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Working Paper No. 1/91

February 1991

DEPARTMENT OF ECONOMETRICS

ISSN 1032-3813

ISBN 0 86746 964 1

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A Generalised Logistic Tobit Model.

Version 1.0

January 1991

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Abstract: This paper proposes the use of a generalisation of the Logistic distribution, the Burr Type II distribution, as the error structure in a Type I (Standard) Tobit model. The use of the Burr II is motivated through both an heterogeneity argument and by noticing the need for potentially non-symmetric distributions in Tobit models. Tests for symmetry (the Logistic distribution) and its implied heterogeneity are also proposed. Finally the model is estimated using Australian data on alcohol expenditure.

¹ Part of this work was supported by ESRC studentship A00428424097. An earlier version of this paper was entitled "Some Aspects of a Tobit Model". I wish to thank Judy Coish for preparing the data for analysis. I also wish to acknowledge the useful comments made on earlier versions of this work by Tim Barmby, Richard Blundell, Len Gill, Grant Hillier, Chris Orme and Richard Smith. All remaining errors are my responsibility.

1. Introduction.

The Tobit (or censored regression model) is widely used in economics (for a survey see Amemiya (1984)). However, in many applications the normality assumption in the standard Tobit model may not be appropriate. For example, Atkinson, Gomulka and Stern (1990) argue that it may not be appropriate to assume normally distributed disturbances in the case of modelling alcohol expenditure. It is well known that if normality is incorrectly assumed then the Tobit maximum likelihood estimates are in general inconsistent. Solutions to this problem fall into two camps. Either the researcher can adopt a semi-parametric approach, making minimal distributional assumptions or the researcher can assume another, more flexible, parametric form for the distribution. It should be pointed out that a potential problem with the latter approach is that if the parametric form is misspecified then inconsistency will still result. This paper chooses to follow the second approach and to use a flexible parametric form for the distribution.

The proposed distribution, the Burr type II (or generalised Logistic) distribution is a potentially non-symmetric, real line distribution, which is computationally tractable. This distribution is motivated by the use of a mixing argument to capture population heterogeneity in section 2. Section 3 then discusses the application of the distribution to the Tobit model and also tests of error distribution symmetry. Section 4 describes the estimation of the generalised Logistic Tobit model and includes an empirical application to data on alcohol expenditure

in New South Wales. Finally section 5 contains some concluding remarks.

2. The generalised Logistic distribution and the Tobit model.

In what follows we will be concerned with the standard (or Type 1) Tobit model given by:

$$y_1^* = \mathbf{x}'_1 \beta + u_1 \quad i = 1, \dots, n.$$
$$y_1 = \begin{cases} y_1^* & \text{if } y_1^* > 0 \\ 0 & \text{if } y_1^* \leq 0 \end{cases}$$

where y_1^* is a latent variable and y_1 is the observed variable. The usual assumption regarding the error is that $\{u_1\}$ are i.i.d. $N(0, \sigma^2)$. Such a normality assumption is common in econometrics. However, Tobit M.L.E. is generally inconsistent when the true distribution is non-normal making tests for distributional misspecification important, Nelson (1981), Bera, Jarque and Lee (1984) and Newey (1987), *inter alia*, have proposed such tests. Goldberger (1983) and Arabmazar and Schmidt (1982) calculated asymptotic bias for certain non-normal distributions.

Further evidence against using normality can be found in Arabmazar and Schmidt (1982), Gomulka (1986) and Atkinson, Gomulka and Stern (1990) where it is found that consideration of non-symmetric distributions makes the normal a potentially inappropriate distribution to use. Gomulka (1986) and Atkinson, Gomulka and Stern (1990) have shown that in the case of modelling alcohol expenditure there is some evidence of the error distribution being non-symmetric. Thus it would seem appropriate to use an error distribution which was potentially non-symmetric and which allowed for a test of symmetry in the error process.

The following list of distributions suggested for the errors in Tobit models includes only one potentially non-symmetric distribution - (the Gamma): (i) Normal; (ii) Laplace; (iii) Student's t ; (iv) Logistic; (v) Log-normal; (vi) Gamma. The Gamma distribution is difficult to deal with, in particular it does not have a closed form for its distribution function. Hence we suggest the use of a generalised Logistic distribution (the Burr type II distribution) which has a computationally tractable form and which can take on a wide range of potentially non-symmetric shapes (see Fry (1988, 1989) for further details of the properties of this distribution).

We may also argue for the use of the Burr II distribution in another way. If we interpret y_1^* as the desired expenditure on alcohol, say, and u_1 as measuring departures from this then we may use an heterogeneity argument to justify the use of the Burr II (note that this argument is an adaptation of that in Morrison and Schmittlein (1980) in a duration context). In our model assume that $u_1 \sim \text{Log-Weibull}$, but that further to variations caused by, say, omitted variables the u_1 differ across individuals. We might choose to model such variation with a suitable distribution. An appropriate choice is the Gamma distribution since it allows for a wide variety of shapes and is tractable in such mixing arguments.

Thus we assume that, conditional on θ , $u_1 \sim \text{Log-Weibull}$:

$$g(u_1 | \theta) = \theta \exp(u_1) \exp(-\theta \exp(-u_1)) \quad -\infty < u_1 < \infty,$$

where $\theta (> 0)$ is a scale parameter. We then argue for θ itself to

vary across the population to capture 'continuous population heterogeneity'. A suitable approach is to have θ follow a Gamma distribution:

$$h(\theta) = \frac{\theta^{\alpha-1} e^{-\theta}}{\Gamma(\alpha)} \quad \theta, \alpha > 0.$$

The resultant density for the error term u_1 is:

$$f(u_1) = \frac{\alpha \exp(-u_1)}{(1 + \exp(-u_1))^{\alpha+1}}$$

This is the p.d.f. for the Burr type II distribution (see Fry (1988, 1989)).

The Burr II distribution is a one parameter generalisation of the Logistic. When $\alpha = 1$ we have the p.d.f. of the Logistic distribution (a symmetric distribution) and this generalisation allows the possibility of non-symmetry in the error distribution. Furthermore, the distribution function of the Burr II given by:

$$F(u_1) = (1 + \exp(-u_1))^{-\alpha}$$

has the convenient property of being closed form thus aiding computation.

As noted above the value $\alpha = 1$ yields the Logistic, a symmetric distribution. Thus a test for $\alpha = 1$ will be a test for symmetry of the error distribution. However, interpreting $h(\theta)$ as modelling population heterogeneity yields a convenient measure of such heterogeneity. That is the coefficient of variation of the θ in the Gamma distribution:

$$d = \frac{\text{s.d.}(\theta)}{E(\theta)} = \alpha^{-1/2}.$$

Hence a test for $\alpha = 1$ will be also be a test for $d = 1$ and population heterogeneity ($h(\theta)$) in the form of a unit exponential

distribution.

3. The Burr Tobit model.

We now consider the specification of a standard Tobit model with Burr II errors. Recall our model is:

$$y_i^* = \mathbf{x}'_i \beta + u_i \quad i = 1, \dots, n.$$

$$y_i = \begin{cases} y_i^* & \text{if } y_i^* > 0 \\ 0 & \text{if } y_i^* \leq 0 \end{cases}$$

and the parameters of interest in this model are β (a $k \times 1$ vector) and α . If we require an estimate of σ^2 (the variance of the error distribution) we are able to recover it from our knowledge that the variance of a B2 distribution equals $\psi'(\alpha) + \pi^2/6$, where $\psi'(\alpha) = d^2 \log(\Gamma(\alpha))/d\alpha^2$ the trigamma function. Hence we may recover an estimate of σ^2 from the estimate of α .

The log-likelihood function for this Burr Tobit model is:

$$\begin{aligned} \ell &= \sum_i (1 - d_i) \log(F(w_i)) + d_i \log(f(u_i)) \\ &= \sum_i \left\{ (1 - d_i) [-\alpha \log(1 + \exp(-w_i))] \right. \\ &\quad \left. + d_i [\log(\alpha) - u_i - (\alpha+1) \log(1 + \exp(-u_i))] \right\}, \end{aligned}$$

where $w_i = -\mathbf{x}'_i \beta$ and $d_i = \begin{cases} 1 & \text{if } y_i^* > 0 \\ 0 & \text{otherwise.} \end{cases}$

It is straightforward to find the first derivatives of the

log-likelihood:

$$\frac{\partial \ell}{\partial \alpha} = - \sum_i \left\{ (1 - d_i) \log(1 + e^{-w_i}) + d_i (-\alpha^{-1} + \varepsilon_i) \right\}$$

$$\frac{\partial \ell}{\partial \beta} = -\alpha \sum_i \left[(1 - d_i) \frac{e^{-w_i}}{1 + e^{-w_i}} + d_i v_i \right] x_i$$

$$\text{where } \varepsilon_i = \log(1 + e^{-u_i}), \quad v_i = \frac{e^{-u_i}}{1 + e^{-u_i}} - \frac{1}{\alpha(1 + e^{-u_i})}$$

may be termed Generalised Residuals, in the sense of Cox and Snell (1968) (see also Gourieroux, Monfort, Renault and Trognon (1987)).

Notice that

$$v_i = 1 - ((\alpha+1)/\alpha) \exp(-\varepsilon_i).$$

Hence we may choose to work with either ε_i or v_i in deriving the Information matrix, and since this is a non-standard maximum likelihood problem, in checking that the expected values of the first order conditions are zero (see Fry (1987) section 3.5 for details of the latter). We choose to use ε_i as this proves more tractable than v_i . It can be shown that if $U \sim \text{Burr II}$ then $Q = \log(1+e^{-U})$ has an exponential(α) distribution. In our Tobit model this means that ε_i has a truncated exponential distribution (see Fry (1987, 1988)). This truncated exponential distribution is particularly helpful in the derivation of the Information matrix.

The Information matrix may be written as

$$I(\alpha, \beta) = \frac{1}{n} \begin{bmatrix} \text{Var}(D_\alpha \ell) & \text{Cov}(D_\alpha \ell, D_\beta \ell) \\ . & \text{Var}(D_\beta \ell) \end{bmatrix},$$

where $D_\alpha \ell = \partial \ell / \partial \alpha$ and $D_\beta \ell = \partial \ell / \partial \beta$. For the Burr Tobit we obtain

the following:

$$\begin{aligned} \text{Var}(D_{\alpha} \ell) &= \sum_i \alpha^{-2} (1 - (1 + e^{-w_i})^{-\alpha}) + 2(\alpha^{-1} - 1) \frac{\log(1 + e^{-w_i})}{(1 + e^{-w_i})^{\alpha}} \\ \text{Var}(D_{\beta} \ell) &= \frac{\alpha}{\alpha+2} \sum_i (1 - (1 + e^{-w_i})^{-(\alpha+2)}) \mathbf{x}_i \mathbf{x}_i' \\ \text{Cov}(D_{\alpha} \ell, D_{\beta} \ell) &= \sum_i \left\{ \frac{-\alpha}{(1 - (1 + e^{-w_i})^{-\alpha})^2} \left(\frac{\log(1 + e^{-w_i}) e^{-w_i}}{(1 + e^{-w_i})^{2\alpha+2}} \right. \right. \\ &\quad \left. \left. + \frac{1}{\alpha(\alpha+1)} \left[1 - (1 + e^{-w_i})^{-(\alpha+1)} - (1 + e^{-w_i})^{-(2\alpha+1)} \right] \mathbf{x}_i \right) \right\} \end{aligned}$$

A hypothesis of particular interest in this model is that $\alpha = 1$. A test of this hypothesis is both a test of symmetry in the form of the Logistic distribution (Logistic Tobit) and for population heterogeneity in the form of a unit exponential. Such a test may be conducted using either Wald, likelihood ratio or Lagrange multiplier statistics. The likelihood ratio test turns out to have a particularly simple form, as we will now show.

We may rewrite the log-likelihood as

$$\ell = \alpha \cdot D_{\alpha} \ell + \sum_i d_i \left[\log(\alpha) - \log(1 + e^{-u_i}) - u_i - 1 \right]$$

Denoting restricted maximum likelihood estimates by \sim and unrestricted ones by $\hat{}$ after some straightforward algebra we obtain:

$$\hat{\ell} - \tilde{\ell} = N^+ (\log(\hat{\alpha}) - 1) + \sum_1 (1 - d_1) \log(1 + e^{-\tilde{w}_1})$$

$$+ \sum_1 d_1 (\tilde{u}_1 - \hat{u}_1) + \sum_1 d_1 (2\tilde{\varepsilon}_1 - \hat{\varepsilon}_1)$$

where $N^+ = \sum_1 d_1$ the number of positive observations.

Fry (1988) finds that the likelihood ratio tests of $\alpha = 1$ in the regression, Tobit and binary choice models all have this distinctive structure, which involves both the difference of residuals and of generalised residuals.

The Wald test is easily obtained from the Information matrix as:

$$\lambda_W = n(\alpha - 1)^2 [\text{Var}(D_\alpha \ell) - \text{Cov}(D_\alpha \ell, D_\beta \ell) \text{Var}(D_\beta \ell)^{-1} \text{Cov}(\cdot)] \approx \chi^2(1)$$

λ_W can be estimated by replacing the unknowns by their unrestricted maximum likelihood estimates.

Note however, that in practice the researcher is unlikely to wish to estimate the 'restricted' model (Logistic Tobit). This is because if symmetry is to be imposed on the error structure, unless there is evidence of "fat tails" the normal distribution would be used. As a consequence, the Wald test is most likely to be of interest to applied workers.

4. Application of the model.

In this section we describe the application of the Burr Tobit model. Firstly, we outline how the model can be estimated in the computer package Limdep (Greene (1990)) and secondly we estimate the model on data for individual household alcohol expenditure in New South Wales, as recorded in the 1984 Household Expenditure Survey. It should be noted that this empirical application is

intended solely as an illustration of the practical use of the Burr Tobit model rather than an econometric study of alcohol expenditure in New South Wales.

Orme (1989) showed how Limdep could be 'tricked' into producing maximum likelihood estimates for the generalised Logistic (Burr II) distribution both in the regression and censored regression (Tobit) models. Exploiting the mixing argument, as used in our heterogeneity motivation earlier, Orme shows that the 'trick' is to regard the observations as minus one times the log of observations on right censored durations, which are generated by a Weibull model subject to multiplicative heterogeneity in the hazard. Thus once the data is read into Limdep, and assuming that the dependent variable is labelled Y, the sequence of Limdep commands to estimate the Burr Tobit model is:

```
CREATE; IF(Y>0)STATUS=1$  
CREATE; Z=-1*Y$  
SURVIVAL; LHS=Z, STATUS; RHS=ONE, ...; MODEL=WEIBULL; HET$
```

where RHS is the list of explanatory variables for the model. It should be noted that since Z is treated as the log of an observation it may be necessary to scale the data by multiplying Y by say -0.01 rather than -1 for the procedure to work.

We use this procedure to estimate a Burr Tobit model for household expenditure on alcohol in New South Wales as recorded in the 1984 Household Expenditure Survey. For this application we might interpret the latent variable y_1^* as being the household's desired expenditure on alcohol and the observed variable y_1 is the actual alcohol expenditure. There are 1009 households (observations) and

350 of these record zero expenditures. That is there are 34.7% censored (zero) observations in this sample. Definitions of the variables used are given in Appendix 1 of this paper.

Table 1 gives the results of the estimation of the Burr Tobit model. Notice that with the exception of occupational status groups Occ4 and Occ9 all the explanatory variables are statistically significant. Notice also that Limdep gives an estimate of σ . This estimate is the same as that which would be obtained using $\sigma^2 = \psi'(\alpha) + \pi^2/6$.

The Wald test of the symmetry assumption ($H_0: \alpha = 1$) is given by a simple t -test based on the estimated value $\hat{\alpha}$ ($= 0.4498$). The value of this test statistic is -3.81 . Thus we would reject the null of a symmetric error distribution implying that the use of the normal distribution in this case would have been questionable. This rejection also implies that the unit exponential heterogeneity assumption is inappropriate. The estimated value for our coefficient of heterogeneity in the Gamma distribution for θ is $\hat{d} = 1.491$ indicating a sizable degree of heterogeneity.

5. Conclusions.

This paper has proposed the use of a generalised Logistic (Burr II) distribution in the Tobit model. This was justified both by an heterogeneity argument and by noticing the need for potentially non-symmetric error distributions in this model. The distribution allows for a test of symmetry (and its associated form of heterogeneity). It also showed how to estimate the model using an existing econometrics package and finally the model was applied to

some Australian data on alcohol expenditure.

The Burr Tobit is an attractive alternative to the semi-parametric approach when the researcher wishes to avoid the normality assumption. It is easy to use and allows for a test of symmetry in the error structure. However, two notes of caution need to be sounded. First, in the parametric approach if there is distributional misspecification then we lose the consistency property of the maximum likelihood estimates. Although since the Burr II does have a wide range of moment coverage this distributional misspecification should not be as great a problem as when the normal distribution is used. Second, if we reject the null hypothesis of symmetry in the error distribution we need to ask whether the underlying cause of the rejection is true non-symmetry or functional misspecification.

This paper adds another variant of the Tobit (censored regression) model to the applied workers 'toolkit'. It is hoped that the paper will encourage researchers to consider the use of the Burr Tobit model whenever the censored regression framework is appropriate to their modelling situation.

References.

- Amemiya, T. (1984), "Tobit Models: A Survey", *Journal of Econometrics*, 24, 3 - 61.
- Arabmazar, A. and Schmidt, P. (1982), "An Investigation of the Robustness of the Tobit Estimator to Non-Normality", *Econometrica*, 50, 1055 - 1063.
- Atkinson, A.B., Gomulka, J. and Stern, N.H. (1990), "Spending on Alcohol: Evidence from the Family Expenditure Survey 1970 - 1983", *Economic Journal*, 100, 808 - 827.
- Bera, A.K., Jarque, C.M. and Lee, L.F. (1984), "Testing for the Normality Assumption in Limited Dependent Variable Models", *International Economic Review*, 25, 563 - 578.
- Cox, D.R. and Snell, E.J. (1968), "A General Definition of Residuals", *Journal of the Royal Statistical Society, Series B*, 30, 248 - 265.
- Fry, T.R.L. (1987), "Some Aspects of a Tobit Model", Discussion paper ES181, Department of Econometrics, University of Manchester.
- Fry, T.R.L. (1988), "On the Use of Some Burr Family Distributions in Econometrics", *Unpublished Ph.D. Thesis*, University of Manchester.
- Fry, T.R.L. (1989), "Univariate and Multivariate Burr Distributions: A Survey", Working paper 9/89, Department of Econometrics, Monash University.
- Goldberger, A.S. (1983), "Abnormal Selection Bias", in *Studies in Econometrics, Time Series and Multivariate Statistics*, eds. S. Karlin, T. Amemiya and L.A. Goodman, London: Academic Press.
- Gomulka, J. (1986), "Gamma-Tobit: A Tobit Type Model with Gamma Distributed Error Term", Discussion paper 67, E.S.R.C. programme on Taxation, Incentives and the Distribution of Income.
- Gourieroux, C., Monfort, A., Renault, E. and Trognon, A. (1987), "Generalised Residuals", *Journal of Econometrics*, 34, 5 - 32.
- Greene, W.H. (1990), "LIMDEP: Version 5.1", Econometric Software Inc., New York.
- Morrison, D.G. and Schmittlein, D.C. (1980), "Jobs, Strikes and Wars: Probability Models for Duration", *Organizational Behavior and Human Performance*, 25, 224 - 251.
- Nelson, F.D. (1981), "A Test for Misspecification in the Censored Regression Model", *Econometrica*, 49, 1317 - 1329.
- Newey, W.K. (1987), "Specification Tests for Distributional Assumptions in the Tobit Model", *Journal of Econometrics*, 34, 125 - 146.

Orme, C. (1989), "Maximum Likelihood Estimation in the Generalised Logistic Model", Mimeo, Department of Economics, University of Nottingham.

Table 1: Maximum likelihood estimates for the Burr Tobit model.
 (1984 Household Expenditure Survey, N.S.W. data, n = 1009)

Variable	
Constant	.1648 (.0179)
Occ1	-.0889 (.0242)
Occ2	-.0930 (.0301)
Occ3	-.0715 (.0284)
Occ4	-.0512 (.0362)
Occ5	-.0718 (.0299)
Occ6	-.1714 (.0712)
Occ7	-.0622 (.0303)
Occ8	-.0749 (.0200)
Occ9	-.0379 (.0324)
Wage	-.0413 (.0097)
Dependents	-.0444 (.0098)
Income	-.000008 (.000003)
Expenditure	-.000011 (.000002)
α	.4498 (.1444)
σ	.1314 (.0056)

log-likelihood value = -151.09.

Notes: (i) standard errors in parentheses.

(ii) dependent variable is -1 times alcohol expenditure.

Appendix 1: The data.

The following variables were used in the analysis. The data came from the 1984 Household Expenditure Survey and concerned households in New South Wales.

Variables Occ1 → Occ9 are occupational status dummy variables for the head of household: Occ1 = managers and administrators; Occ2 = professionals; Occ3 = para-professionals; Occ4 = tradespersons; Occ5 = clerks; Occ6 = salespersons and personal service workers; Occ7 = plant and machine operators; Occ8 = labourers and related workers; Occ9 = unemployed. (Note that Occupational code 10 = other workers was not used in the analysis to avoid linear dependence amongst the variables).

Wage is the number of wage earners in the household; Men is the number of men in the household; Dependents is the number of dependents in the household; Income is the net household income (in cents); Expenditure is the total household expenditure (in cents) on all expenditure groups and Alcohol is household expenditure on alcohol (in hundreds of dollars).

Note that the variable Alcohol was used in hundreds of dollars to avoid having to rescale in the fitting of the Burr Tobit model in Limdep.

