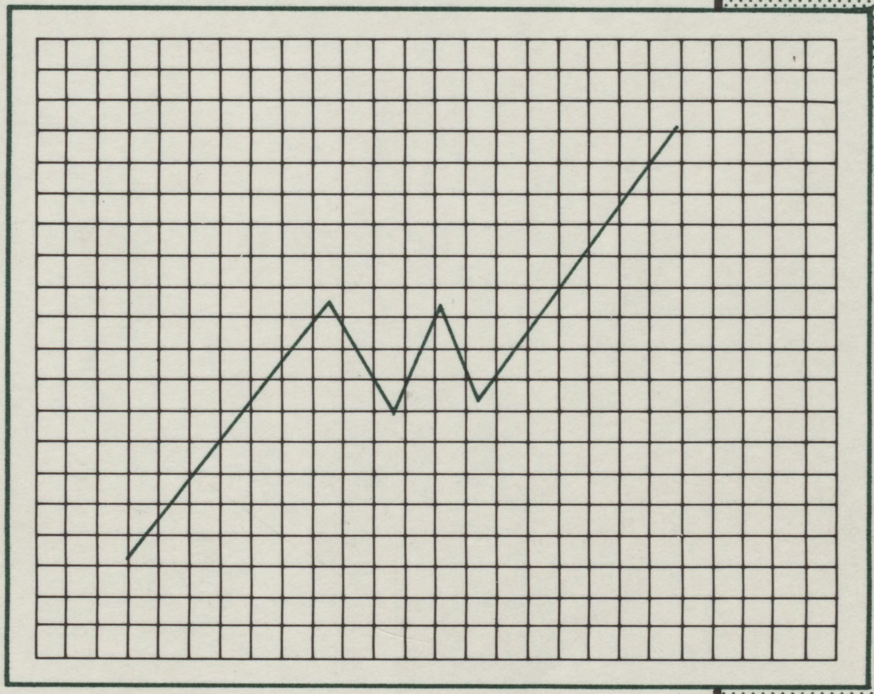

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ANALYTICAL TOOLS IN PRODUCTION ECONOMICS*

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ABSTRACT

Some of the major mathematical programming techniques that have been developed since the application of linear programming in farm planning during the early 1950s are evaluated. Most of these models have attempted to incorporate risk, with varying degrees of success. Quadratic programming (QP) is restricted by limited availability of suitable algorithms. MOTAD is computationally efficient and provides acceptable solutions compared with those derived using QP. Utility-efficient programming appears to have considerable potential.

Of the safety-first models, Target MOTAD seems to be the most useful. Game theory models may lead to farm plans which are conservative. Risk-efficient Monte Carlo programming may be useful when the distribution of risk is non-normal or utility is not quadratic and when a farmer's risk aversion is not known. Models accounting for stochastic input-output coefficients and constrained resources have also been attempted but required relatively large matrices. Goal programming attempts to include farmers' multiple objectives in the objective function. Development of appropriate software for personal computers should be a high priority.

INTRODUCTION

Over the years economists and mathematicians have invested considerable effort in developing techniques to facilitate planning and decision-making on the farm and on a regional basis. Risk and uncertainty in yields, costs and product prices have posed new challenges and numerous attempts have been made to incorporate risk in farm and regional planning models.

Analytical tools in production economics encompass a wide range of techniques or models with varying degrees of sophistication. Broadly, formal techniques range from partial and whole farm budgeting and production, cost and profit functions to mathematical programming techniques such as linear programming (LP), quadratic programming (QP) for risk analysis, linear approximations of QP and utility-efficient programming, safety-first models, game theory models and goal programming.

Development of mathematical programming techniques basically originated from the introduction and use of LP, with Heady (1954) playing an important role (Harle, 1974:153). Although LP gained rapid acceptance and is still widely used, it is the subject of much debate, particularly with regard to its mathematical deficiencies and the assumption underlying it (see Hazel & Norton, 1986: 12 - 14). Nevertheless, LP models have been used extensively in farm planning problems (e.g. maximisation of total farm gross margin) and cost minimisation problems (e.g. formulation of least cost feed rations), and results are inevitably used as a basis for comparison with results of other (more recent) models. Developments of these later models basically stem from dissatisfaction with the LP model.

The object of this paper is to highlight some of the more useful mathematical programming techniques, with emphasis on their applicability to farm and regional planning. All of the more useful models attempt to incorporate risk in the form of income variance or absolute deviations of income from the expected enterprise gross margins. Discussions in this paper will emphasise risk, and knowledge of the basic LP model is assumed.

RISK IN FARM AND REGIONAL PLANNING

The concept of risk is defined variously in the literature as probability of loss, variance of income and size of the maximum possible loss. The probability distributions from which these alternative risk concepts are derived can originate from either subjective expectations of individuals or from objective sources such as historical or experimental data (Young, 1984). Risk can be considered as a cost equal to the difference between the expected monetary value of an outcome and a risk averter's value, i.e. a risk premium necessary to convert the risky expectation into one that is certain (Barry & Fraser, 1976:288).

Agricultural production is generally a risky process, with price, yield and resource risks making farm incomes variable from year to year. Various empirical studies have shown that farmers behave in a risk-averse manner (e.g. Binswanger, 1980; Dillon & Scandizzo, 1978; Lin *et al.*, 1974; Wolgin, 1975). This implies that farmers prefer to sacrifice income for a 'safer' farm plan, for example, through enterprise diversification to spread risks or by producing less of risky enterprises. Neglect of risk-averse behaviour in farm planning models can lead to farm plans which are unacceptable to the farmer owing to specialised cropping patterns, for instance. As a result, several techniques have been developed to incorporate risk-averse behaviour in programming models.

Freund (1956) first developed a farm

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programming model incorporating risk aversion. He assumed an exponential utility function and maximised this function, subject to a set of linear resource constraints, using quadratic programming.

Economic theory suggests that in an uncertain (risky) environment, a decision-maker will choose among alternatives, with outcomes expressed by probability distributions, so as to maximise expected utility (Arrow, 1974). "The utility maximizing choice rests on the decisionmaker's strength of belief, on the relevant characteristics (mean, variance, etc.) of the expected probability distributions, and on his personal valuation of the potential outcomes" (Barry & Fraser, 1976:288). Anderson *et al.* (1977:66 - 69) provide a useful discussion of the expected utility theorem, also known as Bernoulli's principle. This principle provides the means for ranking risky prospects in order of preference, the most preferred being the one with the highest (expected) utility.

The problem facing a researcher is to choose the functional form which best describes a farmer's behaviour. Several successful attempts to elicit individual utility functions have been made (e.g. Lin *et al.*, 1974; Officer & Halter, 1968), and in many cases quadratic utility functions appear to fit the data as well as most other functions (Hazell & Norton, 1986:79). However, since the quadratic utility function is characterised by increasing absolute risk aversion, it has been rejected as untenable by many theorists (e.g. Pratt, 1964). Nevertheless, Levy and Markowitz (1979) have shown that the quadratic utility function can provide a good second-order approximation to more desirable functions.

Markowitz (1952) developed the concept of risk-income efficiency. The expected income-variance (E, V) criterion assumes that a farmer's preferences among alternative farm plans are based on expected income (E) and associated income variance (V). E, V efficiency requires that the decision-maker be risk averse, that the outcome distributions be normal or that the decision-maker's utility function be quadratic (King & Robison, 1984:72). For a risk-averse farmer the iso-utility curves will be convex, i.e. along every iso-utility curve the farmer would prefer a strategy with higher V only if E were also greater (i.e. $|dE'/dV' > 0$), and E increases at an increasing rate with increases in V (i.e. $dE''/dV'' > 0$). With these assumptions, the rational farmer should then only choose a farm plan among those for which the associated income variances are minimal for given expected income levels.

The problem for the analyst is to develop a set of feasible, risk-efficient farm plans. These plans can be derived with the aid of quadratic programming or LP approximation techniques (Hazell & Norton, 1986:80). The efficient E, V pairs lie on the risk-efficient frontier. Since the parameters of the expected utility function are usually not known, the best alternative is to derive the set of efficient farm plans and allow the farmer to make the final choice (*ibid.*:80 - 81). Although the E, V model has theoretical limitations, it has computational advantages.

Quadratic programming

Quadratic programming (QP) is theoretically appealing because it incorporates income variances and covariances of possible enterprise combinations. Data requirements include the mean gross margin for each activity and the corresponding variances and covariances. As the latter are unknown it is necessary to obtain estimates using time series or cross-sectional data of observed gross margins (Hazell, 1971:54 - 55). To obtain the risk-efficient E, V set of farm plans it is necessary to minimise V (the objective function) for each possible level of E, while retaining feasibility with regard to the available resource constraints. Since the objective function is quadratic, it must be solved by a QP algorithm. Some examples of applied QP include work by Camm (1962), McFarquhar (1961) and Stovall (1966).

According to McCarl & Tice (1982:585) QP is both a special case of nonlinear programming and an extended case of LP. Hence, software from both areas has been adapted for quadratic programmes. A useful table summarising alternative QP software is provided by McCarl & Tice (1982:586).

However, since QP algorithms may not be readily available and are only adequate for problems of limited dimensions, several methods have been developed for deriving approximate solutions to the E, V problem through linear programming (Hazell & Norton, 1986:84; McCarl & Tice, 1982). The best known methods include Separable Programming (Thomas *et al.*, 1972), the Marginal Risk Constrained LP Model of Chen and Baker (1974) and MOTAD (Hazell, 1971). MOTAD has been the most widely used and is discussed here.

The MOTAD model

Of the approximation techniques using LP, Hazell's (1971) MOTAD (Minimisation of Total Absolute Deviations) approach has been widely used in farm planning problems (e.g. Atwood *et al.*, 1986a; Mapp *et al.*, 1979; Schurle & Erven, 1979). According to McCarl & Tice (1982:588) MOTAD "works well for risk programming and provides superb computational advantages for large problems".

Hazell & Norton (1986:86) point out that the MOTAD approach is most relevant when the variance of farm income is estimated using time series or cross-sectional sample data. Hazell (1971) used variance estimates based on the sample mean absolute deviation (A).

The objective function in the MOTAD model is the minimisation of the total absolute deviations. The model can be solved by parametric LP (expected income E is parameterised) to obtain the efficient E, A set of farm plans. That is, MOTAD makes use of the expected income (E) - mean absolute deviation (A) criterion as opposed to the E, V criterion of QP. Once the efficient plans have been obtained, an estimate of their income variance can be obtained using the A estimator. Details of the MOTAD model and construction of the LP matrix are provided by Hazell (1971)and Hazel & Norton (1986:86 - 90).

In the absence of a measured E, A utility function, final solution of a farm plan from an efficient E, A set must be left to the individual farmer concerned. According to Hazell (1971:57), when results are presented in probabilistic terms a farmer should not have greater difficulty in considering an efficient E, A set of farm plans than an efficient E, V set.

Hazell (1971) compared MOTAD with QP and found the results (E, V pairs) to be similar, even though A is a less efficient estimator of the population variance than the estimate of sample variance used in QP. However, according to Thomson and Hazell (1972), the essential problem is to find the most efficient farm plan in terms of rank, rather than to estimate the actual numerical value of V. Results from their Monte Carlo study proved A to be only marginally less efficient than the sample variance in ranking equal-income plans. MOTAD is also more appealing empirically than E, V analysis if income distributions are skewed. However, an important difference between QP and MOTAD is that in QP the variance-covariance coefficients are directly incorporated in the model whereas in MOTAD the stochastic gross margins are represented by a set of mutually exclusive vectors of activity deviations (Hardaker & Troncoso, 1979:48).

With regard to data requirements, data on revenue (gross margin) variability for each enterprise considered in the plan are required in addition to the normal LP data requirements. Usually, the only available data on revenue variability are time series data on prices and yields for different enterprises; even so these data are scarce in South Africa. Ideally, for a specific farm planning problem, data on enterprise revenue or gross margin variability for the specific farm in question are needed.

When using variable income data it is necessary to remove any trend or other systematic movement in prices or yields so that variations in the data reflect true stochastic variations. According to Hazell and Norton (1986:235) use of OLS regression on the price and yield series separately is usually best. Detrended revenue is then calculated as the product of detrended yield and price. Deviations of the detrended revenues from their mean should sum to zero for each activity. Should no separate yield and price data, but only time series income or gross margin data for each activity be available, then as a second-best option these income data could be detrended with OLS regression, with deviations from the trend line summing to zero.

A problem with aggregate time series data (aggregated for crop reporting or irrigation districts) is that they understate the variability experienced by individual farms. However, aggregate yield or income data are usually more readily available than individual farm data. Often risk studies are focussed on general risk relationships experienced by a group of farms. In this case, data series from an individual farm may not be representative of the area. Atwood *et al.* (1986b) investigated the merits of pooling cross-sectional and time-series yield data for risk analysis and concluded that this approach is useful when both general area risk studies and individual farm studies are undertaken.

Hardaker & Troncoso (1979) maintain, however, that in a risky environment it is unlikely that historical data alone will be adequate to represent a farmer's beliefs about future risks. They suggest an approach (for use in MOTAD) where subjective information about possible revenue outcomes is elicited from farmers.

Mean-standard deviation (E,σ) analysis

A useful extension of E, V analysis is the mean-standard deviation (E,σ) analysis. Since σ equals the square root of V, the efficient E, σ set of farm plans is identical to the efficient E, V set. The E, σ set can, therefore, also be derived with the aid of QP or approximation techniques such as MOTAD. The value of σ can be computed after the efficient farm plans have been derived.

Baumol (1963) has rationalised the expected gain - confidence limit (E, L) criterion $L = E - \theta\sigma$, where θ is a risk-aversion parameter (positive for a risk-averse person). He argued that a risk-averse person (with risk parameter θ_1) should always select a farm plan that has the maximum value of E for a given level of $L = E - \theta_1\sigma$. This set of farm plans comprises the efficient E, L set (see Hazell & Norton, 1986:91).

A popular adaptation of the E, L criterion is the assumption that a farmer maximises L given his risk aversion parameter θ. One justification for maximising L arises when a farmer has an expected utility function $E(U) = E - \theta\sigma$, which leads to linear indifference (iso-utility) curves in the E, σ axis. If the risk aversion parameter is known, and given the efficient E, σ frontier, then a unique farm plan can be selected (Hazel & Norton, 1986:92 - 93).

The risk aversion parameter θ can be estimated from direct elicitations of farmers' risk preferences. For example, Dillon & Scandizzo (1978), using experimental methods, derived a mean value of 0,9 for a sample of farmers in north-east Brazil, and Moscardi & De Janvry (1977) estimated a mean θ value of 1,12 for farmers in the Pueblo Project in Mexico. However, attempts to elicit individual farmers' risk preferences are expensive and time-consuming, and Hazell (1982:386) maintains that direct elicitation of risk preferences is not likely to become a widely adopted approach in farm advisory work.

Another method used by researchers to impute values of θ is to solve farm models for different values of θ and select the value of θ which produces the closest match between the actual and predicted farm plans. Brink & Mc Carl (1978) reported θ values of less than 0,25 for the majority of Cornbelt farmers in the sample, and concluded that risk preferences were not important. This method of estimating θ has, however, been more widely used with regional models. For example, Hazell *et al.* (1983), Simmons & Pomareda (1975) and Kutcher & Scandizzo (1981) reported θ values ranging from 0,5 to 1,5 for sector models in Mexico and Brazil. Nieuwoudt *et al.* (1976) used a value of 2 because it gave the best solution in simulating peanut production in the USA, while Ortmann & Nieuwoudt (1987) and Nieuwoudt and Frank (1987) reported θ values of 0,25 and 0,5 in simulating regional sugar-cane and maize production, respectively, in South Africa. In these sector models θ is theoretically an aggregate of risk aversion coefficients of all farmers in the region.

The θ value as used above should, however, be seen as a 'fine-tuning' device because it not only captures risk, but also the effects of other criteria such as model constraints, incomplete or inaccurate data risk sharing (e.g. crop insurance) and different objective functions (Hazell, 1982:386; Young, 1979:1066). Nieuwoudt *et al.* (1976), Ortmann & Nieuwoudt (1987) and Nieuwoudt & Frank (1987) attached little significance to the value of θ, which was used only to fine-tune the predictive ability of their models.

LP with endogenous prices and risk

Duloy & Norton (1973, 1975) developed a version of LP for agricultural sector models which incorporates negative-sloping demand functions. This enables product prices and quantities to be generated endogenously within the model. The solution activity levels are such that the sum of producer and consumer surplus is maximised (see Samuelson, 1952). Hazell & Scandizzo (1974) modified the Duloy-Norton method by including risk (E, L) in the model. Neglect of risk-averse behaviour of farmers can result in overstatements of the supply response of high risk enterprises and overestimates of the returns to investment programmes (*ibid.*:235).

In addition to negative-sloping demand functions, positive-sloping supply functions for resources such as labour have been incorporated in LP models by various researchers (e.g. Nieuwoudt *et al.*, 1976; Hazell, 1979; Ortmann & Nieuwoudt, 1987). In these instances prices (wages) of resources (labour) were generated endogenously within the model.

Utility-efficient programming

Patten & Hardaker (1987) have developed a programming model that generates the efficient set of plans for defined classes of utility functions (see Lin & Chang, 1978). This is in response to a situation where no completely satisfactory method of finding the utility-maximising farm plan among the set of possible risky plans is available, owing to the fact that a farmer's utility function is not known and preference elicitation from farmers is difficult.

Farm programming models which maximise expected utility for a specified risk-averse utility function have been described by Lambert & McCarl (1985). Extending this approach to parametric objective programming would generate a 'utility-efficient' (UE) set of solutions (Patten & Hardaker, 1987:5). The model is defined as follows:

$$\begin{aligned} \max E[U] &= \Sigma k\,[Pk\,\{G(Zk) + \lambda H(Zk)\}] \\ \text{subject to:} \quad Zk &= Ck'\,x \text{ for } k=1,\ldots,K \\ Ax &\leqslant b \\ \text{and } x &\geqslant 0 \end{aligned}$$

where Pk = the probability of state k; G and H = two parts of the utility function U; Zk = the total net revenue for state k; λ = a non-negative parameter; Ck = the activity net revenue vector for state k; x = the vector of activity levels; A = matrix of input-output coefficients, and b = vector of right-hand side coefficients (*ibid.*:5 - 6). The two parts of the utility function (G and H) are defined so that the degree of risk aversion varies systematically with λ.

According to Patten & Hardaker (1987:7 - 8), solution of the 'utility-efficient' programming problem requires access to an algorithm which is able to solve mathematical programming formulations with parametric linear combination of two nonlinear objective rows. Practical options include (a) parametric QP routines with quadratic approximation of G and H and (b) a parametric LP routine with linear approximation of G and H. Patten & Hardaker (1987) preferred using the latter because of its simplicity. They demonstrated the technique with a certain functional form of U, the 'sumex', using the Duloy & Norton (1975) approach to linearise the concave functions G and H. Some advantages of UE programming include the following: (a) it is applicable to a large number of types of utility functions; (b) the degree of risk aversion can be limited to a plausible range; (c) the distributional form for activity net revenues is flexible; and (d) it can be performed using parametric LP (Patten & Hardaker, 1987:12 - 13).

Safety-first models

Safety-first models are designed to help a farmer ensure that he achieves the minimum income necessary to cover annual fixed costs and family living expenses. According to Hazell & Norton (1986:100) safety-first models are most appropriate where the risk of a disaster is large owing to an inherently risky environment or because the farmer is in a poor financial state and has minimal financial reserves backing him in a poor year.

Some of the safety-first models include those proposed by Roy (1952), Low (1974), the Focus-Loss model of Boussard & Petit (1967), and Tauer's (1983) Target MOTAD model. Target MOTAD appears to have been the most widely used of these models and will be briefly discussed here. The construction and limitations of the other three models are discussed by Hazell & Norton (1986:100 - 104).

Target MOTAD

Tauer (1983) proposed a Target MOTAD model in which risk is measured as the expected sum of the negative deviations of the farm plan solutions from a target income. Risk is varied parametrically to derive a risk-return frontier. This approach can be useful, according to Tauer (1983:607), because decision-makers often wish to maximise expected returns but are concerned about returns falling below a critical target. It therefore embraces the concept of disaster avoidance. Because the Target MOTAD model has a linear objective function and linear contraints it can be solved with an LP algorithm, as is the case with the other safety-first models. Hazell & Norton (1986:102) provide a useful example of the Target MOTAD matrix.

The Target MOTAD approach has been frequently used in farm planning problems (e.g. Atwood *et al.*, 1986a; Helmers *et al.*, 1985; Mc Camley & Kliebenstein, 1987; Watts *et al.*, 1984; Zimet & Spreen, 1986). According to Helmers *et al.* (1985:28), viewing risk as the occurrence of returns below target incomes rather than as variability from expected income has gained more acceptance recently. However, how risk is perceived would obviously depend on the prevailing circumstances.

Tauer (1983) tested his model for second-degree stochastic dominance (SSD). SSD requires that a farmer's utility function be upward-sloping ($U'(Y)>0$), but at a decreasing rate ($U''(Y)<0$) (see Anderson *et al.*, 1977:284 - 288). Tauer (1983) ran a simple empirical example using both MOTAD and Target MOTAD, and tested the solution farm plans for SSD. He found that Target MOTAD solutions were always SSD and that not all MOTAD solutions were necessarily so, i.e. MOTAD can generate farm plans that no rational risk-averse farmer would consider. This weakness of MOTAD is probably not too serious if the final choice of a farm plan is left to the farmer.

Game theory models

The theory of games was developed by Von Neumann & Morgenstern (1944). After it had initially been hailed as a major breakthrough in economic analysis, game theory as an operational concept did not become established until McInerney (1967) opened the debate again and developed an LP model to derive the maximin solution for a constrained farm planning problem.

Game theory provides an analytical framework for situations of conflict between two or more participants (McInerney, 1967:279). In agriculture, nature can be considered an opponent to the farmer in two-person zero sum games, "who, perhaps randomly rather than willfully, may financially undo a farmer in his selection of a farm plan" (Hazell & Norton, 1986:94). Various decision criteria to aid selection of an appropriate farm plan have been suggested, the most common being the Wald maximin and the minimax (or Savage regret) criteria.

Maximin criterion

This approach assumes that the farmer "is interested in that strategy which secures for him the maximum minimum gain - i.e. that his minimum gain from a play of the game will be as large as possible" (McInerney, 1967:280). This is illustrated in the following payoff matrix:

Farm Plan	State of Nature S1	S2	S3
F1	200	400	500
F2	150	250	300

The farmer may choose between farm plans F1 and F2 and for each plan there are three possible states of nature, e.g. poor, average and good. Under the above circumstances, the maximin strategy for the farmer would be to select the plan that has the largest (maximum) outcome under the worst (minimum) state of nature, namely farm plan F1 since its worst possible payoff is 200.

It is obvious that the maximin approach is inherently conservative, but it may be an appropriate strategy for a beginning farmer (or a risk-averse farmer) with high debt commitments. However, McInerney (1969) has shown that planning to achieve a maximin approach need not necessarily mean making significant sacrifices in expected, or realised, returns over a number of years.

The model requires identification of a finite number of states of nature and is suitable when time series data on the activity gross margins are available, or when the gross margin outcomes for selected bad years can be elicited from farmers (Hazell & Norton, 1986:95). Hazell (1970) made the maximin criterion more useful by adding an expected income (E) equation and developing an efficient E, M (M = worst possible farm income) set of farm plans. Hazell & Norton (1986:96) provide a clear example of how to incorporate the maximin criterion in an LP matrix.

Minimax (Savage regret) criterion

This criterion implies that the decision-maker experiences dissatisfaction (regret or remorse) equal in magnitude to the difference between the return he actually achieved and the (maximum) return he could have achieved had he correctly predicted the state of nature which occurred (McInerney, 1969:271). The minimax criterion is based on the assumption that the decision-maker wishes to minimise this regret.

For example, if the t state of nature prevailed and the farmer could correctly have anticipated all the activity gross margins corresponding to that state, then he would have chosen the farm plan that maximised income, say Y*. However, since the farmer seldom (if ever) has perfect foresight, he will most likely adopt an alternative farm plan yielding income Yt. The difference Y* - Yt is a measure of the regret that the farmer might experience once he knows the consequences of his decision. Given this measure of regret, the minimax criterion considers the largest of these regrets over all states of nature, and calls for selection of the plan with the minimum value of the maximum regret (Hazell & Norton, 1986:97). Hazell (1970) proposed an LP adaptation of the minimax criterion for farm planning, and Hazell & Norton (1986:98) provide a useful tableau showing how the minimax problem is incorporated in an LP model. A disadvantage of the minimax model is that a maximum income must first be derived for each state of nature by solving a series of LP problems.

According to Hazell & Norton (1986:99), game theory decision criteria such as the maximin and minimax rules are attractive because they require less information about possible gross margin outcomes than the E, V models. For example, they do not require information about the relative frequency or probability of occurrence of each state of nature. This implies that once the relevant states of nature have been enumerated, large samples of time series activity gross margins are not important. Finally, McInerney (1969:275 - 277) provides a useful review of game theory models.

Risk-efficient Monte Carlo Programming (Remp)

In Monte Carlo programming, portfolios of activities are selected by random sampling with activity levels expanded to the limits of resource availability. The feasible plans may then be subjected to some quality test, and those that are passed can be displayed and the planner can then choose the most desired plan. This technique can accommodate restrictions to integer activity levels and non-linear production relationships, albeit at the cost of explicit optimisation (Anderson, 1975:94 - 95).

Anderson (1975) extended Monte Carlo programming by including risk. In risk-efficient Monte Carlo programming (REMP) many farm plans are selected partly at random and then reduced by pairwise comparisons to the SSD set, i.e. inefficient plans are sorted out according to the rules of second-degree stochastic dominance. However, REMP does not allow the whole set of efficient plans to be identified, and those identified may, in part, not be truly efficient. Although REMP is conceptually simple and has some desirable flexible features, it is computationally demanding and provides sets of efficient plans which will probably be of a diverse nature and therefore difficult to interpret. However, according to Anderson (1975:105), when risk is non-normal or utility is not quadratic, and when a farmer's risk aversion is not known, the REMP method offers a theoretically acceptable and a practical planning approach.

Risk in the constraint set

Input-output coefficients and resource constraints are usually considered deterministic in a farm model. This may not be the case in an uncertain environment, however. For example, variations in yield may affect hired labour, machinery and capital requirements of farm activities, and the farmer may have to deal with stochastic resource supplies such as livestock feed, hired labour and irrigation water.

The feasibility of a farm plan in any one year can be siginicantly influenced by risk in the constraint set, which ideally should be transferred into the objective function of a model and a single risk decision rule applied (Hazell & Norton, 1986:104). Discrete stochastic programming, developed by Cocks (1968), attempts to achieve this. Rae (1971a, 1971b), for example, has demonstrated how this technique can be used to model detailed sequences of decisions in farming. Major disadvantages of this approach include (a) the substantial data requirements and size of model due to the separate rows and columns required for resources and activities in every state of nature and (b) the problem of unfeasible solutions (Hazell & Norton, 1986:106).

Practical alternatives to discrete stochastic programming include chance-constrained programming and goal programming (*ibid.*:106). The former was developed by Charnes & Cooper (1959), who expressed the feasibility requirements of a model in probabilistic terms: Wicks & Guise (1978) linearised the problem through a MOTAD formulation called RINOCO (Risk in the Input and Output Coefficients). Some major drawbacks of chance constraints are discussed by Hazell & Norton (1986:109 - 110).

GOAL PROGRAMMING

A major weakness of conventional LP is that of a single objective function. In practice, a farmer may want to optimise more than one objective in planning, i.e. he may be interested in achieving an optimal compromise amongst several objectives, some of which may be in conflict. For example, he may wish to maximise gross margin, minimise dependence on hired labour, reduce debts and reduce fixed costs.

The easiest way to account for such multiple goals in a model is to select one goal that will be maximised (or minimised) and to specify the remaining ones as inequality constraints. A shortcoming of this method is that the goals included in the constraint set must be rigidly enforced since the problem will be unfeasible if these goals cannot be met (Hazell & Norton, 1986:71).

In management science several techniques have been developed to deal with multiple criteria decision-making. Of these methods goal programming (GP) and its variants have been applied to a wide range of problems, except in farm planning (Romero & Rehman, 1984:178). The objective of GP is to minimise the deviations between the achievement of goals and their target (or aspiration) levels. "The goals are included in the model by converting inequalities through the addition of positive and negative deviation variables that allow for under-achievement and over-achievement of each goal" (*ibid.*:181). This solves the problem of unfeasibility.

With weighted GP (WGP) all goals are considered simultaneously in a composite objective function which consists of a weighted sum of the deviations between the achievement of goals and their targets, where the deviations are weighted according to the relative importance of each goal to the farmer. This composite objective function is minimised (Romero & Rehman, 1984:185 - 186). If it is a problem to determine an approriate set of weights for the goals, a useful procedure is to solve the problem for different weights and to present the farmer with information about the trade-offs that exist between competing goals (Hazell & Norton, 1986:72). The WGP problem can be solved using an LP algorithm if the components of the objective function are linear.

A detailed review of GP and multiple criteria decision-making in farm planning is given by Romero & Rehman (1984, 1985). The latter article demonstrates how game theoretic principles and the MOTAD approach (including Target MOTAD) can be incorporated in the multiple objective framework when dealing with risk and uncertainty.

SUMMARY AND CONCLUSIONS

This paper considered the more useful mathematical programming techniques which have been developed since the early 1950s, when LP was first applied to farm planning problems. Dissatisfaction with the basic LP model gave rise to the development of alternative models, with varying degrees of success. Most of the models attempt to incorporate risk and uncertainty in order to make model solutions more acceptable to the farmer. Risk-efficient E, V frontiers imply that firms can only obtain higher income levels by accepting proportionately higher risk. According to expected utility theory, a farmer would select a farm plan along the frontier consistent with his risk-income preferences.

Quadratic programming is a useful technique for the derivation of an E, V frontier. However, a problem with QP is that access to suitable software is limited, and software available cannot deal with large problems efficiently. MOTAD overcomes these difficulties because it uses LP algorithms, and solutions compare well with those derived from QP. Efficient E, A farm plans can be derived from which the farmer can select that plan which would maximise his utility. A problem with MOTAD is obtaining time series gross margin data that will adequately represent risk in a particular farm situation. However, this difficulty is applicable to other models as well, for example game theory models and variations of MOTAD such as Target MOTAD and RINOCO. However, the use of subjective information about revenue outcomes elicited from farmers could be a useful supplement to historical data.

A useful modification of the MOTAD objective function is to maximise $L = E - \theta\sigma$, which includes both expected income and risk in the objective function. However, the risk-aversion coefficient θ, which can be parameterised to obtain various farm or regional plans, is basically a fine-tuning device which can be used to improve the predictive ability of the model. This is a useful approach when attempting to simulate present farm or regional enterprise mixes or to test the effects of various policy measures.

A technique that appears to have considerable potential is utility-efficient programming. This technique is still in its infancy, but is a significant contribution towards eliminating some of the drawbacks of the techniques that have evolved since the initiation of LP in farm planning during the 1950s.

Of the safety-first models, Target MOTAD appears to be the most useful. Deviations of income below a target income are taken as measures of risk. This may be an appropriate model where the farmer has a cash flow problem and is highly risk-averse. Game theory models are also applicable here but may lead to farm plans that are conservative. A disadvantage of the Savage regret (minimax) criterion is that a maximum income must first be derived for each state of nature by solving a series of LP problems. Risk-efficient Monte Carlo programming (REMP) may be a useful technique when risk is non-normal or utility is not quadratic, and when a farmer's risk aversion is not known.

The models evaluated above assume that the input-output coefficients and the constrained resources set are deterministic. However, fluctuations in these coefficients are a source of risk and uncertainty. RINOCO appears to be a useful model but its major drawbacks are the size of matrix required even to accommodate a limited number of permutations of input and income risk parameters, and the problem of unfeasibility.

Conceptually, goal programming is attractive since it attempts to account for the multiple goals of farmers. A practical way to incorporate these multiple objectives in the objective function is to weigh the deviations between the achievement of goals and their targets according to the relative importance of each to the farmer.

A challenge for economists and programmers in the future will be not only to develop more suitable models for farm and regional planning but to develop algorithms for use on personal computers. With the increased use of these computers by researchers and farmers, development of suitable algorithms should be a high priority.

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