOW	31,10	23,07	15,03	6,99	-1,04	-9,07
ANY	MEDIAN	33,53	25,49	17,46	9,42	1,39	-6,65
	HIGH	30,16	22,12	14,09	6,05	-1,98	-10,02

In Table 3 the quantity of P and rainfall are varied, and the marginal physical products of N for one particular N application (150 kg/hectare) are shown.

Table 3: Marginal physical products of N for various levels of P and rainfall (N = 150 kg/ha)

RAINFALL LEVEL		MARGINAL PHYSICAL PRODUCTS OF N		
		LEVEL OF P (KG/HECTARE)		
PRE-PLANTING	GROWING-SEASON	40	60	80
	LOW	4,69	6,99	8,94
ANY	MEDIAN	7,12	9,42	11,36
	HIGH	3,75	6,05	7,99

The highest marginal physical product of N is associated with a Median level of growing-season rainfall. In addition, for any particular rainfall level the increment in grain yield is higher for larger applications of P, due to the positive interaction between N and P.

4.3 Response curves for applied P: N held constant

The response of maize grain yields to increasing amounts of P, for five levels of N, is shown in Figure 4. Precipitation was held at the Median level for both the pre-planting and post-planting periods. It is interesting to note the sharp increase in yield at low levels of P.

The joint effect of rainfall and P on yield was determined by holding the quantity of N at 150 kg/hectare and varying the amount of precipitation in the various growth stages. Figure 5 was obtained by using different levels of growing-season rainfall in combination with a Median level of pre-planting rainfall. It can be seen that, for any level of P, predicted grain yields are adversely affected by higher levels of growing-season precipitation.

The influence of pre-planting rainfall on grain yields was determined by using an N rate of 150 kg/hectare, and a Median level of growing-season rainfall in conjunction with the three levels of pre-planting rainfall. The results of this investigation indicate that there is a detrimental effect exerted by increasing amounts of rainfall before the time of sowing (Figure 6).

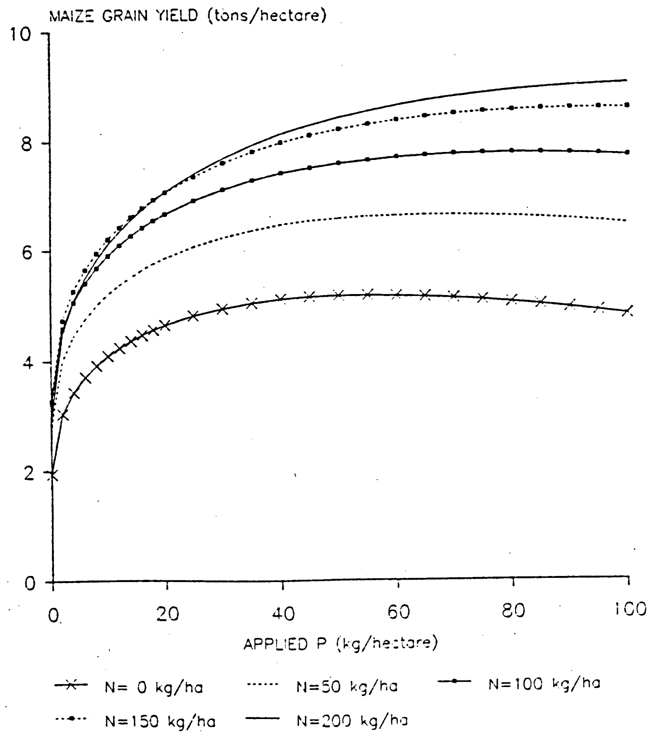

Figure 4: Predicted yield response to applied P for five levels of N and median levels of pre-planting and growing-season rainfall

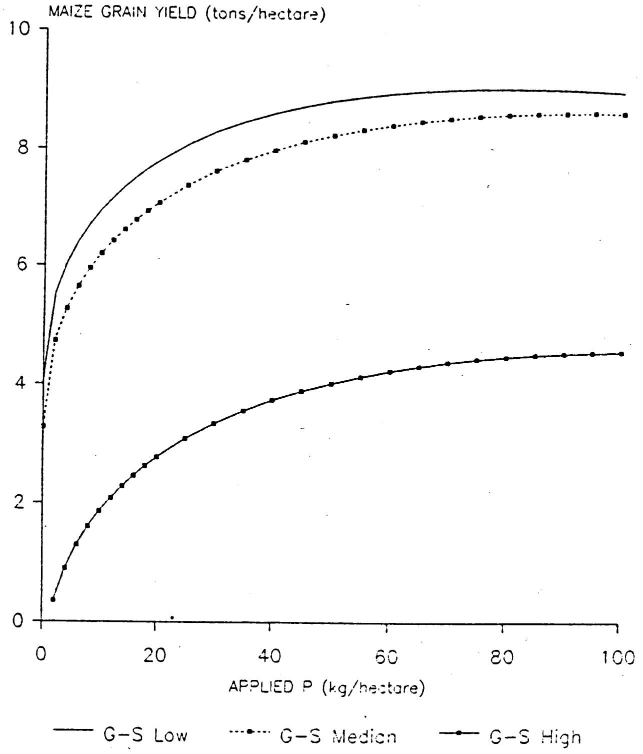

Figure 5: Predicted yield response to applied P and varying amounts of growing-season (G-S) rainfall (N= 150kg/ha and median pre-planting (P-P) rainfall.

4.4 Marginal physical products of applied P

Since the partial derivative dY/dP (equation (3)) involves the pre-planting period the marginal physical product of P will vary with R1. It also varies with the quantity of rainfall occurring between sowing and emergence (R2), and between tassel initiation and the start of grain filling (R9). The marginal physical products listed in Table 4 were calculated by keeping the level of N at 150 kg/hectare.

The trends shown in Table 4 lead to the following conclusions:

(a) Diminishing marginal returns to higher P levels are clearly discernible. Furthermore, the marginal physical products of P decline at a decreasing rate, which is a characteristic of square-root polynomials.

(b) Negative marginal physical products (Stage III) vary with the quantity of P and rainfall.

(c) For a given amount of rainfall before sowing, the marginal products increase as more rainfall occurs during the growing-season. In other words, the yield of maize grain increases by larger amounts, for a given P rate, as more rainfall is received in the relevant growth stages. A similar conclusion can be made concerning increased levels of pre-planting rainfall, for any particular level of growing-season rainfall.

Another set of marginal physical products is contained in Table 5. These were derived for three levels of N and a single P level of 60 kg/hectare. From the data in Table 5 it can be concluded that, irrespective of the amount of rainfall, the response to P improves with higher levels of N due to the positive interaction between N and P. For a particular level of pre-planting rainfall, the marginal physical product of P increases with greater amounts of

Table 4: Marginal physical products of applied P for various levels of N and rainfall (P = 60 kg/ha)

RAINFALL LEVEL		MARGINAL PHYSICAL PRODUCTS OF P					
		P (KG/HECTARE)					
PRE-PLANTING	GROWING-SEASON	5	25	45	65	85	105
LOW	LOW	179,74	42,89	14,70	0,84	-7,77	-13,79
	MEDIAN	184,21	47,36	19,17	5,31	-3,00	-9,32
	HIGH	187,51	50,66	22,47	8,61	0,00	-6,02
MEDIAN	LOW	185,55	48,70	20,51	6,65	-1,97	-7,99
	MEDIAN	190,02	53,17	24,98	11,12	2,50	-3,52
	HIGH	193,32	56,47	28,28	14,42	5,80	-0,22
HIGH	LOW	193,55	56,71	28,52	14,66	6,04	0,02
	MEDIAN	198,02	61,18	32,97	19,13	10,51	4,49
	HIGH	201,32	64,48	36,29	22,43	13,81	7,79

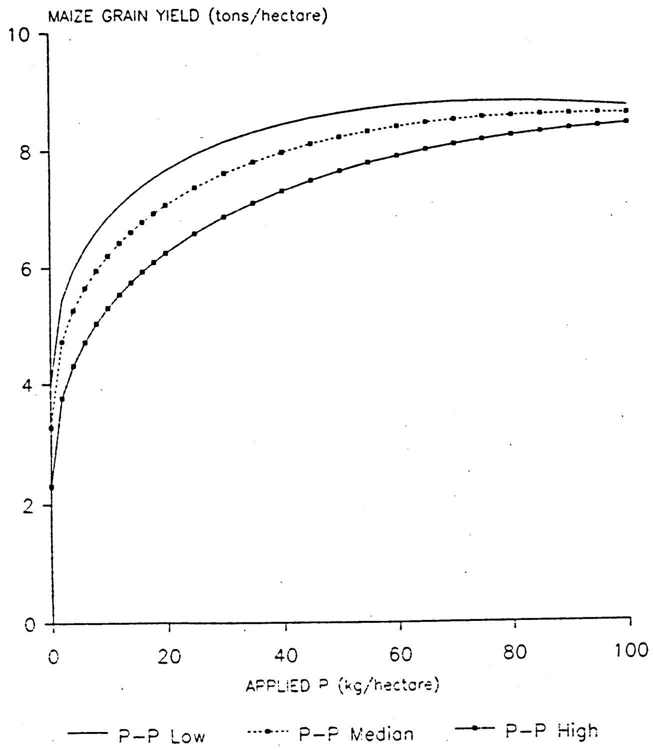

Figure 6: Predicted yield response to applied P and varying amounts of pre-planting (P-P) rainfall. (N=150 kg/ha and median growing season (G-S) rainfall)

growing-season precipitation. If the growing-season rainfall is kept constant, then the marginal product rises with more precipitation prior to planting. These effects are a result of the positive coefficients of R1 and R9, and the negative coefficient of R2 (equation (3)).

Table 5: Marginal physical products of applied P for various levels of N and rainfall (P=60kg/ha)

RAINFALL LEVEL		MARGINAL PHYSICAL PRODUCTS OF P		
		LEVEL OF N (KG/HECTARE)		
PRE-PLANTING	GROWING-SEASON	100	150	200
LOW	LOW	-1,58	3,65	8,88
	MEDIAN	2,89	8,12	13,35
	HIGH	6,19	11,42	16,65
MEDIAN	LOW	4,22	9,45	14,68
	MEDIAN	8,69	13,92	19,15
	HIGH.	11,99	17,22	22,45
HIGH	LOW	12,23	17,46	22,69
	MEDIAN	16,70	21,93	27,16
	HIGH	20,00	25,23	30,46

5. Elasticity of production

The elasticity of production (Ep) measures the degree of responsiveness of output to various levels of input. It can be defined as:

$$Ep = \frac{\text{Percent change in output}}{\text{Percent change in input}}$$

Ep can be determined from the response function (equation (1)) using the following equation:

$$Ep = \frac{\text{Marginal physical product}}{\text{Average physical product}}$$

If rainfall is kept constant at a particular level, elasticity of production coefficients need to be calculated to measure the responsiveness to both N and P.

5.1 Elasticity of production of N

The responsiveness of maize yield to changes in the level of N was determined for one level of P (60 kg/hectare). These results are shown in Table 6. Three elasticities of production are of special interest since they are used to demarcate the "stages of production". In Stage I Ep is greater than unity, in Stage II it is less than unity but greater than zero, and in Stage III Ep is negative. The results show that Ep was never greater than unity, regardless of the quantity of N or the level of rainfall in each growth period. The level of N at which Ep changed from a positive to a negative value (i.e. the transition from Stage II to Stage III) varied with the level of rainfall.

Table 6: Elasticities of production of N for various rainfall levels

RAINFALL LEVEL		ELASTICITIES OF PRODUCTION OF N					
		LEVELS OF N (KG/HECTRE)					
PRE-PLANTING	GROWING-SEASON	5	50	100	150	200	250
LOW	LOW	0,023	0,148	0,172	0,113	-0,022	-0,247
	MEDIAN	0,029	0,182	0,216	0,161	0,031	-0,187
	HIGH	0,073	0,349	0,346	0,198	-0,085	-0,571
MEDIAN	LOW	0,024	0,155	0,179	0,118	-0,023	-0,257
	MEDIAN	0,031	0,192	0,226	0,168	0,032	-0,195
	HIGH	0,089	0,394	0,379	0,215	-0,092	-0,623
HIGH	LOW	0,026	0,166	0,191	0,124	-0,024	-0,272
	MEDIAN	0,034	0,207	0,242	0,179	0,034	-0,207
	HIGH	0,127	0,478	0,438	0,244	-0,104	-0,711

5.2 Elasticity of production of applied P

N was kept constant at 150 kg/hectare to calculate the elasticities of production of P presented in Table 7. The elasticity of production of P exceeds unity for a low level of P and a High level of precipitation prior to planting and during the growing season. Consequently, this section of the production curve is in Stage I. For a specific level of P, Ep increases as the rainfall prior to and after planting increases. The level of P signifying the transition from Stage II to Stage III changes with the amount of rainfall in either or both rainfall periods.

6. Optimum level of maize grain products

For the purposes of this paper the optimum level of production is defined as that yield of maize grain which will maximize profits per hectare. Profits are taken to be the difference between income from maize grain sales and the cost of fertilizer per hectare; i.e.

$$\text{Profit} = \text{Income} - \text{Fertilizer costs}$$

$$= Y^*Pm - (N^*Cn + P^*Cp)$$

where : Y = predicted yield of maize grain (kg/hectare)

Pm = price of maize grain (R/kg)
N = applied N (kg/hectare)
Cn = price of N (R/kg)
P = applied P (kg/hectare)
Cp = price of P (R/kg)

Table 7: Elasticities of production applied P for various rainfall levels (N = 150kg/ha)

RAINFALL LEVEL		ELASTICITIES OF PRODUCTION OF P					
		LEVELS OF P (KG/HECTARE)					
PRE-PLANTING	GROWING-SEASON	5	25	45	65	85	105
LOW	LOW	0,130	0,124	0,072	0,006	-0,072	-0,161
	MEDIAN	0,150	0,149	0,101	0,039	-0,032	-0,113
	HIGH	0,526	0,348	0,233	0,121	0,000	-0,136
MEDIAN	LOW	0,149	0,151	0,106	0,048	-0,019	-0,094
	MEDIAN	0,174	0,180	0,139	0,085	0,025	-0,043
	HIGH	0,877	0,459	0,328	0,218	0,110	-0,005
HIGH	LOW	0,183	0,195	0,159	0,112	0,059	0,000
	MEDIAN	0,219	0,232	0,198	0,155	0,108	0,056
	HIGH	6,134	0,701	0,500	0,380	0,280	0,185

The optimum level of production can be found by equating the marginal physical product of each nutrient to the fertilizer / maize grain price ratio; i.e.

$$\text{Marginal physical product of N} = \frac{dY}{dN} = \frac{Cn}{Pm}$$

$$\text{Marginal physical product of applied P} = \frac{dY}{dP} = \frac{Cp}{Pm}$$

where dY/dN and dY/dP were given in equations (2) and (3) respectively.

At the optimum level of grain production the value of the yield increase caused by the last increment of fertilizer is equal to the cost of that increment (Pesek and Heady, 1957). The price of N (Cn) used in this study is R1,36/kg, and this was derived from an L.A.N. price of R380/ton. A Single Superphosphate price of R380/ton gave a price for P (Cp) of R3,62/kg. The maize grain price (Pm) was set at R240/ton or R0.24/kg.

A change in the price of N, P or maize grain will alter the optimum production level. It has also been seen that pre-planting and growing-season precipitation affected the marginal physical products of N and P. Thus, rainfall will also influence the optimum maize grain yield.

6.1 Optimum levels of N, applied P and maize grain yield

The quantities of N and P required at the optimum level of production were found to increase with higher levels of precipitation before and/or after planting (Table 8). A higher expected price for maize grain resulted in more N and P being needed at the profit-maximizing level of grain production.

Table 8 indicates the variation in optimum maize grain yields for all rainfall combinations and constant prices. Of the nine possible optimum yields, the largest is associated with a Low pre-planting rainfall and a Low growing-season rainfall. All other rainfall combinations result in smaller optimum yields, and they required higher fertilization rates.

Obviously, a higher expected maize price, with constant fertilizer prices, will result in higher optimum yields. This is shown in Figures 7 and 8, which also illustrate the influence of pre-planting and growing-season rainfall. If the pre-planting rainfall is held constant and the growing-season rainfall is al lowed to increase, then it is apparent from Figure 7 that smaller optimum yields are predicted as the growing-season precipitation increases from Low to High. The situation of a Median level of growing-season rainfall is illustrated in Figure 8. It is evident that the optimum yield declines with increasing amounts of precipitation prior to sowing.

Table 8: Variations in optimum predicted grain yields, applied N, applied P and profit for various rainfall levels (Cn = R1,36/kg, Cp = R3,62/kg and Pm = R0,24/kg)

	GROWING-SEASON RAINFALL		
	LOW	MEDIAN	HIGH
A. YIELD (kg/hectare)			
PRE-PLANTING RAINFALL:			
LOW	9134	8814	4494
MEDIAN	8852	8623	4330
HIGH	8618	8566	4334
B. APPLIED N (kg/hectare)			
PRE-PLANTING RAINFALL:			
LOW	147	168	149
MEDIAN	153	175	156
HIGH	163	186	168
C. APPLIED P (kg/hectare)			
PRE-PLANTING RAINFALL:			
LOW	44	53	54
MEDIAN	52	62	65
HIGH	67	81	86
D. PROFIT (R/hectare)			
PRE-PLANTING RAINFALL:			
LOW	1832	1697	681
MEDIAN	1729	1606	593
HIGH	1607	1508	502

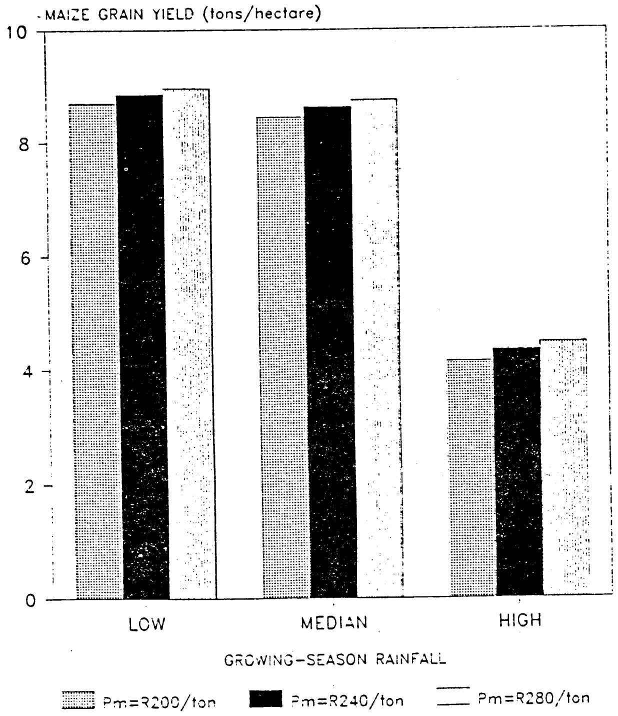

Figure 7: Optimum maize grain yields for various levels of growing-season (G-S) rainfall and three maize grain prices (Pm) (Cn) = R1,36/kg, Cp = R3,62/kg and median pre-planting (P-P) rainfall)

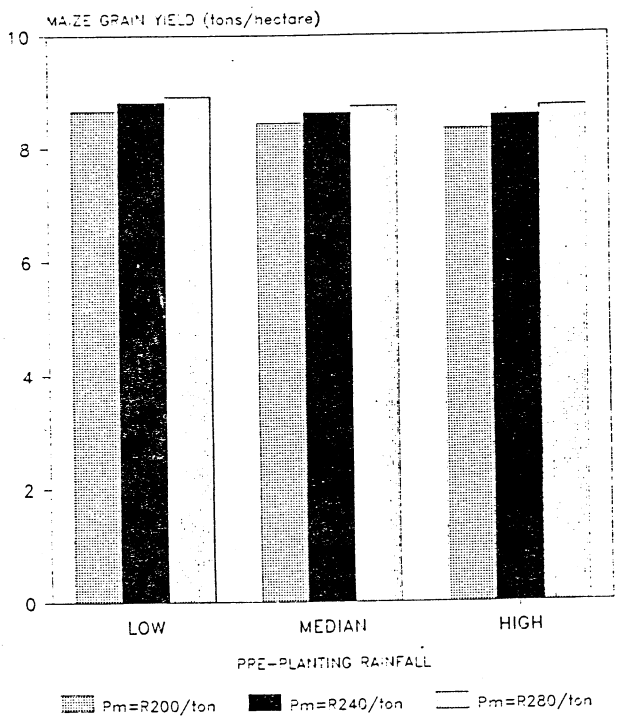

Figure 8: Optimum maize grain yields for various levels of pre-planting (P-P)(Cn = R1,36/kg, Cp = R3,62/kg and median growing season (G-S) rainfall

6.2 Profits at the optimum level of production

The adverse effect exerted on profits by increasing amounts of rainfall is apparent in Table 8 which shows that, for the prices under consideration, the optimum level of production resulting in the highest profit occurred with a Low level of rainfall before and after planting. Every other rainfall combination resulted in an optimum level of production which had a lower profit because (a) the yields themselves are lower resulting in a smaller income and (b) a greater quantity of nutrients (hence increased fertilizer costs) is needed to achieve these optimum production levels.

In the case of Figure 9 the maize price and the growing-season rainfall are changed, whilst pre-planting precipitation is held at the Median level. Costs reach a maximum for a Median level of growing-season rainfall, whilst the largest income is associated with a Low growing-season rainfall. The purpose of Figure 10 is to highlight the influences of higher maize prices and increasing quantities of pre-planting rainfall on the costs, incomes and profits at the optimum level of production. The growing-season rainfall was kept at the Median level in order to derive this Figure. Fertilizer costs increase and incomes decline as pre-planting rainfall increases. Higher levels of pre-planting rainfall offset the effects of a higher relative maize price.

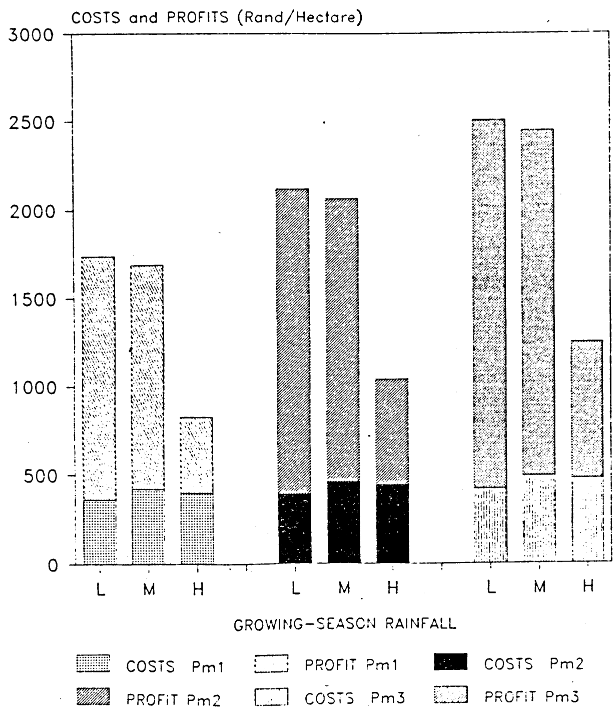

Figure 9: Costs, incomes and profits at the optimum level of production for various levels of growing-season (G-S) rainfall and three maize grain a prices (Pm1 = R200/ton, Pm2 = R240/ton, Pm3 = R280/ton en Cn = R1,36/kg, Cp = R3,62/kg, and medium pre-planting rainfall)

7. Factor-factor analysis

The question of substitution between the two nutrients must be considered, as well as the problem of determining the "right" combination. A factor-factor analysis was performed to determine the different combinations of N and P that would result in a given maize grain yield. These combinations are affected by the level of pre-planting and growing-season precipitation.

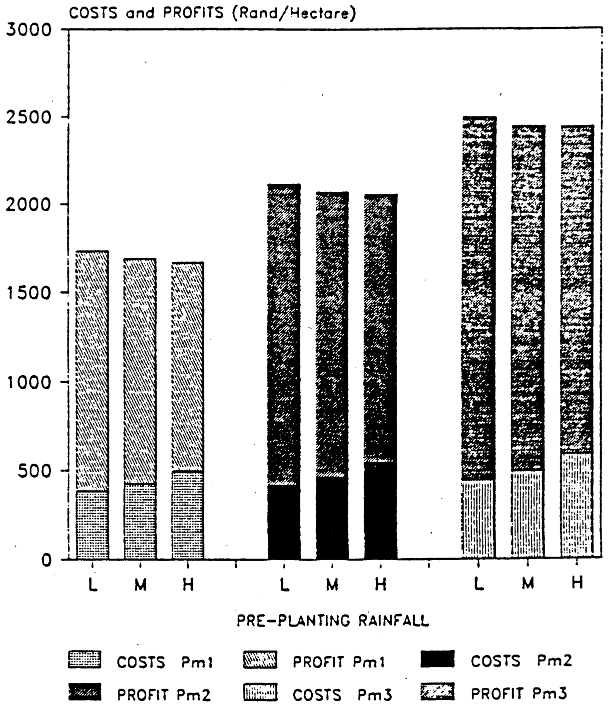

Figure 10: Costs, income and profits at the optimum level of production for various levels of pre-planting (P-P) rainfall and three maize grain prices (Pm1 = R200/ton, Pm2 = R240/ton, Pm3 = R280/ton, Cn = R1,36/kg, Cp = R3,62/kg and median growing-season rainfall)

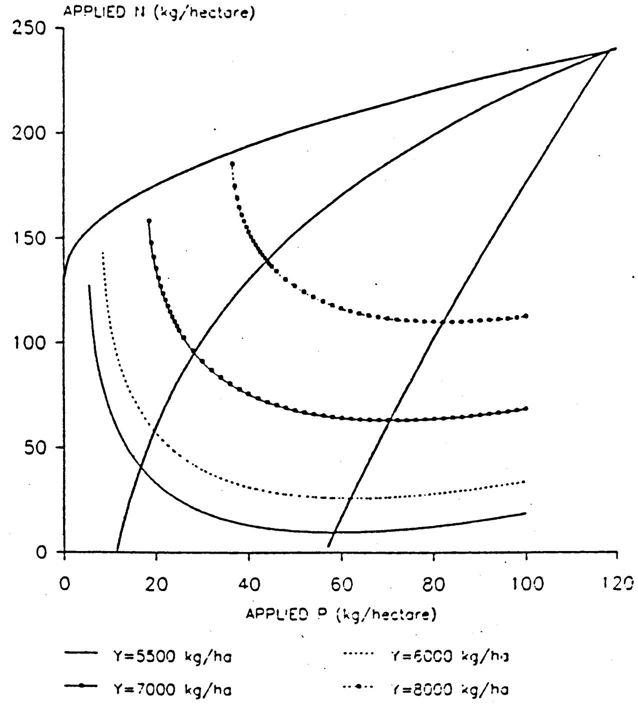

Figure 11: Isoquant map for median levels of rainfall before and after planting (Cn = R1,36/kg and Cp = R3,62/kg)

7.1 Isoquants, ridge lines and expansion paths

In order to develop an isoquant map, it was necessary to express N in terms of grain yield (Y), P and the various rainfall variables (Ri). Different values of Y, P and Ri were substituted into equation (4), and N was determined using the quadratic formula.

$$
\begin{aligned}
&0{,}080{*}N^2 - N{*}(4{,}248 + 1{,}620{*}P^{0{,}5}) \\
&\quad - N{*}(-0{,}276{*}R2 - 0{,}089{*}R5 + 0{,}096{*}R9 + 0{,}037{*}R15) \\
&\quad = 5090{,}477 - Y - 83{,}757{*}P + 864{,}087{*}P^{0{,}5} \\
&\quad - 9{,}474{*}R1 + 0{,}078{*}P{*}R1 \\
&\quad - 4{,}088{*}R2 - 0{,}184{*}P{*}R2 \\
&\quad - 13{,}433{*}R5 \\
&\quad - 32{,}355{*}R7 \\
&\quad - 4{,}049{*}R9 + 0{,}088{*}P{*}R9 \\
&\quad + 2{,}551{*}R15 \\
&\quad - 36{,}254{*}R17 \qquad (4)
\end{aligned}
$$

Figure 11 is the isoquant map obtained using a Median level of rainfall prior to and after planting.

Figure 11 shows that a constant quantity of P must be combined with greater applications of N in order to achieve higher grain yields. Isoquant maps for the other eight rainfall combinations exhibited the same characteristics as Figure 11. Furthermore, it was apparent that if the level of P was kept constant then the amount of N needed to attain a particular yield increased with precipitation levels.

An expansion path is a special type of isocline. Isoclines are lines that pass through points of equal slopes on an isoquant map (Doll and Orazem, 1984: 119). This slope is the marginal rate of substitution at that point on the isoquant. Along the expansion path, the marginal rate of substitution between two inputs must equal the inverse of the price ratio of the two inputs. In other words,

$$\frac{dN}{dP} = \frac{Cp}{Cn}$$

A change in the input price ratio will result in a different expansion path. However, only one expansion path has been plotted in Figure 11. This is based on a N price of R1,36/kg and a P price of R3,62/kg. The curvature of the expansion path implies that the proportion of N and P that must be used to achieve the least cost combination will vary among the yield levels.

Ridge lines are also a special form of isocline. They represent the limits of economic relevance. The upper ridge line represents the points where dY/dN is zero; along the lower ridge line dY/dP is zero. Ridge lines therefore indicate the maximum grain yield from one nutrient, given a fixed quantity of the other nutrient. Inside the ridge lines the isoquants have a negative slope, and N and P are substitutes. The maximum grain yield occurs at the point of intersection of the two ridge lines.

7.2 Marginal rates of substitution between N and applied P

The marginal rate of input substitution (M.R.S.) is represented by the slope (dN/dP) of the isoquant. It indicates the amount by which N must be reduced to maintain grain yield at a constant level when P is increased by one kg/hectare.

Whilst it was possible to calculate the M.R.S. for a variety of yields and rainfall combinations, the analysis was confined to those rainfall combinations that resulted in grain yields of 6000 kg/hectare and 7000 kg/hectare. A High level of growing-season rainfall, with any of the three levels of pre-planting precipitation, did not result in either of these grain yields. These M.R.S. values are presented in Table 9.

For a specific grain yield and for any particular level of rainfall before and after planting it can be seen that the M.R.S. declines up to the boundary of the ridge lines. In other words, each additional unit of P replaces a decreasing quantity of N within the ridge line boundaries. It is also apparent that the M.R.S. changes with the level of rainfall. For a fixed level of pre-planting rainfall and a particular application of P (say 40 kg/hectare), N must be decreased by larger and larger amounts with increases in the level of growing-season rainfall (the M.R.S. associated with this P level increases down Table 9). This is also the situation if varying amounts of pre-planting rainfall are used with a constant level of growing-season precipitation. Thus, it would appear that a given increment in P will replace an amount of N which increases in magnitude with a higher level of rainfall in either or both periods, given a specific grain yield level.

The M.R.S. declines as the quantity of P is increased, implying a decreasing rate of substitution between N and P. In some cases the M.R.S. assumes a positive value. These values represent nutrient combinations that lie beyond the ridge lines, where increasing amounts of P must be accompanied by increasing amounts of N in order to maintain production at a certain yield level.

The zone of economic rational production is that portion of the isoquant which has a negative slope. Since an increase in P (in this zone) permits a smaller application of N, whilst maintaining grain yield, these two nutrients can be termed technical substitutes.

7.3 Least cost combinations of N and applied P

Economic efficiency in factor-factor relationships is achieved when the M.R.S. is equal to or less than zero, and also when it is equal to the input price ratio, i.e.

$$\text{M.R.S.} = \frac{dN}{dP} = \frac{Cp}{Cn}$$

The least cost combination of N and P will depend upon the maize grain yield, the nutrient prices and the amount of precipitation before and after planting. A selection of least cost combinations are shown in Table 10.

Irrespective of the level of rainfall, the quantities of N and P that constitute the least cost combination increase as the grain yield rises from 6000 kg/hectare to 7000 kg/hectare. In the case of a given yield, these quantities increase with the level of precipitation in either or both rainfall periods. This trend implies that the least cost combination becomes progressively more expensive at higher grain yields and higher rainfall levels.

8. Summary and conclusions

The model developed for economic analysis exhibited decreasing marginal returns to both N and P, as well as a positive interaction between N and P. Seven growth stages of the maize plant, which were preceded by a pre-planting period, were included in the model. Three levels of rainfall were determined for each of these stages, namely Low, Median and High. The three levels of pre-planting rainfall were combined with the three levels of growing-season rainfall to determine the effect of rainfall on predicted yields.

The highest predicted maximum maize grain yield (9512 kg/hectare) was found to be associated with a Low pre-planting and a Low growing-season rainfall. All other combinations of rainfall periods resulted in lower maximum yields. The N and P requirements at the maximum predicted grain yield were lowest in the case of Low pre-planting and Low growing-season precipitation.

Maize yields were calculated for each of the rainfall combinations, keeping one nutrient fixed and increasing the other. For a given amount of the fixed nutrient, higher applications of the variable nutrient caused predicted yields to increase at a diminishing rate up to the maximum yield. It was concluded that predicted grain yields were adversely affected by increasing amounts of rainfall. This effect can be partly attributed to the nature of the "experimental" soil, which is prone to waterlogging. However, other studies indicate that rainfall could have an adverse effect on the growth and yield of the maize plant (Berry, 1989). The marginal physical products of N and P were also affected by different levels of precipitation. At Dundee higher levels of rainfall would reduce the marginal response of grain yields to N. However, the marginal yield response to P was improved by higher levels of rainfall.

One determinant of optimum yields is the price ratio between the inputs and output. An increase in the maize grain price relative to fertilizer prices resulted in higher predicted optimum grain yields, and higher applications of N and P. The rainfall level would also affect the optimum yield, due to the effect of precipitation on the marginal physical products. The highest optimum yield resulted from Low levels of pre-planting and growing-season precipitation. For the prices under consideration the largest profit was also predicted to occur for that rainfall combination. Other rainfall combinations resulted in smaller profits, due to lower incomes (smaller optimum yields) and higher total fertilizer costs (higher applications of N and P).

Within the boundaries of the ridge lines the isoquants displayed a diminishing rate of substitution between N and P. For a given grain yield, the M.R.S. increased with greater amounts of precipitation. Furthermore, for a given quantity of P the amount of N needed to achieve a certain yield increased with higher levels of rainfall. Greater amounts of precipitation also resulted in least cost combinations that comprised increased applications of N and P.

The overall conclusion is that nutrient levels and rainfall influence predicted grain yields and profitability. This would indicate a need to fertilize in accordance with expected rainfall levels, as well as with expected prices and marginal physical products. The fact that optimum yields, fertilization rates, costs and profits all varied with the level of rainfall enforces the statement by Thomas and Hanway (1968) that fertilizer recommendations need to take account of the influence of rainfall on maize grain yields. The function discussed in this article could allow the maize producer to determine his fertilization rates based on expected prices and rainfall. The producer could use his average rainfall data, or he could use data representing his expectations as to rainfall.

Table 9: Marginal rates of substitution of applied P for N for predicted maize grain yields (Y) of 6000 kg/ha and 7000 kg/ha for various levels of pre-planting (P-P) and growing season (G-S) rainfall (L = low, M = median and H = high)

YIELD AND LEVELS OF RAINFALL		MARGINAL RATES OF SUBSTITUTION OF APPLIED P FOR N						
		LEVEL OF APPLIED P (KG/HECTARE)						
		10	20	30	40	50	60	70
Y = 6000 KG/HA								
P-P	G-S							
L	L	-3,09	-0,96	-0,28	0,06			
M	L	-4,82	-1,52	-0,62	-0,20	0,04		
H	L	-26,23	-2,92	-1,31	-0,67	-0,33	-0,11	0,03
L	M	-5,29	-1,64	-0,69	-0,23	0,02		
M	M	-12,99	-2,60	-1,16	-0,56	-0,24	-0,03	0,11
H	M		-6,77	-2,28	-1,21	-0,70	-0,41	-0,22
Y = 7000 KG/HA								
P-P	G-S							
L	L	-6,03	-1,61	-0,57	-0,10	0,18		
M	L		-2,86	-1,13	-0,47	-0,11	0,11	
H	L			-2,77	-1,27	-0,66	-0,32	-0,11
L	M		-3,14	-1,22	-0,51	-0,14	0,09	
M	M		-9,91	-2,30	-1,07	-0,53	-0,21	-0,01
H	M				-2,73	-1,37	-0,79	-0,47

Table 10: Least cost combinations of N and applied P for predicted maize grain yields of 6000 kg/ha and 7000 kg/ha using various rainfall levels (Cn = R1,36/kg and Cp = R3,62/kg)

	GROWING-SEASON RAINFALL			
	LOW		MEDIAN	
	N	P	N	P
	(Kg/Hectare)			
YIELD = 6000 KG/HECTARE				
PRE-PLANTING RAINFALL:				
LOW	2,7	11,1	39,4	15,2
MEDIAN	22,3	14,5	57,9	19,7
HIGH	47,6	21,0	80,4	28,0
YIELD = 7000 KG/HECTARE				
PRE-PLANTING RAINFALL:				
LOW	38,4	15,5	76,5	21,5
MEDIAN	59,3	20,6	95,4	28,4
HIGH	85,0	30,4	116,6	40,3

However, it must be remembered that the function was based on a hydromorphic soil type, so it might have limited application in other areas. This observation does not detract from the importance of including precipitation in maize grain yield predictions. There seems to be a need to develop a series of yield functions for different environmental conditions. Furthermore, future studies could investigate the suitability of other climatic measures as explanatory variables.

Farmers with soils similar to those at the Dundee Research Station are advised to apply N at a rate of 160 kg/hectare, and this recommendation is based on long term trials (Farina, 1989). It is interesting to note that the optimum level of N associated with Median levels of rainfall is 175 kg/hectare (Table 8), and this is acceptably close to the above recommendation. P recommendations are made only in respective of the amount of P required to raise the P soil test reading to the optimum level of 30 mg/l. The required P application will depend upon the soil test reading prior to fertilization. The least cost combination of N and P associated with Median levels of rainfall are similar to those suggested by other long term field experiments (Farina, 1989).

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