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DERIVATION OF APPROPRIATE FUNCTIONS FOR THE ECONOMIC ANALYSIS OF MAIZE YIELD RESPONSES TO FERTILIZER AND RAINFALL VARIATIONS AT DUNDEE

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ABSTRACT

This paper describes the derivation of two production functions showing the dependence of maize grain yields on fertilization and rainfall. The data were taken from maize fertilizer trials conducted for eleven years at the Dundee Research Station. The models lend themselves to economic analysis. Fertilization was represented by either applied nitrogen (N) and applied phosphate (Pa), or N and post-fertilization P soil test (Pt) readings. The functions conform to the Law of Diminishing Returns, and show a positive interaction between the two nutrients. Total rainfall was determined for a pre-planting period and a sequence of consecutive growth stages, extending from sowing to physiological maturity. The models include linear rainfall terms and linear rainfall-nutrient interaction terms. The adjusted R^2 values were 0,897 for the model using P applied, and 0,805 for the P soil test model.

OPSOMMING

Hierdie referaat beskryf die afleiding van twee produksiefunksies wat mieliegraanopbrengste se afhanklikheid van bemesting en reënval aantoon. Die data is geneem van mieliebemestingproewe wat oor elf jaar by die Dundee Navorsingstasie uitgevoer is. Die modelle leen hulself tot ekonomiese ontleding. Bemesting is verteenwoordig deur of toegediende stikstof (N) en toegediende fosfaat (Pa), of N en na-bemesting P-grondtoets- (Pt-) lesings. Die funksies is in ooreenstemming met die Wet van Dalende Meeropbrengs, en toon 'n positiewe interaksie tussen die twee voedingstowwe. Totale reënval is vasgestel vir 'n vooraanplantingstydperk en 'n reeks opeenvolgende groeistadia vanaf saai tot fisiologiese wasdom. Die modelle sluit lineêre reënvalterme en lineêre reënval-voedingstof-interaksieterme in. Die aangepaste R^2-waardes was 0,897 vir die model wat P toegedien gebruik, en 0,805 vir die P-grontoetsmodel.

1. INTRODUCTION

Maize is a major South African cash and fodder crop, and extensive research has been conducted to find ways of improving the quality and quantity of grain yields. A primary aspect of the research centres on the fertilization of the maize crop, and the response of maize to different applications of nitrogen (N) and phosphates (P).

However, the growth and yield of the maize plant is not determined solely by the level of fertilization and soil nutrient status. There is ample evidence that the growth of the maize plant and ultimately the grain yield are influenced by climatic conditions. Moreover, the effect of weather depends upon the stage of development of the maize plant. Consequently, the tendency has been to relate climatic factors to a number of physiological growth stages. A variety of growth stages have been utilized by plant physiologists (Hanway, 1971; Waldren 1983). Jones *et al* (1986) favour the following growth stages, which are preceded by a period before planting:

(a) pre-sowing;
(b) sowing to germination;
(c) germination to emergence;
(d) emergence to the end of the juvenile stage;
(e) end of the juvenile stage to tassel initiation;
(f) tassel initiation to silking;
(g) silking to the beginning of grain filling;
(h) grain filling; and
(i) end of grain filling to physiological maturity.

Some researchers recommend the inclusion of climatic factors in yield-nutrient response functions. Thomas and Hanway (1968) state that fertilizer recommendations based solely on yield-nutrient relationships fail to recognize that climate is a contributor to the final crop yield. Climate can affect the optimum fertilization rate in two ways. The first is by rainfall and temperature variations which influence crop growth. Secondly, rainfall affects the availability of N, whilst P availability is temperature dependent. Consequently, as Asghari and Hanson (1984) mention, climate will influence the effect that fertilizers have on plant growth and yields. Furthermore, climatic influences should be related to the various developmental phases of the maize plant (Ramadas, 1970).

A decision of profound consequence in maize production centres on the determination of fertilization rates. Kassier and Mallet (1966) point out that this involves two sets of relationships, namely physical production relationships and those economic principles governing production and resource allocation. The correct amount of fertilizer, for a specific set of conditions, will depend on: (a) the expected yield; (b) the expected price of fertilizer and maize; (c) the capital position of the farmer; and (d) the probable returns from alternative uses of this capital (Farina *et al*, 1975). The calculation of optimal fertilizer rates requires the existence of a mathematical function that describes the relationship between fertilization and yields. In estimating crop production functions, the economic analyst is interested in deriving the best response surface. The estimated yield function is used to determine the value of the marginal product of the input, and thereby the optimum level of production. There are many models to choose from, each with its advantages and disadvantages (Colwell, 1978 and 1981). When developing models, Fitts (1974) considers it essential that two important aspects be kept in mind; (a) the ability of the model to predict yields, and (b) the ease of usage of the model for economic analysis.

The objective of this study was to develop two economically useful production functions relating maize grain yields to (a) the amount of applied N and P and (b) the amount of applied N and post-fertilization P soil test readings. An attempt was also made to include the effect of rainfall on grain yields. Rainfall was related to various non-overlapping growth stages of the maize plant.

2. CHOICE OF PRODUCTION FUNCTION

The choice of a mathematical function may be initially restricted by the availability of estimation procedures. Consequently, functions that can be estimated using ordinary least squares regression methods have been popular (Upton, 1979). As far as fertilizer experiments are concerned, the choice of production functions is restricted to those equations which display diminishing returns (Munson and Doll, 1959; Sparrow, 1979). In addition to this requirement, experimental evidence may indicate that the function should exhibit a maximum yield, at a unique combination of inputs, and the yield should decline after the maximum is reached. Taking this feature into account, the choice is limited to the ordinary and inverse quadratic equations, the transcendental, and the power parabola. For a detailed exposition of various functions see Berry (1989).

A review of overseas literature indicates that the majority of these functions have been utilized by agronomists and economists (for example, Abraham and Rao 1966; Engelstad and Doll, 1961; Heady *et al*, 1961; Hexem *et al*, 1976; Jain and Goel 1980; Jonsson 1974; Pesek *et al*, 1967; Swanson *et al*, 1973). The consensus of opinion is that quadratic and square-root polynomials compare favourably with other types of functions such as the Cobb-Douglas, the transcendental, and inverse polynomials. The question of simplicity is considered to be important by some authors. As Jonsson (1974) points out, the power parabola requires numerous iterative steps to determine the exact form of the equation. The inverse functions are also more complex to fit than the ordinary polynomials. It seems reasonable to expect a clear advantage in terms of goodness of fit if the disadvantages of cumbersome fitting and interpretation are to be warranted (Jonsson, 1974). As far as computational convenience is concerned, polynomials are often preferred because (Colwell, 1978; Heady and Dillon, 1961):

(a) they are easy to fit using standard multiple regression techniques;
(b) they can easily accommodate interaction effects; and
(c) they allow direct calculations of optimal application rates under a variety of circumstances.

South African researchers have also employed the quadratic and square-root functions. Comparative studies show that these polynomials can adequately represent the relationship between maize grain yields and the level of fertilization (Allison, 1977; Farina *et al*, 1975; Farina *et al*, 1980; Farina and Mapham, 1973; Kassier and Mallet, 1966; Korentajer *et al*, 1987; Mapham, 1975; Mapham and Farina, 1974; Nienaber and Groenewald, 1979; Nieuwoudt and Behrmann, 1976).

Following various evaluations it was decided to limit the choice of a response function to polynomial models. Other functions based on the Law of the Minimum were ignored because of the difficulties of fitting them. Furthermore, the selected model would be used to determine various economic characteristics, so computational convenience was an important consideration.

3. FERTILIZATION RATES, MAIZE GRAIN YIELDS AND CLIMATIC DATA

The data used in the study were obtained from maize fertilizer trials conducted at the Dundee Research Station from 1973/74 to 1985/86. As a consequence, the characteristics of both cross-sectional and time-series data must be considered. However, it can be assumed that the experimental plots are similar and the same external conditions applied to each plot.

The Dundee Research Station in Northern Natal falls into Phillip's bioclimatic group 8. The "experimental" soil was an Avalon sandy loam. This type of soil is usually moderately deep and has, at best, a weakly developed structure and a low organic matter content. Hydromorphism becomes progressively more marked. Hydromorphic soils do not drain well, and this can result in denitrification in the presence of excess moisture (Farina, 1970). Furthermore, due to its low clay percentage, the soil at Dundee would be expected to have a relatively high nitrogen requirement (Farina and Venter, 1984). The response to nitrogen is generally poorer the higher the clay content of the soil (Mengel and Kirkby, 1987).

3.1 Fertilization rates

The experiment was a twice replicated 5^2 N, P factorial. The nutrient application rates are shown in Table 1. All the P and half the N were broadcast and disced into the soil immediately prior to planting. The balance of N was applied as a topdressing. Topsoil samples were collected 14 days after fertilization and analysed for P. Maize seed was planted at populations of 38000 per hectare throughout the trial. Consequently, plant population was taken as constant.

Table 1: Application rates of N and P (kg/hectare) for maize trials at the Dundee research station

SEASON OF TRIAL							
1973/74		1974/75-1982/83		1983/84		1984/85-1985/86	
N	P	N	P	N	P	N	P
50	0	50	0	0	0	50	0
100	20	100	17,5	50	17,5	100	17,5
150	40	150	35	100	35	150	35
200	80	200	70	150	70	200	70
250	160	250	140	200	140	250	140

3.2 Maize grain yield data

Plants were harvested at maturity and grain yields were expressed at a moisture content of 12,5%. The annual mean yields of maize grain over all plots are listed in Table 2, together with the coefficient of variation (C.V.) for each year.

Table 2: Annual mean maize grain yields (kg/hectare) at the Dundee research station

YEAR	MEAN YIELD	C.V. (%)	YEAR	MEAN YIELD	C.V. (%)
1973/74	7276,2	13,14	1980/81	5955,0	13,18
1974/75	4986,2	16,24	1981/82	2918,4	34,69
1975/76	5507,6	16,85	1982/83	3593,4	25,17
1976/77	6138,8	8,23	1983/84	4166,4	10,25
1977/78	6876,9	8,29	1984/85	6482,8	14,52
1978/79	5208,1	9,51	1985/86	6945,6	13,99
1979/80	7351,5	10,29			

3.3 Rainfall data for various physiological growth stages of maize

The physiological growth phases listed in section 1 (Introduction) were combined to form other growth periods. For example, (a) sowing to emergence and emergence to the end of the juvenile stage were coupled to give a period from sowing to the end of the juvenile stage and (b) emergence to the end of the juvenile stage and the end of the juvenile stage to tassel initiation were combined to form a period extending from emergence to tassel initiation.

Daily rainfall records for the Dundee Research Station were used to determine the total rainfall for each of the possible growth stages. Table 3 lists the highest, lowest and mean rainfall in each of the growth periods, for the 13 years of the trial.

Table 3: Rainfall at the Dundee Research Station (1973/74 to 1985/86 for various physiological growth stages of maize

PHYSIOLOGICAL GROWTH STAGE AND RAINFALL PERIOD (Ri)	TOTAL RAINFALL (MM)		
	LOWEST	HIGHEST	MEAN
R1 : PRE-PLANTING	30,2	226,6	129,5
R2 : SOWING - EMERGENCE	0,0	101,7	24,9
R3 : SOWING - END OF THE JUVENILE STAGE	49,3	236,1	104,3
R4 : SOWING - TASSEL INITIATION	69,0	256,8	125,8
R5 : EMERGENCE - END OF THE JUVENILE STAGE	30,8	139,2	79,4
R6 : EMERGENCE - TASSEL INITIATION	40,5	203,3	101,0
R7 : END OF THE JUVENILE STAGE - TASSEL INITIATION	0,0	96,8	21,5
R8 : TASSEL INITIATION - SILKING	96,7	348,8	203,5
R9 : TASSEL INITIATION - BEGINNING OF GRAIN FILLING	124,6	460,8	257,7
R10 : TASSEL INITIATION - END OF GRAIN FILLING	224,2	674,0	420,9
R11 : TASSEL INITIATION - PHYSIOLOGICAL MATURITY	228,3	676,7	427,0
R12 : SILKING - BEGINNING OF GRAIN FILLING	4,3	153,5	54,2
R13 : SILKING - END OF GRAIN FILLING	120,1	350,0	217,4
R14 : SILKING - PHYSIOLOGICAL MATURITY	124,2	350,0	223,5
R15 : BEGINNING OF GRAIN FILLING - END OF GRAIN FILLING	70,7	274,8	163,2
R16 : BEGINNING OF GRAIN FILLING - PHYSIOLOGICAL MATURITY	71,6	277,5	169,3
R17 : END OF GRAIN FILLING - PHYSIOLOGICAL MATURITY	0,0	20,4	6,1
SOWING - PHYSIOLOGICAL MATURITY	353,4	899,7	552,8
PRE-PLANTING - PHYSIOLOGICAL MATURITY	451,5	980,3	682,3

3.4 A comparison between maize grain yields and rainfall for the Dundee research station

Figure 1 summarizes the mean maize yield data in Table 2 and the total mean rainfall in Table 3. Each of the years (1 to 13) represents a season; i.e Year 1 is 1973/74, Year 2 is 1974/75 and so forth. For each year there are three columns. The left-most column represents the mean grain yield for that year. The total amount of rainfall from sowing to maturity (RA) is indicated in the middle column. The last column represents the total rainfall from pre-planting to maturity (RB).

4. DEVELOPMENT OF RESPONSE FUNCTIONS

The choice of a response function was restricted to quadratic or square-root models, or a combination of these two forms.

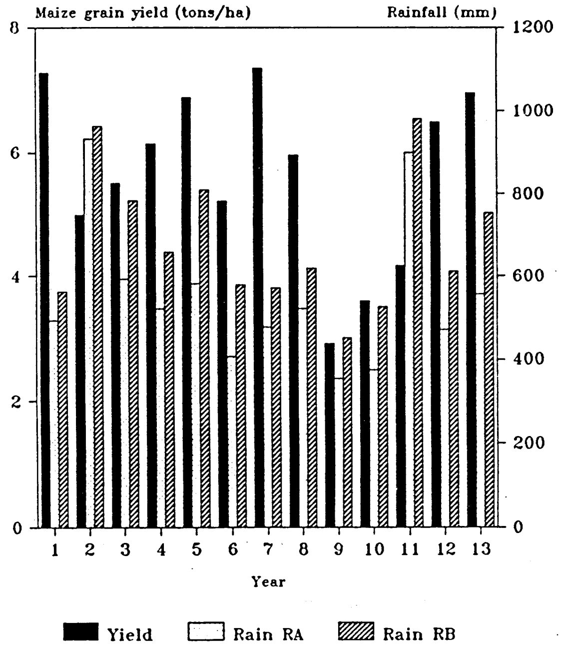

Figure 1: Annual mean maize grain yields for all experimental plots and total rainfall for the growing season (RA) and from pre-planting and maturity (RB) for the Dundee Research Station

4.1 Polynomial functions: Choice of fertilizer terms

The variables are denoted as follows:

N = applied nitrogen (kg/hectare);
Pa = applied phosphate (kg/hectare);
Pt = post-fertilization phosphate soil test value (mg/l); and
Y = maize grain yield (kg/hectare).

Yields were regressed on N and Pa or Pt, using the following combinations:

(a) N, N^2, and Pa, Pa^2, or Pt, Pt^2
(b) N, N^2, and $Pa, Pa^{0.5}$, or $Pt, Pt^{0.5}$
(c) $N, N^{0.5}$, and Pa, Pa^2, or Pt, Pt^2
(d) $N, N^{0.5}$, and $Pa, Pa^{0.5}$, or $Pt, Pt^{0.5}$

These initial regressions were also used to determine the best interactive term. The results of these regressions indicated that maize grain yields should be related to:

(a) $N, N^2, Pa, Pa^{0.5}$, and $N*Pa^{0.5}$ (Model A)
(b) $N, N^2, Pt, Pt^{0.5}$, and $N*Pt^{0.5}$ (Model B).

4.2 Yearly regression analysis

The mean grain yields in 1981/82 and 1982/83 were 2918,4 kg/hectare and 3593,4 kg/hectare respectively. These yields were significantly lower than the 13-year average of

5646,7 kg/hectare. The adjusted R^2 values shown in Table 4 were obtained when Model A and Model B were applied to the annual maize yield data.

Table 4: Adjusted R^2 (%) values for annual maize grain yields and fertilizer data

YEAR	MODEL A	MODEL B	YEAR	MODEL A	MODEL B
1973/74	94,8	90,4	1980/81	86,0	63,6
1974/75	89,9	84,6	1981/82	25,7	10,0
1975/76	94,6	91,7	1982/83	60,2	38,3
1976/77	95,7	88,4	1983/84	93,9	88,1
1977/78	97,5	93,9	1984/85	90,3	78,4
1978/79	95,8	83,0	1985/86	90,2	70,7
1979/80	93,7	83,6	OVERALL	57,4	46,6

In light of the very low adjusted R^2 values for 1981/82 and 1982/83 it was decided to eliminate these years from the study. The overall adjusted R^2 values were re-calculated using the data for the remaining 11 years, and they increased to 76,3% for Model A and 65,0% for Model B.

4.3 Inclusion of rainfall periods

The next stage of the analysis entailed finding which combination of rainfall periods (Ri) resulted in the best function when they were added to Model A and Model B. The 17 possible rainfall periods were combined into a non-overlapping sequence, extending from pre-planting to maturity; for example, R1, R2, R5, R7, R8, R12, R15, and R17, or R1, R2, R5, R7, R9, and R16. This resulted in 32 possible regressions.

The adjusted R^2 values of the regressions based on Model A ranged from 87,4% to 83,0%. The 32 possible regressions in the case of Model B had adjusted R^2 values ranging from 76,2% to 70,3%. At this stage the adjusted R^2 values were used to chose 10 Pa models and 10 Pt models for further investigation.

4.4 Temperature and heat units

The suitability of temperature and heat units as explanatory variables was investigated in the early stages of the analysis. Two temperature variables are considered by plant physiologist to be relevant, namely, daily maximum temperature and daily mean temperature. The following method was used to calculate heat units:

$$\text{Heat units} = \sum_{i=1}^{n} \frac{\text{(Max. + Min.) daily temp.}}{2} - \text{Base temp.}$$

where n is the number of days in a particular growth stage (Coelhs and Dale, 1980; Crane *et al*, 1977; Cross and Zuber, 1972). The base temperature from sowing to emergence was assumed to be 10°C. For the remainder of the growing season the base temperature was taken as 8°C (Jones *et al*, 1986). Temperature and heat units prior to planting were not included in the regression models. The adjusted R^2 values and Residual Mean Square (R.M.S.) values of the regressions using either temperature variables or heat units were lower than those obtained with rainfall. Consequently, it was felt that neither temperature nor heat units were as good explanatory variables as rainfall when their respective regression statistics were compared. Furthermore, it would seem that many more farmers maintain rainfall records rather than detailed daily temperatures (Clemence, 1987). For these reasons, the remainder of the study concentrated on the rainfall models discussed so far.

4.5 Selection of rainfall - fertilizer interaction terms

The next step in the analysis involved selecting significant interaction terms between the various precipitation variables and N, Pa, and Pt. A step-wise regression was considered to be an appropriate means of determining the most suitable interaction terms. This implied that not all the interaction terms would necessarily be included in the new models.

4.5.1 Interaction terms and models using applied P

Step-wise regressions were performed for each of the 10 models that were retained for further study. Whilst the inclusion of interaction terms improved the goodness of fit of all the models, it was decided to retain the five best models (on the basis of adjusted R^2 values and R.M.S. values) for further analysis. Table 5 contains some of the statistical information given in the Analysis of Variance for these five models.

Table 5: Analysis of variance for selected models using applied N and P

	MODEL				
	A1	A2	A3	A4	A5
RAINFALL PERIOD (Ri)	1, 2, 5, 7, 8, 12, 15, 17	1, 2, 5, 7, 9, 15, 17	1, 2, 6, 8, 12, 15, 17	1, 2, 6, 9, 15, 17	1, 2, 5, 7, 8, 13, 17
N*Ri	1, 2, 5, 7, 8,12	2, 5, 9, 15	2, 8	2, 9	2, 5, 7, 8
Pa*Ri	1, 2, 8, 15	1, 2, 9	1, 2, 8, 15	1, 2, 9	1, 2, 8, 13
DEGREES OF FREEDOM					
REGRESSION	23	19	18	16	20
RESIDUAL	251	255	256	258	254
TOTAL	274	274	274	274	274
REGRESSION M.S.	69901145	84436660	88814483	99713556	79853496
RESIDUAL M.S.	662917	665968	685382	692642	697041
TOTAL M.S.	6474885	6474885	6474885	6474885	6474885
ADJUSTED R^2 (%)	89,76	89,71	89,41	89,30	89,23
F-VALUE	105,45	126,79	129,58	143,96	114,56

4.5.2 Interaction terms and models using P soil test

The step-wise method of regression was used to select the best sub-set of interaction terms. After this exercise was completed, five of the 10 possible models were retained on the basis of adjusted R^2 and R.M.S. values. Table 6 shows details extracted from the Analysis of Variance, together with the best interaction terms, for these five models.

4.6 Final choice of models

The models using Pa all had fairly similar adjusted R^2 and R.M.S. values, indicating that no one model fitted the data significantly better than the other models. A similar conclusion was made concerning the five models using Pt.

It was decided to calculate the PRESS statistic for each of the models. The PRESS statistic, or predicted residual sum-of-squares, is a measure of a model's predictive ability. A model with a lower PRESS statistic is preferred to one that has a higher PRESS statistic.

For the chosen models, predicted Y values ($\hat{Y}$) were plotted against the error terms. These plots did not show any trends. Derivation of the Durbin-Watson statistic to test for autocorrelation in the residuals was not possible because the data were of a cross-sectional / time-series nature.

Table 6: Analysis of variance for selected models using applied N and P soil test

	MODEL				
	B1	B2	B3	B4	B5
RAINFALL PERIOD (Ri)	1, 2, 5, 7, 8, 13, 17	1, 2, 5, 7, 9, 15, 17	1, 2, 5, 7, 8, 12, 15, 17	1, 3, 7, 8, 13, 17	1, 2, 6, 8, 13, 17
N*Ri	2, 7, 8, 13	1, 2, 5, 7, 9	1, 2, 7, 8, 15	1, 7, 8, 13	2, 8
Pt*Ri	2, 7, 8, 13	2, 7, 9, 15	7, 15, 17	3, 7, 8, 13	1, 8, 13
DEGREES OF FREEDOM					
REGRESSION	20	21	21	19	16
RESIDUAL	254	253	253	255	258
TOTAL	274	274	274	274	274
REGRESSION M.S.	72661016	69192388	68883841	75395999	88477288
RESIDUAL M.S.	1263378	1269084	1294695	1339586	1389464
TOTAL M.S.	6474885	6474885	6474885	6474885	6474885
ADJUSTED R^2 (%)	80,49	80,40	80,00	79,31	78,54
F-VALUE	57,51	54,52	53,21	56,28	63,68

4.6.1 Models using applied P

Model A1 had an adjusted R^2 value of 89,76% and a PRESS statistic of 201025643. The adjusted R^2 value of Model A2 was slightly lower at 89,71%. However, of all the models, this Model had the lowest PRESS statistic of 198833691. Furthermore, Model A2 had fewer explanatory variables than Model A1. Consequently, Model A2 was selected as the model most suitable for representing the relationship between maize grain yield, nitrogen applied (N), phosphate applied (Pa) and rainfall. Table 7 lists all the relevant details of Model A2. The coefficient of the linear term for N is not significant. However, the coefficients of N^2, Pa, and $Pa^{0.5}$ are significant at the 1% level. The signs of these four coefficients conform to the Law of Diminishing Returns, upon which this study was based. The highly significant coefficient of the cross-product term ($N*Pa^{0.5}$) is positive, implying a positive reaction between the two nutrients.

Table 7: Regression analysis and analysis of variance for the selected model using applied N and P

	REGRESSION COEFFICIENTS			
	ESTIMATE	STANDARD ERROR	T-VALUE	PROB > |T|
CONSTANT	5090,4771	852,1680	5,974	0,0001
N	4,2477	5,4500	0,779	0,4365
N^2	-0,0804	0,0114	-7,025	0,0001
Pa	-83,7567	6,5186	-12,849	0,0001
$Pa^{0.5}$	864,0873	46,0139	18,779	0,0001
$N*(Pa^{0.5})$	1,6201	0,1725	9,390	0,0001
R1	-9,4736	1,7359	-5,457	0,0001
R2	-4,0877	8,2556	-0,495	0,6209
R5	-13,4327	4,7749	-2,813	0,0053
R7	-32,3548	5,0714	-6,380	0,0001
R9	-4,0487	3,1477	-1,286	0,1995
R15	2,5513	2,5639	0,995	0,3206
R17	-36,2538	10,2436	-3,539	0,0005
N*R2	-0,2760	0,0480	-5,749	0,0001
N*R5	-0,0887	0,0279	-3,176	0,0017
N*R9	0,0955	0,0178	5,367	0,0001
N*R15	0,0368	0,0137	2,689	0,0076
Pa*R1	0,0775	0,0189	4,100	0,0001
Pa*R2	-0,1840	0,0501	-3,670	0,0003
Pa*R9	0,0876	0,0181	4,832	0,0001
RESIDUAL MEAN SQUARE				665968
DEGREES OF FREEDOM				255
ADJUSTED R^2 (%)				89,71
F-VALUE				126,788

The following rainfall periods have negative coefficients which are significant at the 1% level:

R1 :	Pre-planting
R5 :	Emergence to the end of the juvenile stage
R7 :	End of the juvenile stage to tassel initiation
R17 :	End of grain filling to physiological maturity.

Three rainfall periods have associated coefficients which are not significant, namely R2 (sowing to emergence), R9 (tassel initiation to the beginning of grain filling) and R15 (beginning to the end of grain filling). The cross-product terms N*R2, N*R5 and Pa*R2 have negative coefficients, whilst those of N*R9, N*R15, Pa*R1 and Pa*R9 are positive. All coefficients of the rainfall-fertilizer interaction terms are significant at the 1% level.

4.6.2 MODELS USING P SOIL TEST

Model B1 had an adjusted R^2 value of 80,49%, a PRESS statistic of 372419858, and 20 explanatory variables. Model B2, with 21 variables, gave an adjusted R^2 value of 80,40% and a PRESS statistic of 377533176. Model B3 also had 21 variables, whilst B4 and B5 had 19 and 16 variables respectively. However, these last three models all had lower adjusted R^2 values and higher PRESS statistics. Hence Model B1 was chosen as the model best able to explain the dependence of maize grain yields on nitrogen applied (N), phosphate soil test values (Pt) and rainfall. Table 8 lists all the relevant details of Model B1.

Table 8: Regression analysis and analysis of variance for the selected model using applied N and P soil test

	REGRESSION COEFFICIENTS			
	ESTIMATE	STANDARD ERROR	T-VALUE	PROB > |T|
CONSTANT	2611,9541	1299,8370	2,009	0,0455
N	21,2847	8,4364	2,523	0,0122
N^2	-0,0904	0,0158	-5,729	0,0001
Pt	-187,8854	18,4383	-10,190	0,0001
$Pt^{0.5}$	1859,0463	111,9161	16,611	0,0001
$N*(Pt^{0.5})$	2,1706	0,3248	6,682	0,0001
R1	-6,6433	1,7707	-3,752	0,0002
R2	-13,6066	11,7378	-1,159	0,2475
R5	-27,6877	3,4303	-8,072	0,0001
R7	-21,4235	13,3436	-1,606	0,1096
R8	0,9474	3,9550	0,240	0,8109
R13	0,1986	4,9738	0,040	0,9682
R17	-80,0857	17,1541	-4,669	0,0001
N*R2	-0,1090	0,0632	-1,723	0,0861
N*R7	0,1934	0,0711	2,720	0,0070
N*R8	0,0645	0,0207	3,114	0,0021
N*R13	-0,0557	0,0261	-2,137	0,0336
Pt*R2	-0,4192	0,1206	-3,476	0,0006
Pt*R7	-0,8513	0,1490	-5,715	0,0001
Pt*R8	0,1223	0,0455	2,688	0,0077
Pt*R13	0,2362	0,0538	4,391	0,0001
RESIDUAL MEAN SQUARE				1263378
DEGREES OF FREEDOM				254
ADJUSTED R^2 (%)				80,49
F-VALUE				57,513

The coefficients of N^2, Pt and $Pt^{0.5}$ are significant at the 1% level. The Law of Diminishing Returns is evident in the signs of these coefficients and that of N, which is not significant. The interaction between N and Pt is positive, and the coefficient is significant at the 1% level. Three rainfall periods have coefficients which are significant at the 1% level, namely R1 (preplanting), R5 (emergence to the end of the juvenile stage) and R17 (end of grain filling to physiological maturity). The remaining rainfall periods do not have significant coefficients, namely R2 (sowing to emergence), R7 (end of the juvenile stage to tassel initiation), R8 (tassel initiation to silking) and R13 (silking to the end of grain filling). The coefficients of the interaction terms N*R2, N*R13, Pt*R2 and Pt*R7 are negative and those of N*R7, N*R8, Pt*R8 and Pt*R13 are positive. Two interactions terms were not significant at the 1% level, namely N*R2 (10% level) and N*R13 (5% level).

5. SUMMARY AND CONCLUSIONS

Initial regressions indicated that the model using applied P should contain the terms N, N^2, Pa, $Pa^{0.5}$, and $N*Pa^{0.5}$. The best terms for the P soil test model were N, N^2, Pt, $Pt^{0.5}$, and $N*Pt^{0.5}$. The lower adjusted R^2 value of the Pt model can be partly attributed to the experimental errors involved in soil sampling and soil tests. The positive coefficient of the nutrient cross-product term implies that, for a given amount of a nutrient, a higher yield is predicted for increased levels of the other nutrient. These observations verify claims that the beneficial effects of a nutrient are dependent upon, and can be enhanced by, adequate concentrations of other nutrients. Furthermore, they highlight the need for a balanced fertilization policy.

Rainfall was a superior explanatory variable when compared with maximum and mean temperatures, and with heat units. The model using applied P included the following rainfall periods: pre-planting, sowing to emergence, emergence to the end of the juvenile stage, end of the juvenile stage to tassel initiation, tassel initiation to the beginning of grain filling, the stage of grain filling, and the end of grain filling to physiological maturity. The model based on applied N and P soil test also contained seven rainfall periods. The first four periods were the same as the first four periods of the model using applied P. The remaining periods were: tassel initiation to silking, silking to the end of grain filling, and the end of grain filling to physiological maturity. Interaction terms between the different rainfall periods of each model and the relevant nutrients were selected using a step-wise regression. In comparison to similar studies the adjusted R^2 values of the two models are acceptable, taking into account the complex nature of maize production.

The negative coefficients of the linear rainfall variables would seem, at first glance, to be questionable. However, an examination of the data supports the results of the study. The mean maize grain yield for the 11-year period was 6081,4 kg/hectare, and the mean rainfall for the growing season (from sowing to maturity) was 587,1 mm. Six seasons produced above-average mean yields, and five of these seasons had a growing-season rainfall that was below the mean for this period. The remaining five seasons had grain yields greater than the 11-year average, and in three of these instances the rainfall during the growing-season exceeded 587,1 mm. Thus, the initial indications were that above average rainfall during the growing season would result in below-average maize grain yields. Other studies indicate that excessive rainfall can result in reduced grain yields (Aldrich *et al*, 1976; Allison and Wilson, 1976; Brown, 1977; Eck, 1984; Lembake *et al*, 1982; Mengel and Kirkby, 1987; Nel and Smit, 1978; Shaw, 1977; Thomas and Hanway, 1968; Thompson, 1969; van der Paauw, 1962).

A further explanation relates to the hydromorphic soils at the Dundee Research Station. This type of soil becomes waterlogged if excessive amounts of precipitation occur, and this hampers root formation. In addition, there are the hazards of denitrification and leaching at times when the nutrient requirements of the maize plant are high.

The usefulness of the two models lies in the possibility that they can be used to determine fertilization rates that will result in maximum profits. These rates can be predicted for any given combination of fertilizer and maize prices by equating the value of the marginal physical product of fertilizer to the price of fertilizer. The models would also allow for the farmer's expectations as regards the probability of poor, average or good rainfall prior to and during the growing season. Rainfall can also be accounted for in the determination of the least cost combination of the two nutrients, the elasticities of production, and so forth.

The determination of applied N and applied P is obtained directly from either of the models. The use of the P soil test function is more complex. The difference between the optimum Pt level and the current Pt level will indicate the amount by which the Pt level must be raised. It is then possible to determine the quantity of P that must be applied to raise the soil test reading by the required amount. Although the function using Pt had a lower R^2 value, the use of soil test readings is advocated by soil scientists. As early as 1933, Spillman (Jensen and Pesek, 1959) concluded that a production model which excluded soil fertility levels could be misleading. In order to make realistic recommendations specific to farmers' fields, it is desirable that the level of soil nutrients be included in a yield equation (Koch, 1970; Mombiela *et al*, 1981; Sumner and Farina, 1986). Furthermore, P can be accumulated in the soil, and this raises the question of the economics of carry-over effects. The use of soil test readings takes into account the level of P already present in the soil and, consequently, the farmer would be advised to apply only that amount of P needed to raise the soil test level to the optimum reading. As far as this study is concerned, nitrogen soil test values are not available because of the lack of a method of estimating either residual fertilizer nitrogen or native soil nitrogen (Farina and Venter, 1984).

Whilst acceptable results were obtained in this study, it is possible that better predictive functions could be obtained using other, more complex forms instead of a mixed quadratic and square-root function. Furthermore, rainfall is only one aspect of climate, and other studies have obtained good results using climatic measures such as evaporation (Bates, 1955; Chen and da Fonseca 1980), daylength (Chang, 1981), solar radiation (Hatfield, 1975; Phipps *et al*, 1975), rainfall-temperature indices (Dubey, 1970; Oury, 1965), and drought-days (Parks and Knetsch, 1959 and 1960; Sopher *et al*, 1973).

It is also important that the nature of the experimental soils be taken into account. Different functions might be needed for different soil types and climatic conditions to adequately describe the complex interactions between fertilization, climate and maize growth and yields. However, for all practical purposes, the models derived here lend themselves to economic analysis, which is the subject of another article (Berry and Ortmann, 1989).

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