UNIVERSITY OF NOTTINGHAM
DEPARTMENT OF AGRICULTURAL ECONOMICS

Production Function Analyses of British and Irish Farm Accounts

With Calculations of "Managerial" and "Random" Variations in Gross Product

by

KNUD RASMUSSEN

with

M. M. SANDILANDS

June 1962

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PREFACE

It is now almost a hundred years since the beginning of the unbroken series of annual censuses of British agriculture. During this period the statistical approach, which pervades modern life, has spread to all aspects of agricultural activity and has now reached right into the detail of management of the individual farm. The accumulation of data, both physical and financial, is impressive and feeling grows that much of it has been inadequately explored.

Dr. K. Rasmussen, my predecessor as Head of this Department, was convinced that one particular stock of data relating to British agriculture - namely the Farm Management Survey records collected annually from some 2,400 farms by Agricultural Economics Departments of the universities - could be made to yield much more valuable information about the essential economic relationships on the farm than had hitherto been revealed. In deciding to make some pioneer investigations of this material he was further encouraged by the knowledge that comparable data of a very comprehensive kind had been collected in the Republic of Ireland.

This study presents the results of the author's statistical analysis and a description of the methods used. It opens up new possibilities of further research and is likely to be the forerunner of many subsequent studies which will consolidate and build upon this pioneer work.

The author's publication entitled *Variance and Production Function Analyses of Farm Accounts*, (Oxford, Basil Blackwell, 1962) may be regarded as a companion volume to this study.

D. K. Britton.

Professor of Agricultural Economics

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FOREWORD

The present study was planned some years ago but the actual calculations were not started till the spring of 1959. The senior author left his employment at Nottingham University during the summer of 1959, when only some of the calculations had been done, but Miss M. M. Sandilands carried the subsequent calculations through according to plan. The methods for this study were developed in the senior author's previous work on farm accounts.[1] Thanks are due to the Ministry of Agriculture, Fisheries and Food for making the Farm Management Survey accounts available and for paying the costs involved in this study. Thanks are also due to Dr. M. D. McCarthy and Mr. T. P. Linehan of the Central Statistics Office, Dublin for making the Irish farm accounts available and for being kind enough to adapt the accounts to conform with the British data.

Thanks are also due to Dr. F. Yates of Rothamsted Experimental Station for giving permission for the use of the electronic computor at Rothamsted, and to Mr. M. J. R. Healy and Mr. J. C. Gower for planning the computations and carrying them out.

The great care with which the clerks and typists of the Agricultural Economics Department of Nottingham University checked the farm accounts and carried out all the necessary work connected with this study deserves a special word of thanks.

Last but not least the senior author, who was responsible for the planning, the selection of methods and the text, wishes to thank Miss M. M. Sandilands for the meticulous care and great accuracy with which she has carried out the vast amount of calculation involved and for the drawing up of the tables and graphs. He also wishes to thank her for the great assistance he has received in editing the text. On the other hand if the language may in places be a little quaint the senior author accepts all the blame, as he may have insisted on the use of certain expressions.

Knud Rasmussen.

[1] KNUD RASMUSSEN, *Variance and Production Function Analyses of Farm Accounts, with special reference to inter-farm and random variation.* Oxford, Basil Blackwell, 1962.

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INTRODUCTION

CHAPTER 1

PRINCIPAL PROBLEMS ENCOUNTERED IN THE CALCULATION OF PRODUCTION FUNCTIONS BASED ON FARM ACCOUNTS

This study is chiefly concerned with the results obtained from an analysis of British farm accounts and includes the presentation of new methods designed to derive more information about the farms analysed than has been possible from those previously applied.

It is not intended to deal in detail with previous work done on the subject nor with all the problems facing the student in this field of applied economics. However, a brief discussion of the more important problems encountered in earlier studies may be appropriate. All earlier studies[1] known to the author have used data for a single year only, and the fact that this study had available 4 years' accounts for each of the 1646 British farms made it possible to carry out new types of analyses. This has thrown fresh light on several problems in this field of study and, in particular, has made possible further analyses of the differences between actual and calculated products.

The principal problems encountered in the calculation of production functions based upon farm accounts are:

(1) Choice of function,

(2) Choice of variables and difficulties arising from the high degree of inter-correlation between them,

(3) Difficulties arising from possible errors of measurement of the input variables,

(4) The problem of interfarm and intrafarm relationships.

[1] For a review of previous literature and a discussion of problems involved, see:

PARISH, R.M. and DILLON, J.L., - "Recent Applications of the Production Function in Farm Management Research" - *Review of Marketing and Agricultural Economics*, Dec. 1955 - Dept. of Agriculture, N.S.W., Australia.

Resource Productivity, Returns to Scale, and Farm Size - ed. by HEADY, EARL O., JOHNSON, GLENN L. and HARDIN, LOWELL S. - Iowa State College Press, 1956.

and HEADY, EARL O., and DILLON, JOHN L., - *Agricultural Production Functions* - Iowa State College Press, 1961.

(1) In the choice of function to be used in this study, the author was guided by previous experience as to the goodness of fit which might be expected from a Cobb-Douglas function. The results of the regressions confirmed the expectation of goodness of fit, since in the Between Farms regressions the multiple correlation coefficient ranged from 0.98 to 0.99, and, as a further test, the distributions round the regression surfaces all proved to be very near to normal. In addition, the breakdown by thirds according to size of farm (p.14) shows, with the possible exception of Group 1 (Dairy), the same picture in each third.

Some workers in this field have criticised the Cobb-Douglas function because it implies constant elasticities and linear Least Cost Combinations over the whole range. The acid test of a production function must be its agreement with reality and it does not seem that the Cobb-Douglas function can be discarded on this score. Another criticism frequently made about this function, that it is not defined for an input of zero for any of the independent variables, may not be very serious for practical studies but may in these put a limit to the extent to which the inputs may be broken down into independent variables. Fortunately, one is not normally interested in farm management problems relating to inputs which are unusual for a given type of farm. If new inputs are to be introduced, one cannot expect to get empirical evidence for the productivity of such new inputs from farm accounts; the most reasonable approach would be by means of controlled experiments, the results of which might be used in a budgeting procedure. A production function study based upon farm accounts, as well as other similar forms of statistical analyses, should not be used for extrapolations. The Cobb-Douglas function has also been criticised[1] on the grounds that it implies that a farmer with a fixed amount of labour available can increase infinitely the use of capital in the form of machinery and livestock and, according to this function, continue to increase gross income, while in reality the physical capacity of a man limits the amount of machinery and livestock he can handle. In an economic production function study, one is hardly interested in such extremes, which would bring negative marginal productivities of an input, and it is the experience of the author that few farmers would use anything like such uneconomic combinations of inputs. For this reason, the author is satisfied with a function which is able to describe relevant economic relationships within the range of combinations of inputs found in the data.

[1] CARTER, H.O., - "Modification of the Cobb-Douglas function to destroy Constant Elasticity and Symmetry" - *Resource Productivity, Returns to Scale, and Farm Size;* - op. cit.

(2) The choice of variables must, as previous studies have clearly shown, be to a great extent arbitrary. For this reason, alternative calculations, using different breakdowns of the inputs, to assess the importance of alternative subdivisions of the variables may be valuable.

In the present studies, Gross Product is defined as Gross Farm Output (as used in the Farm Management Survey) adjusted for changes in Tenantright and *less* depreciation on machinery and equipment. Some production function studies have considered depreciation as an input, but it was preferred here to consider the assets, in value terms, as the inputs and depreciation as a negative item of production.

Regarding the inputs, the general principle has been followed of including *all* inputs which can be considered to determine the costs of achieving the given output. At the same time, it was thought preferable to group the inputs in such a way that they represented the main cost items to which British farmers are accustomed. This led to 4 independent variables in the first instance:

Rent & Rates (£),

Total Tenant's Capital (£),

Total cost of Labour, including farmer and wife (£),

Total Purchases of Raw Materials, etc. (£).

This list covers comprehensively the cost to the farmer of achieving the Gross Product since the interest on Tenant's Capital added to the other 3 variables directly gives the costs incurred.

In the second instance, a more detailed breakdown was made by dividing Total Tenant's Capital into 2 components: Tenant's Capital invested in machinery and equipment, and Other Tenant's Capital, and Total Purchases into the 3 components: Seeds and Feeding-stuffs, Fertilisers & Lime, and Other Purchases. This further breakdown in no way alters the fact that all costs incurred are included.

In order to examine the relationships between the variables considered in pairs, all the variables were averaged in their logarithmic form over the 4 years for each farm, and the simple correlations were calculated within each of the 4 farm-type groups. These are shown in Table 1.[1]

An inspection of the table shows that the variables are more highly correlated than might be desired for multiple correlation analysis. However, in spite of the high correlations between the independent variables, there is still a very substantial variation in the use of any individual input for fixed other inputs. Using the variables in their logarithmic form averaged over the 4 years, a regression was calculated for each independent variable on the remaining 3 independent variables in the 4 independent variable case,

[1] Tables 1-44 will be found in the Appendix. For definitions of groups, see page 8; for definition of symbols used, see pages 9 and 10.

and a similar regression was calculated for each of the 5 remaining variables which only appear in the 7 independent variable case. The standard deviations (the square roots of the residual variances) from these calculations are shown in Table 2. *These standard deviations indicate that, when other variables are fixed, the variation in a given variable is such that its largest value may be expected to be at least about 4 times the smallest value.*

The fact that the Between Farms regressions show individual regression coefficients which are *all* significantly different from zero, even at the 1% level, indicates that the independent variables are not too highly correlated for the use of multiple regression techniques. Because the standard deviations of the individual regression coefficients are small, the production functions may be accepted as useful tools for farm management purposes.

The fact that the independent variables in a multiple regression are highly correlated does not give any systematic bias to the calculated regression coefficients, but results in larger variances for the individual regression coefficients at a given level of goodness of fit (same residual sum of squares) than would have been the case had the independent variables been less highly correlated. At the same time, this high degree of positive inter-correlation causes the various regression coefficients to be in general negatively correlated, such that, if one coefficient happens to be an over-estimate of its "true" value, then one or more of the other coefficients are likely to be undervalued in relation to their "true" values.[1] The result is that the variance of the sum of the coefficients is smaller than the sum of the variances of the individual coefficients. In the present study, the variances were, in general, only about one tenth of the sums of the variances of the individual coefficients. In other words, the standard deviation of the sum of the coefficients is much smaller than might have been expected from a casual inspection of the standard deviations of the individual coefficients. The fact that the standard deviations of the sums of the regression coefficients are small is of great economic importance since the sums indicate increasing, constant or decreasing returns to scale, and small standard deviations mean that, even if these sums deviate only slightly from 1, it is almost certain that the differences are significant, thus indicating increasing, or decreasing, returns to scale.

[1] HALD, A., - *Statistical Theory with Engineering Applications* - Chapman & Hall, London, 1952, p. 631 and p. 639.

(3) The problem of errors of measurement in the input variables is a difficult one. When the independent variables in regression analysis are subject to errors of measurement, it follows that there is a systematic downward bias in the size of the regression coefficients.[1] In multiple regression analysis, this means that the individual coefficients, and therefore their sum, tend to be smaller than would have been the case had there been no errors of measurement. Various suggestions have been made to deal with this problem.[2] In the present study, see Chapter IV, an attempt to minimise this systematic error has been made by means of regression using Variance and Co-variance Components. This was made possible by the fact that accounts were available for 4 years for each farm. When the variables are averaged over the 4-year period, the importance of errors of measurement in input variables is likely to be small. On the other hand, one would expect that the product resulting from a given combination of inputs would show a large variation. This variation is divided later in this study into "Managerial" or "Ability" variance and a "Random" variance. The joint variation caused by these two elements has the same effect on the analysis of the association between the variables as would have substantial errors of measurement in the product. This reasoning suggests that the use of regression techniques may be permissible, and, further, that the use of Variance and Co-variance Components can add to the reliability of the results.

(4) A difficulty, particularly hard to resolve, which faces the student of production functions is the one mentioned but *not discussed* in detail by Heady and du Toit[3]:

> "We use a cross-sectional sample of farm firms to provide a mean intrafirm estimate of resource productivity."

In an earlier study, Bronfenbrenner[4] stated:

> "......that the use of any interfirm production function is illegitimate in the determination of the elasticity of demand for a productive service, however legitimate this use may be for other purposes."

[1] HALD, A.G., - *ibid.* pp. 615 - 616.

[2] ANTILL, A.G., - Farm Economist, Vol. VIII, No. 1. 1955, pp. 1/11.
TINTNER, G., - *Econometrics*, Wiley, 1952.
MARSCHAK, JACOB and ANDREWS, WILLIAM H., Jr. "Random Simultaneous Equations and the Theory of Production" - *Econometrica*, Vol. 12, Nos.3 & 4, pp. 143 - 205.

[3] HEADY, EARL O. and SCHALK du TOIT, "Marginal Resource Productivity for Agriculture in Selected Areas of South Africa and the United States." *Journal of Political Economy*, Vol. LXII, 1954, pp. 494 - 505.

[4] BRONFENBRENNER, M., "Production Functions: Cobb-Douglas, Interfirm, Intrafirm." *Econometrica*, Vol. 12. No. 1. 1944. pp. 35 - 44.

Marschak and Andrews[1] also discussed the problem that different firms have different production functions for various reasons and they expressed strong doubts as to the value of a "mongrel" surface fitted to interfirm data:

> "For example, the production function........will change, even within the same industry, from firm to firm and from year to year, depending on the technical knowledge, the will, effort, and luck of a given entrepreneur: these factors can be summarised as "technical efficiency", and may be represented by one or more random parameters."
>
> ....
>
> "......a scatter diagram in the $(X_0, X_1, X_2,)$-space, and a "mongrel" surface fitted to it by some method would serve the purpose of "meteorological" prediction. It will tell us what likely production we shall expect from a firm whose technical and economic efficiency and other characteristics are such as to make it hire a given amount of manpower and capital. This is different from the "engineering" type of question: how much will a firm produce, if a certain amount of labor and capital is forced upon it. The latter question is answered by the parameters of the random equation......taken by itself, not by the "mongrel" function......"

It is quite true that no economist would expect to be able to *force* certain amounts of labour and capital upon farmers and to study the resulting products. In farming there are, however, great gaps in our knowledge about which combinations of resources constitute the optimum combination(s?). Accordingly, any farmer must use a given combination which, due to the lack of knowledge, is bound to be to a very great extent experimental, and, as previously mentioned, the input combinations do in fact vary substantially so that, for fixed other inputs, a given input may be expected to vary in such a way that the maximum use is about four times the minimum use. As one cannot expect to carry out planned experiments in this field, one may yet be justified in considering the various choices of resource combinations from the sample of farm accounts to be an *approximation* to such an experiment. Experience has taught the author that farmers can be successful (or unsuccessful) at very different resource combinations, and it might be acceptable as a working hypothesis to assume that farmers choose the *global* com-

[1] MARSCHAK, JACOB and ANDREWS, WILLIAM H. Jr. *op. cit.*

bination of resources[1] in a way which is independent to a great extent of their ability. In this sense, *global* combinations of resources mean the combinations of input *groups* only and not the detailed composition of an individual input group, the latter being undoubtedly dependent on the *husbandry skill* of the farmer which shows itself in the choice of the right *kinds* of fertilisers, feedingstuffs etc. for a given expenditure, and also in the right *time* of application.

This reasoning leads the author to believe that it is justifiable to use an *interfarm* production function as the basis for *intrafarm* conclusions about individual farms. This point will be discussed further in Chapter IV and Chapter VI.

[1] "The same global combination" means a combination where the several variables used in the function (Labour, Tenant's Capital, etc.) have the same values but not necessarily the same composition. Since, in the regressions, only the "global" amount of each input is used, the deviations from the regression surfaces arise partly because of differences in the detailed composition within input groups (this composition being determined by what is called the "Managerial" element) and partly because of good or bad luck ("Random" element).

SECTION I

REGRESSION ANALYSES APPLIED TO BRITISH FARM ACCOUNTS

Four Independent Variable Case

CHAPTER II

THE FUNCTIONS CALCULATED

Data

The accounts for 1648 farms over the 4-year period 1954/55 to 1957/58 were made available by the Ministry of Agriculture, Fisheries and Food from those collected for the Farm Management Survey. These farms were deemed to be "identical" for the period considered having undergone no change in occupier, nor substantial change in acreage.

After examination of the accounts before starting the analyses, it became apparent that those from two farms contained, in all probability, large errors; it was therefore decided to exclude these two farms from the analyses.

The final sample, as a result, was composed of the accounts from 1646 farms. These were divided, according to the Farm Management Survey classification, into four farm-type groups:

Group 1 - Dairy farms, containing 406 accounts for each of 4 years

Group 2 - Livestock farms, containing 416 accounts for each of 4 years

Group 3 - Mixed farms, containing 587 accounts for each of 4 years

Group 4 - Arable farms, containing 237 accounts for each of 4 years.

The term acreage as used in this study is defined as "adjusted acreage", that is, the acreage of crops and grass plus the equivalent pasture acreage of rough grazing as estimated by the Provincial Agricultural Economists. No restriction was made regarding acreage and the number of farms falling into each size group is shown in Table 3 for each of the four farm-type groups and for the whole sample.

A study of this table shows that there are few farms in Group 1 (Dairy) of over 300 adjusted acres. This is undoubtedly a consequence of the present concept of Dairying used in the Farm Management Survey which accepts only those farms on which dairying is the main enterprise. These farms are chosen from certain recognised dairying districts where dairy farms over 300 acres would not be typical.

To some extent the classification into the four broad farming groups is somewhat arbitrary and, as mentioned later, an attempt was made to consider all farms as one group. However, it was found that the grouping nevertheless contained some differences which, although not large, led to the decision that all the analyses should be carried out for each of the 4 groups separately.

Type of Analysis

Gross Product or Output, defined as Gross Farm Output adjusted for changes in Tenantright and net of depreciation on machinery and equipment, has been considered as a function of a number of items of Costs or Inputs, and production functions of the Cobb-Douglas[1] type have been fitted using the method of least squares.

In the analysis the following expressions[2] have been used:

Z = Gross Product	$z = \log Z$
X_1 = Acres (Adjusted)	$x_1 = \log X_1$
X_2 = Rent & Rates	$x_2 = \log X_2$
X_3 = Tenant's Capital invested in machinery & equipment	$x_3 = \log X_3$
X_4 = Other Tenant's Capital[3]	$x_4 = \log X_4$

[1] Gross Product $= K_i X_1^{b_{1i}} X_2^{b_{2i}} \ldots\ldots X_n^{b_{n_i}}$, where i indicates the year.

Expressed in logarithms this becomes:
$\log \text{Gross Product} = k_i + b_{1_i} \log X_1 + b_{2_i} \log X_2 + \ldots\ldots + b_{n_i} \log X_n$

[2] The means of all variables are given in Tables 29 (a) and 29 (b).

[3] The approximate composition of "Other Tenant's Capital" per farm is shown below. The figures are those from the Farm Management Survey averaged for the 4 years 1954/55 to 1957/58.

Farm-type Group	Group 1 (Dairy)	Group 2 (Livestock)	Group 3 (Mixed)	Group 4 (Arable)
Average farm acreage	122	239	229	197
	£	£	£	£
Crops	271	114	563	1068
Livestock	2073	1259	1475	986
Total	2344	1373	2038	2054

$X_5 = X_3 + X_4$ = Total Tenant's Capital $\quad x_5 = \log X_5$

X_6 = Total Cost of Labour (including Farmer & Wife) $\quad x_6 = \log X_6$

X_7 = Seeds & Feed $\quad x_7 = \log X_7$

X_8 = Fertilisers & Lime $\quad x_8 = \log X_8$

X_9 = Other Purchases[1] $\quad x_9 = \log X_9$

$X_{10} = X_7 + X_8 + X_9$ = Total Purchases $\quad x_{10} = \log X_{10}$

$X_{11} = X_2 + X_6 + X_{10}$ = Total Costs exclusive of interest on Tenant's Capital

Two cases have been considered, the first with Gross Product expressed as a function of 4 independent variables, X_2, X_5, X_6, and X_{10}; the second with Gross Product expressed as a function of 7 independent variables, X_2, X_3, X_4, X_6, X_7, X_8 and X_9.

In view of the fact that X_1, Adjusted Acres, was thought to contain a greater element of quality variation and arbitrary assessment than X_2, Rent & Rates, it seemed likely that X_2 would be a better criterion of size than X_1 as regards capacity for production. This, in fact, proved to be the case and, as shown in Table 4, more of the variation between farms was explained in the 4 independent variable case when x_2 was used in place of x_1. *As a result, it was decided to use x_2 (log Rent & Rates) and not x_1 (log Adjusted Acres) in all the production functions.*

4 Independent Variable Case

Regression analyses were carried out within each farm-type group:

[1] The approximate composition of "Other Purchases" per farm is shown below. The figures are from the same source.

Farm-type Group	Group 1 (Dairy)	Group 2 (Livestock)	Group 3 (Mixed)	Group 4 (Arable)
Average farm acreage	122	239	229	197
	£	£	£	£
Fuel	185	152	323	356
Electricity	37	13	60	37
Vehicle Taxes & Insurance	35	32	48	53
Contract Services	74	63	159	181
Twine & Wire	8	11	27	24
Machinery repairs	145	106	325	348
Haulage & Transport	17	36	49	73
Vet. & Medicine	54	46	70	32
Minor Repairs & Small Tools	121	88	192	133
Sundry	149	81	258	221
Total	825	628	1511	1458

(1) for each of the 4 years separately,

(2) considering the regression planes for the individual years to be parallel, hereafter called the Within Years regression,

(3) for the averages of each variable (logs) for the 4-year period for each farm, hereafter called the Between Farms regression.

A further regression was calculated considering all the farms as belonging to one group. This regression was only carried out on the averages for the 4-year period.

The variance analyses of the regressions are shown in Table 5, and the values of the regression coefficients in Table 6. The latter table also shows the sum of the coefficients and the multiple correlation coefficient (R) in each case, as well as the standard errors of the coefficients from the Between Farms regression. In all cases the multiple correlation coefficient is very high (0.96 to 0.99).

The sums of the regression coefficients are all greater than unity for the Between Farms regression, which for this purpose should be the most reliable. The smallest is 1.03 and the largest 1.07. The t-test shows that they are all significantly greater than 1 at the 2½ % level. This indicates increasing returns to scale for all 4 groups of farms. When considering the sums of the regression coefficients for individual years, it is only in Year 3 for Group 4 that Σb_i has a value of less than 1 (0.999). It should be remembered that any errors of measurement in the values of the "independent" variables, especially using one-year figures, are bound to result in a downward bias of the regression coefficients.

In order to discover whether there was justification for considering the regression coefficients for each of the 4 years to be random variations from single values of each of the coefficients b_2, b_5, b_6, b_{10}, the variation was further analysed and the results are shown in Table 7. In Groups 1 and 3, there seems little doubt but that the hypothesis may be accepted. In Groups 2 and 4, however, the position appears to be rather doubtful. It was, nevertheless, decided to consider the regression planes for individual years to be parallel, even in the case of the latter two groups, since the gain in simplicity of presentation would seem to outweigh the probable error made in accepting identical coefficients for all four years. Furthermore, it would seem that the main interest would be in a production function which would indicate the relationships "normally" existing, and for this reason an average function would seem to be preferable to 4 almost identical functions, one for each of the 4 years.

The third analysis in this section was on the averages for each farm for the 4-year period. The results, as shown in Tables 5 and 6, are not very different from those of the Within Years regressions. However, the residual variances are in all cases somewhat lower than in the individual years and in the Within Years regressions. This is what might be expected and this characteristic is dealt with in more detail later when an attempt is made to explain the residuals round the regression surfaces as the sum of an "ability" or "managerial" element, constant for each farm over the 4 years, and a "random" element which each year affects the product of any farm in its respective group.

The Within Years regressions give a good description of the production relationships and although, as argued later, they cannot be expected to give the best basis for the calculations of "Marginal Products" and Least Cost Combinations, they will be used here for a special analysis of (1) the "ability" or "managerial" variations in Gross Product achieved by farmers at given global combinations of inputs, and (2) the "random" fluctuations in Gross Product affecting all farmers in the group.

The equations from the *Within Years* regressions for each of the 4 groups are as follows:

(a) Group 1: $z = 0.150\ x_2 + 0.145\ x_5 + 0.170\ x_6 + 0.567\ x_{10} + 0.3168$

Group 2: $z = 0.140\ x_2 + 0.314\ x_5 + 0.184\ x_6 + 0.425\ x_{10} + 0.1499$

Group 3: $z = 0.078\ x_2 + 0.179\ x_5 + 0.283\ x_6 + 0.508\ x_{10} + 0.2310$

Group 4: $z = 0.130\ x_2 + 0.163\ x_5 + 0.283\ x_6 + 0.456\ x_{10} + 0.3546$

The constant in each case is the average for 4 years.

(b) Group 1: $Z = 2.074\ X_2^{0.150}\ X_5^{0.145}\ X_6^{0.170}\ X_{10}^{0.567}$

Group 2: $Z = 1.412\ X_2^{0.140}\ X_5^{0.314}\ X_6^{0.184}\ X_{10}^{0.425}$

Group 3: $Z = 1.702\ X_2^{0.078}\ X_5^{0.179}\ X_6^{0.283}\ X_{10}^{0.508}$

Group 4: $Z = 2.262\ X_2^{0.130}\ X_5^{0.163}\ X_6^{0.283}\ X_{10}^{0.456}$

The variation in the constant term, in logs, from year to year is shown for each group in Table 8. In this table, a high constant for a given year is an indication that that year has given a larger Gross Product than other years at given inputs, both expressed in monetary terms, and *vice versa.*

In Group 1 (Dairying), Year 2 (55/56) was the worst year and Year 4 (57/58) the best, showing a 5% higher Gross Product for the same inputs.

In Group 2 (Livestock), Year 2 (55/56) was the worst year and Year 4 (57/58) the best, showing a 21% higher Gross Product for the same inputs.

In Group 3 (Mixed), Year 3 (56/57) was the worst year and Year 4 (57/58) the best, the latter having a Gross Product 2% higher for the same inputs.

In Group 4 (Arable), Year 3 (56/57) was again the worst with Year 2 (55/56) the best, the latter having a Gross Product higher by 8% for the same inputs.

In Table 9 where a single regression using parallel planes for the 4 groups has been calculated based upon the 4 *Between Farms* regressions, some indication is given for the 4-year period as to the relative Gross Products achieved from given inputs in the different farm-type groups. Although it is not strictly correct to consider the functions for the 4 type groups as being parallel, there is little doubt that it is justified for this specific purpose. The table then indicates that there is little difference in products achieved from given inputs during the 4-year period for the first two groups, Dairying and Livestock. Mixed farms show a Gross Product 1% to 2% greater than Dairying and Livestock, and Arable farms as much as 8% greater from the same inputs. Since the costs must be the same for given inputs, the 8% higher Gross Product for the Arable farms indicates a substantial difference in net income for farmers belonging to different type groups but incurring the same costs in their businesses.

The fact that the parallel planes regression for all 4 groups gives higher constants - a greater production for given inputs - for the 2 groups which on average consist of the largest farms (Mixed and Arable) agrees with the fact that a regression (Between Farms) for all farms without division into the 4 type groups gives a higher value for Σb_i (1.053) than the parallel planes regression between the 4 groups (1.044).

Goodness of Fit of the Regressions

The multiple correlation coefficients for Within Years (parallel planes) regressions are, for the 4 groups, 0.981, 0.966, 0.981 and 0.980 respectively. The corresponding multiple correlation coefficients for the Between Farms regressions are 0.988, 0.980, 0.989 and 0.990. These high values of the coefficients in themselves indicate a very good fit.

To analyse further the degree to which the regressions fit the data, the residuals - the logs of the actual Gross Products *minus* the regression values - were calculated. The distributions of these residuals are shown in Tables 10 and 11 for the Within Years (parallel planes) regressions and for the Between Farms regressions respectively. All these distributions are fairly close approximations to normal distributions, though they show a tendency to have larger negative than positive deviations.

As a final test of the Between Farms regressions, an analysis was made within each group of farms by carrying out separate regressions for each of the three thirds obtained by subdivision according to the size of $X_{11} = X_2 + X_6 + X_{10}$ (The sum of Rent & Rates, Total Cost of Labour including that of farmer and wife, and Total Purchases; for convenience, no account was taken of the interest charge on Tenant's Capital which represents a very small proportion of the costs).

X_{11} provides, undoubtedly, a reasonably good measure of the size of farm (according to costs incurred), and the 3 thirds therefore constitute groupings of the smallest, the medium-sized and the largest farms.

The results of these regressions are shown in Table 12. One point to notice in this table is the tendency towards a somewhat smaller residual variance among the large farms, but the quantative difference is not large and, although it should be noted, it would seem that there is little reason to discard the regression functions on this score.

As shown in Table 12, there is a tendency towards a slight systematic decline in the sum of the regression coefficients (Σb_i) in Groups 1 and 2, and the same tendency, though less marked, may exist in Groups 3 and 4.

From the Between Farms regressions the residual sums of squares are obtained. Similarly from the regressions for the 3 thirds a residual sum of squares is obtained by adding the individual residual sums of squares for the 3 thirds. In Table 13, these two types of residual sums of squares are shown with their respective degrees of freedom. The table further shows the differences between these two sums of squares with their respective degrees of freedom. These differences represent the total sums of squares due to differences between the positions of the 3 planes and to differences between the regression coefficients. If a single regression gives an equally good fit as 3 regressions, one for each third, then these 3 variances should be identical apart from random deviations. A test for this shows that for Group 1 it is statistically doubtful whether the hypothesis is valid that the whole range can be accepted as represented by one regression. From an economic point of view, the difference, though statistically significant, is so small that it is unlikely to lead to erroneous economic conclusions, and for the further analysis it will be considered that one regression for the whole range is justifiable. For Groups 2, 3 and 4, it appears that a single regression gives as good a fit as the 3 regressions. It should be noted that even for the Dairy group the statistically significant discrepancy between the 3 regression planes and the treatment as one group is no more important than to give smaller multiple regression coefficients for each of the 3 thirds (0.950, 0.884 and 0.958) than for the group as a whole (0.988).

Finally, an attempt was made to consider a Between Farms regression for all farms without grouping by type. The variance breakdown is shown in Table 5. The regression coefficients, their sum and the multiple correlation coefficients are shown in Table 6. There is, however, no doubt that it is not permissible to consider all the farms as belonging to a single group. A simple test for this is as follows:

	Sum of squares	Degrees of freedom	Variance
Residual variation from single Between Farms regression	82.0634	1641	0.050008
Residual variation from 4 Between Farms regressions	74.3450	1626	0.045723
Difference	7.7184	15	0.514560

Variance ratio: $\frac{0.514560}{0.045723}$ = 11.2539 with 15 and 1626 degrees of freedom.

Indeed, the figures appearing in this table have been calculated using the sums of the variables instead of the averages. The sums of squares and variances are accordingly about 16 times those in Table 5.

There cannot be any doubt that significant differences exist between the regressions within farm-type groups and that these are so large that it is not permissible to disregard the type grouping. This might also have been expected from Table 9 which shows that Arable farms (Group 4) obtain about 8% greater Gross Product from the same inputs than Dairy farms (Group 1) and Livestock farms (Group 2). It is interesting, however, to note that, in spite of this, the multiple correlation coefficient for the single regression is as high as 0.988, but this alone is not enough to permit the acceptance of the hypothesis that all farms belong to one group.

CHAPTER III

AN ESTIMATE OF THE "RANDOM" VARIANCE AND A "MANAGERIAL" VARIANCE IN GROSS PRODUCT ACHIEVED FROM GIVEN GLOBAL COMBINATIONS OF INPUTS

A farmer's job may be said to consist of two parts:

(1) decisions regarding the global combinations of the main input categories - land, buildings, etc., represented in this work by their costs: Rent & Rates, Total Tenant's Capital, Total Labour, and Total Purchased Raw Materials,

(2) the achievement, at the given global combination of inputs, that is at given costs, of the greatest possible value of production.

(1) The production functions as such analyse the consequences of using various *global* combinations of inputs. This is dealt with in Chapter IV on "Marginal Productivities" and in Chapter V on Least Cost Combinations. In these calculations the detailed composition of the individual input categories is disregarded in so far as each variable enters into the calculations only as a given amount.

(2) If there were a large number of farms all with exactly the same global combinations of inputs, one would not expect that they would all achieve the same value of production.

It would seem reasonable to consider two causes for the differences in products achieved by various farmers:

(a) Some farmers are better managers than others,

and (b) All farmers are, in any one year, subject to the influences of good and bad luck, or, in other words, to a "Random" element.

(a) "Managerial" Variance

It would be a mistake to believe that the total costs alone, or even various global combinations of inputs (of the input *groups* used in the regressions) all leading to the same total costs, completely determine the level of production in farming. At any given global combination of resources, farmers vary substantially in achievement because of varying ability to make the best use of the resources in detail. Two farmers, each using fertilisers of the same monetary value, may buy, in detail, very different plant nutrients and may also apply them to crops with very different degrees of husbandry skill. Some farmers appear "always" to do the right thing at the right time, others "always" at the wrong time. Similarly, a given amount spent on purchased feedingstuffs may be correctly or incorrectly balanced and may be fed to the individual animals in the right way, and *vice versa*. The presence of pesticides and fungicides means that the good husbandman, by noticing attacks on crops early and taking appropriate action, may save his crops, while less able neighbours may spend the same amount too late and thus lose a substantial portion of the crop. Similar reasoning could be applied *ad nauseam* to other input categories such as livestock, machinery, labour etc.

The position of Rent & Rates in this respect is slightly different from other input categories in so far as there are large variations in rents for farms of equivalent productive capacity. It has, however, been shown in Chapter II that Rent & Rates is nevertheless a better variable to use in the regressions than Adjusted Acres (see Table 4). In spite of the large element of variation in rents on British farms, which is due to historical developments and is not always related to the productive capacities of the farms, there still remains some element of ability regarding choice of farm, and, due to this in combination with the fact that land is a relatively unimportant factor of production on British farms, rent forming only a small proportion of total farm costs, it seems reasonable to expect that the peculiar rent situation will not invalidate to any great extent the reasoning in this chapter.

It must also be mentioned in this section that even within farm-type groups, substantial scope remains for farmers regarding choice of product. A better choice may be of great importance as regards the Gross Product achieved from a given combination of inputs. In this study no analysis has been made of this aspect of management, but the author does not believe that it would explain a very large part of the "Managerial" variance calculated in this chapter.

Lastly, it is necessary to state that any systematic upwards or downwards error in the valuation of the production for individual farms will enter into the concept which is here called "Managerial" variation. A systematic error in the valuation of the production affecting *all* farms, by about the same percentage, will enter into the constant in the Cobb-Douglas function.

(b) "Random" Variance

All farmers are subject to good and bad luck. Annual variations in climate for individual farms, biological variations in the quality of seed and livestock, damage by pests and disease in livestock as well as in crops, annual variations in timeliness of cultivations, sowing, fertiliser application, etc. would cause a variation in any *individual* farm's production from year to year even if its inputs and the overall farming conditions remained the same. Also, most farmers have annual deviations from their general stocking and cropping programmes. Further, the production in any one year may be slightly affected by errors of measurement due to incomplete records or to the arbitrary nature of valuations.

The joint effect of this multitude of influences may be termed "Random" variation, and it seems reasonable to expect that this forms a near normal distribution in the logarithms of Gross Product, thus affecting large Gross Products by a larger absolute amount than smaller Gross Products.

A First Estimate of "Random" and "Managerial" Variance

In Chapter II, the hypothesis that the individual years' regression coefficients could be considered as random deviations from one set of coefficients for each group was accepted, although with some hesitation for Groups 2 and 4.

In Table 5, the Within Years (parallel planes) regression[1] shows, as an example, the following breakdown of the total variation in z (log Gross Product) for Group 1:

	Sum of squares	Degrees of freedom	Variance
Variation due to regression	150.5538	4	–
Residual (I)	5.7595	1616	0.003564
Total	156.3133	1620	–

The residual sum of squares (I), 5.7595 with 1616 degrees of freedom, gives the variance 0.003564. Since each year's log Gross Product must be influenced by the "Managerial" as well as by the "Random" variance, the residual variance may be considered to be the sum of the 2 variances ($s_2^2 + s_1^2$). From the remarks at the beginning of this chapter under (a) it may be expected that a given farm is affected each year by the same "Managerial" influence, whereas each year's result has been influenced by the "Random" element.

[1] This regression takes account of the *average* changes from year to year because, for each year, the sum of the deviations from the regression values is zero.

The sum of squares (I) above is calculated as:

$$_{i}\Sigma_{j} \left[(z_{ij} - z_{i.}) - b_2 (x_{2\,ij} - x_{2\,i.}) - \ldots\ldots - b_n (x_{n_{ij}} - x_{n_{i.}}) \right]^2$$

where i = 1,2,3,4, indicates the year

j = 1,2,......N indicates the farm (N = 406 for Group 1).

x_n(n = 1,2,......n) indicates the various input categories.

b_n (n = 1,2,......n) indicates the Within Years regression coefficients.

z_{ij} = log Gross Product in year i for farm j (i=1,2,3,4) (j=1,2,......N)

$$z_{.j} = \frac{1}{4} \sum_i z_{ij}$$

$$z_{i.} = \frac{1}{N} \sum_j z_{ij}$$

$$z_{..} = \frac{1}{4 \times N} {}_{i}\Sigma_{j}\; z_{ij}$$

$x_{n_{ij}}$ = log (n^{th} input) in year i for farm j (i = 1,2,3,4) (j = 1,2,......N)

$$x_{n.j} = \frac{1}{4} \sum_i x_{n_{ij}}$$

$$x_{n_{i.}} = \frac{1}{N} \sum_j x_{n_{ij}}$$

$$x_{n..} = \frac{1}{4 \times N} {}_{i}\Sigma_{j}\; x_{n\,ij}$$

The components of this sum of squares may be written as:

$$_{i}\Sigma_{j} (z_{ij} - z_{i.})^2 = {}_{i}\Sigma_{j} (z_{.j} - z_{..})^2 + {}_{i}\Sigma_{j} (z_{ij} - z_{i.} - z_{.j} + z_{..})^2 \quad \text{and}$$

$$_{i}\Sigma_{j} (x_{n_{ij}} - x_{n_{i.}})^2 = {}_{i}\Sigma_{j} (x_{n_{.j}} - x_{n_{..}})^2 + {}_{i}\Sigma_{j} (x_{n_{ij}} - x_{n_{i.}} - x_{n_{.j}} + x_{n_{..}})^2$$

The sum of squares, (I), in the above table, may then be broken down into the two sums of squares (II) and (III):

$$(II) = 4 \times \sum_j \left[(z_{.j} - z_{..}) - b_2 (x_{2\,.j} - x_{2\,..}) - \ldots\ldots - b_n (x_{n.j} - x_{n..}) \right]^2$$

and

$$(III) = {}_i\Sigma_j \left[(z_{ij} - z_{i.} - z_{.j} + z_{..}) - b_2 (x_{2_{ij}} - x_{2_{i.}} - x_{2_{.j}} + x_{2_{..}}) - \ldots\ldots - b_n (x_{n_{ij}} - x_{n_{i.}} - x_{n_{.j}} + x_{n_{..}}) \right]^2$$

The calculation of both these sums of squares is somewhat cumbersome since the normal equations cannot be used for simplification.

The resulting breakdown is as follows. The sum of squares (II) shows the Between Farms variation, the corresponding variance being an estimate of $(4s_2^2 + s_1^2)$. The sum of squares (III) shows the "Random" variation, the corresponding variance being an estimate of s_1^2. An estimate of s_2^2 is obtained by subtracting the variance, s_1^2, from the Between Farms variance and dividing by 4.

These calculations were carried out and Table 14 shows, for each farm-type group, the analysis of the total variance from the Within Years regressions whereby variance (I) is broken down into variance (II), $(4s_2^2 + s_1^2)$ referred to as Between Farms, and variance (III), s_1^2 referred to as "Random".

Table 15 shows the calculated values of s_1^2, the "Random" element, and s_2^2, the "Managerial" element.

A Second Estimate of "Managerial" and "Random" Variances

The residual variance (round the regression planes) from the Between Farms regression (see Table 5) provides a joint variance influenced mainly by the "Managerial" variance but also to some extent by the "Random" variance. This variance may be expected to be an estimate of $(s_2^2 + \frac{1}{4} s_1^2)$. The estimates obtained in this way are in almost complete agreement with those of the previous section.

Another estimate can also be obtained for the "Random" variance by itself by carrying out a multiple regression for each individual value of z, corrected for row (farm) and column (year) deviations, upon the similarly corrected independent variables. Using the notation on page 19 these variables may be written as:

$$(z_{ij} - z_{i.} - z_{.j} + z_{..})$$

and

$$(x_{n_{ij}} - x_{n_{i.}} - x_{n_{.j}} + x_{n_{..}})$$

The regression of these z-variables on the corresponding x-variables gives a residual sum of squares, with a corresponding variance which is an estimate of s_1^2. This new estimate of s_1^2, as a result of the minimising process applied, is bound to be a minimum.

The variances calculated in this section are shown below for each farm-type group, together with the resulting new estimates of s_2^2.

Estimate of	Group 1	Group 2	Group 3	Group 4
$s_2^2 + \frac{1}{4}s_1^2$	0.002262	0.003544	0.002671	0.003139
s_1^2	0.001666	0.003074	0.002335	0.003626
s_2^2	0.001846	0.002775	0.002087	0.002233

Conclusions regarding "Managerial" and "Random" variation

In the previous two sections, two estimates have been obtained of the "Managerial" and "Random" variances. The two estimates of each agree very well as will be seen below where the corresponding standard deviations are shown. These standard deviations indicate, for each particular kind of variation, the limits for the deviations from the regression values of log Gross Product within which the actual values may be expected to lie in 68% of all cases.

Standard Deviations for "Managerial" (s_2) and "Random" (s_1) Variation

	Group 1		Group 2		Group 3		Group 4	
	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1
First Estimate	0.043	0.042	0.052	0.058	0.045	0.049	0.046	0.061
Second Estimate	0.043	0.041	0.053	0.055	0.046	0.048	0.047	0.060

For the "Random" element, the largest variations are found in the Arable and Livestock groups and the smallest in the Dairy group. For the "Managerial" element, the largest variation is found in the Livestock group and the smallest again in the Dairy group. The following illustrations are all based on the standard deviations of the first estimate in the table above.

"Random" Variation

The consequence of the "Random" element is that, even under the same farming conditions, a given farm could only expect 68% of all years' log Gross Products to fall between its average log Gross Product *plus* or *minus* one standard deviation. This means that, due purely to this "Random" element, a variation of the actual Gross Products round their regression values may be expected as shown below:

Group 1 (Dairy)

68 % of actual values between 91 % and 110 % of regression values
95 % of actual values between 82 % and 121 % of regression values

Group 2 (Livestock)

68 % of actual values between 88 % and 114 % of regression values
95 % of actual values between 77 % and 131 % of regression values

Group 3 (Mixed)

68 % of actual values between 89 % and 112 % of regression values
95 % of actual values between 80 % and 125 % of regression values

Group 4 (Arable)

68 % of actual values between 87 % and 115 % of regression values
95 % of actual values between 76 % and 132 % of regression values

In other words, a dairy farmer having a Gross Product of about £5000 might well have his Gross Product influenced in an upwards or downwards direction by £500 in any particular year due purely to the "Random" element (good and bad luck). This would also have the effect of increasing or decreasing his income by £500 since the costs would be the same.

"Managerial" variation

The consequence of the "Managerial" element is that, even if a large number of farmers operated for a period of several years under the same conditions and with the same global inputs, then only 68 % of them might expect to have an average log Gross Product falling between the overall average log Gross Product *plus* or *minus* one standard deviation. This means that, due purely to this "Managerial" element, a variation of the average actual Gross Products round the average regression values may be expected as shown below:

Group 1 (Dairy)

68 % of actual values between 91 % and 110 % of regression values
95 % of actual values between 82 % and 121 % of regression values

Group 2 (Livestock)

68 % of actual values between 89 % and 113 % of regression values
95 % of actual values between 79 % and 127 % of regression values

Group 3 (Mixed)

68% of actual values between 90% and 111% of regression values
95% of actual values between 81% and 123% of regression values

Group 4 (Arable)

68% of actual values between 90% and 111% of regression values
95% of actual values between 81% and 123% of regression values

In other words, even if a large number of dairy farmers operated for a period of several years under the same conditions and with the same global inputs giving an average annual Gross Product of £5000, there would be a substantial variation between the farms in the average Gross Products so that only about 68% would achieve an average Gross Product falling between £4550 and £5500. The "Managerial" variation alone would, in fact, cause an average annual Gross Product of about £4600 for the less able half of the farmers in the group, and an average annual Gross Product of £5400 for the more able half.[1] These two halves would thus show a difference of £800 in average annual income as inputs, and therefore costs, would be the same for all farms.

For the other farm-type groups, the variation would be even greater.

Importance of "Random" Variance and "Managerial" Variance in the Total Variance

For farm accounts averaged over different periods, it is obvious that the "Managerial" variance will remain the same while the "Random" variance will be reduced through being averaged over the period for which the accounts have been considered. This is shown in Table 16.

From an inspection of the table, it is obvious that a single year's accounts have a variance which is about equally influenced by "Random" (good and bad luck) elements and by "Managerial" elements. By averaging the accounts for three to four years, it is shown that the "Random" element is substantially reduced, and *it is the author's firm opinion that far too many farm management analyses are being carried out using accounts from a single year only where it would be possible, and much more reliable, to use accounts averaged over, say, a 4-year period.* It is no doubt of little importance whether arithmetic or geometric means are used.

In particular, if accounts for a single year are divided into the "best" and the "worst" halves according to results, then this divi-

[1] The mean positive (and negative) deviations being 0.8 times the standard deviation (Geary's ratio).

sion is to a great extent a grouping according to good and bad luck. *Such groupings can hardly provide useful information for farm management purposes.*

A Comparison between Farms with Positive and Negative Deviations from Regression Values in the Between Farms Regression

The original accounts were divided, within each type group, into two groups: those with larger average log Gross Product than that given by the regression (Between Farms) and those with smaller average log Gross Product.

The two groups did not show exactly the same Total Costs (inclusive of 9% interest on Tenant's Capital) but in Table 17 the figures for Inputs and Gross Product are shown for £5000 Total Costs.

It is evident that the two groups show very similar combinations of inputs and differ chiefly in Gross Product, the positive deviation group showing:

Group 1 (Dairy)

£703 higher Gross Product than negative deviation group

Group 2 (Livestock)

£885 higher Gross Product than negative deviation group

Group 3 (Mixed)

£858 higher Gross Product than negative deviation group

Group 4 (Arable)

£1007 higher Gross Product than negative deviation group

As the variances of the deviations which have been used for this grouping (see Table 16) are of the type $(s_2^2 + \frac{1}{4} s_1^2)$, it is obvious that the grouping is mainly according to "Managerial" ability.

CHAPTER IV

"MARGINAL PRODUCTIVITIES"

Introduction

While in the previous chapter the interest was centred on the variations *round* the production surfaces, this chapter and the following one on Least Cost Combinations will be concerned with the functions themselves and their derivatives, the "Marginal Productivity" functions.

In the previous chapter, three main regressions were carried out:

(1) *Residuals* regressions on variables of the form

$(z_{ij} - z_{i.} - z_{.j} + z_{..})$ and $(x_{n_{ij}} - x_{n_{i.}} - x_{n_{.j}} + x_{n_{..}})$.

(2) *Within Years* regressions on variables of the form $(z_{ij} - z_{i.})$ etc. Four such regressions may be calculated, one for each year, but the pooling of the sums of squares and sums of products to provide 4 parallel regression surfaces was accepted (in some cases with a little hesitation).

(3) *Between Farms* regressions, the regressions on the 4-year averages of each variable for each farm, variables of the type: $(z_{.j} - z_{..})$.

The results of these 3 regressions are shown in Table 18 together with the results from another regression, the *Variance Components* regression which is explained below.

There is a great difference between the *Residuals* regression on the one hand, and the *Within Years* and *Between Farms* regressions on the other.

The first regression, on Residuals, is essentially *Intrafarm*, whereas the other two are *Interfarm*.

The Residuals regression does not take into account differences in structure between farms but only uses variables corrected for *Year* and *Farm* means. For this reason the range of the variables is very much smaller in this regression than in the other two. This is shown in Table 19. Because of the very small range of the independent variables for the Residuals regression compared with the Between Farms regression, a lower multiple correlation coefficient might be expected. This is clearly the case as is shown in Table 18.

The residuals used in the *Residuals* regression may be due to:

(1) Deliberate decisions by farmers to change the inputs out of step with the average changes on other farms.

(2) Minor changes due to particular features of the season as it affects the individual farm.

(3) More or less haphazard, and not carefully considered, changes in inputs.

(4) Actual mistakes and inconsistencies in the accounts. This is probably a relatively important component of these small variables, much more important than for the larger variables in the Within Years regression and especially for those in the Between Farms regression in which the use of 4-year averages tends to even out a great part of errors of measurement.

The *Within Years* and *Between Farms* regressions are essentially *interfarm*. The regressions are calculated in a way which gives very great weight to conditions on farms where the independent variables differ most from the average values. This means that the regression surfaces are estimates of the Gross Products found by moving *from* farms with one input combination *to* farms with different input combinations.

This naturally leads to a discussion of whether *interfarm* production functions can be expected to give valuable information about *intrafarm* functions.

This problem was discussed in Chapter I, where the conclusion was that, as farmers in fact differ very much in their input combinations, the various combinations actually found may in the event be considered to be approximations to experimental data. Thus the production functions can be expected to provide good estimates of the Gross Product *resulting* from the various combinations. On these grounds it would seem to the author that it is reasonable to expect that the *interfarm* functions do in fact represent *intrafarm* functions.

In the previous chapter, it was seen that a given farm may in fact have a separate production function which differs from the regression function only because its actual log Gross Product tends to be, say, 0.041 above (or below) the regression value of log Gross Product. In fact, some 16% of farmers could be expected to obtain a Gross Product at least 10% higher than the average for farmers using the same global input combinations, and some 16 % of farmers an equally lower Gross Product.

It must be admitted that it is conceivable, *though in the author's opinion not probable*, that able farmers tend to have certain input combinations and less able farmers other combinations. In this case, the estimated functions would be distorted and would no longer measure the *consequences* of the input combinations by themselves.

Two arguments may be presented, both against the likelihood of this kind of distortion:

(1) It is the author's experience that successful farmers use very different input combinations. The same is true of farmers who are not successful.

(2) As shown in Chapter VI, the *global combination of inputs* (i.e. the combination of the four *groups* of inputs: Rent & Rates, Tenant's Capital, Labour and Purchased Raw Materials) is of less importance for efficient production than are the *varying abilities of farmers* to achieve large Gross Products at given global combinations of inputs.

In *Chapter VI* a breakdown of the variation of

$E = \log \frac{\text{Gross Product}}{\text{Total Costs}}$ is carried out. This measure, the logarithm of the Gross Product per £1 Total Costs, is a measure which is suitable for comparisons of production efficiency at *different* global combinations of inputs as well as at the *same* global combinations.

It is evident in *Chapter VI* that *efficiency in British farming is influenced to a greater extent by the "Managerial" variance, the variation in husbandry skill (and choice of products) than by the various global combinations of inputs ("Allocation of Resources")*.

Therefore, it seems that one would have to refute the hypothesis that able farmers do, in fact, choose very different combinations of inputs than do less able farmers. If this happened then it would have to be assumed that the able farmers choose *better* combinations of inputs than the less able (the reverse would hardly be tenable) and *if* better farmers choose better combinations, then the influence of input combinations would be expected to be substantial, the more so since, on *a priori* reasoning, one would expect some economies of scale.

To sum up: It is accepted that the regressions do, in fact, give good estimates of Gross Products *resulting* from the input combinations, and, what amounts to the same thing, it is accepted that the *interfarm* production functions do, in fact, show, or at least give good estimates of, *intrafarm* functions which, however, for any individual farm may have to be corrected by a constant added to or subtracted from the log Gross Product to take account of the farmer's "Managerial" ability (his husbandry skill).

Choice of Regression for Calculation of "Marginal Productivities"

Table 18 shows the results of the Residuals, the Within Years and the Between Farms regressions together with the new type of regression on Variance components.

The Residuals regression distinguishes itself from the other three regressions in two important respects:

(1) The multiple correlation coefficients are much smaller, and

(2) The sums of the regression coefficients are also much smaller.

The table also shows the type of variances (co-variances) which are used for the regressions (multiplied by the degrees of freedom giving sums of squares and products), and here probably lies the explanation of the small coefficients in the Residuals regression.

If the variables taken from single years' accounts have only a small element of errors of measurement, then the same errors of measurement must also affect the residuals, and as these are all very small then the element of error will be of much greater relative importance.

The errors of measurement then affect, particularly in the Residuals regression, the multiple correlation coefficient and also the sum of the regression coefficients in a downward direction.

The same is true for the Within Years regression, but here to a lesser extent because these variables are much larger than those in the Residuals regression.

On the other hand, the errors of measurement are likely to cancel each other out to a great extent when the variables for the Between Farms regressions are considered, because these variables are averages for a 4-year period. At the same time, the variables and their ranges are much larger (see Table 19) than those in the Residuals regression.

Although the Residuals regression formally represents *intrafarm* relations, it must be discarded for "Marginal Productivity" calculations because of the relatively large element of errors of measurement in the independent variables and the resulting bias.

From a study of Table 18, which shows the type of variances (and co-variances) used for the regressions, it may be seen that:

The *Residuals* regression uses variances (co-variances) of the type s_1^2

The *Within Years* regression uses variances (co-variances) of the type $(s_2^2 + s_1^2)$

The *Between Farms* regression uses variances (co-variances) of the type $(s_2^2 + \frac{1}{4} s_1^2)$

If it is accepted that errors of measurement are part of the variances (co-variances) of the type s_1^2, then it would seem natural, as an attempt to avoid the bias introduced into the regression when the "independent" (the "explaining") variables are subject to errors of measurement, to carry out a regression on variables of the type s_2^2 alone. The technique for this is the use of Variance and Co-variance components.[1]

The Variance and Co-variance components are obtained from the two-sided (years and farms) variance breakdown of each variable and from the corresponding breakdown of the co-variances as illustrated in the table below in which, for simplicity, the Between Farms variance is represented by the variance of the sums of the variables over the 4 years instead of by the variance of the averages over the 4 years.

	Type of variance (co-variance) used	z	zx_2	
Between Farms	$4s_2^2 + s_1^2$	0.377713	0.358090	etc.
Residuals	s_1^2	0.002747	0.000412	
Variance (Co-variance) components	$4s_2^2$	0.374966	0.357678	

It has been argued previously that the Variance and Co-variance components, of the type s_2^2, may be assumed to be even less influenced by errors of measurement regarding the independent variables than are the variables used for the Between Farms regressions, of the type $(s_2^2 + \frac{1}{4} s_1^2)$. This would lead to an expectation of a slightly higher value of Σb_i for the Variance Components regression than for the Between Farms regression. An inspection of Table 18 shows that this is the case, and when comparing the 4 different regressions presented in the table it may be seen that, as the Residuals element, s_1^2, in the variables used for the regression becomes smaller, then

[1] See M.H. QUENOUILLE, *Associated Measurements*, Butterworth, London, 1952. pp. 134 - 138.

both Σb_i and R increase. The most marked jump is from Residuals to Within Years, but also from Within Years to Between Farms and further to Variance Components an increase is shown in the values of Σb_i and R for all 4 farm-type groups. Attention should be drawn to the fact that in the 7 Independent Variable Case dealt with in Chapter VI, similar increases in Σb_i do not occur. On the contrary, in moving from the Within Years to the Variance Components regression there is a slight decrease in the value of Σb_i, a phenomenon for which the author can offer no explanation.

In the following calculations of "Marginal Productivities" (and also in the calculations of the Least Cost Combinations in the next chapter), the *Variance Components* regressions[1] are preferred throughout, partly because they show the highest multiple correlation coefficients (varying from 0.985 in the Livestock to 0.993 in the Arable group), but principally because it can be assumed that errors of measurement have been, for practical purposes, eliminated from the independent variables so that the regression coefficients can be expected to be free from the systematic downward bias so apparent in the Residuals regressions but also of importance in the Within Years and to a lesser extent in the Between Farms regressions.

Resulting "Marginal Productivities"

By differentiation, the production functions give the "Marginal Productivities" functions:

$$\text{If } Z = k\, X_2^{b_2}\, X_5^{b_5}\, X_6^{b_6}\, X_{10}^{b_{10}}$$

then

$$\frac{\delta Z}{\delta X_2} = b_2 \frac{Z}{X_2}$$

$$\frac{\delta Z}{\delta X_5} = b_5 \frac{Z}{X_5}$$

$$\frac{\delta Z}{\delta X_6} = b_6 \frac{Z}{X_6}$$

$$\frac{\delta Z}{\delta X_{10}} = b_{10} \frac{Z}{X_{10}}$$

The "Marginal Productivities" for all variables at geometric mean values are shown in Table 20. In the calculations for the table the fact is disregarded that the regression values of log Gross

[1] As no satisfactory method of obtaining degrees of freedom for the Variance Components regression is known to the author, no standard deviations have been calculated for the regression coefficients. However, those calculated for the Between Farms regression should give some indication.

Product do not in fact correspond to the mean values of the *actual* Gross Products. This follows from the lognormal distribution of Gross Product round the regression values.[1] The mean values of the Gross Products can be calculated by adding (for the Between Farms regression) a constant of about 0.0030 to the regression values of log Gross Product, and the same correction can no doubt be used as a good approximation for the Variance Components regression. This correction differs a little between the groups:

0.0026 for Group 1	0.0031 for Group 3
0.0041 for Group 2	0.0036 for Group 4

This correction means that to obtain the mean values of expected Gross Products or the mean values of "Marginal Productivities" *about ¾ of 1%* should be added to the values obtained directly from the regressions. As this correction is so very small and as it is easy to calculate, it has been omitted here. It should be emphasised, however, that the omission is hardly warranted, the more so as the author has not seen so much as a mention of this correction in any previous study.

The table shows 3 calculated "Marginal Productivities" for each variable, one for each of the 3 regressions: Within Years, Between Farms and Variance Components. Of these 3 calculations, as previously stated, *those based on the Variance Components regression are definitely preferred.*

Rent & Rates

For all farm-type groups an extra £1 spent on Rent & Rates, for other inputs fixed at their geometric means, can be expected to be associated with more than £1 extra Gross Product, varying from £1.19 to £2.66. This indicates that, at given other inputs, most farms could be operated more efficiently if they had more land and/or better equipped farms.

Tenant's Capital

For the 4 groups the extra Gross Product from £1 extra Tenant's Capital, with other inputs at their geometric means, is shown to vary between £0.16 and £0.24. In other words, the return to Tenant's Capital on the margin is about 16% to 24%, which is a very good return on capital invested, showing that British farms tend to have too little Tenant's Capital for the given other inputs. Probably the values used for Tenant's Capital in the Farm Management Survey are under-estimates of the market value of these assets, but, even

[1] HALD, A., *op. cit.* p. 161.

at an undervaluation of about 30%, the "Marginal Productivities" correspond to about 11% - 17% returns on these market values. It should be remembered that depreciation has already been accounted for as a deduction from Gross Product.

Labour

The extra Gross Product from £1 extra spent on Labour, with other inputs at their geometric means, is just about £1 for Mixed farms, thus indicating an economic use of Labour. For the 3 other groups the "Marginal Productivities" are £0.55 in the Livestock group, £0.61 in the Dairy group, and £0.83 in the Arable group.

This indicates the use of far too much labour for the given other inputs in all these 3 groups, this excessive use of labour being of the most importance in the Livestock and Dairy groups.

Total Purchases

The "Marginal Productivities", the extra Gross Product for £1 extra spent on this input with other inputs at their geometric means, are all very close to £1, ranging from £1.01 for Livestock farms to £1.06 for Arable farms.

Regarding these "Marginal Productivities", a reference must however be made to the next section where, instead of using a single variable, Total Purchases, a breakdown is made into 3 variables: Seeds & Feed, Fertilisers & Lime, and Other Purchases. The separate "Marginal Productivities" for these new variables differ from those using a single variable, with Fertilisers & Lime showing the highest "Marginal Productivity" and Other Purchases showing the smallest, less than £1 in all groups.

It has previously been argued that the same production functions apply to farms with able managers (farmers) as well as to farms with less able managers, with the sole difference that a constant is added to (or subtracted from) the log Gross Product. In the last chapter it was shown that, due to different managerial abilities (husbandry skill), about 16% of all farmers can expect to achieve a Gross Product at least 10% above the average Gross Product from the same input combination. Also, about 16% of all farmers can expect a Gross Product at least 10% below the average Gross Product from the same input combination. This same difference will exist for the "Marginal Productivities". The 16% best managers (farmers) will have "Marginal Productivities" at least 20% higher than the 16% worst managers, all at the same input combinations.

CHAPTER V

LEAST COST COMBINATIONS

From the discussion about the type of production function to use for this study (in Chapter I) and from the subsequent tests for the goodness of fit of the Cobb-Douglas function chosen, it must be apparent that the author is not seriously worried over the linearity of the Least Cost Combinations which is the consequence of choosing this type of function.

The author thus accepts that the Least Cost Combinations are a good approximation to conditions on British farms and are the same for all farms in a given type-group, *irrespective* of the scale of operation and *irrespective* of the managerial ability of the farmer.

The only two things which must be remembered in this connection are:

(1) All types of farms show increasing returns to scale so that twice a given Gross Product will not require twice the previous inputs but somewhat (about 10%) less.

(2) As a result of better management, some farmers achieve, from given inputs, much larger Gross Products than other farmers. About 16% of the farmers (the best) will achieve from given inputs at least 20% larger Gross Products than the 16% worst farmers.

It should also, perhaps, be mentioned that, as all type-groups show increasing returns to scale, no optimum combination can be calculated. At Least Cost Combinations, or at any other combinations using fixed proportions of inputs, the production of large farms is more efficient than that of small.

The Calculation of Least Cost Combinations

In the previous chapter the "Marginal Productivities" were given as:

$$\frac{\delta Z}{\delta X_2} = b_2 \frac{Z}{X_2}$$

$$\frac{\delta Z}{\delta X_5} = b_5 \frac{Z}{X_5}$$

$$\frac{\delta Z}{\delta X_6} = b_6 \frac{Z}{X_6}$$

$$\frac{\delta Z}{\delta X_{10}} = b_{10} \frac{Z}{X_{10}}$$

The condition for a Least Cost Combination is that the "Marginal Productivities" should all bear the same relation to the unit cost of the variables concerned.

In this case all variables are measured in monetary terms, but even so one of the variables differs somewhat from the 3 others, i.e. Tenant's Capital. Whereas £1 extra for the three variables means £1 extra cost, for Tenant's Capital £1 extra (as depreciations have been considered as a negative item of Gross Product) means an extra interest charge, which for this purpose is chosen to be 9%.[1]

The condition for a Least Cost Combination is thus:

$$b_2 \frac{Z}{X_2} = v$$

$$b_5 \frac{Z}{X_5} = 0.09 \ v$$

$$b_6 \frac{Z}{X_6} = v$$

$$b_{10} \frac{Z}{X_{10}} = v$$

The variable, v, because of the increasing returns to scale, is, in the relevant region, greater than unity, and, further, for a proportionate increase in all inputs the variable, v, will increase.

The above conditions for Least Cost Combinations may also be written as:

$$b_2 \frac{Z}{X_2} = b_5 \frac{Z}{0.09X_5} = b_6 \frac{Z}{X_6} = b_{10} \frac{Z}{X_{10}}$$

or

$$\frac{b_2}{X_2} = \frac{b_5}{0.09X_5} = \frac{b_6}{X_6} = \frac{b_{10}}{X_{10}}$$

It is clearly seen that the values of b_i and the chosen rate of interest

[1] As Tenant's Capital is no doubt substantially undervalued in the accounts used, this rather high percentage rate is chosen. It probably corresponds to about 6% - 7% on the market value of Tenant's Capital.

(9%) determine the proportion of inputs which form Least Cost Combinations.

For the 4 farm-type groups, using the regression coefficients from the Variance Components regression, the following Least Cost Combinations were calculated and, for comparison, the actual relative use of inputs on the farms studied are also shown in Table 21.

In order to compare the Least Cost Combinations with the actual combinations found in the data, it was decided to show, for each type-group, the actual average inputs and the resulting Gross Product. The Least Cost Combinations giving the same Total Costs (including 9% interest on Tenant's Capital) were calculated, and from these inputs the expected Gross Product, achieved at the same Total Costs as the actual Gross Product, was calculated. These calculations are shown in Table 21.

It will be seen that the Least Cost Combinations would, apart from Group 3 (Mixed), require much larger farms. It must be emphasized that this relates to a situation where it is a *condition* for the calculated Least Cost Combination that the Total Costs (including 9% interest on Tenant's Capital) should be the same as the actual average Total Costs.

The amount of Tenant's Capital should also be much larger in all groups.

The Labour cost should be much lower in Groups 1 & 2 (Dairy and Livestock) and somewhat lower in Groups 3 and 4 (Mixed and Arable).

Total Purchases should be a little lower in all groups.

A warning must be given in connection with the Least Cost Combinations. For some of the variables the changes required from the present combinations are so great that the Least Cost Combinations are, or at least are very close to being, *extrapolations* outside the range found in the data. For this reason some caution must be exercised in using the calculated Least Cost Combinations. The author does not distrust the realities behind these calculations, *but,* as most British farmers appear to operate with combinations of inputs far away from the Least Cost Combinations, it follows that the accuracy of the calculated Least Cost Combinations is bound to be less than it would have been had British farmers on the whole operated with input combinations closer to the Least Cost Combinations.

Per £1000 Total Costs (including 9% interest on Tenant's Capital), the Least Cost Combinations would give Gross Products (*and profits*) greater than those from the present average combinations of inputs[1] by the following amounts:

[1] It should be emphasised once more that the Least Cost Combination means farming at the *same Total Costs per farm (same scale)* as the present average combination of inputs.

Group 1	(Dairy)	£107
Group 2	(Livestock)	£197
Group 3	(Mixed)	£ 41
Group 4	(Arable)	£ 81

It would thus appear that on average the Mixed farms (Group 3), and also to a lesser extent the Arable farms (Group 4), are much closer to the Least Cost Combinations than Dairy farms (Group 1) or particularly, Livestock farms (Group 2). (See, however, Chapter VI where a similar calculation has been carried out for the 7 independent variable case).

It might be argued that the size of British farms is so unchangeable - so sacred - that it is unrealistic to present the above calculated Least Cost Combinations. These imply the same total costs as the present average total costs, but allow for changes in farm size (Rent & Rates) as well as in other inputs in reaching the Least Cost Combination.

If the farm size must remain unchanged, another conditional Least Cost Combination may be calculated under this restraint. This has been done and the results in Table 22 show that the new *conditional* Least Cost Combination compared with the previous Least Cost Combination would require a little more of all other inputs. At the same time the calculated new Gross Product is of course somewhat lower than that obtained from the unconditional Least Cost Combination.

In Figures 1-4 the variable, v, is shown for various Least Cost Combinations for the 4 farm-type groups.

In these figures, v indicates the "Marginal Productivities" of the inputs, Rent & Rates, Labour and Purchased Raw Materials directly while for the last variable, Tenant's Capital, the values of v should be multiplied by 9% to give the "Marginal Productivities" of Tenant's Capital as an interest rate.

CHAPTER VI

CAUSES OF VARIATION IN PRODUCTION/COST RATIOS

Many Agricultural Economists have in recent years calculated production functions based upon farm accounts. All of those calculations known to the author have been calculated using accounts from a single year only, which is a weakness due to the systematic downward bias in the regression coefficients in which such calculations are bound to result (see p. 28).

But these regressions based upon a single year's accounts have yet another weakness, which is not so obvious. As no breakdown of the variation round the regression surfaces, of the kind shown in Chapter III, has been possible, many readers (and writers?) may have been led to believe that the variation round the regression surfaces, on the whole, represented the unavoidable random element in farm production.

In this study the benefit of having 4 years' accounts for each farm has made it possible to show that, at least for a period of a few years, which is of necessity of greater importance in farming than a single year, the main cause of variation round the regression surfaces has been the "Managerial" ability which was analysed in Chapter III.

This "Managerial" variation has two main components:

(1) The detailed composition of the input groups (husbandry skills), and

(2) The choice of products.

It is with regret that the author is unable to offer a breakdown into these two components. It is important to emphasise that the "Managerial" variation shows a variation at given global combinations of resources, in other words, at given Total Costs.

Because of the use of 4-year accounts for each farm in this study, it is possible to obtain a breakdown of the causes of variation in efficiency which is very detailed.

For this analysis it is necessary to choose a measure of efficiency which can measure efficiency at the *same* as well as at *different* combinations of resources.

Such a measure is

$$E = \log \frac{\text{Gross Product}}{\text{Total Costs}},$$

the logarithm of Gross Product per £1 Total Costs.

Total Costs were defined as the sum of Rent and Rates, Labour, Total Purchases and 9% of Total Tenant's Capital. $z_{.j}$, has been used previously for average log Gross Product (over the 4 years). Let $T_{.j}$ indicate average log Total Costs (over the 4 years), and let $z^*_{.j}$ indicate the average regression value of log Gross Product (over the 4 years). In this case the Between Farms regression was used.

$(z_{.j} - T_{.j}) = E_{.j}$ indicates, for farm j, the logarithm

of the average actual value of Gross Product per £1 Total Costs.

$(z^*_{.j} - T_{.j}) = E^*_{.j}$ indicates, for farm j, the logarithm

of the average regression value of Gross Product per £1 Total Costs.

As shown in Chapter III and illustrated in Table 16, the 4-year average deviation in log Gross Product from the regression value of log Gross Product can be considered as the sum of two components: the "Managerial" and the "Random" elements.

Let this average deviation be called:

$$w_{.j} = z_{.j} - z^*_{.j}$$

This means that:

$$z_{.j} = z^*_{.j} + w_{.j}$$

and accordingly also that:

$$(z_{.j} - T_{.j}) = (z^*_{.j} - T_{.j}) + w_{.j}\text{, or:}$$

$$\underline{E_{.j} = E^*_{.j} + w_{.j}.}$$

As the two variables $E^*_{.j}$ and $w_{.j}$ are, for all practical purposes, uncorrelated, or at least show so little correlation that the error in considering them to be uncorrelated is very slight, it follows that:

$$\sum_j E^2_{.j} \text{ is almost equal to } \sum_j E^{*2}_{.j} + \sum_j w^2_{.j}.$$

This provides an opportunity to break down $\sum_j E^2_{.j}$ into four different components.

From the regression of $E^*_{.j}$ upon $T_{.j}$ the total variation $\sum_j E^{*2}_{.j}$ may be broken down into two components:

(1) Scale of farming (defined by $T_{.j}$),

(2) Global combinations of inputs at given scale of farming, i.e. (1) + (2) equals

(3) Total global combination of inputs.

From Table 16, the total variation $\sum_j w^2_{.j}$ may be broken down into two components:

(4) "Managerial" variance at given global combination of inputs,

(5) "Random" variation.

These sums of squares, and their totals (6) are shown in Table I together with the actual sum of squares $\sum_j E^2_{.j}$, (7). In the last line (8) of Table I are shown the correlation coefficients between $E^*_{.j}$ and $w_{.j}$. In agreement with the slight, and for the first 3 groups negative, correlation, the figures for these 3 groups in line (6) are slightly larger than those in line (7), but the difference is so small that one can for all practical purposes accept lines (1), (2), (4) and (5) as a breakdown of (7) as well as of (6).

RELATIVE IMPORTANCE OF DIFFERENT FACTORS FOR VARIATION IN E.

TABLE I

TYPE OF VARIATION	SUMS OF SQUARES			
	Group 1	Group 2	Group 3	Group 4
1. Scale of farming *	0.135	0.467	0.192	0.052
2. Global combination of inputs at given scale of farming	0.227	0.732	0.148	0.104
3. Total global combination of inputs	0.362	1.199	0.340	0.156
4. "Managerial" variance at given global combination of inputs	0.732	1.112	1.201	0.506
5. "Random" variation	0.177 **	0.347 **	0.356 **	0.223 **
6. TOTAL	1.271	2.658	1.897	0.885
7. Actual variation in E	1.229	2.612	1.759	0.885
8. Correlation coefficient between E* and w	-0.035	-0.016	-0.094	+0.001

* Scale of farming defined as log Total Costs.

** These figures refer to averages for 4 years.

From the sums of squares in Table I, it is possible, by dividing each sum of squares by one less than the number of farms in the group, to derive corresponding variances, which, in contrast to the sums of squares, may be compared between the different groups. These variances are shown in Table II together with similar variances from analysis of Danish farm accounts.[1]

It has previously been shown that variations(4)and(5)have been normal distributions; and if, as seems reasonable, it may also be assumed that the distributions of variations(1)and(2)are near normal, then the contents of Table II may be illustrated in a way which may be preferred by some readers.

In normal distributions, the mean of the positive (and negative)

[1] KNUD RASMUSSEN, *Variance and Production Function Analyses of Farm Accounts*, Basil Blackwell, Oxford, 1962.

VARIANCES OF LOG E (E = PRODUCTION PER £1 TOTAL COSTS).

TABLE II

TYPE OF VARIATION	Group 1	Group 2	Group 3	Group 4	Corresponding figures from Danish farm accounts *
1. Scale of farming	0.000334	0.001125	0.000328	0.000220	0.000086
2. Global combination of inputs at given scale of farming	0.000559	0.001763	0.000252	0.000440	0.000166
3. Total global combination of inputs	0.000893	0.002888	0.000580	0.000660	0.000252
4. "Managerial" variance at given global combination of inputs	0.001807	0.002680	0.002050	0.002146	0.001597
5. "Random" variation	0.000438 **	0.000837 **	0.000607 **	0.000945 **	0.000482 **
6. TOTAL	0.003138	0.006405	0.003237	0.003751	0.002331
7. Actual variation in E	0.003034	0.006294	0.003001	0.003750	0.002313

* The Danish Analysis was restricted to farms from 10 ha - 100 ha and most of the Danish farms correspond to farms in Group 3 (Mixed Farms).

** These figures refer to 4-year average accounts - for one-year accounts the figures would be 4 times as large.

deviations lies at about 0.8 of a standard deviation (Geary's ratio). This means that the types of variation may be compared by showing how great a difference in production each variation by itself would cause between the more successful half and the less successful half of the farms per £1000 Total Costs. These differences are shown in Table III.

It will be seen from Tables II and III that, for all groups except Group 2 (Livestock farms), the clearly dominating influence for the Production/Cost ratio is (4) ("Managerial" variance at given global combination of inputs). For group 2 the tables show, in relation to the other groups, the very great importance of scale of farming (1) and also of (2) (Global combination of inputs at given scale of farming).

But even for Group 2 the type of variation (4) is more important than the type of variation (2), and indeed nearly as important as the total global combination of inputs.

To refer back to the discussion on pp. 26-28 it must be emphasised that the "Managerial" variation (4) even for Group 2 shows a

DIFFERENCES IN PRODUCTION PER £1000 TOTAL COSTS BETWEEN THE MORE SUCCESSFUL HALF AND THE LESS SUCCESSFUL HALF OF THE FARMS, ACCORDING TO CONTRIBUTORY CAUSE.

TABLE III

TYPE OF VARIATION	Group 1	Group 2	Group 3	Group 4	Corresponding figures from Danish farm accounts
	£	£	£	£	£
1. Scale of farming	67	124	67	55	32
2. Global combination of inputs at given scale of farming	87	154	59	78	45
3. Total global combination of inputs	110	198	89	94	59
4. "Managerial" variance at given global combination of inputs	156	191	166	171	144
5. "Random" variation	78 *	106 *	91 *	113 *	81 *
6. TOTAL	207	296	209	226	178

* These figures relate to 4-year average accounts; for one-year accounts they would be twice as large, and also the figures in line 6 would be slightly larger.

variance (Table II) which is about 2.4 times as great as the variance due to scale of farming variation (1). For Groups 1, 3 and 4 the corresponding ratios are : 5.4, 6.3 and 9.8.

Similarly the ratio of "Managerial" variance (4) to the variance (2), Global combination of inputs at given scale of farming, for the Groups 1, 2, 3 and 4 are: 3.2, 1.5, 8.1 and 4.9.

Because of the relatively small importance of combination of resources in relation to "Managerial" variation it seems to the author that one is justified in rejecting the hypothesis that the better farmers do in fact to a significant degree choose other combinations than less able farmers. Had this been the case, one would have to assume that *better* farmers choose *better* combinations of resources and then one would have expected the influence of combination of resources to dominate that of "Managerial" variance. *As it is, the author feels, justified in assuming that there is no strong correlation between the farmers' "Managerial" ability and their choice of combination of resources, and therefore that the estimated production functions, which have been derived from* INTERFARM *regressions, do in fact give a reasonable estimate of* CONSEQUENCES *of* INTRA-FARM *changes in use of inputs.*

SECTION II

REGRESSION ANALYSES APPLIED TO BRITISH FARM ACCOUNTS

Seven Independent Variable Case

CHAPTER VII
THE RESULTS OBTAINED

The analysis of the farm accounts data was planned in a way which made possible two alternative regressions:

(1) *Gross Product* upon Rent & Rates, Tenant's Capital, Labour and Total Purchases,

and

(2) *Gross Product* upon the same actual inputs *but* with the differences that *Tenant's Capital* was divided into:

- (a) Machinery & Equipment, and
- (b) Other Tenant's Capital,

and *Total Purchases* were divided into:

- (a) Seeds & Feed,
- (b) Fertilisers & Lime,
- (c) Other Purchases.

In the previous chapters the analysis using 4 independent variables was treated in some detail. In this chapter, the analysis using 7 independent variables will be dealt with much more briefly as the methods used and the problems encountered are the same.

The Functions Calculated

The calculated regression coefficients, their sums and the multiple correlation coefficients are shown in Table 23, which also shows the standard errors for the regression coefficients and for Σb_i from the Between Farms regression. An inspection of the table shows that in this case also the multiple correlation coefficients increase when moving from the regression on Residuals to the Within Years, Between Farms and Variance Components regressions. On the other hand, the value of Σb_i decreases slightly from the Within Years to the Variance Components regression, a feature which the author cannot explain.

The residual variances are slightly larger than those from the regressions using 4 independent variables, the *Gross Product* being the same in each case. This accords with the slightly smaller multiple correlation coefficients.

RESIDUAL VARIANCES AND MULTIPLE CORRELATION COEFFICIENTS

Group	Type of Regression	4 Independent Variables		7 Independent Variables	
		Residual Variance	R	Residual Variance	R
Group 1 (Dairy)	Residuals	0.001666	0.629	0.001665	0.630
	Within Years	0.003564	0.981	0.003818	0.980
	Between Farms	0.002262	0.988	0.002481	0.987
	Variance Components	-	0.990	-	0.989
Group 2 (Livestock)	Residuals	0.003074	0.315	0.003049	0.329
	Within Years	0.006032	0.966	0.006374	0.964
	Between Farms	0.003544	0.980	0.003752	0.979
	Variance Components	-	0.985	-	0.984
Group 3 (Mixed)	Residuals	0.002335	0.530	0.002464	0.492
	Within Years	0.004479	0.981	0.005024	0.979
	Between Farms	0.002671	0.989	0.003005	0.987
	Variance Components	-	0.991	-	0.990
Group 4 (Arable)	Residuals	0.003626	0.401	0.003711	0.381
	Within Years	0.005934	0.980	0.006245	0.979
	Between Farms	0.003140	0.990	0.003122	0.990
	Variance Components	-	0.993	-	0.994

It might seem surprising at first glance that the 7 independent variable case gives slightly larger residual variances than the 4 independent variable case, except in Group 4 (Arable) for Between Farms. It should be remembered, however, that in this case more independent variables are not used *in addition to* the previous 4, but 2 of the 4 variables were subdivided, one into two components and the other into three.

The fact that the 7 independent variables do in fact give a slightly less good fit than the 4 is nevertheless a warning that about 4 to 7 independent variables may be near to the limit of the number of variables into which the inputs may usefully be divided for the regression. This does not of course make impossible many *alternative* subdivisions of inputs for the same data. On the contrary this method will probably in time prove to be the best way of obtaining estimates of "Marginal Productivities" for many small input items.

A test for the possibility of considering the 4 individual years' regressions as having the same slopes shows that this may be accepted without difficulty for Group 1 and Group 3, but with a little hesitation for Group 2 and Group 4.

Using parallel planes within years for each group, the following relative Gross Products are obtained from given inputs for the various years (compare Table 8):

Group 1 (Dairy)

Year 1	100
Year 2	99
Year 3	101
Year 4	104

Group 2 (Livestock)

Year 1	100
Year 2	94
Year 3	104
Year 4	114

Group 3 (Mixed)

Year 1	100
Year 2	101
Year 3	100
Year 4	101

Group 4 (Arable)

Year 1	100
Year 2	105
Year 3	96
Year 4	98

Using parallel planes for the Between Farms regressions for the 4 groups, the following relative Gross Products are obtained from given inputs for the 4 type-groups (compare Table 9):

Group 1 (Dairy)	104
Group 2 (Livestock)	100
Group 3 (Mixed)	103
Group 4 (Arable)	110

The Variance Components Regression Coefficients

A comparison between the regression coefficients from this regression with those obtained from using only 4 independent variables (see Table 18) may be of interest.

Firstly, an inspection of the standard errors from the Between Farms regression given in Table 23 for each regression coefficient and for Σb_i shows that all the regression coefficients are determined with a high degree of accuracy, which indicates that the independent variables are not too highly correlated (see Table 1) to make possible the use of regression techniques. The Between Farms regression is so closely related to the Variance Components regression that these estimates of standard errors must give valuable information about the latter regression's regression coefficients.

In Table 24 a comparison is made between the various regression coefficients. The main discrepancy between the 7 and the 4 independent variable cases appears when comparing b_{10} with the corresponding sum ($b_7 + b_8 + b_9$).

An analysis was made of the changes in the composition of Total Purchases (X_{10}) as it increases, and it was shown that the three components on average do not increase by the same percentage as X_{10} increases:

Group 1 (Dairy)

An increase of 1% in Total Purchases (X_{10}) was on an average associated with:

1.14% increase in Seeds & Feed	(X_7)
1.38% increase in Fertilisers & Lime	(X_8)
0.69% increase in Other Purchases	(X_9)

Group 2 (Livestock)

An increase of 1% in Total Purchases (X_{10}) was on average associated with:

1.32% increase in Seeds & Feed	(X_7)
0.93% increase in Fertilisers & Lime	(X_8)
0.68% increase in Other Purchases	(X_9)

Group 3 (Mixed)

An increase of 1% in Total Purchases (X_{10}) was on average associated with:

1.21% increase in Seeds & Feed	(X_7)
1.27% increase in Fertilisers & Lime	(X_8)
0.71% increase in Other Purchases	(X_9)

Group 4 (Arable)

An increase of 1% in Total Purchases (X_{10}) was on average associated with:

1.19% increase in Seeds & Feed	(X_7)
1.35% increase in Fertilisers & Lime	(X_8)
0.72% increase in Other Purchases	(X_9)

It is obvious that when the most productive constituents of X_{10} increase by a greater percentage than the least productive element, then the value of b_{10} would be expected to be larger than the sum ($b_7 + b_8 + b_9$).

The last column of Table 24 shows the weighted sum of (b_7 + b_8 + b_9) calculated as the sum of the individual coefficients, each multiplied by the percentage increase in the corresponding variable for an increase of 1% in X_{10}. It will be seen that in all groups the weighted value of (b_7 + b_8 + b_9) is larger than the unweighted value and, for all groups except the Arable farms, the weighting brings the value of (b_7 + b_8 + b_9) closer to b_{10}.

Estimated "Random" Variance and a "Managerial" Variance in Gross Product Achieved from Given Global Combinations of Inputs

Calculations exactly similar to those in Chapter III were also carried out for the regressions using 7 independent variables. The resulting estimates of the "Random" and the "Managerial" variances agree very well indeed with those in Chapter III as will be seen by comparison of the estimates (the corresponding standard deviations) in Table 25 where the similar estimates from Chapter III are also shown.

"Marginal Productivities"

As in Chapter IV the "Marginal Productivities" were calculated for each variable at its geometric mean value with all other variables also at their geometric mean values.

The estimated "Marginal Productivities" in Table 26, to the extent to which they may be compared, agree fairly well with the estimates shown in Chapter IV.

The two constituent parts of Tenant's Capital for each farm-type group show the same relative importance for increasing Tenant's Capital. For the 4 groups the Total Tenant's Capital is composed as follows:

	Machinery & Equipment.	Other Tenant's Capital.
Group 1 (Dairy)	33%	67%
Group 2 (Livestock)	26%	74%
Group 3 (Mixed)	34%	66%
Group 4 (Arable)	34%	66%

These weights were applied to the two "Marginal Productivities" and the results are shown below compared with the "Marginal Productivity" of Total Tenant's Capital from the 4 independent variable case. (Variance Components Regressions used throughout).

	7 Independent Variable Case	4 Independent Variable Case
Group 1 (Dairy)	0.165	0.155
Group 2 (Livestock)	0.257	0.243
Group 3 (Mixed)	0.205	0.189
Group 4 (Arable)	0.175	0.203

The differences are certainly not large.

In both cases the "Marginal Productivities" indicate from about 16% to about 25% interest on Tenant's Capital as valued in the Farm Management Survey. As mentioned in Chapter IV this probably corresponds to about 11% to 17% interest on market values of Tenant's Capital.

It is interesting to note - in view of so much previous discussion about the possibility of over-mechanisation on British farms - that the estimates obtained here show a very high "Marginal Productivity" of Machinery & Equipment.

For Total Purchases also the agreement is fairly good for all groups, with the exception of the Arable farms. The comparison is made here in such a way that for each farm-type group the different increases of the three variables in percentages are used to calculate the total increase in these three variables corresponding to an increase of 1% in Total Purchases. Using the previously calculated weighted (b_7 + b_8 + b_9) the estimated increase in Gross Product is then calculated and the increase per £1 extra Total Purchases is obtained by division. These calculations use the geometric means of the variables and the regression coefficients from the Variance Components regression:

	7 Independent Variable Case	4 Independent Variable Case
Group 1 (Dairy)	£1.023	£1.032
Group 2 (Livestock)	£1.012	£1.010
Group 3 (Mixed)	£1.045	£1.028
Group 4 (Arable)	£1.288	£1.059

The variable X_7 *(Seeds & Feed)* appears to be used very nearly to the right extent (for given other inputs) in Groups 1 and 3, while too little is used in Groups 2 and 4.

The variable X_8 *(Fertilisers & Lime)* in all groups shows a "Marginal Productivity" greater than 1.0. Particularly in Group 4 it appears that the use of more fertilisers would pay handsomely, and the same is also true to a lesser - though still great - extent for Groups 2 and 3, all at given other inputs.

The variable X_9 *(Other Purchases)* shows "Marginal Productivities" of less than 1.0 in all 4 groups. As this variable contains a number of items of costs which are hardly very productive, this is what the author would have expected.

Least Cost Combinations

The Least Cost Combinations were calculated in exactly the same way as in Chapter V, and the results are shown in Table 27.

There is on the whole very good agreement between these calculations and those of Chapter V.

The table shows that for all groups the Least Cost Combinations would require more of both the components of Tenant's Capital.

Seeds & Feed should in all groups be somewhat reduced.

Less Fertilisers & Lime should be used in the Dairy group, about the same in the Livestock and Mixed groups, and much more in the Arable group.

Less Other Purchases should be used in all groups.

Per £1000 Total Costs (inclusive of 9% interest on Tenant's Capital) the Least Cost Combinations would give the following amounts of larger Gross Products *(and Profits)* than the present average combinations of inputs:

Group 1	(Dairy)	£144
Group 2	(Livestock)	£385
Group 3	(Mixed)	£109
Group 4	(Arable)	£326

These figures are all higher than the corresponding ones in Chapter V, but in this case the Least Cost Combinations also take into account the breakdown of Tenant's Capital into two groups and Total Purchases into three. The position of the various farm-type groups is the same apart from the Arable farms which, no doubt due to the effect of the variable Fertilisers & Lime, now appear further away from Least Cost Combinations than in Chapter V.

It might be argued that the size of British farms is so unchangeable - so sacred - that it is unrealistic to present the above calculated Least Cost Combinations. These imply the same total costs as the present average total costs, *but allow* for changes in farm size (Rent & Rates) as well as in other inputs to reach the Least Cost Combinations.

If the farm size *must* remain unchanged another *conditional* Least Cost Combination may be calculated under this restraint. This has been done and the results in Table 28 show that the new conditional Least Cost Combination compared with the previous Least Cost Combination would require a little more of all other inputs. At the same time the new calculated Gross Product is of course somewhat lower than that obtained from the unconditional Least Cost Combination.

Causes of Variation in Production/Cost Ratio

Similar calculations to those in Chapter VI were carried out. The results are given in Table IV and V below.

VARIANCES OF LOG E (E = PRODUCTION PER £1 TOTAL COSTS)

TABLE IV

TYPE OF VARIATION	Group 1	Group 2	Group 3	Group 4
1. Scale of farming	0.000302	0.001019	0.000296	0.000200
2. Global combination of inputs at given scale of farming	0.000890	0.002532	0.000844	0.001022
3. Total global combination of inputs	0.001192	0.003551	0.001140	0.001222
4. "Managerial" variance at given global combination of inputs	0.002002	0.002853	0.002329	0.002115
5. "Random" variation	0.000452 *	0.000877 *	0.000672 *	0.001029 *
6. TOTAL	0.003646	0.007281	0.004141	0.004366
7. Actual variation in E	0.003034	0.006294	0.003001	0.003750

* These figures refer to 4-year average accounts ; for one-year accounts the figures would be 4 times as large.

DIFFERENCES IN PRODUCTION PER £1000 TOTAL COSTS BETWEEN THE MORE SUCCESSFUL HALF AND THE LESS SUCCESSFUL HALF OF THE FARMS, BY CONTRIBUTORY CAUSE

TABLE V

TYPE OF VARIATION	Group 1	Group 2	Group 3	Group 4
	£	£	£	£
1. Scale of farming	63	117	63	53
2. Global combination of inputs at given scale of farming	109	185	107	117
3. Total global combination of inputs	126	220	124	129
4. "Managerial" variance at given global combination of inputs	165	198	177	169
5. "Random" variation	78 *	109 *	95 *	118 *
6. TOTAL	223	315	238	244

* These figures relate to 4-year average accounts; for one-year accounts they would be about twice as large and also the figures in line 6 would be slightly larger.

It will be seen that these results do not differ very much from those of the 4 independent variable case given in Chapter VI (Tables II and III). It will be noticed, however, that in Table IV the differences between the calculated and actual variations in E (lines 6 and 7) are somewhat greater. This agrees with the fact that there is a higher degree of correlation between $E^*_{\cdot j}$ and $w_{\cdot j}$ in the 7 independent variable case.

Although the main picture is exactly the same as that obtained in the 4 independent variable case, in Group 3 and Group 4 the relative importance of global combination of inputs at given scale of farming is somewhat greater. In all probability this is due to the importance of fertilisers as an input, associated with the high "Marginal Productivity" of this input, in these two groups.

SECTION III

REGRESSION ANALYSES APPLIED TO IRISH FARM ACCOUNTS

CHAPTER VIII

THE RESULTS OBTAINED

Data

The Central Statistics Office in Dublin kindly made available for this study three years' accounts for each of 1139 farms. These accounts, which exclude the small group of 35 farms classified as "Other", are those analysed in tabular form in:

> "National Farm Survey, 1955/56 - 1957/58, Financial Results for Farms Included Throughout the Three Years", compiled by the Central Statistics Office, Dublin, and published as a supplement to the Irish Trade and Statistical Bulletin, December, 1959.

The farms were divided initially into 6 farm-type groups according to the classification of the Central Statistics Office:

Group 11 - Mainly Dairying, containing 71 }
Group 12 - Dairying Mixed without Cash Crops, containing 218 } 289 accounts for each of 3 years

Group 13 - Dairying Mixed with Cash Crops, containing 217 accounts for each of 3 years

Group 20 - Crops Mixed, containing 200 accounts for each of 3 years

Group 30 - Cattle Mixed, containing 280 accounts for each of 3 years

Group 00 - Subsistence, containing 153 accounts for each of 3 years.

After the calculations had been carried out in the 6 groups, the possibility of pooling all or some groups was tested. The only pooling which was quite justifiable was of Groups 11 and 12. All the results shown relate, therefore, to 5 groups; the 4 main groups containing from 200 to 289 farms each and the Subsistence group 153.

The Central Statistics Office kindly amended the individual accounts so that the variables are almost identical in content to the corresponding variables in the British data. The precise definitions of the variables[1] as used in the Irish data are given below:

Z = Total Ouput *less* depreciation *plus* crops purchased from other farmers.

X_1 = Area farmed (adjusted acres).

X_2 = Annuities of own land farmed *plus* rates of own land farmed *plus* rent of conacre etc.

X_3 = End of year inventory of machinery.

X_4 = End of year inventory of livestock *plus* end of year inventory of crops.

X_5 = $X_3 + X_4$ = Total Tenant's Capital.

X_6 = Total Labour, family and hired, in Labour Units. 1 Labour Unit in this study is the equivalent of 1/100 of the Labour Units used in the National Farm Survey. For economic conclusions, 1 Labour Unit was valued at £2.7.

X_7 = Feed purchased from merchants *plus* seed purchased from merchants *plus* crops purchased from other farmers.

X_8 = Fertilisers & Lime.

X_9 = Other current farm expenses.

X_{10} = $X_7 + X_8 + X_9$ = Total Purchases.

Table 30 shows the distribution of the farms according to adjusted acres for each farm-type group.

There are, however, two points to note. Firstly, in the British Farm Management Survey, Farm Family Labour is charged at current rates for hired Labour; in the Irish National Farm Survey no such imputation is made. In the latter (see the publication mentioned above), Labour has been measured in Labour Units and for the economic analyses following, the average cost of hired Labour has been applied to all Labour. This average, over the three years was, £270 per annum, and as the unit chosen in this study was 1/100 of the Irish Labour Unit, the cost of one Labour Unit may be assessed at £2.7. It would be well to emphasise the predominance of Family Labour. For the groups used, the proportion which non-paid Labour formed of total Labour was approximately 85%, 70%, 70%, 90% and 99% respectively. These figures compare with 36%, 50%, 17% and 14% respectively for the British farms (Farm Management Survey 1957-58).

[1] The means of all variables are given in Tables 44 (a) and 44 (b).

Secondly, in some instances no expenditure on a given input item appeared in the accounts for any of the 3 years. This was most noticeable in the Subsistence group, and there were also a few cases in 3 of the other groups. In these cases an input of £1 was inserted in the accounts, a procedure which is not likely to have seriously influenced the results of the analysis.[1]

4 Independent Variable Case

Exactly the same calculations were carried out for the Irish farms as for the British.

In Table 31 it is shown that for the Irish farms also there are great variations in the use of any single input for all other inputs fixed. If anything, there are greater variations for Irish farms than for British.

The variables are also, as in the British data, highly intercorrelated, as may be seen in Table 32. It appears that on the whole these correlations are not quite as high as the corresponding correlations in the British data.

As in the British data, Rent & Rates (X_2) was chosen instead of Adjusted Acres (X_1) as providing a better guide to the size or productive capacity of the farms. Here again the residual variances from the regression using log X_2 with log X_5, log X_6 and log X_{10} were smaller than those for the regressions using log X_1 with the same 3 other variables, except in the case of Group 00 (Subsistence) where the difference was very slight.

The Regression Coefficients and Multiple Correlation Coefficients

In Table 33 the various coefficients calculated are shown, as well as the standard errors of the individual regression coefficients and of their sums for all the regressions except Variance Components.

[1] For the following numbers of farms no input appeared for a given variable *over the three year period :*

		X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
Groups 11 & 12 combined	(Mainly Dairying and Dairying Mixed Without Cash Crops)	-	2	-	-	-	-	-	-	-
Group 13	(Dairying Mixed with Cash Crops)	-	-	-	-	-	-	-	-	-
Group 20	(Crops Mixed)	-	2	-	-	-	-	-	-	-
Group 30	(Cattle Mixed)	-	6	-	-	-	1	2	-	-
Group 00	(Subsistence)	-	23	-	-	-	1	6	-	-

In all but Group 00 (Subsistence) the values of Σb_i show a steady increase when moving from the Residuals to the Variance Components regression. In all groups the multiple correlation coefficients show a similar increase. Though all the multiple correlation coefficients for the Between Farms and the Variance Components regressions are very high, they are all a little lower than those for the British data.

An inspection of the standard errors of the individual regression coefficients shows that, disregarding Group 00 (Subsistence) where the standard errors are much higher, the other groups show a picture very similar to that for the British data, though with a tendency towards slightly higher standard errors for the coefficients.

All groups, with the exception of Group 00 (Subsistence), show values of Σb_i fairly close to 1.00, and only Group 30 (Cattle Mixed) shows a value of Σb_i which is significantly different from 1.00, indicating diminishing returns to scale for this group.

Table 34 shows the deviations of actual values of Gross Product from the regression values for the Within Years (parallel planes) regressions. These distributions are fairly close to normal.

Table 35 shows the corresponding deviations from the Between Farms regressions. These distributions are also fairly close to normal.

Estimates of the "Random" Variance and a "Managerial" Variance

Estimates of "Random" and "Managerial" variances were made in exactly the same way as for the British data. The results are shown in Table 36 in the form of the corresponding standard deviations.

As one might expect, Group 00 (Subsistence) shows very large variations due to the "Managerial" ability of the farmers as well as to "Random" elements.

In the other groups, the Irish farms show larger variances than those of the most comparable British groups. The "Random" variation gives standard deviations which are from 15% to 57% larger than those of British farms, and the "Managerial" variation 30% to 50% larger. The larger "Managerial" variations are probably partly, but not fully, explained by the differences in methods of selection of the farms in the two samples.

"Marginal Productivities"

The estimated "Marginal Productivities" are shown in Table 37 for all variables at their geometric mean values.

In this table the estimated "Marginal Productivities" for Labour assume that all Labour, inclusive of Family Labour, has been valued at £2.7 per unit (corresponding to £270 per year).

Tenant's Capital:

The table shows the peculiar picture of Group 00 (Subsistence) where all inputs, with the exception of Tenant's Capital, have very low "Marginal Productivities". Tenant's Capital on the other hand has a "Marginal Productivity" of about 36 %, which is indeed a very good return on capital invested.

In the other groups also, Tenant's Capital shows a good return varying from 13 % for Group 20 (Crops Mixed) to 33 % for Groups 11 & 12 combined (Dairy, and Dairy Mixed Without Cash Crops), for all other variables at their geometric mean values.

Labour:[1]

Leaving out Group 00 (Subsistence) which in this respect is also unique ("Marginal Productivity" of only £0.07), the 4 remaining groups vary from £0.25 to £0.86 extra Gross Product per £1 extra input of Labour at given inputs for the other variables. These figures indeed support the general impression that Labour is not very efficiently employed in Irish farming, because of a very high degree of over-employment on farms which really is disguised unemployment.

Group 13 and Group 20 (Dairying mixed with Cash Crops, and Crops Mixed) show the highest "Marginal Productivities", but no higher than £0.47 and £0.86 respectively.

Total Purchases:

All 4 main type-groups show very high "Marginal Productivities" for Total Purchases at given other inputs, varying from £1.42 to £1.55 per £1 extra spent on this input. It seems obvious that most Irish farmers are using too little Purchased Raw Materials for the most economic production with the given use of other inputs.

Least Cost Combinations

As in the British Study, the Least Cost Combinations were calculated in a way which gives, for each type-group, the same Total Costs (inclusive of 9 % interest on Tenant's Capital) as the actual average Total Costs for the group. Further, for the Least Cost Combinations the expected Gross Products were calculated according to the production function (Variance Components). In these calculations, which are shown in Table 38, the cost of Labour was throughout taken to be £2.7 per unit.

An inspection of the table shows that at given Total Costs (inclusive of 9 % interest on Tenant's Capital), the Least Cost Combinations, in comparison with the actual average combinations of inputs, would require:

[1] For the relative importance of unpaid family Labour, see page 53

Not very great changes in the size of the farm as measured by Rent & Rates.

Much more Tenant's Capital in all groups, except in Group 20.

Only a small fraction of the present amount of Labour, again except in Group 20 where the amount required is about ⅞ of that actually used.

Somewhat more Purchased Raw Materials in all groups.

The same cautious remarks as were made about Least Cost Combinations for the British data are even more strongly needed here as the Least Cost Combinations would in nearly all cases be *Extrapolations* outside the range of observations. There can be no doubt, however, that substantial changes in the directions indicated would be needed to improve the efficiency of Irish farming.

Per £1,000 Total Costs (inclusive of 9% interest on Tenant's Capital) the Least Cost Combinations would give the following amounts of larger Gross Products *(and Profits)* than the present average combinations of inputs.

Groups 11 & 12 combined	(Mainly Dairying, & Dairying Mixed Without Cash Crops)	£ 896
Group 13	(Dairying Mixed with Cash Crops)	£ 532
Group 20	(Crops Mixed)	£ 573
Group 30	(Cattle Mixed)	£ 733
Group 00	(Subsistence)	£1028

For these figures again the warning must be given that, as most Irish farmers appear to operate very far away from Least Cost Combinations, the accuracy of these calculations cannot be nearly as high as it would have been had Irish farmers operated nearer the Least Cost Combinations, that is more efficiently.

It is nevertheless interesting to compare the figures above with the corresponding calculations for British farms which varied from £41 to £197. This comparison certainly indicates that British farmers on average operate much nearer to Least Cost Combinations (much more efficiently) than Irish farmers.

The British Farm Management Survey is confined to full-time commercial farms, one of its main objectives being to supply production standards and other information for farm advisory purposes. No one has ever contended that it is a representative sample, even of the full-time commercial farms and, *a priori*, one would expect that the "Managerial" variation derived from these farm records would be lower than for a more representative sample. Equally one would expect better use of resources i.e. nearer to Least Cost Combination on these farms.

In the Irish National Farm Survey the fundamental objective was to include a representative sample of farms, stratified by size, holdings down to 5 acres being included, and there was no restriction to full-time commercial farms. Particular care was taken to ensure random selection of the sample as it was desired to use the results to provide national estimates of many constituents of expenditure etc. in the agricultural sector. Although in the event the sample covered was somewhat above average, *a priori*, one would expect that the results of such a (representative) sample would show a greater variation than a sample similar to the British one. It may be argued that the Irish group of "subsistence" farms has been treated separately. As the farms so classed were "those under 50 acres, where *no commercial farming activity* could be said to exist", it follows that the other groups include farms which could not be considered as being anything like full-time commercial farms.

Causes of Variation in Production/Cost Ratio

Similar calculations to those described in Chapter VI were carried out on the Irish data. The results for the 4 independent variable case are given in Tables VI and VII below.

VARIANCES OF LOG E (E = PRODUCTION PER £1 TOTAL COSTS)

TABLE VI

TYPE OF VARIATION	Groups 11 & 12 combined	Group 13	Group 20	Group 30	Group 00
1. Scale of farming	0.002134	0.001298	0.001386	0.000633	0.002868
2. Global combination of inputs at given scale of farming	0.006596	0.002901	0.001729	0.007379	0.007326
3. Total global combination of inputs	0.008730	0.004199	0.003115	0.008012	0.010194
4. "Managerial" variance at given global combination of inputs	0.003221	0.003566	0.004720	0.002939	0.008311
5. "Random" variation	0.001450 *	0.001489 *	0.001655 *	0.002612 *	0.004175 *
6. TOTAL	0.013401	0.009254	0.009490	0.013563	0.022680
7. Actual variation in E	0.012888	0.008945	0.008236	0.013656	0.022289

* These figures refer to 3-year average accounts ; for one-year accounts the figures would be 3 times as large.

DIFFERENCES IN PRODUCTION PER £1000 TOTAL COSTS BETWEEN THE MORE SUCCESSFUL HALF AND THE LESS SUCCESSFUL HALF OF THE FARMS, ACCORDING TO CONTRIBUTORY CAUSE.

TABLE VII

TYPE OF VARIATION	Group 11 & 12 combined	Group 13	Group 20	Group 30	Group 00
	£	£	£	£	£
1. Scale of farming	171	133	137	92	198
2. Global combination of inputs at given scale of farming	300	198	154	317	316
3. Total global combination of inputs	346	239	207	332	374
4. "Managerial" variance at given global combination of inputs	209	220	254	200	338
5. "Random" variation	141 *	143 *	149 *	188 *	239 *
6. TOTAL	430	356	360	433	562

* These figures relate to 3-year average accounts; for one-year accounts they would be 1.7 times as large and also the figures in line 6 would be slightly larger.

In agreement with the fact that the typical combinations of resources are much further from the Least Cost Combinations than those of the British data, the relative importance of combination of resources is greater here than in the corresponding results for British farms.

7 Independent Variable Case

In Table 39 the various regression coefficients, their sums and the multiple correlation coefficients are shown. Further, for the Between Farms regressions the standard errors of the individual regression coefficients and of their sums are given.

There is on the whole close agreement between the two regressions using 4 and 7 independent variables respectively. A comparison for some groups of coefficients is shown in Table 40.

There is however one regression coefficient (Machinery & Equipment for Groups 11 & 12 combined) which is negative. As the standard error of this coefficient clearly shows that it is not a significantly negative coefficient no more comments will be made on this point.

The multiple correlation coefficients for the Variance Components regression range (for the four main type-groups) from 0.978 to 0.985, and for all groups the coefficient is slightly higher than in the regressions using only 4 independent variables.

Estimates of the "Random" Variance and a "Managerial" Variance

These estimates were made in exactly the same way as in previous sections and Table 41 shows the corresponding estimated standard deviations. These agree very well with those obtained from the regressions using only 4 independent variables.

"Marginal Productivities"

In Table 42 the estimated "Marginal Productivities" are shown for all variables at their geometric mean values, and assuming the cost of one unit of Labour to be £2.7.

Tenant's Capital:

The breakdown of Total Tenant's Capital into its two components: Machinery & Equipment and Other Tenant's Capital (to a great extent Livestock), shows that for the four main type-groups the latter variable is the more productive with a return varying from 13% for Group 20 (Crops Mixed) to 45% for Groups 11 & 12 combined (Dairy, and Dairying Mixed without Cash Crops). The very high return on this variable probably indicates some under-stocking of much Irish farmland, for other inputs fixed.

The negative sign for the coefficient for Machinery & Equipment in Groups 11 & 12 combined need not worry the reader unduly. This is a very insignificant input on this type of farm - on average for all farms in this group only 13% of Total Tenant's Capital. For the other three main type-groups the return on the Machinery & Equipment Capital is low varying from 5% for Group 20 (Crops Mixed) to 8% for Group 30 (Cattle Mixed) for given other inputs.

Labour:

The "Marginal Productivities" for Labour show the same picture as that from the regression using 4 independent variables: £0.05 return for £1 extra spent on Labour in Group 00 (Subsistence) and values in the four main type-groups ranging from £0.19 in Group 30 (Cattle Mixed) and in Groups 11 & 12 combined (Mainly Dairying, and Dairying Mixed without Cash Crops) to £0.81 in Group 20 (Crops Mixed). As in the 4 independent variable case the two Groups 13 & 20 (Dairying Mixed with Cash Crops, and Crops Mixed), show the highest "Marginal Productivities" of Labour, but even so only £0.46 and £0.81 respectively.

Seeds & Feed:

This input, for all the four main type-groups shows a high "Marginal Productivity" ranging from £1.21 in Group 20 (Crops Mixed) to £1.75, £1.85 and £1.89 in the other 3 groups, all at given other inputs.

Fertilisers & Lime:

This input shows a very poor return in Groups 11 & 12 combined (Dairy, and Dairying Mixed Without Cash Crops), but for the other three main type-groups, which are also the largest users of Fertilisers & Lime, the "Marginal Productivity" varies from £2.09 to £2.66 indicating a very good return for - and too little use of - this input for given other inputs.

Other Purchases:

For the 4 main type-groups this variable shows a good return varying from £1.08 in Group 20 (Crops Mixed) to £2.12 in Groups 11 & 12 combined for given other inputs.

Least Cost Combinations

As in the 4 independent variable case, those Least Cost Combinations were calculated which gave the same Total Costs (inclusive of 9% interest on Tenant's Capital)[1] as the average Total Costs for the various groups. In this calculation, the small negative regression coefficient for Machinery & Equipment in Groups 11 & 12 combined was replaced by zero. The Least Cost Combinations are shown in Table 43.

This table on the whole shows the same picture as the corresponding table for the 4 independent variable case.

It is seen here that the increase required in Tenant's Capital is mainly - or solely - in Other Tenant's Capital, no doubt in particular in Livestock.

> Only a fraction of present Labour should be used, except again in Group 20 where the amount required is about 5/6 of that actually used.
>
> More Seeds & Feed should be used in all groups.
>
> Substantially more Fertilisers & Lime should be used in the last 3 groups.

[1] The same (high) interest rate is used here as was previously used for British farms. This is not because of an expected undervaluation of the Irish figures for Tenant's Capital, but because many Irish farmers may find it difficult to borrow at low interest rates.

A little more Other Purchases should be used in all groups, except in Group 20 where the amount should be slightly less.

Per £1000 Total Costs (inclusive of 9% interest on Tenant's Capital) the Least Cost Combinations would give the following amounts of larger Gross Products (*and Profits*) than the present average combinations of inputs:

Groups 11 & 12 combined (Mainly Dairying, & Dairying Mixed Without Cash Crops)	£1673
Group 13 (Dairying Mixed with Cash Crops)	£ 684
Group 20 (Crops Mixed)	£ 676
Group 30 (Cattle Mixed)	£ 909
Group 00 (Subsistence)	£1099

The same comments on the dangers of extrapolation apply here as in the 4 independent variable case.

Causes of Variation in Production/Cost Ratio

As in the 4 independent variable case, calculations to show the causes of variation in the Production/Cost Ratio were carried out. The results are shown in Tables VIII and IX below.

VARIANCES OF LOG E (E = PRODUCTION PER £1 TOTAL COSTS)

TABLE VIII

TYPE OF VARIATION	Group 11 & 12 combined	Group 13	Group 20	Group 30	Group 00
1. Scale of farming	0.002098	0.001255	0.001337	0.000578	0.002838
2. Global combination of inputs at given scale of farming	0.007172	0.003546	0.002468	0.008321	0.007672
3. Total global combination of inputs	0.009270	0.004801	0.003805	0.008899	0.010510
4. "Managerial" variance at given global combination of inputs	0.002888	0.003151	0.004417	0.002593	0.007737
5. "Random" variation	0.001415 *	0.001714 *	0.001645 *	0.002629 *	0.004342 *
6. TOTAL	0.013573	0.009666	0.009867	0.014121	0.022589
7. Actual variation in E	0.012888	0.008945	0.008236	0.013656	0.022289

* These figures refer to 3-year average accounts; for one-year accounts the figures would be 3 times as large.

DIFFERENCES IN PRODUCTION PER £1000 TOTAL COSTS BETWEEN THE MORE SUCCESSFUL HALF AND THE LESS SUCCESSFUL HALF OF THE FARMS, ACCORDING TO CONTRIBUTORY CAUSE.

TABLE IX

TYPE OF VARIATION	Groups 11 & 12 combined	Group 13	Group 20	Group 30	Group 00
	£	£	£	£	£
1. Scale of farming	168	131	135	88	197
2. Global combination of inputs at given scale of farming	313	220	184	338	324
3. Total global combination of inputs	356	256	227	349	380
4. "Managerial" variance at given global combination of inputs	198	207	246	187	326
5. "Random" variation	139 *	152 *	149 *	189 *	243 *
6. TOTAL	433	364	367	441	561

* These figures relate to 3-year average accounts; for one-year accounts they would be 1.7 times as large and also the figures in line 6 would be slightly larger.

Similar remarks to those made in the 4 independent variable case apply equally here: that it is not surprising that combination of resources has a greater relative importance than in the British results since typical Irish combinations of resources are so much further from the Least Cost Combinations than the British data.

CHAPTER IX

CONCLUSIONS

1. Methodology in Farm Production Function Studies

(a) If agronomists published results of fertiliser treatments based upon only one year's results they would normally be heavily criticised; in most cases such studies cover the average results over several years, while at the same time the individual years' results are also given, partly to indicate what great differences there may be in the results based upon a single year.

With a single exception[1] farm production functions have been based upon accounts for a single year only, and in Hildebrand's study he considers parallel planes only, having shown great variations in the annually calculated coefficients (on very small samples). He does not attempt to use averages of the variables over a period of years.

In this study where, for large samples of farms, four (or three) years accounts were available for each farm it has been possible:

(1) To calculate one regression for each year.

(2) To consider parallel planes for the years, with only a difference in the constants.

(3) To average the variables before the regressions are carried out.

(4) To calculate variance (co-variance) components for use in the regressions.

(5) As a special case of (4) to calculate regressions upon variables after corrections for year and farm ("Residuals" regressions).

[1] JOHN R. HILDEBRAND, "Some Difficulties with Empirical Results from Whole-Farm Cobb-Douglas-Type Production Functions." *Journal of Farm Economics*, November, 1960.

By this method it has been shown, that due to a (small?) element of error of measurement in the independent variables, a downward bias in the regression coefficients may be expected. This bias may be expected (and appear to be) especially large for the "Residuals" regression but is also noticeable for the regressions using accounts for a single year only.

For the regression using averages of the variables, this bias may be expected to be but small, and a special kind of regression, the "Variance Components" regression, is suggested as probably providing the best method.

(b) By the reasoning above, the "Residuals" regression, which is intrafarm, regrettably has to be rejected due to the above mentioned bias, and the problem arises whether an interfarm production function can be expected to give good guidance for intrafarm production decisions.

The problem is here whether there does exist a (strong) correlation between the farmer's ability as a manager and his chosen combination of resources, especially whether large farms are more efficient not only because of the many *a priori* reasons for increasing returns to scale but also because the farmers on large farms are more able managers than those on small farms.

After some arguments, which may not be quite conclusive, it is assumed that farmers do in fact choose very different input combinations - partly due to lack of knowledge about which combinations are the best. The chosen input combinations are thus to a great extent experimental and it then follows that such correlation between managerial ability and choice of input combinations which may exist is not great enough to cause any substantial bias in the calculated interfarm regressions - *and* that therefore these regressions may usefully be used for intrafarm guidance.

(c) The fact that several years accounts were available for each farm made it possible to estimate the "Random" element in farm production and an element of "Managerial" variation, which is the variation net of the "Random" element in average product obtained by different farmers all using the same global combinations of inputs i.e. the same combinations of inputs as measured by the independent variables used in the regression. This "Managerial" variation is caused by:

(1) The manager's better choice of inputs within the input groups used; his better choice of fertilisers - at the same fertiliser expenditure - than the neighbours; his better choice of farm - at the same rent - than the neighbours, etc.

(2) Better choice of products. The importance of this element is not specifically investigated in this study.

(d) While most previous farm production function studies have given their final management recommendations with reference to the "Marginal Products" for the various inputs (for all other inputs at their geometric mean values), this study emphasises Least Cost Combinations. This must be preferable since a *given input* might very well have a "paying" "Marginal Product" for other inputs at their geometric mean values, which would apparently indicate the recommendation of greater use of *this* input, while the correct picture is that far too much is used of *other* inputs, and only for this reason is the "Marginal Product" of the first input so high. Least Cost Combinations, *which consider simultaneously all inputs used,* are much to be preferred for the purpose of farm management advice to "Marginal Products" which use arbitrary (geometric mean) values of other inputs.

(e) The Cobb-Douglas production function has often been criticised for having constant elasticities and therefore linear Least Cost Combinations.

The author can see no strong *a priori* reasons for criticising these qualities of the Cobb-Douglas function for use in farming. The test must be whether the function gives a good fit to the actual farm data. For the British data (4 independent variables) the Variance Components regressions gave, for the four farm-type groups, the following multiple correlation coefficients: 0.990, 0.985, 0.991, 0.993; and for the Irish groups the corresponding coefficients were: 0.978, 0.976, 0.977, 0.982, 0.845 - this last coefficient being for the Subsistence group. These multiple correlation coefficients are indeed so high that a criticism of the type of function on the score of poor fit cannot be substantiated by them.

As a further test of the goodness of fit of the functions the British farms were divided in thirds according to Total Costs.

For all but the Dairy group (Group 1), the regressions for the three thirds could not give a better explanation than one regression. For the Dairy group, the sum of the regression coefficients decreases from the first third (small farms) to the last third (large farms). Here it might be of interest to point out that a large and very efficiently managed farm would hardly be classified as a Dairy farm in the Farm Management Survey (see page 8).

All things considered, the Cobb-Douglas production function appears to fit the data well and it cannot be discarded.

(f) It should be noted that the use of 4 independent variables

gives a better fit, a smaller residual, than the use of 7 independent variables. This might indicate that somewhere between 4 and 7 is the optimum number of variables for farm production functions, and that, if one is interested in information about more variables, then a method of *alternative* further breakdown of *some* of the groups of inputs might be preferable to a simultaneous breakdown into many more variables.

(g) A special analysis - to the author's knowledge not previously attempted by other research workers - of the total variation in efficiency of production (measured by log E where E = Production per £1 Total Costs) was carried out, whereby the relative importance for efficiency was broken down into the contributory factors:

(1) Scale of farming.

(2) Global combination of inputs at given scale of farming.

(3) Total global combination of inputs (Total allocation of resources).

(4) "Managerial" variance at given global combination of inputs.

(5) "Random" variation.

This analysis (a) induces the author to accept the interfarm estimates as valid for intrafarm management decisions, and (b) emphasises the great importance of the "Managerial" variance, in other words the great importance of the many detailed husbandry decisions in comparison with the "Allocation of Resources", about which the production function as such can give information. The value of the production function as such (the Allocation of Resources) should not however be disregarded. In as far as there are good reasons for accepting linear (or at least near linear) Least Cost Combinations, advice based upon production functions can be used for very large groups of farms and it can at least indicate in which directions the farmers could look for changes in input combinations to improve their farming efficiency.

(h) The large "Random" element in farm production should warn Agricultural Economists against the often used breakdown of farm accounts based upon a single year only according to the results achieved. This amounts, to a very great extent, to a breakdown of the accounts according to good or bad luck.

2. The British Results

One of the main conclusions regarding the British farms is the indication of increasing returns to scale for all types of farms- 1% extra of all inputs would give from 1.03% to 1.07% increase in the product obtained. This means that, for the range of farms studied, there are no optimum combinations of inputs but only a suggestion to increase the average size of farms.

While no optimum size of farm can be indicated, it is possible to recommend Least Cost Combinations which, according to the Cobb-Douglas production function, and also most probably in actual fact, are linear.

For the same "size" of farm (defined as a farm with the same Total Costs) as the present average, the Least Cost Combination would give, in the 7 independent variable case, a production of:

	£144	for Group 1 (Dairy),
	£385	for Group 2 (Livestock),
	£109	for Group 3 (Mixed)
and	£326	for Group 4 (Arable)

more per £1000 Total Costs than the present actual combination of inputs, giving the same Total Costs per farm as the Least Cost Combinations.

These Least Cost Combinations would in comparison with present average use require:

(1) At least twice as large farms (Rent and Rates) for all groups apart from Mixed farms (Group 3) which should only be increased by about 20%.

(2) About 100% more Machinery and Equipment on all farms.

(3) About 100% more Other Tenant's Capital (mainly livestock and stores of feedingstuffs, etc.) on all farms except Arable farms (Group 4) where the increase should only be about 30%.

(4) About 30% and 45% less Labour on Dairy (Group 1) and Livestock (Group 2) farms but "only" 4% and 17% less Labour on Mixed (Group 3) and Arable (Group 4) farms.

(5) About 7% and 16% less Seeds and Feed on Dairy (Group 1) and Livestock (Group 2) farms and about 20% and 7% less on Mixed (Group 3) and Arable (Group 4) farms.

(6) About 40% and 5% *less* Fertilisers and Lime on Dairy (Group 1) and Livestock (Group 2) farms but about 2% and 220% *more* on Mixed (Group 3) and Arable (Group 4) farms.

(7) For all farm types less of the variable Other Purchases, (see page 10 for the composition of this variable).

3. The Irish Results

Among the Irish farm groups the Subsistence farms (Group 00) comes in a class of its own. For this group there *appear* to be very marked decreasing returns to scale, 1% more of all inputs giving about 0.75% greater product.

For the Dairy farms (Groups 11 & 12 combined, and 13) there appear to be nearly constant returns to scale.

For the Crops Mixed farms (Group 20) there are increasing returns to scale, 1% more of all inputs giving about 1.06% greater product.

The Cattle Mixed farms (Group 30) shows a slight tendency to decreasing returns to scale, 1% more of all inputs giving about 0.94% greater product.

As the coefficient for Machinery and Equipment in the 7 independent variable case for the group Dairy, and Dairying Mixed without Cash Crops (11 & 12 combined) gave a small negative coefficient (a very insignificant input on this type of farm), the Least Cost Combinations will here be dealt with mainly from the 4 independent variable case.

For the same "size" of farm (defined as a farm with the same Total Costs) as the present average, the Least Cost Combination would give, in the 4 independent variable case, a production of:

	£ 896	for Groups 11 & 12 combined	(Mainly Dairying, and Dairying Mixed without Cash Crops),
	£ 532	for Group 13	(Dairying Mixed with Cash Crops),
	£ 573	for Group 20	(Crops Mixed),
	£ 733	for Group 30	(Cattle Mixed)
and	£1028	for Group 00	(Subsistence)

more per £1000 Total Costs than the present actual combination of inputs, giving the same Total Costs per farm as the Least Cost Combination.

Some of these Least Cost Combinations are clear *extrapolations* from the data and should be treated with some care - though the main "recommendations" may still be very much to the point as suggestions of the *directions* of change required.

These Least Cost Combinations would in comparison with the present average require:

(1) Little change in size of farms for all groups apart from Cattle Mixed farms (Group 30) where it should be halved.

(2) Tenant's Capital should be very much increased. For the Crops Mixed farms (Group 20) by about 14% and for all other main groups from 2¼ to 3½ times as much should be used (Subsistence 9 times).
This increase in Tenant's Capital should not be an increase in Machinery and Equipment but in Other Tenant's Capital, mainly Livestock.

(3) Labour should be reduced very much for all groups: for the Crops Mixed farms (Group 20) down to about 87% of the present use and for the other main groups down to about 30% to 50% of the present use; for the Subsistence farms (Group 00) Labour should be reduced to about 16% of the present use.
The warning about *extrapolation* must be repeated here but there can hardly be any doubt that Irish farming is using far too much Labour - mainly Family Labour.

(4) Total Purchases should be increased for all groups by from 15% to 55%.
This increase in Total Purchases should about equally affect Seeds and Feed, Fertilisers and Lime, and Other Purchases.

APPENDIX

TABLE 1 BRITISH

CORRELATION COEFFICIENTS (r) BETWEEN ALL VARIABLES USED (LOGS)

Group 1 (Dairy)

	z	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_{10}	0.96	0.76	(0.80)	(0.89)	0.90	0.82	(0.96)	(0.66)	(0.87)
x_9	0.91	0.84	0.85	0.88	(0.91)	0.86	0.72	0.72	
x_8	0.70	0.60	0.73	0.72	(0.75)	0.67	0.52		
x_7	0.89	0.65	0.68	0.80	(0.80)	0.70			
x_6	0.89	0.82	0.77	0.87	0.87				
x_5	0.94	0.85	(0.91)	(0.98)					
x_4	0.93	0.84	0.81						
x_3	0.84	0.74							
x_2	0.86								

Group 2 (Livestock)

	x_9	x_8	x_7	x_6	x_5	x_4	x_3	x_2	z
x_2									0.87
x_3								0.74	0.81
x_4							0.76	0.87	0.90
x_5						(0.98)	(0.87)	0.88	0.92
x_6					0.80	0.79	0.69	0.77	0.84
x_7				0.42	(0.54)	0.52	0.49	0.47	0.72
x_8			0.39	0.63	(0.69)	0.65	0.70	0.61	0.71
x_9		0.70	0.60	0.80	(0.89)	0.86	0.81	0.86	0.91
x_{10}	(0.88)	(0.64)	(0.89)	0.74	0.81	(0.78)	(0.73)	0.75	0.93

Group 3 (Mixed)

	z	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_{10}	0.97	0.81	(0.84)	(0.88)	0.89	0.88	(0.93)	(0.79)	(0.92)
x_9	0.94	0.88	0.87	0.90	(0.92)	0.92	0.72	0.81	
x_8	0.83	0.71	0.80	0.78	(0.82)	0.79	0.60		
x_7	0.84	0.64	0.67	0.71	(0.72)	0.71			
x_6	0.94	0.86	0.83	0.90	0.91				
x_5	0.94	0.89	(0.93)	(0.98)					
x_4	0.92	0.88	0.85						
x_3	0.88	0.82							
x_2	0.88								

Group 4 (Arable)

	x_9	x_8	x_7	x_6	x_5	x_4	x_3	x_2	z
x_2									0.92
x_3								0.84	0.89
x_4							0.78	0.85	0.89
x_5						(0.96)	(0.92)	0.90	0.95
x_6					0.91	0.86	0.84	0.90	0.96
x_7				0.82	(0.80)	0.77	0.74	0.75	0.87
x_8			0.75	0.89	(0.90)	0.81	0.88	0.87	0.93
x_9		0.92	0.82	0.93	(0.93)	0.88	0.86	0.91	0.96
x_{10}	(0.96)	(0.92)	(0.93)	0.93	0.92	(0.87)	(0.87)	0.88	0.97

N.B. The figures in brackets relate to pairs of variables which are not used together in any one regression.

TABLE 2 BRITISH

STANDARD DEVIATIONS OF RESIDUAL VARIATIONS FROM REGRESSIONS OF EACH INDEPENDENT VARIABLE ON ALL OTHER INDEPENDENT VARIABLES

"Dependent" Variable	Standard Deviations of Residual Variations.			
	Group 1	Group 2	Group 3	Group 4
4 Independent Variable Case				
x_2	0.1719	0.1543	0.1463	0.1406
x_5	0.1096	0.1184	0.1179	0.1386
x_6	0.1206	0.1198	0.1166	0.1136
x_{10}	0.1358	0.1796	0.1458	0.1259
7 Independent Variable Case				
x_3	0.1988	0.2038	0.1831	0.2052
x_4	0.1110	0.1240	0.1273	0.2012
x_7	0.1998	0.3027	0.2702	0.2328
x_8	0.4156	0.3750	0.2961	0.1743
x_9	0.1162	0.1172	0.1060	0.1043

TABLE 3 BRITISH

DISTRIBUTION BY SIZE (ADJUSTED ACRES) AND FARM TYPE OF THE 1646 F.M.S. FARMS

Size (Adjusted acres)	Group 1	Group 2	Group 3	Group 4	All Groups
	Number of Farms				
5 -	2	-	2	2	6
15 -	90	20	36	45	191
50 -	135	103	108	59	405
100 -	72	93	104	28	297
150 -	45	70	85	23	223
200 -	30	45	74	19	168
250 -	21	17	42	10	90
300 -	8	56	97	38	199
500 -	3	11	36	13	63
1000 & Over	-	1	3	-	4
Total	406	416	587	237	1646

TABLE 4 BRITISH

ANALYSIS OF VARIANCE IN THE 4 INDEPENDENT VARIABLE CASE

FOR EACH OF THE 4 FARM-TYPE GROUPS

Variation	Using x_1 with x_5, x_6, x_{10}			Using x_2 with x_5, x_6, x_{10}		
	Sum of squares	Degrees of freedom	Estimated variance	Sum of squares	Degrees of freedom	Estimated variance
GROUP 1						
Due to regression	37.1039	4	9.2760	37.3369	4	9.3342
Residual	1.1401	401	0.0028	0.9071	401	0.0023
Total	38.2440	405	-	38.2440	405	-
GROUP 2						
Due to regression	34.8359	4	8.7090	35.0090	4	8.7523
Residual	1.6295	411	0.0040	1.4564	411	0.0035
Total	36.4654	415	-	36.4654	415	-
GROUP 3						
Due to regression	67.1349	4	16.7837	67.1674	4	16.7919
Residual	1.5872	582	0.0027	1.5547	582	0.0027
Total	68.7221	586	-	68.7221	586	-
GROUP 4						
Due to regression	34.2784	4	8.5696	34.3429	4	8.5857
Residual	0.7929	232	0.0034	0.7284	232	0.0031
Total	35.0713	236	-	35.0713	236	-

TABLE 5 BRITISH

VARIANCE ANALYSES BY FARM–TYPE GROUPS IN FOUR INDEPENDENT VARIABLE CASE (x_2, x_5, x_6, x_{10})

	GROUP 1			GROUP 2			GROUP 3			GROUP 4		
	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance
YEAR 1												
Variation due to regression	37.7933	4	-	35.4334	4	-	66.2462	4	-	34.0921	4	-
Residual	1.8116	401	0.0045	2.8531	411	0.0069	2.8076	582	0.0048	1.2989	232	0.0056
Total	39.6049	405	-	38.2865	415	-	69.0538	586	-	35.3910	236	-
YEAR 2												
Variation due to regression	37.9570	4	-	34.8321	4	-	68.6813	4	-	35.4966	4	-
Residual	1.4472	401	0.0036	3.1274	411	0.0076	2.5158	582	0.0043	1.4380	232	0.0062
Total	39.4042	405	-	37.9595	415	-	71.1971	586	-	36.9346	236	-
YEAR 3												
Variation due to regression	37.6094	4	-	36.5510	4	-	67.0765	4	-	33.4249	4	-
Residual	1.2163	401	0.0030	1.8646	411	0.0045	2.5674	582	0.0044	1.2640	232	0.0054
Total	38.8257	405	-	38.4516	415	-	69.6439	586	-	34.6889	236	-
YEAR 4												
Variation due to regression	37.2345	4	-	33.5162	4	-	68.1720	4	-	34.9370	4	-
Residual	1.2440	401	0.0031	1.9175	411	0.0047	2.5146	582	0.0043	1.3753	232	0.0059
Total	38.4785	405	-	35.4337	415	-	70.6866	586	-	36.3123	236	-
// PLANES WITHIN YEARS												
Variation due to regression	150.5538	4	-	140.1068	4	-	270.0994	4	-	137.7493	4	-
Residual	5.7595	1616	0.0036	9.9885	1656	0.0060	10.4820	2340	0.0045	5.5775	940	0.0059
Total	156.3133	1620	-	150.0953	1660	-	280.5814	2344	-	143.3268	944	-
BETWEEN FARMS												
Variation due to regression	37.3369	4	-	35.0090	4	-	67.1674	4	-	34.3429	4	-
Residual	0.9071	401	0.0023	1.4564	411	0.0035	1.5547	582	0.0027	0.7284	232	0.0031
Total	38.2440	405	-	36.4654	415	-	68.7221	586	-	35.0713	236	-
CONSIDERING ALL FARMS AS ONE GROUP												
Variation due to regression							203.5503	4	-			
Residual							5.1277	1641	0.0031			
Total							208.6780	1645	-			

REGRESSION COEFFICIENTS, THEIR SUMS AND MULTIPLE CORRELATION COEFFICIENTS - 4 INDEPENDENT VARIABLE CASE (x_2, x_5, x_6, x_{10})

Type of Regression	b_2	b_5	b_6	b_{10}	Sum of coeffi-cients	R
GROUP 1						
Year 1	0.176	0.147	0.177	0.541	1.041	0.977
Year 2	0.131	0.134	0.161	0.604	1.030	0.981
Year 3	0.155	0.142	0.168	0.571	1.036	0.984
Year 4	0.140	0.156	0.175	0.553	1.024	0.984
// Planes Within Years	0.150	0.145	0.170	0.567	1.032	0.981
Between Farms	0.151	0.135	0.164	0.584	1.034	0.988
(Standard Errors)	(0.014)	(0.022)	(0.020)	(0.017)	(0.009)	-
GROUP 2						
Year 1	0.141	0.331	0.209	0.398	1.079	0.962
Year 2	0.144	0.255	0.161	0.499	1.059	0.958
Year 3	0.163	0.288	0.195	0.443	1.089	0.975
Year 4	0.112	0.389	0.173	0.358	1.032	0.973
// Planes Within Years	0.140	0.314	0.184	0.425	1.063	0.966
Between Farms	0.135	0.312	0.183	0.438	1.068	0.980
(Standard Errors)	(0.019)	(0.025)	(0.024)	(0.016)	(0.014)	-
GROUP 3						
Year 1	0.107	0.163	0.283	0.496	1.049	0.979
Year 2	0.060	0.149	0.317	0.531	1.057	0.982
Year 3	0.092	0.176	0.250	0.524	1.042	0.981
Year 4	0.058	0.228	0.276	0.478	1.040	0.982
// Planes Within Years	0.078	0.179	0.283	0.508	1.048	0.981
Between Farms	0.069	0.179	0.284	0.515	1.047	0.989
(Standard Errors)	(0.015)	(0.018)	(0.018)	(0.015)	(0.007)	-
GROUP 4						
Year 1	0.065	0.177	0.338	0.452	1.032	0.981
Year 2	0.224	0.146	0.236	0.449	1.055	0.980
Year 3	0.120	0.212	0.145	0.522	0.999	0.982
Year 4	0.118	0.117	0.393	0.412	1.040	0.981
// Planes Within Years	0.130	0.163	0.283	0.456	1.032	0.980
Between Farms	0.124	0.166	0.272	0.469	1.031	0.990
(Standard Errors)	(0.026)	(0.026)	(0.032)	(0.029)	(0.011)	-
// Planes from the 4 Between Farms Regressions	0.125	0.206	0.211	0.502	1.044	0.986
(Standard Errors)	(0.009)	(0.011)	(0.011)	(0.009)	(0.005)	-
Considering all Farms as one Group	0.129	0.184	0.237	0.503	1.053	0.988
(Standard Errors)	(0.009)	(0.010)	(0.011)	(0.008)	(0.004)	-

TABLE 7

BRITISH

WITHIN YEARS (// PLANES) REGRESSION FOR THE 4 INDEPENDENT VARIABLE CASE
COMPARED WITH INDIVIDUAL YEARS' REGRESSIONS

Type of Variation	GROUP 1			GROUP 2			GROUP 3			GROUP 4		
	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance
Due to // planes regression	150.5538	4	-	140.1068	4	-	270.0994	4	-	137.7493	4	-
Due to difference in co-efficients	0.0404	12	0.003367	0.2259	12	0.018825	0.0766	12	0.006383	0.2013	12	0.016775
Due to sum of regressions	150.5942	16	-	140.3327	16	-	270.1760	16	-	137.9506	16	-
Residual	5.7191	1604	0.003566	9.7626	1644	0.005938	10.4054	2328	0.004470	5.3762	928	0.005793
Total	156.3133	1620	-	150.0953	1660	-	280.5814	2344	-	143.3268	944	-
Variance Ratio	$\frac{0.003566}{0.003367} = 1.06$			$\frac{0.018825}{0.005938} = 3.17$			$\frac{0.006383}{0.004470} = 1.43$			$\frac{0.016775}{0.005793} = 2.90$		
Degrees of freedom	1604 and 12			12 and 1644			12 and 2328			12 and 928		
Level of probability	50%			1%			20%			1%		

TABLE 8 BRITISH

CONSTANT TERMS (IN LOGS) FROM WITHIN YEARS REGRESSIONS

	Constant term	Relative values of production from given costs - Year 1 = 100
GROUP 1		
Year 1	0.3115	100
Year 2	0.3088	99
Year 3	0.3168	101
Year 4	0.3299	104
GROUP 2		
Year 1	0.1376	100
Year 2	0.1121	94
Year 3	0.1544	104
Year 4	0.1955	114
GROUP 3		
Year 1	0.2300	100
Year 2	0.2325	101
Year 3	0.2271	99
Year 4	0.2344	101
GROUP 4		
Year 1	0.3548	100
Year 2	0.3761	105
Year 3	0.3397	97
Year 4	0.3478	98

TABLE 9 BRITISH

CONSTANT TERMS (IN LOGS) FROM BETWEEN FARMS REGRESSIONS USING // PLANES FOR THE 4 GROUPS

	Constant term	Relative values of production from given costs - Group 2 = 100
Group 1	0.2549	101
Group 2	0.2517	100
Group 3	0.2600	102
Group 4	0.2856	108

TABLE 10 BRITISH

ACTUAL VALUES OF LOG GROSS PRODUCT *MINUS* CALCULATED VALUES FROM WITHIN YEARS REGRESSIONS (CONSTANTS FOR EACH YEAR)

Range of deviations	Group 1	Group 2	Group 3	Group 4
	Number of farms			
Less than - 0.360 -	-	1	-	-
- 0.360 -	-	1	1	1
- 0.320 -	2	-	4	3
- 0.280 -	3	6	2	2
- 0.240 -	2	7	3	8
- 0.200 -	8	34	25	10
- 0.160 -	33	52	63	23
- 0.120 -	90	128	141	93
- 0.080 -	216	220	337	121
- 0.040 -	418	357	564	193
0 -	480	361	576	207
+ 0.040 -	252	265	369	166
+ 0.080 -	86	147	192	73
+ 0.120 -	24	64	59	37
+ 0.160 -	9	15	10	8
+ 0.200 -	1	3	2	3
+ 0.240 - + 0.279	-	3	-	-
Total negative deviations	772	806	1140	454
Total positive deviations	852	858	1208	494
TOTAL	1624	1664	2348	948

TABLE 11 BRITISH

ACTUAL VALUES OF LOG GROSS PRODUCT *MINUS* CALCULATED VALUES FROM BETWEEN FARMS REGRESSIONS

Range of deviations	Group 1	Group 2	Group 3	Group 4	All Groups
	Number of farms				
- 0.275 -	-	-	1	-	1
- 0.250 -	-	-	-	-	-
- 0.225 -	1	1	1	-	3
- 0.200 -	-	-	-	-	-
- 0.175 -	1	1	1	1	4
- 0.150 -	4	9	3	4	20
- 0.125 -	7	11	10	4	32
- 0.100 -	9	24	23	15	71
- 0.075 -	22	29	46	14	111
- 0.050 -	60	54	82	42	238
- 0.025 -	87	67	128	35	317
0 -	97	86	120	38	341
+ 0.025 -	70	53	72	42	237
+ 0.050 -	33	46	57	25	161
+ 0.075 -	10	16	27	13	66
+ 0.100 -	2	12	11	2	27
+ 0.125 -	2	4	5	2	13
+ 0.150 -	-	2	-	-	2
+ 0.175 - + 0.199	1	1	-	-	2
Total negative deviations	191	196	295	115	797
Total positive deviations	215	220	292	122	849
TOTAL	406	416	587	237	1646

TABLE 12 BRITISH

BETWEEN FARMS REGRESSIONS (SUB-DIVISION BY THIRDS ON X_{11}) AND SINGLE REGRESSION FOR EACH GROUP

Type of Regression	b_2	b_5	b_6	b_{10}	Sum of Coeffi-cients	Residual Variances	R
GROUP 1							
1st third	0.210	0.144	0.105	0.581	1.040	0.0028	0.950
2nd third	0.107	0.120	0.194	0.543	0.964	0.0019	0.884
3rd third	0.080	0.128	0.207	0.534	0.949	0.0017	0.958
Between Farms in One Group	0.151	0.135	0.164	0.584	1.034	0.0023	0.988
GROUP 2							
1st third	0.180	0.327	0.206	0.434	1.147	0.0045	0.915
2nd third	0.116	0.375	0.171	0.450	1.112	0.0034	0.845
3rd third	0.094	0.215	0.285	0.419	1.013	0.0025	0.956
Between Farms in One Group	0.135	0.312	0.183	0.438	1.068	0.0035	0.980
GROUP 3							
1st third	0.059	0.165	0.324	0.514	1.062	0.0026	0.971
2nd third	0.104	0.172	0.266	0.471	1.013	0.0032	0.836
3rd third	0.052	0.182	0.257	0.532	1.023	0.0022	0.965
Between Farms in One Group	0.069	0.179	0.284	0.515	1.047	0.0027	0.989
GROUP 4							
1st third	0.082	0.139	0.317	0.545	1.083	0.0035	0.945
2nd third	0.156	0.172	0.251	0.416	0.995	0.0031	0.912
3rd third	0.109	0.177	0.290	0.449	1.025	0.0029	0.966
Between Farms in One Group	0.124	0.166	0.272	0.469	1.031	0.0031	0.990

RESIDUAL SUMS OF SQUARES FROM ONE REGRESSION AND FROM THREE REGRESSIONS BY THIRDS ACCORDING TO "TOTAL COSTS" FOR EACH FARM-TYPE GROUP

Type of Variation	GROUP 1			GROUP 2			GROUP 3			GROUP 4		
	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance
Residual variance from Between Farms regression	0.9071	401	0.002262	1.4564	411	0.003544	1.5547	582	0.002671	0.7284	232	0.003140
Sum of residual variances from regressions on thirds	0.8306	391	0.002124	1.3808	401	0.003443	1.5318	572	0.002678	0.7100	222	0.003198
Difference	0.0765	10	0.007650	0.0756	10	0.007560	0.0229	10	0.002290	0.0184	10	0.001840
Variance Ratio	$\frac{0.007650}{0.002124} = 3.6017$			$\frac{0.007560}{0.003443} = 2.1958$			$\frac{0.002678}{0.002290} = 1.1694$			$\frac{0.003198}{0.001840} = 1.7380$		
Degrees of freedom	10 & 391			10 & 401			572 & 10			222 & 10		
Significance level	1%			10%			40%			30%		

TABLE 14 BRITISH

FURTHER BREAKDOWN OF WITHIN YEARS SUMS OF SQUARES

Variation	Variances, estimates of	GROUP 1			GROUP 2			GROUP 3			GROUP 4		
		Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance	Sum of squares	D.f.	Variance
Due to // planes regression	-	150.5538	4	-	140.1068	4	-	270.0994	4	-	137.7493	4	-
Due to Between Farms (II)	$4s_2^2 + s_1^2$	3.6373	405	0.008981	5.8367	415	0.014064	6.2265	586	0.010625	2.9175	236	0.012362
Random (III)	s_1^2	2.1222	1211	0.001752	4.1518	1241	0.003346	4.2555	1754	0.002426	2.6600	704	0.003778
Total	-	156.3133	1620	-	150.0953	1660	-	280.5814	2344	-	143.3268	944	-

TABLE 15 BRITISH

ESTIMATED VALUES OF "MANAGERIAL" AND "RANDOM" VARIANCES

Type of variance estimate	GROUP 1	GROUP 2	GROUP 3	GROUP 4
	Variance	Variance	Variance	Variance
$4s_2^2 + s_1^2$	0.008981	0.014064	0.010625	0.012362
s_1^2	0.001752	0.003346	0.002426	0.003778
s_2^2	0.001807	0.002680	0.002050	0.002146

TABLE 16 BRITISH

RESIDUAL VARIANCES FROM BETWEEN FARMS REGRESSIONS FOR VARYING PERIODS

Number of years for which accounts are averaged	GROUP 1			GROUP 2			GROUP 3			GROUP 4		
	"Random" variance	"Managerial" variance	Total variance	"Random" variance	"Managerial" variance	Total variance	"Random" variance	"Managerial" variance	Total variance	"Random" variance	"Managerial" variance	Total variance
1	0.001752	0.001807	0.003559	0.003346	0.002680	0.006026	0.002426	0.002050	0.004476	0.003778	0.002146	0.005924
2	0.000876	0.001807	0.002683	0.001673	0.002680	0.004353	0.001213	0.002050	0.003263	0.001889	0.002146	0.004035
3	0.000584	0.001807	0.002391	0.001115	0.002680	0.003795	0.000809	0.002050	0.002859	0.001259	0.002146	0.003405
4	0.000438	0.001807	0.002245	0.000837	0.002680	0.003517	0.000607	0.002050	0.002657	0.000945	0.002146	0.003091
∞	-	0.001807	0.001807	-	0.002680	0.002680	-	0.002050	0.002050	-	0.002146	0.002146

TABLE 17 BRITISH

GROSS PRODUCT & COSTS FOR £5,000 TOTAL COSTS FOR FARMS HAVING LARGER AVERAGE GROSS PRODUCT THAN REGRESSION VALUES (POSITIVE DEVIATIONS) & FOR THOSE HAVING SMALLER AVERAGE GROSS PRODUCT (NEGATIVE DEVIATIONS) WITHIN EACH FARM-TYPE GROUP

	Gross Product	Acres	Rent & Rates	Tenant's Capital			Labour	Seeds & Feed	Ferts. & Lime	Other Purchases	Total Purchases	"Total Costs"*
				Machinery & Equipt.	Other	Total						
	Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	
	£	£	£	£	£	£	£	£	£	£	£	£
GROUP 1 (Dairy)												
Positive Deviations (215 farms)	5489	116	299	1488	2956	4444	1327	1997	212	765	2974	5000
Negative Deviations (191 farms)	4786	116	324	1408	3011	4419	1282	2003	193	800	2996	5000
Difference (+ ve *minus* - ve)	+703	-	-25	+ 80	- 55	+ 25	+ 45	- 6	+19	-35	- 22	-
GROUP 2 (Livestock)												
Positive Deviations (220 farms)	5766	258	375	1753	4853	6606	1619	1301	258	852	2411	5000
Negative Deviations (196 farms)	4881	268	383	1670	5152	6822	1567	1214	272	950	2436	5000
Difference (+ ve *minus* - ve)	+885	-10	- 8	+ 83	-299	-216	+ 52	+ 87	-14	-98	- 25	-
GROUP 3 (Mixed)												
Positive Deviations (292 farms)	5901	144	301	1796	3377	5173	1460	1478	380	915	2773	5000
Negative Deviations (295 farms)	5043	143	298	1708	3554	5262	1477	1416	352	983	2751	5000
Difference (+ ve *minus* - ve)	+858	+ 1	+ 3	+ 88	-177	- 89	- 17	+ 62	+28	-68	+ 22	-
GROUP 4 (Arable)												
Positive Deviations (122 farms)	6134	141	313	1619	3215	4834	1741	1014	503	994	2511	5000
Negative Deviations (115 farms)	5127	128	303	1583	3112	4695	1714	1060	469	1031	2560	5000
Difference (+ ve *minus* - ve)	+1007	+13	+10	+ 36	+103	+139	+ 27	- 46	+34	-37	- 49	-

* "Total Costs" = $X_2 + X_6 + X_{10}$ + 9% interest on X_5

TABLE 18 BRITISH

REGRESSION OF LOG G ON LOGS OF X_2, X_5, X_6, X_{10}

Type of Regression	Type of Variances (Co-variances) used	b_2 (Rent & Rates)	b_5 (Total Tenant's Capital)	b_6 (Labour)	b_{10} (Total Purchases)	Σb_i	Differences	R	Differences
GROUP 1									
Residuals	s_1^2	0.106	0.068	0.204	0.442	0.8202		0.629	
(Standard Errors)		(0.025)	(0.035)	(0.029)	(0.020)		0.2120		0.352
Within Years	$s_2^2 + s_1^2$	0.150	0.145	0.170	0.567	1.0322		0.981	
(Standard Errors)		(0.008)	(0.013)	(0.012)	(0.010)		0.0018		0.007
Between Farms	$s_2^2 + \frac{1}{4} s_1^2$	0.151	0.135	0.164	0.584	1.0340		0.988	
(Standard Errors)		(0.014)	(0.022)	(0.020)	(0.017)		0.0006		0.002
Variance Components	s_2^2	0.152	0.130	0.161	0.591	1.0346		0.990	
GROUP 2									
Residuals	s_1^2	0.049	0.070	0.093	0.253	0.4660		0.315	
(Standard Errors)		(0.036)	(0.047)	(0.036)	(0.024)		0.5977		0.651
Within Years	$s_2^2 + s_1^2$	0.140	0.314	0.184	0.425	1.0637		0.966	
(Standard Errors)		(0.012)	(0.015)	(0.015)	(0.010)		0.0043		0.014
Between Farms	$s_2^2 + \frac{1}{4} s_1^2$	0.135	0.312	0.183	0.438	1.0680		0.980	
(Standard Errors)		(0.019)	(0.025)	(0.024)	(0.016)		0.0016		0.005
Variance Components	s_2^2	0.132	0.311	0.183	0.443	1.0696		0.985	
GROUP 3									
Residuals	s_1^2	0.097	0.073	0.196	0.416	0.7827		0.530	
(Standard Errors)		(0.025)	(0.031)	(0.029)	(0.020)		0.2642		0.451
Within Years	$s_2^2 + s_1^2$	0.078	0.179	0.283	0.508	1.0469		0.981	
(Standard Errors)		(0.009)	(0.011)	(0.011)	(0.009)		0.0015		0.008
Between Farms	$s_2^2 + \frac{1}{4} s_1^2$	0.069	0.179	0.284	0.515	1.0484		0.989	
(Standard Errors)		(0.015)	(0.018)	(0.018)	(0.015)		0.0006		0.002
Variance Components	s_2^2	0.066	0.179	0.285	0.518	1.0490		0.991	
GROUP 4									
Residuals	s_1^2	0.084	0.027	0.229	0.338	0.6782		0.401	
(Standard Errors)		(0.042)	(0.043)	(0.054)	(0.037)		0.3531		0.579
Within Years	$s_2^2 + s_1^2$	0.130	0.163	0.283	0.456	1.0313		0.980	
(Standard Errors)		(0.017)	(0.017)	(0.021)	(0.018)		0.0003		0.010
Between Farms	$s_2^2 + \frac{1}{4} s_1^2$	0.124	0.166	0.272	0.469	1.0316		0.990	
(Standard Errors)		(0.026)	(0.026)	(0.032)	(0.029)		0.0002		0.003
Variance Components	s_2^2	0.122	0.167	0.267	0.475	1.0318		0.993	

TABLE 19 BRITISH

STANDARD DEVIATIONS FOR VARIABLES (IN LOGS)

	z	x_2	x_5	x_6	x_{10}
GROUP 1					
Residuals regression	0.052	0.047	0.035	0.042	0.064
Between Farms regression	0.307	0.340	0.325	0.264	0.312
GROUP 2					
Residuals regression	0.058	0.043	0.034	0.044	0.067
Between Farms regression	0.296	0.334	0.296	0.210	0.314
GROUP 3					
Residuals regression	0.057	0.047	0.040	0.041	0.061
Between Farms regression	0.342	0.334	0.356	0.308	0.345
GROUP 4					
Residuals regression	0.066	0.054	0.054	0.043	0.064
Between Farms regression	0.385	0.364	0.414	0.354	0.388

TABLE 20 BRITISH

"MARGINAL PRODUCTIVITIES" WITH ALL VARIABLES AT GEOMETRIC MEAN VALUES (4 INDEPENDENT VARIABLES)

Type of regression used	X_2 (Rent & Rates)	X_5 (Total Tenant's Capital)	X_6 (Labour)	X_{10} (Total Purchases)
	(£)	(£)	(£)	(£)
GROUP 1 (Dairy)				
Within Years	2.625	0.172	0.642	0.990
Between Farms	2.646	0.160	0.617	1.020
Variance Components	2.659	0.155	0.608	1.032
Considering all farms as one group and using Group 1 means	2.261	0.219	0.894	0.879
GROUP 2 (Livestock)				
Within Years	2.070	0.246	0.550	0.969
Between Farms	1.986	0.244	0.548	0.999
Variance Components	1.955	0.243	0.547	1.010
Considering all farms as one group and using Group 2 means	1.908	0.144	0.708	1.148
GROUP 3 (Mixed)				
Within Years	1.402	0.188	1.007	1.008
Between Farms	1.250	0.189	1.014	1.023
Variance Components	1.194	0.189	1.015	1.028
Considering all farms as one group and using Group 3 means	2.329	0.194	0.844	0.999
GROUP 4 (Arable)				
Within Years	2.239	0.197	0.874	1.018
Between Farms	2.146	0.202	0.840	1.047
Variance Components	2.112	0.203	0.826	1.059
Considering all farms as one group and using Group 4 means	2.235	0.223	0.732	1.124
Considering all farms as one group and using overall means	2.186	0.189	0.802	1.020

TABLE 21 BRITISH

ACTUAL AVERAGE COMBINATIONS OF INPUTS COMPARED WITH LEAST COST COMBINATIONS GIVING THE SAME AVERAGE TOTAL COSTS

	Estimated & Actual Gross Product Z	Rent & Rates X_2	Total Tenant's Capital X_5	Labour X_6	Total Purchases X_{10}	Total Costs (including 9% interest on Tenant's Capital)
	£	£	£	£	£	£
GROUP 1						
Least Cost Combination	5650	729	6946	772	2834	4960
Actual average (arithmetic means)	5118	308	4396	1296	2960	4960
GROUP 2						
Least Cost Combination	4321	424	11073	585	1417	3423
Actual average (arithmetic means)	3646	260	4596	1090	1659	3423
GROUP 3						
Least Cost Combination	8688	485	14591	2085	3791	7674
Actual average (arithmetic means)	8376	461	8009	2254	4238	7674
GROUP 4						
Least Cost Combination	8559	846	12882	1850	3287	7142
Actual average (arithmetic means)	7983	440	6796	2466	3624	7142

TABLE 22 BRITISH

ACTUAL AVERAGE COMBINATIONS OF INPUTS COMPARED WITH LEAST COST COMBINATIONS

GIVING THE SAME AVERAGE TOTAL COSTS ON THE CONDITION OF FIXED RENT & RATES (FIXED FARM SIZE)

	Estimated & Actual Gross Product Z	Rent & Rates X_2	Total Tenant's Capital X_5	Labour X_6	Total Purchases X_{10}	Total Costs (including 9% interest on Tenant's Capital)
	£	£	£	£	£	£
GROUP 1						
Least Cost Combination	5391	308	7637	849	3116	4960
Actual average (arithmetic means)	5118	308	4396	1296	2960	4960
GROUP 2						
Least Cost Combination	4259	260	11678	617	1495	3423
Actual average (arithmetic means)	3646	260	4596	1090	1659	3423
GROUP 3						
Least Cost Combination	8686	461	14640	2092	3803	7674
Actual average (arithmetic means)	8376	461	8009	2254	4238	7674
GROUP 4						
Least Cost Combination	8362	440	13713	1970	3498	7142
Actual average (arithmetic means)	7983	440	6796	2466	3624	7142

TABLE 23 BRITISH

REGRESSION OF LOG G ON LOGS OF X_2, X_3, X_4, X_6, X_7, X_8, X_9

Type of Regression	Type of Variances (Co-variances) used	b_2 (Rent & Rates)	b_3 Tenant's Capital (Machinery & Equipt.)	b_4 Tenant's Capital (Other)	b_6 (Labour)	b_7 (Seeds & Feed)	b_8 (Ferts. & Lime)	b_9 (Other Purchases)	Σb_i	R
GROUP 1										
Residuals	s_1^2	0.100	-0.040	0.220	0.211	0.268	0.009	0.097	0.8659	0.630
Within Years	$s_2^2 + s_1^2$	0.148	0.046	0.125	0.199	0.359	0.017	0.131	1.0245	0.980
Between Farms	$s_2^2 + \frac{1}{4} s_1^2$	0.152	0.048	0.099	0.195	0.375	0.022	0.134	1.0242	0.987
(Standard Errors)		(0.015)	(0.013)	(0.022)	(0.021)	(0.012)	(0.006)	(0.021)	(0.010)	-
Variance Components	s_2^2	0.155	0.047	0.087	0.193	0.382	0.025	0.135	1.0232	0.989
GROUP 2										
Residuals	s_1^2	0.040	-0.065	0.186	0.094	0.118	0.004	0.075	0.4526	0.329
Within Years	$s_2^2 + s_1^2$	0.181	0.070	0.262	0.188	0.210	0.031	0.120	1.0615	0.964
Between Farms	$s_2^2 + \frac{1}{4} s_1^2$	0.186	0.074	0.251	0.182	0.220	0.044	0.103	1.0599	0.979
(Standard Errors)		(0.021)	(0.015)	(0.024)	(0.026)	(0.010)	(0.008)	(0.026)	(0.015)	-
Variance Components	s_2^2	0.190	0.074	0.246	0.178	0.224	0.053	0.092	1.0576	0.984
GROUP 3										
Residuals	s_1^2	0.106	-0.011	0.072	0.230	0.173	0.014	0.123	0.7073	0.492
Within Years	$s_2^2 + s_1^2$	0.080	0.079	0.129	0.301	0.230	0.048	0.163	1.0299	0.979
Between Farms	$s_2^2 + \frac{1}{4} s_1^2$	0.075	0.071	0.126	0.293	0.236	0.065	0.160	1.0262	0.987
(Standard Errors)		(0.016)	(0.012)	(0.018)	(0.021)	(0.008)	(0.008)	(0.021)	(0.008)	-
Variance Components	s_2^2	0.074	0.066	0.124	0.287	0.239	0.077	0.157	1.0232	0.990
GROUP 4										
Residuals	s_1^2	0.080	-0.041	0.052	0.242	0.118	0.014	0.155	0.6187	0.381
Within Years	$s_2^2 + s_1^2$	0.127	0.089	0.065	0.315	0.170	0.109	0.141	1.0158	0.979
Between Farms	$s_2^2 + \frac{1}{4} s_1^2$	0.125	0.076	0.069	0.297	0.184	0.168	0.086	1.0046	0.990
(Standard Errors)		(0.027)	(0.018)	(0.018)	(0.032)	(0.016)	(0.021)	(0.035)	(0.013)	-
Variance Components	s_2^2	0.126	0.063	0.074	0.285	0.193	0.126	0.038	0.9950	0.994

TABLE 24 BRITISH

A COMPARISON BETWEEN REGRESSION COEFFICIENTS FROM VARIANCE COMPONENTS REGRESSIONS IN 4 AND 7 INDEPENDENT VARIABLE CASES

Type of Regression	$(b_2 + b_6)$	b_5 or $(b_3 + b_4)$	b_{10} or $(b_7 + b_8 + b_9)$	Weighted $(b_7 + b_8 + b_9)$
GROUP 1 (Dairy)				
4 Independent Variable case	0.313	0.130	0.591	-
7 Independent Variable case	0.348	0.134	0.542	0.563
GROUP 2 (Livestock)				
4 Independent Variable case	0.315	0.311	0.443	-
7 Independent Variable case	0.368	0.320	0.369	0.408
GROUP 3 (Mixed)				
4 Independent Variable case	0.351	0.179	0.518	-
7 Independent Variable case	0.361	0.190	0.473	0.498
GROUP 4 (Arable)				
4 Independent Variable case	0.389	0.167	0.475	-
7 Independent Variable case	0.411	0.137	0.447	0.549

TABLE 25 BRITISH

STANDARD DEVIATIONS s_1 AND s_2

	Group 1		Group 2		Group 3		Group 4	
	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1
7 Independent Variable Case								
1st Estimate	0.045	0.043	0.053	0.059	0.048	0.052	0.046	0.064
2nd Estimate	0.045	0.041	0.055	0.055	0.049	0.050	0.047	0.061
4 Independent Variable Case								
1st Estimate	0.043	0.042	0.052	0.058	0.045	0.049	0.046	0.061
2nd Estimate	0.043	0.041	0.053	0.055	0.046	0.048	0.047	0.060

TABLE 26 BRITISH

"MARGINAL PRODUCTIVITIES" WITH ALL VARIABLES AT GEOMETRIC MEAN VALUES (7 INDEPENDENT VARIABLES)

Type of Regression	X_2 (Rent & Rates)	X_3 (Tenant's Capital) (Machinery & Equipt.)	X_4 (Tenant's Capital) (Other)	X_6 (Labour)	X_7 (Seeds & Feed)	X_8 (Ferts. & Lime)	X_9 (Other Purchases)
	£	£	£	£	£	£	£
GROUP 1 (Dairy)							
Within Years	2.592	0.183	0.221	0.750	0.979	0.790	0.880
Between Farms	2.666	0.189	0.176	0.735	1.021	1.028	0.900
Variance Components	2.710	0.186	0.155	0.727	1.040	1.194	0.904
GROUP 2 (Livestock)							
Within Years	2.669	0.235	0.277	0.563	1.057	0.947	0.731
Between Farms	2.744	0.248	0.265	0.545	1.108	1.374	0.630
Variance Components	2.805	0.248	0.260	0.533	1.129	1.639	0.562
GROUP 3 (Mixed)							
Within Years	1.441	0.269	0.205	1.074	0.957	0.987	0.931
Between Farms	1.344	0.243	0.200	1.043	0.983	1.347	0.914
Variance Components	1.341	0.224	0.196	1.024	0.994	1.582	0.895
GROUP 4 (Arable)							
Within Years	2.199	0.338	0.125	0.975	1.009	1.530	0.780
Between Farms	2.158	0.289	0.133	0.918	1.092	2.362	0.474
Variance Components	2.179	0.240	0.142	0.881	1.147	3.036	0.212

TABLE 27 BRITISH

ACTUAL AVERAGE COMBINATIONS OF INPUTS COMPARED WITH LEAST COST COMBINATIONS
GIVING THE SAME AVERAGE TOTAL COSTS

	Estimated and Actual Gross Product	Rent & Rates	Tenant's Capital			Labour	Seeds & Feed	Ferts. & Lime	Other Purchases	Total Purchases	Total Costs (including 9% on Tenant's Capital)
			Machinery & Equipt.	Other	Total						
	Z	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	
	£	£	£	£	£	£	£	£	£	£	£
GROUP 1											
Least Cost Combination	5830	751	2526	4697	7223	934	1850	122	653	2625	4960
Actual average (arithmetic means)	5118	308	1439	2957	4396	1296	1984	201	775	2960	4960
GROUP 2											
Least Cost Combination	4963	615	2647	8850	11497	577	725	172	299	1196	3423
Actual average (arithmetic means)	3646	260	1171	3425	4596	1090	861	181	617	1659	3423
GROUP 3											
Least Cost Combination	9210	558	5467	10317	15784	2155	1789	574	1177	3540	7674
Actual average (arithmetic means)	8376	461	2687	5322	8009	2254	2220	561	1457	4238	7674
GROUP 4											
Least Cost Combination	10310	904	5016	5886	10902	2049	1385	1547	276	3208	7142
Actual average (arithmetic means)	7983	440	2284	4512	6796	2466	1484	692	1448	3624	7142

TABLE 28

BRITISH

ACTUAL AVERAGE COMBINATIONS OF INPUTS COMPARED WITH LEAST COST COMBINATIONS GIVING THE SAME AVERAGE TOTAL COSTS ON THE CONDITION OF FIXED RENT & RATES (FIXED FARM SIZE)

	Estimated & Actual Gross Product	Rent & Rates	Tenant's Capital			Labour	Seeds & Feed	Ferts. & Lime	Other Purchases	Total Purchases	Total Costs (including 9% on Tenant's Capital)
			Machinery & Equipt.	Other	Total						
	Z	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	
	£	£	£	£	£	£	£	£	£	£	£
GROUP 1											
Least Cost Combination	5540	308	2792	5191	7983	1033	2045	134	722	2901	4960
Actual average (arithmetic means)	5118	308	1439	2957	4396	1296	1984	201	775	2960	4960
GROUP 2											
Least Cost Combination	4671	260	2982	9970	12952	650	817	194	336	1347	3423
Actual average (arithmetic means)	3646	260	1171	3425	4596	1090	861	181	617	1659	3423
GROUP 3											
Least Cost Combination	9200	461	5541	10457	15998	2185	1813	582	1193	3588	7674
Actual average (arithmetic means)	8376	461	2687	5322	8009	2254	2220	561	1457	4238	7674
GROUP 4											
Least Cost Combination	10022	440	5390	6324	11714	2201	1488	1663	296	3447	7142
Actual average (arithmetic means)	7983	440	2284	4512	6796	2466	1484	692	1448	3624	7142

TABLE 29 (a) BRITISH

GEOMETRIC MEAN VALUES OF VARIABLES USED (LOGS)

Farm – Type Group	Gross Product	Rent & Rates	Tenant's Capital			Labour	Seeds & Feed	Ferts. & Lime	Other Purchases	Total Purchases
			Machinery & Equipt.	Other	Total					
	z	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Group 1 (Dairy)	3.6023	2.3596	3.0047	3.3529	3.5276	3.0256	3.1671	1.9249	2.7752	3.3602
Group 2 (Livestock)	3.4541	2.2851	2.9272	3.4301	3.5614	2.9786	2.7516	1.9647	2.6686	3.0961
Group 3 (Mixed)	3.7919	2.5363	3.2593	3.5913	3.7698	3.2401	3.1721	2.4768	3.0359	3.4942
Group 4 (Arable)	3.7287	2.4911	3.1481	3.4435	3.6455	3.2387	2.9552	2.5801	2.9860	3.3801

TABLE 29 (b) BRITISH

GEOMETRIC MEAN VALUES OF VARIABLES USED (ACTUAL VALUES)

Farm – Type Group	Gross Product	Rent & Rates	Tenant's Capital			Labour	Seeds & Feed	Ferts. & Lime	Other Purchases	Total Purchases
			Machinery & Equipt.	Other	Total					
	Z	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
	£	£	£	£	£	£	£	£	£	£
Group 1 (Dairy)	4002	229	1011	2254	3370	1060	1469	84	596	2292
Group 2 (Livestock)	2845	193	846	2693	3642	952	564	92	466	1247
Group 3 (Mixed)	6193	344	1817	3902	5884	1738	1486	300	1086	3120
Group 4 (Arable)	5355	310	1406	2776	4421	1733	902	380	968	2400

TABLE 30 IRISH

DISTRIBUTION BY SIZE (ADJUSTED ACRES) AND FARM TYPE OF THE 1139 N.F.S. FARMS

Size (adjusted acres)	Groups 11 & 12 combined	Group 13	Group 20	Group 30	Group 00	All Groups
	Number of Farms					
Less than 5	-	-	-	-	1	1
5 -	9	2	9	6	58	84
15 -	77	16	40	71	78	282
30 -	87	52	44	96	12	291
50 -	80	76	54	53	4	267
100 -	24	36	24	23	-	107
150 -	7	19	18	16	-	60
200 -	4	14	5	3	-	26
250 -	1	2	5	-	-	8
300 -	-	-	1	10	-	11
500 - 1000	-	-	-	2	-	2
Total	289	217	200	280	153	1139

TABLE 31

IRISH

STANDARD DEVIATIONS OF RESIDUAL VARIATIONS FROM REGRESSIONS OF EACH INDEPENDENT VARIABLE ON ALL OTHER INDEPENDENT VARIABLES

"Dependent" Variable	Standard Deviations of Residual Variations				
	Groups 11 & 12 combined	Group 13	Group 20	Group 30	Group 00
4 Independent Variable Case					
x_2	0.2571	0.2247	0.2383	0.2228	0.2669
x_5	0.1423	0.1368	0.1819	0.1360	0.1891
x_6	0.1209	0.1103	0.1245	0.1513	0.1717
x_{10}	0.2459	0.1819	0.1903	0.1863	0.1915
7 Independent Variable Case					
x_3	0.3160	0.2595	0.2983	0.3781	0.4798
x_4	0.1271	0.1419	0.2021	0.1365	0.1910
x_7	0.3071	0.2310	0.2133	0.2961	0.3250
x_8	0.3106	0.1856	0.1903	0.3155	0.3407
x_9	0.1804	0.1243	0.1472	0.1815	0.2274

TABLE 32 IRISH

CORRELATION COEFFICIENTS (r) BETWEEN ALL VARIABLES USED (LOGS)

Groups 11 & 12 combined
(Mainly Dairying & Dairying mixed without cash crops)

	z	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	
x_{10}	0.87	0.48	(0.71)	(0.69)	0.72	0.50	(0.93)	(0.73)	(0.85)	
x_9	0.83	0.66	0.75	0.74	(0.78)	0.50	0.64	0.67		
x_8	0.66	0.36	0.59	0.60	(0.62)	0.40	0.55			
x_7	0.75	0.30	0.56	0.53	(0.55)	0.43			0.78	x_2
x_6	0.67	0.48	0.54	0.68	0.67			0.64	0.75	x_3
x_5	0.92	0.74	(0.83)	(0.99)			0.71	0.80	0.87	x_4
x_4	0.91	0.74	0.77			(0.98)	(0.83)	0.81	0.90	x_5
x_3	0.78	0.57			0.72	0.69	0.62	0.60	0.76	x_6
x_2	0.69			0.59	(0.64)	0.62	0.53	0.56	0.82	x_7
			0.75	0.64	(0.77)	0.72	0.71	0.61	0.86	x_8
		0.85	0.70	0.66	(0.84)	0.78	0.82	0.77	0.89	x_9
	(0.92)	(0.91)	(0.91)	0.69	0.81	(0.77)	(0.74)	0.71	0.93	x_{10}
	x_9	x_8	x_7	x_6	x_5	x_4	x_3	x_2	z	

Group 13
(Dairying mixed with cash crops)

N.B. The figures in brackets relate to pairs of variables which are not used together in any one regression.

Group 20 (Crops mixed)

	z	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_{10}	0.93	0.79	(0.81)	(0.76)	0.83	0.73	(0.94)	(0.92)	(0.96)
x_9	0.89	0.79	0.85	0.71	(0.81)	0.70	0.83	0.85	
x_8	0.90	0.71	0.74	0.76	(0.79)	0.72	0.81		
x_7	0.85	0.69	0.68	0.70	(0.74)	0.66			
x_6	0.84	0.65	0.67	0.77	0.79				
x_5	0.89	0.79	(0.87)	(0.96)					
x_4	0.85	0.73	0.74						
x_3	0.81	0.75							
x_2	0.81								

x_9	x_8	x_7	x_6	x_5	x_4	x_3	x_2	z	
								0.83	x_2
							0.65	0.73	x_3
						0.69	0.86	0.93	x_4
					(0.99)	(0.75)	0.87	0.94	x_5
				0.60	0.60	0.51	0.47	0.67	x_6
			0.51	(0.47)	0.45	0.45	0.39	0.61	x_7
		0.45	0.50	(0.68)	0.67	0.58	0.52	0.73	x_8
	0.68	0.57	0.55	(0.84)	0.80	0.76	0.78	0.86	x_9
(0.93)	(0.75)	(0.77)	0.61	0.83	(0.80)	(0.74)	0.73	0.89	x_{10}

Group 30 (Cattle mixed)

x_9	x_8	x_7	x_6	x_5	x_4	x_3	x_2	z	
								0.29	x_2
							0.29	0.45	x_3
						0.43	0.27	0.74	x_4
					(0.99)	(0.51)	0.29	0.75	x_5
				0.59	0.60	0.29	0.16	0.54	x_6
			0.12	(0.20)	0.18	0.25	-0.01	0.29	x_7
		0.16	0.21	(0.37)	0.35	0.27	0.19	0.39	x_8
	0.41	0.13	0.35	(0.41)	0.38	0.40	0.33	0.49	x_9
(0.64)	(0.51)	(0.76)	0.25	0.38	(0.35)	(0.39)	0.21	0.47	x_{10}

Group 00 (Subsistence)

N.B. The figures in brackets relate to pairs of variables which are not used together in any one regression.

TABLE 33 IRISH

REGRESSION OF LOG G ON LOGS OF X_2, X_5, X_6, X_{10}.

Type of Regression	Type of Variances (Co-variances) used	b_2 (Rent & Rates)	b_5 (Tenant's Capital)	b_6 (Labour)	b_{10} (Total Purchases)	Σb_i	R
GROUPS 11 & 12 COMBINED							
Residuals	s_1^2	0.055	0.413	0.105	0.158	0.7303	0.427
(Standard Errors)		(0.030)	(0.053)	(0.045)	(0.027)	-	-
Within Years	$s_2^2 + s_1^2$	0.056	0.451	0.165	0.320	0.9917	0.950
(Standard Errors)		(0.011)	(0.019)	(0.023)	(0.011)	-	-
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.056	0.435	0.174	0.337	1.0013	0.968
(Standard Errors)		(0.016)	(0.029)	(0.034)	(0.017)	(0.026)	-
Variance Components	s_2^2	0.057	0.424	0.180	0.347	1.0074	0.978
GROUP 13							
Residuals	s_1^2	0.126	0.414	0.060	0.216	0.8170	0.437
(Standard Errors)		(0.034)	(0.065)	(0.064)	(0.040)	-	-
Within Years	$s_2^2 + s_1^2$	0.056	0.312	0.181	0.433	0.9816	0.947
(Standard Errors)		(0.015)	(0.024)	(0.030)	(0.018)	-	-
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.048	0.291	0.190	0.460	0.9888	0.966
(Standard Errors)		(0.022)	(0.036)	(0.044)	(0.027)	(0.031)	-
Variance Components	s_2^2	0.043	0.279	0.195	0.476	0.9928	0.976
GROUP 20							
Residuals	s_1^2	0.111	0.253	-0.055	0.376	0.6842	0.506
(Standard Errors)		(0.030)	(0.044)	(0.056)	(0.042)	-	-
Within Years	$s_2^2 + s_1^2$	0.069	0.193	0.306	0.434	1.0009	0.953
(Standard Errors)		(0.016)	(0.020)	(0.030)	(0.020)	-	-
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.065	0.165	0.364	0.441	1.0343	0.968
(Standard Errors)		(0.024)	(0.032)	(0.046)	(0.030)	(0.030)	-
Variance Components	s_2^2	0.063	0.145	0.402	0.446	1.0559	0.977
GROUP 30							
Residuals	s_1^2	-0.010	0.775	0.018	0.133	0.9155	0.476
(Standard Errors)		(0.039)	(0.068)	(0.060)	(0.035)	-	-
Within Years	$s_2^2 + s_1^2$	0.020	0.505	0.149	0.254	0.9281	0.937
(Standard Errors)		(0.015)	(0.024)	(0.023)	(0.017)	-	-
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.031	0.460	0.162	0.284	0.9366	0.966
(Standard Errors)		(0.020)	(0.033)	(0.030)	(0.024)	(0.022)	-
Variance Components	s_2^2	0.039	0.430	0.168	0.305	0.9413	0.982
GROUP 00							
Residuals	s_1^2	-0.010	0.198	0.142	0.258	0.5885	0.410
(Standard Errors)		(0.012)	(0.050)	(0.059)	(0.045)	-	-
Within Years	$s_2^2 + s_1^2$	0.043	0.370	0.140	0.210	0.7631	0.715
(Standard Errors)		(0.025)	(0.031)	(0.035)	(0.030)	-	-
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.039	0.422	0.117	0.179	0.7576	0.790
(Standard Errors)		(0.035)	(0.049)	(0.054)	(0.048)	(0.059)	-
Variance Components	s_2^2	0.035	0.468	0.094	0.149	0.7456	0.845

TABLE 34 IRISH

ACTUAL VALUES OF LOG GROSS PRODUCT *MINUS* CALCULATED VALUES
FROM WITHIN YEARS REGRESSIONS (CONSTANTS FOR EACH YEAR)

Range of deviations	Groups 11 & 12 combined	Group 13	Group 20	Group 30	Group 00
	Number of farms				
- 1.000 -	2	-	-	-	2
- 0.800 -	10	-	-	-	-
- 0.600 -	13	-	-	1	-
- 0.540 -	9	1	-	1	1
- 0.480 -	12	1	1	1	2
- 0.420 -	26	2	1	2	2
- 0.360 -	35	-	3	8	5
- 0.300 -	43	4	4	6	10
- 0.240 -	50	15	14	23	23
- 0.180 -	51	30	31	52	30
- 0.120 -	82	86	92	108	50
- 0.060 -	79	165	133	179	83
0 -	76	183	168	208	95
+ 0.060 -	102	118	103	174	74
+ 0.120 -	82	39	36	62	56
+ 0.180 -	53	7	7	14	20
+ 0.240 -	44	-	2	1	3
+ 0.300 -	35	-	3	-	2
+ 0.360 -	26	-	2	-	-
+ 0.420 -	14	-	-	-	-
+ 0.480 -	12	-	-	-	-
+ 0.540 -	6	-	-	-	-
+ 0.600 -	2	-	-	-	1
+ 0.660 - + 0.720	3	-	-	-	-
Total negative deviations	412	304	279	381	208
Total positive deviations	455	347	321	459	251
TOTAL	867	651	600	840	459

TABLE 35 IRISH

ACTUAL VALUES OF LOG GROSS PRODUCT *MINUS* CALCULATED VALUES FROM BETWEEN FARMS REGRESSIONS

Range of deviations	Groups 11 & 12 combined	Group 13	Group 20	Group 30	Group 00	All Groups
	Number of farms					
- 0.600 -	-	-	-	-	1	1
- 0.550 -	-	-	-	-	-	-
- 0.500 -	-	-	-	-	-	-
- 0.450 -	-	-	-	-	-	-
- 0.400 -	-	-	-	-	1	1
- 0.350 -	-	2	-	-	-	2
- 0.300 -	-	-	-	2	-	2
- 0.250 -	3	-	3	-	4	10
- 0.200 -	2	5	4	9	3	23
- 0.150 -	13	7	17	14	16	67
- 0.100 -	43	36	22	41	15	157
- 0.050 -	79	51	49	59	39	277
0 -	92	66	56	78	21	313
+ 0.050 -	38	39	35	59	28	199
+ 0.100 -	14	8	10	18	15	65
+ 0.150 -	3	3	1	-	7	14
+ 0.200 -	2	-	2	-	3	7
+ 0.250 -	-	-	-	-	-	-
+ 0.300 - + 0.324	-	-	1	-	-	1
Total negative deviations	140	101	95	125	79	540
Total positive deviations	149	116	105	155	74	599
TOTAL	289	217	200	280	153	1139

TABLE 36 IRISH

STANDARD DEVIATIONS s_1 AND s_2

4 INDEPENDENT VARIABLE CASE

	Groups 11 & 12 combined		Group 13		Group 20		Group 30		Group 00	
	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1
1st Estimate	0.057	0.066	0.060	0.067	0.069	0.070	0.054	0.089	0.091	0.112
2nd Estimate	0.058	0.064	0.061	0.064	0.071	0.066	0.056	0.086	0.093	0.110

TABLE 37 IRISH

"MARGINAL PRODUCTIVITIES" WITH ALL VARIABLES AT GEOMETRIC MEAN VALUES (4 INDEPENDENT VARIABLES)

Type of regression used	X_2 (Rent & Rates)	X_5 (Total Tenant's Capital)	X_6 (Labour)	X_{10} (Total Purchases)
	£	£	£	£
GROUPS 11 & 12 COMBINED (Mainly Dairy and Dairying mixed without cash crops)				
Within Years	1.296	0.356	0.267	1.332
Between Farms	1.298	0.343	0.282	1.402
Variance Components	1.310	0.334	0.293	1.443
GROUP 13 (Dairying mixed with cash crops)				
Within Years	1.109	0.268	0.434	1.409
Between Farms	0.942	0.251	0.457	1.495
Variance Components	0.846	0.240	0.469	1.547
GROUP 20 (Crops mixed)				
Within Years	1.268	0.174	0.651	1.385
Between Farms	1.199	0.149	0.774	1.407
Variance Components	1.169	0.131	0.855	1.423
GROUP 30 (Cattle mixed)				
Within Years	0.281	0.304	0.225	1.251
Between Farms	0.441	0.277	0.244	1.399
Varience Components	0.547	0.258	0.254	1.503
GROUP 00 (Subsistence)				
Within Years	0.836	0.283	0.106	0.940
Betweens Farms	0.754	0.323	0.089	0.802
Variance Components	0.681	0.358	0.071	0.668

TABLE 33

ACTUAL AVERAGE COMBINATIONS OF INPUTS COMPARED WITH LEAST COST COMBINATIONS GIVING THE SAME AVERAGE TOTAL COSTS

	Estimated & Actual Gross Product Z	Rent & Rates X_2	Total Tenant's Capital X_5	Labour X_6	Total Purchases X_{10}	Total Costs (including 9% interest on Tenant's Capital)
	£	£	£	£	£	£
GROUPS 11 & 12 COMBINED						
Least Cost Combination	1876	55	4576	175	337	979
Actual average (arithmetic means)	999	56	1296	529	277	979
GROUP 13						
Least Cost Combination	2813	72	5180	326	796	1660
Actual average (arithmetic means)	1930	116	2263	699	641	1660
GROUP 20						
Least Cost Combination	2434	88	2228	555	616	1460
Actual Average (arithmetic means)	1597	106	1948	637	542	1460
GROUP 30						
Least Cost Combination	1612	39	4826	171	308	952
Actual average (arithmetic means)	914	94	1685	500	206	952
GROUP 00						
Least Cost Combination	630	19	2746	49	79	394
Actual average (arithmetic means)	225	13	308	302	51	394

TABLE 39 IRISH

REGRESSION OF LOG G ON LOGS OF X_2, X_3, X_4, X_6, X_7, X_8, X_9

Type of Regression	Type of Variances (Co-variances) used	b_2 (Rent & Rates)	b_3 Tenant's Capital (Machinery & Equipt.)	b_4 Tenant's Capital (Other)	b_6 (Labour)	b_7 (Seeds & Feed)	b_8 (Fertilisers & Lime)	b_9 (Other Purchases)	Σb_i	R
GROUPS 11 & 12										
Residuals	s_1^2	0.053	-0.040	0.495	0.105	0.095	0.008	0.034	0.7492	0.482
Within years	$s_2^2 + s_1^2$	0.047	-0.002	0.518	0.118	0.194	0.011	0.110	0.9963	0.953
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.042	-0.005	0.511	0.118	0.203	0.006	0.128	1.0023	0.971
(Standard Errors)		(0.017)	(0.012)	(0.031)	(0.033)	(0.013)	(0.013)	(0.022)	(0.026)	-
Variance Components	s_2^2	0.035	-0.008	0.509	0.119	0.207	-0.002	0.146	1.0061	0.980
GROUP 13										
Residuals	s_1^2	0.132	-0.001	0.420	0.058	0.100	0.012	0.074	0.7947	0.456
Within Years	$s_2^2 + s_1^2$	0.058	0.013	0.318	0.194	0.157	0.102	0.155	0.9966	0.946
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.051	0.015	0.293	0.187	0.203	0.110	0.140	0.9985	0.968
(Standard Errors)		(0.023)	(0.019)	(0.034)	(0.044)	(0.021)	(0.026)	(0.039)	(0.032)	-
Variance Components	s_2^2	0.051	0.013	0.279	0.189	0.206	0.131	0.130	0.9981	0.978
GROUP 20										
Residuals	s_1^2	0.101	-0.011	0.241	-0.026	0.131	0.086	0.134	0.6547	0.524
Within Years	$s_2^2 + s_1^2$	0.082	0.013	0.162	0.296	0.132	0.162	0.140	0.9863	0.955
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.082	0.011	0.133	0.346	0.125	0.184	0.134	1.0153	0.970
(Standard Errors)		(0.024)	(0.019)	(0.028)	(0.045)	(0.027)	(0.030)	(0.039)	(0.030)	-
Variance Components	s_2^2	0.083	0.009	0.113	0.378	0.118	0.203	0.129	1.0336	0.978
GROUP 30										
Residuals	s_1^2	0.001	-0.009	0.685	0.030	0.079	0.018	0.013	0.8158	0.481
Within Years	$s_2^2 + s_1^2$	0.035	0.014	0.516	0.116	0.114	0.048	0.087	0.9290	0.939
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.044	0.011	0.469	0.123	0.118	0.062	0.106	0.9335	0.968
(Standard Errors)		(0.021)	(0.012)	(0.032)	(0.030)	(0.015)	(0.014)	(0.024)	(0.023)	-
Variance Components	s_2^2	0.052	0.009	0.432	0.128	0.118	0.077	0.119	0.9350	0.985
GROUP 00										
Residuals	s_1^2	-0.038	0.013	0.158	0.159	0.094	0.041	0.083	0.5098	0.378
Within Years	$s_2^2 + s_1^2$	0.038	0.026	0.314	0.131	0.080	0.032	0.106	0.7261	0.718
Between Farms	$s_2^2 + \frac{1}{3} s_1^2$	0.032	0.022	0.369	0.099	0.072	0.030	0.108	0.7331	0.798
(Standard Errors)		(0.036)	(0.019)	(0.049)	(0.054)	(0.029)	(0.027)	(0.041)	(0.058)	-
Variance Components	s_2^2	0.027	0.018	0.419	0.068	0.064	0.029	0.108	0.7329	0.856

TABLE 40 IRISH

A COMPARISON BETWEEN REGRESSION COEFFICIENTS FROM VARIANCE COMPONENTS REGRESSIONS IN 4 & 7 INDEPENDENT VARIABLE CASES

Type of Regression	$(b_2 + b_6)$	b_5 or $(b_3 + b_4)$	b_{10} or $(b_7 + b_8 + b_9)$	Σb_i
GROUPS 11 & 12 COMBINED				
4 Independent Variable case	0.237	0.424	0.347	1.007
7 Independent Variable case	0.154	0.501	0.351	1.006
GROUP 13				
4 Independent Variable case	0.238	0.279	0.476	0.993
7 Independent Variable case	0.240	0.292	0.467	0.998
GROUP 20				
4 Independent Variable case	0.465	0.145	0.446	1.056
7 Independent Variable case	0.461	0.122	0.450	1.034
GROUP 30				
4 Independent Variable case	0.207	0.430	0.305	0.941
7 Independent Variable case	0.180	0.441	0.314	0.935
GROUP 00				
4 Independent Variable case	0.129	0.468	0.149	0.746
7 Independent Variable case	0.095	0.437	0.201	0.733

TABLE 41 IRISH

STANDARD DEVIATIONS s_1 AND s_2

7 INDEPENDENT VARIABLE CASE

	Groups 11 & 12 combined		Group 13		Group 20		Group 30		Group 00	
	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1
1st Estimate	0.054	0.065	0.056	0.072	0.066	0.070	0.051	0.089	0.088	0.114
2nd Estimate	0.056	0.062	0.059	0.064	0.069	0.066	0.053	0.086	0.092	0.112

"MARGINAL PRODUCTIVITIES" WITH ALL VARIABLES AT GEOMETRIC MEAN VALUES
(7 INDEPENDENT VARIABLES)

Type of Regression	X_2 (Rent & Rates)	X_3 Tenant's Capital (Machinery & equipt.)	X_4 Tenant's Capital (Other)	X_6 (Labour)	X_7 (Seeds & Feed)	X_8 (Ferts. & Lime)	X_9 (Other Purchases)
	£	£	£	£	£	£	£
GROUPS 11 & 12 COMBINED (Mainly Dairy and Dairying mixed without cash crops)							
Within Years	1.088	- ve	0.461	0.192	1.643	0.418	1.597
Between Farms	0.964	- ve	0.455	0.192	1.715	0.213	1.867
Variance Components	0.826	- ve	0.453	0.193	1.750	- ve	2.120
GROUP 13 (Dairying with cash crops)							
Within Years	1.134	0.072	0.341	0.467	1.410	1.617	1.373
Between Farms	1.001	0.081	0.315	0.449	1.824	1.742	1.241
Variance Components	0.993	0.072	0.299	0.455	1.850	2.086	1.148
GROUP 20 (Crops mixed)							
Within Years	1.511	0.067	0.192	0.630	1.352	2.111	1.163
Between Farms	1.511	0.057	0.158	0.737	1.275	2.408	1.119
Variance Components	1.533	0.048	0.134	0.806	1.206	2.656	1.078
GROUP 30 (Cattle mixed)							
Within Years	0.497	0.125	0.344	0.175	1.821	1.334	1.106
Between Farms	0.623	0.104	0.313	0.186	1.880	1.740	1.349
Variance Components	0.742	0.081	0.288	0.193	1.887	2.150	1.517
GROUP 00 (Subsistence)							
Within Years	0.727	0.606	0.256	0.099	0.816	1.312	1.537
Between Farms	0.629	0.496	0.301	0.075	0.740	1.227	1.568
Variance Components	0.520	0.411	0.343	0.052	0.660	1.190	1.560

ACTUAL AVERAGE COMBINATIONS OF INPUTS COMPARED WITH LEAST COST COMBINATIONS GIVING THE SAME AVERAGE TOTAL COSTS

	Estimated and Actual Gross Product	Rent & Rates	Tenant's Capital			Labour	Seeds & Feed	Ferts. & Lime	Other Purchases	Total Purchases	"Total Costs"*
			Machinery & Equipt.	Other	Total						
	Z	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	
GROUPS 11 & 12 COMBINED	£	£	£	£	£	£	£	£	£	£	£
Least Cost Combination	2637	35	-	5447	5447	114	199	-	141	340	979
Actual average (arithmetic means)	999	56	172	1124	1296	529	159	38	80	271	979
GROUP 13											
Least Cost Combination	3065	84	238	5150	5388	314	342	219	216	777	1660
Actual average (arithmetic means)	1930	116	479	1784	2263	699	252	147	242	641	1660
GROUP 20											
Least Cost Combination	2584	117	141	1769	1910	534	167	287	183	637	1460
Actual average (arithmetic means)	1597	106	482	1466	1948	637	184	141	217	542	1460
GROUP 30											
Least Cost Combination	1779	54	101	4886	4987	130	120	78	121	319	952
Actual average (arithmetic means)	914	94	191	1494	1685	500	68	45	93	206	952
GROUP 00											
Least Cost Combination	658	14	107	2502	2609	36	35	16	58	109	394
Actual average (arithmetic means)	225	13	17	291	308	302	26	8	17	51	394

* "Total Costs" = $X_2 + X_6 + X_{10}$ + 9% interest on X_5

TABLE 44 (a)

GEOMETRIC MEAN VALUES OF VARIABLES USED (LOGS)

Farm—Type Group	Gross Product	Rent & Rates	Tenant's Capital			Labour	Seeds & Feed	Ferts. & Lime	Other Purchases	Total Purchases
			Machinery & Equipt.	Other	Total					
	z	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Groups 11 & 12 combined (Mainly Dairying, and Dairying Mixed without Cash Crops)	2.9020	1.5391	1.9234	2.9526	3.0050	2.6910	1.9746	1.3376	1.7394	2.2829
Group 13 (Dairying Mixed with Cash Crops)	3.1906	1.8977	2.4442	3.1597	3.2554	2.8096	2.2365	1.9898	2.2438	2.6782
Group 20 (Crops Mixed)	3.0798	1.8138	2.3562	3.0055	3.1246	2.7518	2.0703	1.9635	2.1590	2.5757
Group 30 (Cattle Mixed)	2.8260	1.6759	1.8661	3.0023	3.0467	2.6473	1.6227	1.3794	1.7209	2.1329
Group 00 (Subsistence)	2.3102	1.0237	0.9491	2.3988	2.4267	2.4314	1.2987	0.7000	1.1490	1.6587

TABLE 44 (b)

GEOMETRIC MEAN VALUES OF VARIABLES USED (ACTUAL VALUES)

Farm —Type Group	Gross Product	Rent & Rates	Tenant's Capital			Labour	Seeds & Feed	Ferts. & Lime	Other Purchases	Total Purchases
			Machinery & Equipt.	Other	Total					
	Z	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
	£	£	£	£	£	£	£	£	£	£
Groups 11 & 12 combined (Mainly Dairying, and Dairying Mixed without Cash Crops)	798	35	84	897	1012	491	94	22	55	192
Group 13 (Dairying Mixed with Cash Crops)	1551	79	278	1444	1801	645	172	98	175	477
Group 20 (Crops Mixed)	1201	65	227	1013	1332	565	118	92	144	376
Group 30 (Cattle Mixed)	670	47	73	1006	1114	444	42	24	53	136
Group 00 (Subsistence)	204	11	9	251	267	270	20	5	14	46

FIGURE 1

BRITISH

"MARGINAL PRODUCTIVITIES" AT LEAST COST COMBINATIONS

GROUP 1

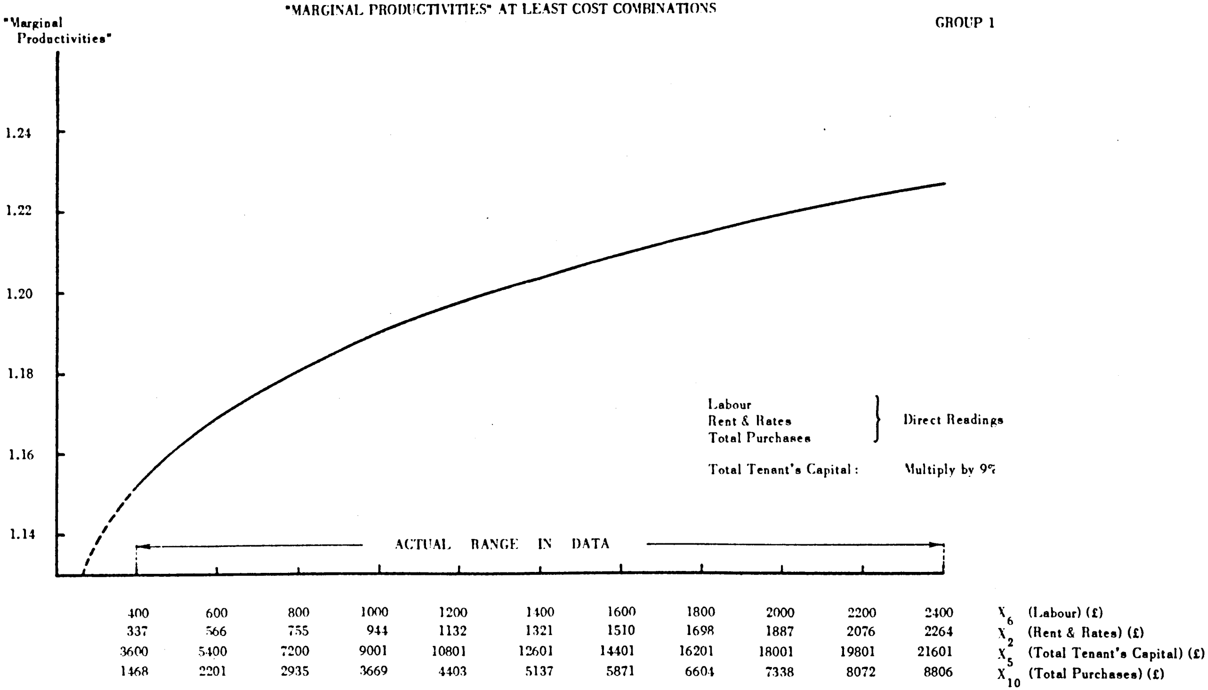


FIGURE 2

BRITISH

"MARGINAL PRODUCTIVITIES" AT LEAST COST COMBINATIONS

GROUP 2

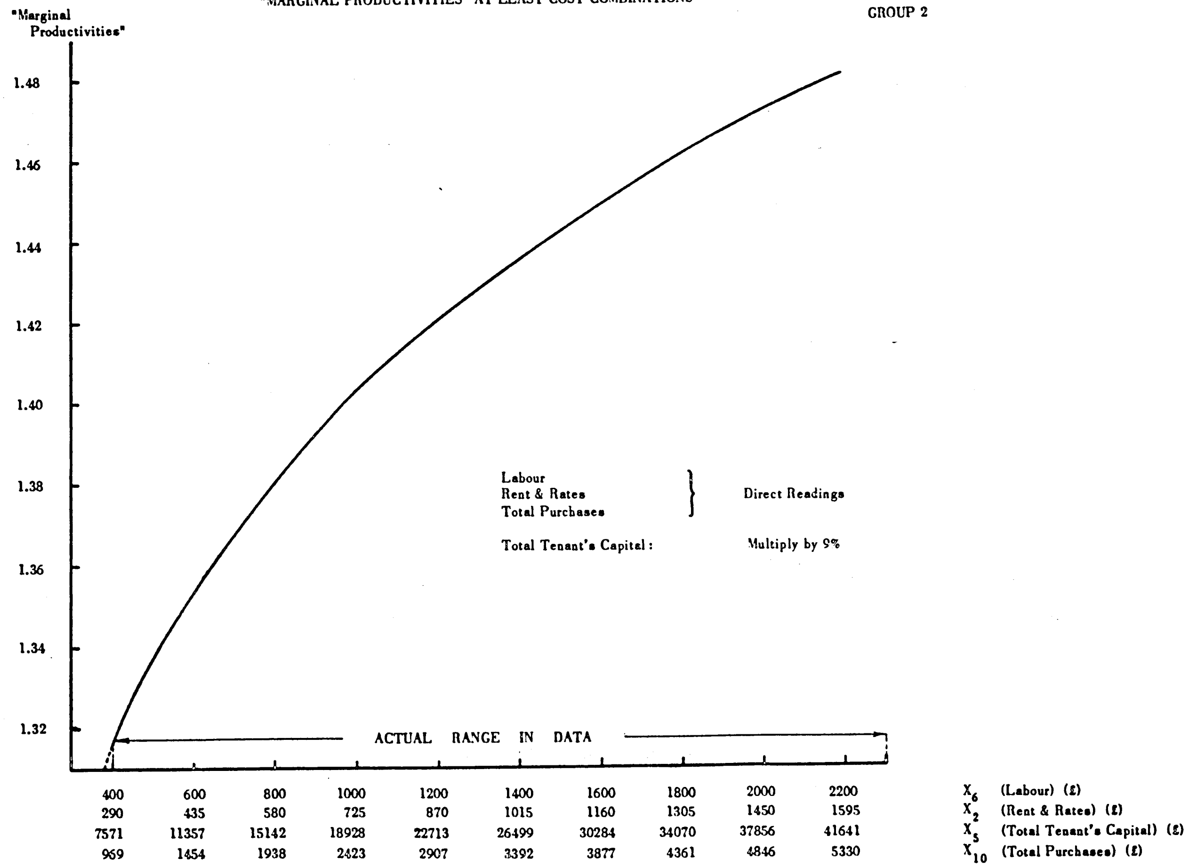


FIGURE 3

BRITISH

GROUP 3

"MARGINAL PRODUCTIVITIES" AT LEAST COST COMBINATIONS

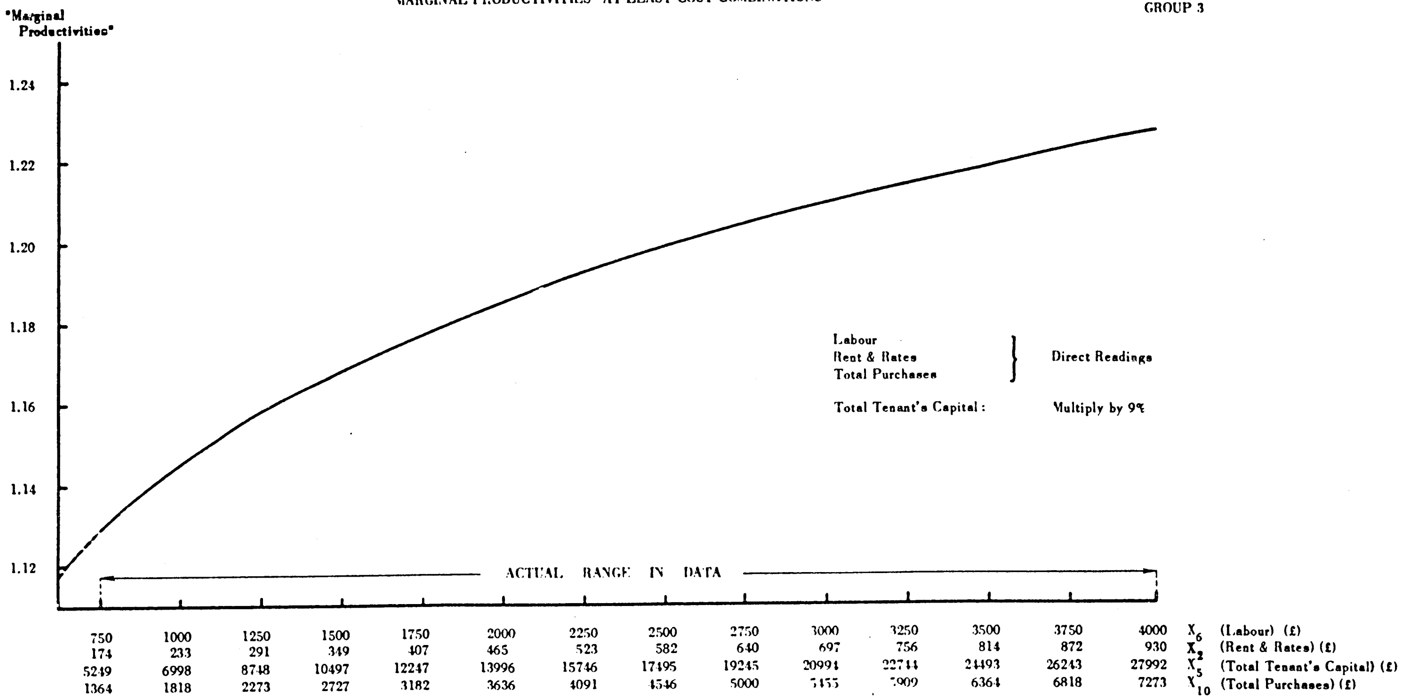
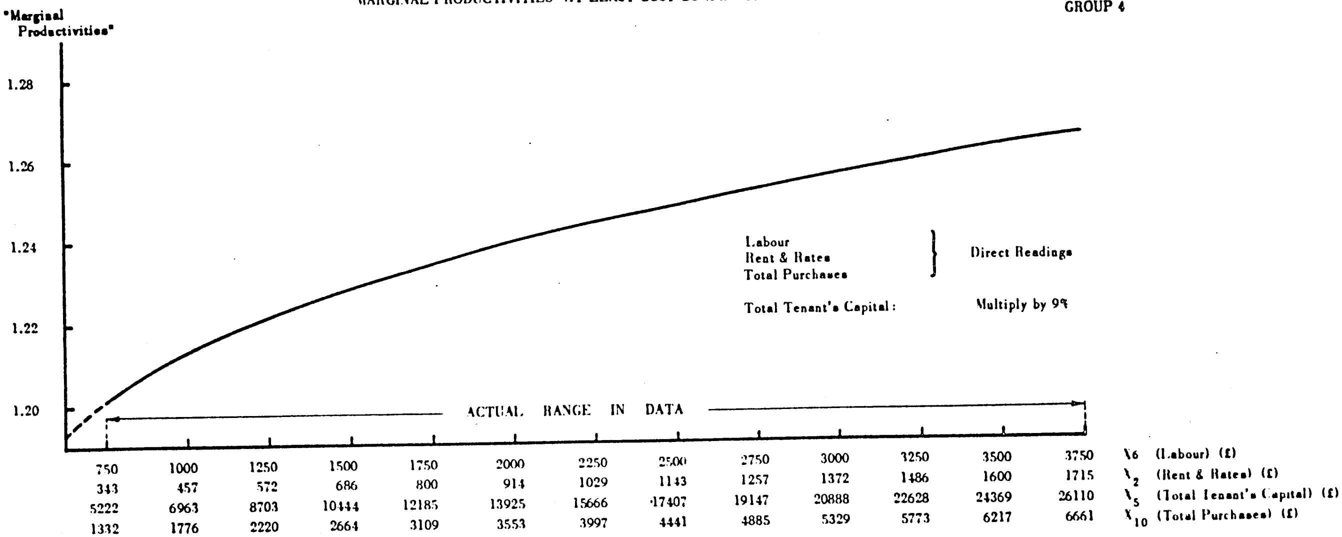


FIGURE 4

BRITISH

GROUP 4

"MARGINAL PRODUCTIVITIES" AT LEAST COST COMBINATIONS



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