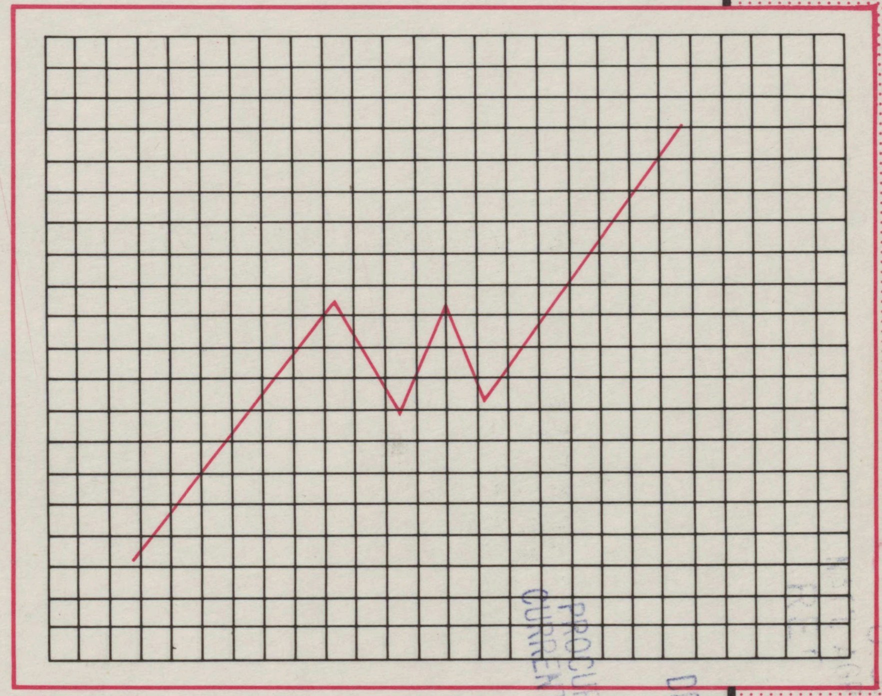



# AGREKON

FOUR-MONTHLY JOURNAL
ON AGRICULTURAL ECONOMICS

Vol. 27 No. 1
FEBRUARY 1988

Price R2,67
(plus GST)

# OPTIMUM RESOURCE ALLOCATION: ESTIMATES FOR FRESH-MILK PRODUCERS IN NATAL*

by R.J. GORDIJN and G.F. ORTMANN**

## ABSTRACT

The allocation of some of the more important inputs in the production of fresh milk in three regions of Natal is assessed with the use of production functions. The marginal rate of substitution criteria is used to establish least cost combinations of concentrates and pastures in East Griqualand and in the Natal Midlands, and concentrates and forages in the Ixopo area. Estimates of profit-maximizing combinations of inputs are also presented for the three regions. The results indicate that fresh-milk farmers need to reassess their combinations of inputs to improve profits.

## INTRODUCTION

Gordijn and Ortmann (1986) reported that fresh-milk farmers in Natal need to reallocate their resources if economic optima are to be achieved. In this study, this finding is further analysed on the assumption that the farmer is a profit maximizer, utilizing deterministic models with no restrictions on capital and knowledge. Under these circumstances it is possible to use marginal rates of substitution to determine least cost combinations of feed inputs and profit-maximizing combinations of resources. It is recognized that in his environment the farmer is a rational decision-maker and that the difference between his use of resources and that which economic theory suggests can be attributed to factors such as risk, imperfect knowledge and limited capital. This study is intended to be an indication of the extent to which actual resource allocation deviates from static economic optima.

Since feed is the most important input in milk production, the marginal rate of substitution of concentrates for pasture (forage) is analysed for three areas of Natal, namely East Griqualand, the Natal Midlands and Ixopo. The profit-maximizing combinations of resources are also estimated for minimum, average and maximum sample herd sizes. Cobb-Douglas (double-log) production functions were developed, using cross-sectional and time series data extracted from the Mail-In Record Scheme of the Department of Agriculture and Water Supply, Directorate of Agricultural Production Economics. The Mail-In Record Scheme provided information for the five years from 1978/79 to 1982/83 on farmers who had kept the records necessary for the analysis.

The results, although applicable to this selected group of farmers, might have wider implications. Prices (costs) were based on the period 1982/83.

## PRODUCTION FUNCTIONS

The derived production functions used in this analysis are summarized in Table 1.

**TABLE 1 - Cobb-Douglas production functions for three regions in Natal, 1978/79 to 1982/83. (The dependent variable is the total litres of milk per farm per annum)**

| Variable | Region | | |
|---|---|---|---|
| | East Griqualand | Natal Midlands | Ixopo |
| Constant | 441,795 | 312,204 | 195,091 |
| Cows | 0,4514** | 0,6086** | 0,5805** |
| | (6,214) | (12,569) | (8,795) |
| Concentrates | 0,2227** | 0,3434** | 0,1896** |
| | (6,916) | (13,677) | (4,339) |
| Pastures | 0,0290* | 0,0461** | - |
| | (2,325) | (4,845) | |
| Forage | - | - | 0,2053** |
| | | | (4,241) |
| Labour | 0,1482** | 0,0238 | 0,1077* |
| | (4,221) | (1,134) | (2,125) |
| Veterinary | 0,0947** | - | - |
| | (3,247) | | |
| $\Sigma b_i$ | 0,9460 | 1,0219 | 1,0831 |
| | (-0,718) | (0,7604) | (1,6687) |
| $\bar{R}^2$ | 0,86 | 0,91 | 0,93 |
| df | 83 | 190 | 40 |

* = significant at the 5% level
** = significant at the 1% level
$\bar{R}^2$ = adjusted multiple correlation coefficient
df = degrees of freedom
t-statistic in parentheses

Table 1 shows the estimated elasticities of production of inputs significant at either the 5% or the 1% level. Cows show the highest elasticity of production of all inputs, the coefficient varying from 0,45 in East Griqualand to 0,61 in the Natal Midlands.

The sum of coefficients was higher than unity for the Natal Midlands and Ixopo and less than one for East Griqualand. However, t-tests indicated that the deviations of the sum of coefficients from unity were not statistically significant for the three areas (t = -0,718 for East Griqualand, t = 0,7604 for Natal Midlands and t = 1,6687 for Ixopo).

## LEAST COST FEED ALLOCATIONS

The least cost criteria used below do not necessarily imply a profit-maximizing combination of inputs. To

*Results are based on R.J. Gordijn's M.Sc. (Agric.) thesis entitled "A Production Function Analysis of Fresh Milk Production in Natal"

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Article submitted: October 1986
Article received back from authors: August 1987

determine a profit-maximizing combination, the value of the marginal product of each input (VMPx) must equal the unit price of that input (Px) and this equality must be satisfied simultaneously for all inputs (Doll and Orazen, 1978, p. 123). The least cost combination of feed inputs is estimated by using marginal rates of substitution.

The transformed power functions used to calculate isoquants for fresh milk in three areas of Natal are given below. In conjunction with these, the marginal rates of substitution (MRS) were determined for points along the isoquants.

Transformed function 1: (East Griqualand)

$$\text{Past} = (M/\text{conc}^{0,2227}\,K_1))^{1/0,0290}$$

Transformed function 2: (Natal Midlands)

$$\text{Past} = (M/(\text{conc}^{0,3434}\,K_2))^{1/0,0461}$$

Transformed function 3: (Ixopo)

$$\text{Forage} = (M/(\text{conc}^{0,1896}\,K_3))^{1/0,2053}$$

Where:
$K_1 = 441{,}975\ \text{Vet}^{0,0947}\ \text{Lab}^{0,1482}\ \text{Cows}^{0,4514}$

$K_2 = 312{,}204\ \text{Lab}^{0,0238}\ \text{Cows}^{0,6086}$

$K_3 = 195{,}091\ \text{Lab}^{0,1077}\ \text{Cows}^{0,5805}$

M = milk yield per annum

Past = pastures

conc = concentrates

Vet = veterinary

Lab = labour

Cows = number of cows

The isoquants derived are asymptotic to the input axis and therefore show decreasing rates of substitution. Having calculated the MRS, the least cost combination of inputs can be determined. Any isoquant comprises an infinite number of points and only one represents the minimum cost combination of inputs. Minimum cost is achieved where the MRS equals the inverse price ratio i.e.

$$MRS_{x2\ \text{for}\ x1} = \frac{-P_{x2}}{P_{x1}}$$

In the following sections the marginal rates of substitution and least cost criteria are discussed for the East Griqualand, Natal Midlands and Ixopo areas.

## East Griqualand

The isoquants and marginal rates of substitution between pastures and concentrates given in Table 2 are determined for milk yields 10% above and 10% below the geometric mean yield. The geometric mean yield is taken as the total annual milk production for the geometric mean herd in the sample. The isoquants are determined while keeping all other factors constant at their geometric means.

To calculate the MRS of concentrates for pastures, the following formula is used:

$$\text{MRS of concentrates for pastures} = \frac{dx_1}{dx_2} = \frac{-b_2x_1}{b_1x_2}$$

$$= -7{,}6793\ \frac{\text{pastures}}{\text{concentrates}}$$

From Table 2 the least cost combination of concentrates and pastures can be assessed for the various milk yields while other factors are kept constant. Since the unit price/cost of concentrates and pastures are both equal to one rand, the least cost criterion is satisfied where MRS = 1. At the mean milk production of 363 194 litres per annum, the least cost combination occurs when R34 120 and R4 443 per annum are spent on concentrates and pastures respectively. The geometric (actual) means of concentrates and pastures are R35 134 and R3 321 respectively. Since the difference between actual and least cost combinations is insubstantial, farmers in East Griqualand appear to be allocating concentrates and pastures on a least cost basis. The overall increased cost of R108 may be the result of rounding errors in the equations used.

The isoquants calculated in Table 2 are illustrated in Figure 1. The line OE is the expansion path. A rational and efficient farmer wishing to increase his production would move from point B to point C on the expansion path. To deviate from the expansion path would not be cost effective.

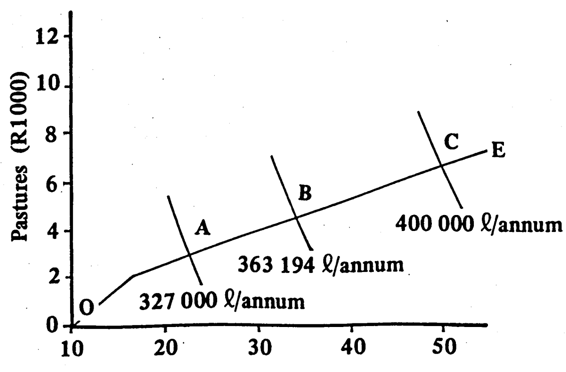


**FIG. 1 - Expansion path OE and isoquants for three milk yields for the East Griqualand area, 1978/79 to 1982/83**

**TABLE 2 - Marginal rates of substitution for three milk yields for the East Griqualand area, 1978/79 to 1982/83**

| 327 000 ℓ/annum | | | 363 194 ℓ/annum | | | 400 000 ℓ/annum | | |
|---|---|---|---|---|---|---|---|---|
| Concentrates R | Pastures R | MRS | Concentrates R | Pastures R | MRS | Concentrates R | Pastures R | MRS |
| 20 000 | 7 195 | 2,763 | 32 000 | 7 272 | 1,745 | 47 000 | 10 597 | 1,731 |
| 21 000 | 4 947 | 1,809 | 33 000 | 5 742 | 1,336 | 48 500 | 8 325 | 0,172 |
| 22 485 | 2 927 | 1,000 | 34 120 | 4 443 | 1,000 | 50 070 | 6 518 | 1,000 |
| 23 500 | 2 086 | 0,682 | 35 000 | 3 654 | 0,802 | 51 000 | 5 659 | 1,853 |
| 24 000 | 1 774 | 0,568 | 35 500 | 3 277 | 0,709 | 52 000 | 4 875 | 1,720 |

## Natal Midlands

The marginal rates of substitution (MRS) of concentrates for pastures for three annual milk yields are given in Table 3, keeping the other factors constant at their geometric means. The MRS figures are calculated for the geometric mean milk yield, and for milk yields 10% above and 10% below the mean milk yield. From the VMPs (see Gordijn, 1985, p. 61) it is evident that the use of concentrates and pastures is such that a reallocation of these two inputs would benefit the farmer, *ceteris paribus.*

The geometric means of concentrates and pastures are R49 767 and R8 437 respectively. The geometric mean milk yield is 429 328 litres per annum. At the mean milk yield the least cost combination is where R51 065 is spent on concentrates and R6 858 on pastures. This means a reduction of R1 579 on pastures and an increase of R1 298 on concentrates or a gain of R281. The net gain is small since the use of concentrates and pastures is nearly optimal. The above result is contrary to the general belief that more farm-produced feeds should be used relative to concentrates.

The isoquants are given in Figure 2. The line OE is the expansion path. A movement along this line would ensure least cost combinations of concentrates and pastures. A farmer wishing to increase production by 10% from the geometric mean would move from point B to point C.

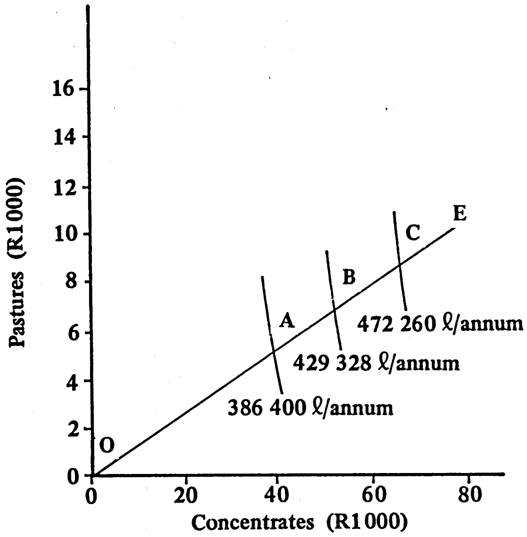


**FIG. 2 - Expansion path OE and isoquants for three different milk yields for the Natal Midlands, 1978/79 to 1982/83**

## Ixopo

Gordijn (1985, p. 73) showed that, in accordance with economic criteria, there was an over-utilization of concentrates and an under-utilization of forages in this region. Table 4 gives the least cost combinations of concentrates and forages in producing specified levels of fresh milk. It is assumed that the fresh-milk producer can increase production by 10% from the geometric mean without increasing the amount of labour used and the number of cows. The forages variable includes maize silage, pastures (kikuyu, Italian rye-grass and eragrostis hay) and, to a lesser extent, green feed (Japanese radish).

For 672 218 litres per annum the optimum feeding strategy occurs when concentrate expenditure is reduced from R71 987 to R40 480 and forage expenditure is increased from R25 848 to R43 825. This involves a decrease of 44% in concentrate expenditure and an increase of 69% in forage expenditure. At the least cost combination of these two feeds, this will constitute a net gain of R13 530 in the production of the mean milk yield.

The isoquants of the three milk yields are given in Figure 3. The isocline OE is the expansion path. If the fresh-milk producer is to expand his operation to achieve a higher yield, he would have to move along the expansion path to give him the least cost combination of inputs, while keeping other factors constant.

## PROFIT-MAXIMIZING COMBINATIONS OF INPUTS

To produce any output at minimum cost, the ratio

**TABLE 3 - Marginal rates of substitution for three milk yields in the Natal Midlands, 1978/79 to 1982/83**

| 386 400 ℓ/annum | | | 429 328 ℓ/annum | | | 472 260 ℓ/annum | | |
|---|---|---|---|---|---|---|---|---|
| Concentrates R | Pastures R | MRS | Concentrates R | Pastures R | MRS | Concentrates R | Pastures R | MRS |
| 35 000 | 11 635 | 2,475 | 46 000 | 14 932 | 2,415 | 62 000 | 12 774 | 1,534 |
| 36 000 | 9 433 | 1,953 | 47 000 | 12 721 | 2,016 | 62 500 | 12 032 | 1,435 |
| 37 000 | 7 691 | 1,548 | 49 000 | 9 327 | 1,418 | 63 000 | 11 339 | 1,340 |
| 38 965 | 5 231 | 1,000 | 51 065 | 6 858 | 1,000 | 65 225 | 8 756 | 1,000 |
| 40 000 | 4 303 | 0,801 | 52 000 | 5 991 | 0,858 | 66 000 | 8 018 | 0,905 |
| 42 000 | 2 992 | 0,531 | - | - | - | - | - | - |

**TABLE 4 - Marginal rates of substitution of concentrates for forages for three milk production levels in the Ixopo area, 1978/79 to 1982/83**

| 604 996 ℓ/annum | | | 672 218 ℓ/annum | | | 739 440 ℓ/annum | | |
|---|---|---|---|---|---|---|---|---|
| Concentrates R | Forage R | MRS | Concentrates R | Forage R | MRS | Concentrates R | Forage R | MRS |
| 27 500 | 38 208 | 1,280 | 38 000 | 47 410 | 1,151 | 50 000 | 58 601 | 1,081 |
| 28 000 | 37 579 | 1,238 | 39 500 | 45 748 | 1,067 | 51 500 | 57 026 | 1,020 |
| 31 280 | 33 930 | 1,000 | 40 880 | 44 322 | 1,000 | 52 050 | 56 470 | 1,000 |
| 32 000 | 33 226 | 0,957 | 42 000 | 43 232 | 0,952 | 53 000 | 55 536 | 0,966 |
| 33 000 | 32 296 | 0,902 | 42 500 | 42 762 | 0,928 | - | - | - |

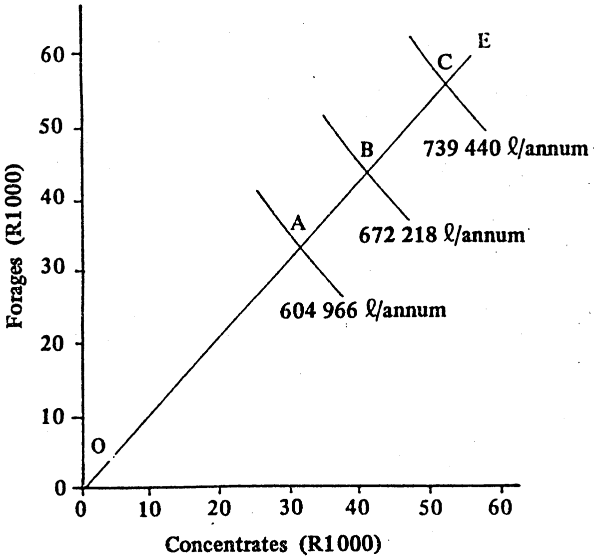


**FIG. 3 - Expansion path OE and isoquants for three milk yields in the Ixopo area of Natal, 1978/79 to 1982/83**

of the marginal product of an input to the price of that input must be equal for all inputs. However, if the farmer is also concerned with maximizing profits, the ratio of the value of the marginal product of each input (VMPx) to the unit cost of that input (Px) should equal unity and this must be satisfied simultaneously for all inputs, i.e.

$$\frac{VMP_{x1}}{P_{x1}} = \frac{VMP_{x2}}{P_{x2}} = \frac{VMP_{xn}}{P_{xn}} = 1$$

To calculate profit-maximizing combinations of resources, a necessary condition is that the returns to scale be constant or decreasing. In all three regions t-tests indicated constant returns to scale.

Profit-maximizing combinations of inputs for the East Griqualand, Natal Midlands and Ixopo areas of Natal are presented below:

### East Griqualand

Actual and profit-maximizing combinations of inputs for various herd sizes in East Griqualand are given in Table 5. The annual cost of a dairy cow is taken as the rent value of a cow plus a 4% mortality factor. For the purposes of this analysis, a value of 14% of the market value of a cow is taken to represent costs in each region (Gordijn and Ortmann, 1986).

Table 5 summarizes the actual, mean combination of inputs and the profit-maximizing combinations of inputs for the mean, the smallest and the largest cow herd in the sample.

From Table 5 it is evident that the average fresh-milk producer participating in the East Griqualand Mail-In Record scheme should improve his yield from 363 194 litres per annum to 528 318 litres per annum to maximize profits, i.e. an increase of 45%. This may not be genetically possible with the existing cows in the short term. The increase in total expenditure is estimated at 54% and the increase in total margin at 35%. The fact that the margin to total cost ratio for the actual case (86%) is higher than for the profit-maximizing combination (76% on average) may indicate that farmers are rational in what they do owing to the risk involved.

Table 5 shows that all profit-maximizing combinations have similar margins per litre, while the margin per cow and margin per hectare of feed crop change substantially. If the number of cows is the most limiting factor then comparisons among herds should be based on this limiting resource. In the above situation a small high-producing herd shows the highest margin per cow and per hectare of feed crop.

### Natal Midlands

The profit-maximizing combinations of inputs for three different situations in the Natal Midlands, namely the geometric mean herd size, the minimum herd size and the maximum herd size, are given in Table 6. The actual input combinations are also given.

The estimated figures indicate the importance of comparing profits on the basis of the most limiting factor, as indicated previously for East Griqualand. A comparison of the profit-maximizing

**TABLE 5 - Actual and profit-maximizing combinations of inputs for dairy farms in East Griqualand, 1978/79 to 1982/83**

| Input/output | Actual combination (mean herd) | Profit-maximizing combination | | |
|---|---|---|---|---|
| | | Mean herd | Minimum herd | Maximum herd |
| Cows | 103 | 103 | 51 | 176 |
| ℓ/annum | 363 194 | 528 318 | 282 000 | 852 522 |
| Concentrates (R) | 35 134 | 36 438 | 19 449 | 58 799 |
| Veterinary (R) | 2 165 | 15 494 | 8 271 | 25 003 |
| Pastures (R) | 3 321 | 4 745 | 2 533 | 7 657 |
| Labour (R) | 7 604 | 24 248 | 12 943 | 39 129 |
| Cows (R)* | 12 257 | 12 257 | 6 069 | 20 944 |
| **Total cost (R)** | **60 481** | **93 182** | **49 265** | **151 532** |
| **Total income (R)** | **112 481** | **163 620** | **87 335** | **264 026** |
| Margin (R) | 52 000 | 70 438 | 38 070 | 112 494 |
| Margin/cow (R) | 505 | 683 | 746 | 639 |
| Margin/ℓ (c) | 14 | 13 | 13 | 13 |
| Margin/ha feed crop (R) | 277 | 376 | 1 082 | 926 |
| ℓ/cow | 3 526 | 5 129 | 5 529 | 4 844 |
| Ha feed crop/cow | 1,82 | 1,82 | 0,69 | 0,69 |

*Cost of cow per annum = R119

**TABLE 6 - Actual and profit-maximizing combinations of inputs for the Natal Midlands, 1978/79 to 1982/83**

| Input/output | Actual combination (mean herd) | Profit-maximizing combination | | |
|---|---|---|---|---|
| | | Mean herd | Minimum herd | Maximum herd |
| Cows | 111 | 111 | 44 | 361 |
| ℓ/annum | 429 328 | 384 315 | 147 173 | 1 306 097 |
| Concentrates (R) | 49 767 | 42 627 | 16 324 | 144 870 |
| Pastures (R) | 8 437 | 5 723 | 2 191 | 19 448 |
| Labour (R) | 15 833 | 2 954 | 1 131 | 10 040 |
| Cows (R)* | 16 410 | 16 410 | 6 505 | 53 370 |
| **Total cost (R)** | **90 447** | **67 714** | **26 151** | **227 728** |
| **Total income (R)** | **138 673** | **124 134** | **47 537** | **421 869** |
| Margin (R) | 48 226 | 56 420 | 21 386 | 194 141 |
| Margin/cow (R) | 434 | 508 | 486 | 538 |
| Margin/ℓ (c) | 11 | 15 | 15 | 15 |
| Margin/ha feed crop (R) | 511 | 598 | 810 | 708 |
| ℓ/cow | 3 868 | 3 462 | 3 345 | 3 618 |
| Ha feed crop/cow | 0,85 | 0,85 | 0,60 | 0,76 |

*Cost of cow per annum = R147,84

**TABLE 7 - Actual and profit-maximizing combinations of inputs for the Ixopo area, 1978/79 to 1982/83**

| Input/output | Actual combination (mean herd) | Profit-maximizing combination | | |
|---|---|---|---|---|
| | | Mean herd | Minimum herd | Maximum herd |
| Cows | 147 | 147 | 56 | 225 |
| ℓ/annum | 67 218 | 746 349 | 241 988 | 1 226 572 |
| Concentrates (R) | 71 987 | 46 103 | 14 948 | 75 767 |
| Forage (R) | 25 848 | 49 921 | 16 186 | 82 041 |
| Labour (R) | 15 961 | 26 188 | 8 491 | 43 039 |
| Cows (R)* | 24 696 | 24 696 | 9 408 | 37 800 |
| **Total cost (R)** | **138 492** | **146 908** | **49 033** | **238 647** |
| **Total income (R)** | **219 009** | **243 160** | **78 840** | **399 617** |
| Margin (R) | 80 517 | 96 252 | 29 807 | 160 970 |
| Margin/cow (R) | 548 | 655 | 532 | 715 |
| Margin/ℓ (c) | 12 | 13 | 12 | 13 |
| Margin/ha feed crop (R) | 898 | 1 073 | 634 | 917 |
| ℓ/cow | 4 573 | 5 077 | 4 321 | 5 451 |
| Ha feed crop/cow | 0,61 | 0,61 | 0,84 | 0,78 |

*Cow cost per annum = R168

cases in Table 6 shows that the margin per litre for all cases is 15c. The maximum herd size indicates the largest margin per cow while the margin per hectare of feed crop is largest for the the smallest cow herd.

From a comparison of the actual input mix with the profit-maximizing input combination for an average herd it is evident that input reallocation could improve the total margin by 17%.

With regard to the mean cow herd, a comparison between the actual expenses and those under the mean herd size column in Table 6 shows that all expenses have been reduced and milk yield has dropped by 10%. However, the net gain in margin is 17% owing to the reallocation of resources.

For the mean herd size a large difference in labour costs between the actual and profit-maximizing cases is evident. This may be due to a possible error in the initial allocation of labour costs to the dairy enterprise.

## Ixopo

Table 7 summarizes actual expenditure on concentrates, forage and labour as well as profit-maximizing combinations of inputs for three herd sizes in the Ixopo area.

The actual margin is R80 517 per annum. The profit-maximizing combination at the geometric mean herd size improves the fresh-milk farmer's margin by 20%, the margin per cow by 19% and the margin per hectare of feed crop by 19%. Concentrate expenditure has been reduced and forage expenses and labour expenses have increased for the mean herd size of 147 cows.

So, by an overall improvement in the management of the dairy enterprise, farmers in the Ixopo study group are able to improve milk production per cow and increase profits.

The relatively high unit cost of a cow may be attributed to the better-than-average dairy cow found among the Ixopo study group farmers.

For the three profit-maximizing cases, the margin per cow is highest for the maximum herd size while the mean herd size indicates the greatest margin per hectare of feed crop.

The relatively high reductions and/or increases in the inputs may be a result of having omitted some variables, e.g. management. Griliches (1957, p. 11) points out that if the omitted variable is correlated with one or more of the variables included, this will bias the estimates of at least one coefficient upwards.

## DISCUSSION AND CONCLUSIONS

The foregoing analysis of fresh-milk farmers in Natal who participate in the Mail-In Record Scheme indicates that these farmers can improve their resource allocation to achieve higher profits.

According to the marginal rate of substitution criteria for least cost resource combinations the Natal Midlands was the only area in which farmers should be using more concentrates and less pasture. However, the net gain is small. East Griqualand farmers appear to allocate concentrates and pastures at a least cost level. For the Ixopo farmers it was estimated that there would be a net gain if the use of concentrates was reduced and the use of forage was increased.

Allowing all factors except cows to vary, it appears that East Griqualand farmers need to improve on the way in which they allocate factors of production and milk yield. The milk yield of an average-sized herd may be improved by 45% by means of improved resource allocation, i.e. from 3 526 litres/cow/annum to 5 129 litres/cow/annum. It is possible to achieve this increase in the long run. Considering that the East Griqualand farmers also engage in other farming activities it may not be economical and/or practical for them to have large herds. A small herd and an estimated 5 529 litres per cow per annum would maximize profits per cow and per hectare of feed crop.

In the Natal Midlands the analysis indicates that for a mean herd size the use of inputs should decrease for profits to be maximized. Concentrate expenditure is expected to decrease by R7 140, pasture expenditure by R2 714 and labour by R12 879. The high reduction in labour costs may be attributed to the labour-intensive nature of fresh-milk farming in the Natal Midlands. A reduction in the number of labourers may thus improve profits. This large reduction may also be the result of incorrect allocation of labour to the dairy enterprise.

Of the three areas considered, Ixopo fresh-milk farmers produce the most milk per cow per annum. However, there still appears to be room for improvement. For a mean herd size and profit-maximization it is estimated that milk production per cow per annum could be improved by a reduction in concentrate use and an estimated 93% increase in the use of forages and labour. It would appear that Ixopo farmers need to reassess their feeding strategies carefully.

The above analysis, although based on Mail-In Record farmers only, has indicated that fresh-milk farmers in Natal need to reassess their input mix, particularly the relationship between purchased and farm-produced feeds. Used with discretion, the results reported in this article might be useful to farmers, extension officers and researchers in dairy management.

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