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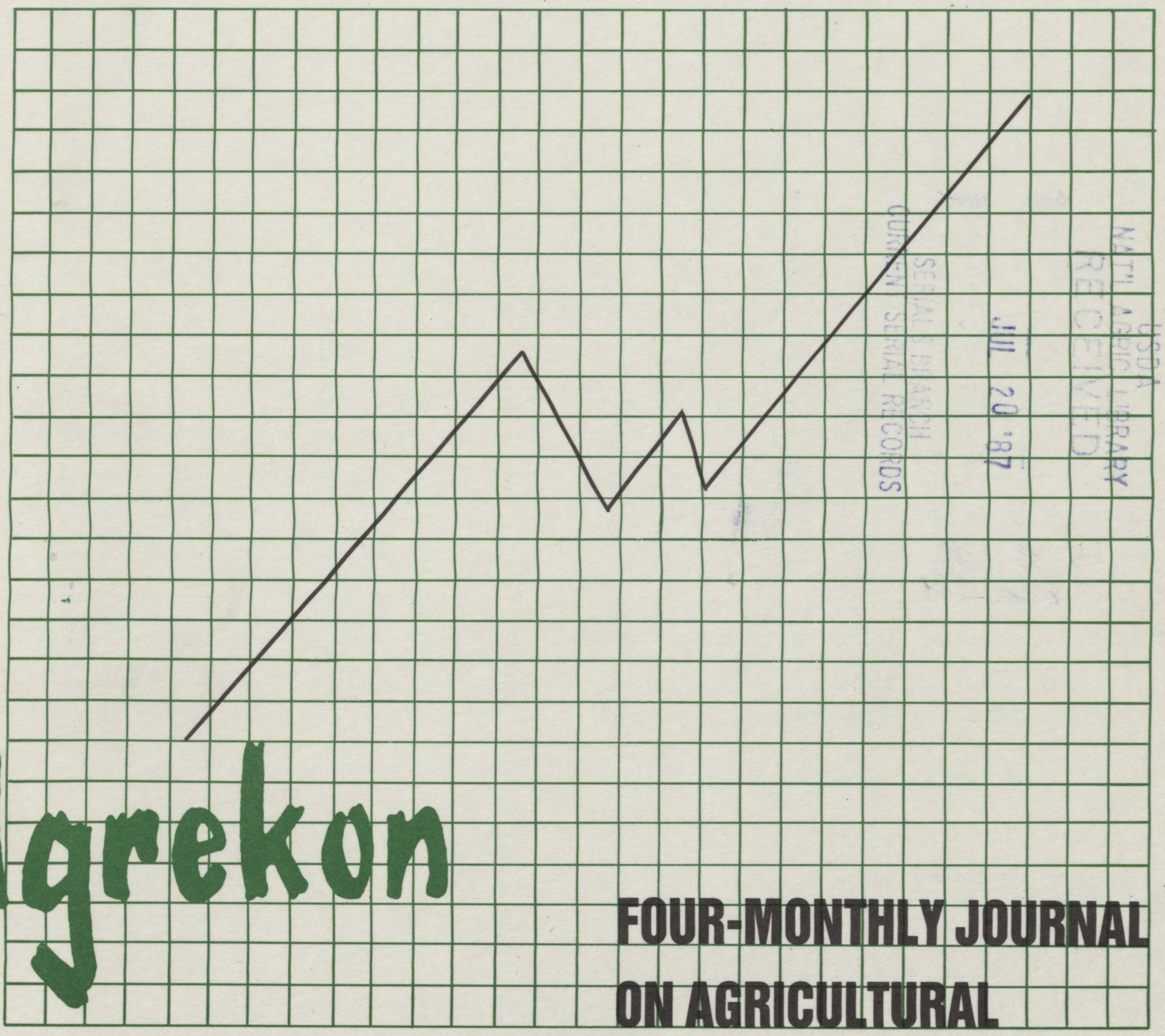

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DUALITY AND ELASTICITIES OF SUBSTITUTION II: AN EMPIRICAL APPLICATION*

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ABSTRACT

An analysis was made of potential differences in elasticities of substitution between agricultural inputs over time in different sectors of South African agricultural production.

The elasticity of substitution between input pairs in South African agriculture has declined considerably from 1960-1972 to 1973-1985. Total agricultural production seems to have more flexibility in dealing with input price variation than either wheat or maize production as distinct sub-sectors.

U.S. agriculture exhibits much more flexibility in dealing with especially machinery price variations. The relative rigidity of the South African agricultural production process may at least partially be attributed to present competitive structures.

INTRODUCTION

The empirical illustration of the application of theory presented in a previous publication (Van Zyl, 1986) is concerned with the potential differences in elasticities of substitution between agricultural inputs over time, and in different sectors of South African agriculture.

Fuss, McFadden and Mundlak (1978) refer to technological change which impacts the partial elasticities of substitution between input pairs as substitution augmenting technological change. Substitution augmenting technological change that increases the elasticity of substitution between input pairs is desirable in that the producer is given additional flexibility in dealing with changes in the relative prices of the inputs that might occur due to shocks within the factor markets. Suppose for example, that the elasticity of substitution between capital and labour within an economy is near zero. The firm is then faced with a situation in which capital and labour will be used in nearly fixed proportions to each other irrespective of relative price levels. Moreover, the firm has little flexibility for dealing with short-run variability in input prices. Estimates of elasticities of substitution among input pairs must necessarily rely on data series for a number of years. If there are changes in elasticities of substitution over time due to technological change, then the data series for a long period of time can not be relied upon to measure these shifts (Debertin and Pagoulatos, 1985). However, if the data series are too short, degrees of freedom problems, multicollinearity between input vectors and instability of regression coefficients used for derivation of elasticity estimates become issues.

EMPIRICAL APPROACH

The calculation of substitution coefficients, using factor shares from cost data - which are often more readily available than physical data - by means of translog specifications of production functions is a viable contemporary approach (Van Zyl, 1983).

A translog cost function for derivation purposes is specified as follows (Aoun, 1983):

$$\ln C^* = \alpha_o + \alpha_y \ln Y + \Sigma_i \alpha_i \ln P_i + {}^1\!/_2\, \beta_{yy} (\ln Y)^2 + {}^1\!/_2\, \Sigma_i \Sigma_j \beta_{ij} \ln P_i \ln P_j + \Sigma \gamma_{yi} \ln Y \ln P_i + \phi_t t + {}^1\!/_2\, \phi_{tt} t^2 + \phi_{ty} t \ln Y + \Sigma_i \phi_{ti} t \ln P_i \;..... (1)$$

where:

C^* = minimum total cost
i, j = n, l, m, f, e
Y = output
n = land
l = labour
m = machinery
f = fertiliser
e = energy
t = annual time trend variable
P_i, P_j = input prices on n, l, m, f and e

The translog cost function is assumed to be continuous, monotonically increasing, concave and homogeneous of degree one with respect to factor prices. Following Brown and Christensen (1981), it is assumed that the translog cost function represents a constant returns to scale technology. This implies the following restrictions:

$\alpha_y = 1;$
$\Sigma\gamma_{yi} = O$ for i = n, l, m, f and e;
$\beta_{yy} = O;$
$\phi_{ty} = O.$

By partial differentiation of Equation 1 with respect to the i-th input price, assuming that the imposed restrictions hold:

$$d \ln C^*/d \ln P_i = \alpha_i + \Sigma\beta_{ij} \ln P_i + \gamma_{yi} \ln Y + \phi_{ti}\, t$$

Invoking Shephards lemma:

$$d \ln C^*/d\ln P_i = d\, C^*/dP_i\, P_i/C^* = (X_iP_i)/C^* = S_i$$

where: S_I = the cost share for the i-th input and:

*This article is the second in a series of two and represents an empirical application of the theory discussed in the first article, also by Van Zyl (1986)

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$$S_i = \alpha_i + \Sigma\beta_{ij} \ln P_j + \gamma_{yi} \ln Y + \phi_{ti}\, t \; ..... (2)$$

for i = n, l, m, f en e

The restrictions imposed on the estimation were:

$$\Sigma\alpha_i = 1$$
$$\Sigma\beta_{ij} = \Sigma\beta_{ji} = \Sigma\Sigma\beta_{ij} = O$$
$$\Sigma\gamma_{yi} = O$$
$$\Sigma\phi_{ti} = O$$
$$\beta_{ij} = d \ln C^*/(d\ln P_j) = \beta_{ji} = d \ln C^*/(d \ln P_j \ln P_i)$$

The Allen elasticity of substitution (AES) is derived by substituting the parameter estimates of the cost share equation into:

$$\sigma^A_{ij} = (\beta_{ij} + S_i S_j)/S_i S_j \; ..... (3)$$

where:

σ^A_{ij} = Allen elasticity of substitution (AES)

The mean of the cost shares ($\bar{S}_i$) for each input category in the data for the sample period is inserted into Equation 3 in order to obtain the AES estimates. Once the AES estimates are obtained, the corresponding Morishima (MES) and Shadow (SES) elasticities of substitution can be obtained by using Equations 4 and 5 respectively:

$$\sigma^M_{ij} = S_j (\sigma^A_{ij} - \sigma^A_{jj}) .... (4)$$

$$\sigma^S_{ij} = \left[(S_i S_j)/(S_i + S_j)\right]\left[2\, \sigma^A_{ij} - \sigma^A_{ii} - \sigma^A_{jj}\right] \; ..... (5)$$

where:

σ^M_{ij} = Morishima elasticity of substitution (MES)

σ^S_{ij} = Shadow elasticity of substitution (SES)

Estimates of elasticities of substitution for the Allen (AES), Morishima (MES) and Shadow (SES) measures were obtained for South African agriculture for the two distinct periods 1960-1972 and 1972-1985, and for the entire period comprising 26 years from 1960-1985. The same elasticity measures were also separately estimated for maize production (1973-1984) and wheat production (1974-1985).

Restricted three-stage least squares was used. When equations are not interdependent, but "seemingly unrelated", joint generalized least square or seemingly unrelated regression uses estimates of the covariance of residuals across equations to increase the efficiency of the estimates. This was handled by a SYSTEM card in SAS (Carry, 1982). The restrictions on the estimations were imposed via the RESTRICT procedure in SAS.

Price indexes for the various input categories were obtained from the Directorate of Agricultural Economic Trends (1986), except for labour where the index was constructed from data obtained from the Department of Statistics (1986). Processed production cost survey data (Directorate of Agricultural Economic Trends, 1973-1985) were used to estimate elasticities of substitution for maize and wheat production.

Estimates of Allen elasticities of substitution (AES) for total agricultural production in South Africa for the periods 1960-1972, 1972-1985 and 1960-1985, as well as for maize production in the period 1973-1984 and wheat production in the period 1974-1985 are shown in Table 1. The corresponding Morishima (MES) and Shadow (SES) elasticities of substitution are respectively shown in Tables 2 and 3.

EMPIRICAL RESULTS

The Allen elasticity of substitution (AES) is in reality the cross price factor demand elasticity, while the Morishima measure is the difference between the cross and own price elasticity of factor demand evaluated at constant output. The Shadow elasticity of substitution (SES) is closer to the original definition of Hicks (1932) than both AES and MES, and can thus be thought of as a long run elasticity of substitution.

Estimates of SES for most input pairs (Table 3) differ significantly from 1,0 suggesting that the Cobb-Douglas function is not the appropriate production function to represent South African agriculture.

TABLE 1 - Estimates of Allen cross elasticities of substitution (AES) for total agricultural production, maize production and wheat production, Republic of South Africa

Sector	Period	Item	NL	NM	NF	NE	LM	LF	LE	MF	ME	FE
			Input pairs									
Total agricultural production	1960-1985	σ-value	+1,616	-2,871	-0,811	-0,524	-3,278	+ 2,942	-0,595	+20,551	- 9,137	+25,284
		t-value	(1,964)	(-8,419)	(-3,219)	(-8,339)	(-2,413)	(0,866)	(-2,014)	(3,606)	(-13,649)	(15,083)
		p>/t/	0,065	0,000	0,005	0,000	0,027	0,398	0,059	0,002	0,000	0,000
	1960-1972	σ-value	+5,250	+1,786	+7,810	+1,467	-0,844	-13,910	-7,326	+ 4,940	-18,058	+ 8,275
		t-value	(6,699)	(2,719)	(8,673)	(2,076)	(-2,013)	(- 7,648)	(-5,186)	(1,530)	(-16,109)	(2,807)
		p>/t/	0,001	0,042	0,000	0,081	0,100	0,001	0,004	0,187	0,000	0,038
	1973-1985	σ-value	-0,290	+3,071	-2,018	+0,112	+6,035	- 2,237	-0,452	+16,004	- 0,828	+16,903
		t-value	(-4,265)	(3,438)	(-3,504)	(-2,929)	(2,617)	(- 1,161)	(-1,213)	2,578)	(- 4,778)	(7,099)
		p>/t/	0,008	0,019	0,017	0,033	0,067	0,298	0,279	0,050	0,005	0,001
Maize	1973-1984	σ-value	+3,766	+1,017	+1,175	+0,904	+4,669	- 1,842	-0,018	0,820	- 1,560	+ 0,730
		t-value	(1,975)	(0,428)	(0,166)	(-0,132)	(3,317)	(- 1,924)	(-2,496)	(- 2,866)	(- 3,671)	(- 0,270)
		p>/t/	0,118	0,691	0,877	0,901	0,030	0,127	0,067	0,046	0,021	0,801
Wheat	1974-1985	σ-value	+2,566	+4,297	+1,575	+1,562	+0,355	- 0,795	-7,016	- 7,018	+ 0,357	- 0,784
		t-value	(0,791)	(2,908)	(0,562)	(0,907)	(-0,556)	(- 1,275)	(-7,737)	(- 4,083)	(- 1,536)	(- 2,301)
		p>/t/	0,473	0,044	0,604	0,416	0,608	0,272	0,002	0,015	0,199	0,083

*N = land; L = labour; M = machinery; F = fertiliser; E = energy

** $\sigma_{ij} > O$: factor i and factor j are substitutes $\sigma_{ij} < O$: factor i and factor j are complements

TABLE 2 - Morishima elasticities of substitution (MES) for total agricultural production, maize production and wheat production, Republic of South Africa

Sector	Period	Input	Land	Labour	Machinery	Fertiliser	Energy
			Input				
Total agricultural production	1960-1985	Land	0	-0,317	-0,470	+2,180	-0,353
		Labour	-0,234	0	-0,506	+2,371	-0,400
		Machinery	-1,838	-0,692	0	+3,267	-0,764
		Fertiliser	-1,101	-0,216	+1,612	0	+0,701
		Energy	-0,999	-0,486	-1,207	+3,687	0
	1960-1972	Land	0	+0,926	+0,478	+1,887	-1,811
		Labour	+2,041	0	-0,251	+0,947	-2,139
		Machinery	+0,955	-0,455	0	+1,763	-2,539
		Fertiliser	+2,843	-0,556	+0,750	0	-1,558
		Energy	+0,855	-0,046	-1,236	+1,907	0
	1973-1985	Land	0	-0,571	+0,447	+0,584	-0,112
		Labour	-1,368	0	+0,718	+0,571	-0,139
		Machinery	+0,777	+0,093	0	+1,638	-0,348
		Fertiliser	-1,268	-0,718	+1,629	0	+0,691
		Energy	-0,412	-0,583	-0,275	+1,690	0
Maize	1973-1984	Land	0	+0,811	-0,149	-0,408	-0,663
		Labour	+0,950	0	+0,374	-1,048	-0,745
		Machinery	+0,236	+0,876	0	-0,831	-0,882
		Fertiliser	-0,553	+0,377	+2,000	0	+0,759
		Energy	-0,001	+0,506	-0,988	+0,073	0
Wheat	1974-1985	Land	0	+0,805	-0,100	+0,216	-0,798
		Labour	+1,245	0	-0,830	-0,176	-1,534
		Machinery	+1,664	+0,649	0	-1,206	-0,901
		Fertiliser	+0,762	+0,567	-2,196	0	-0,999
		Energy	+1,001	+0,127	-0,830	-0,174	0

South African agriculture, 1960-1985

During the period 1960-1985 estimates of AES, MES and SES between land and machinery, land and energy, labour and machinery, labour and energy, and machinery and energy were negative, thus indicating that the input pairs were complements. Estimates of AES, MES and SES for the same period between machinery and fertiliser, and between fertiliser and energy were positive, indicating substitutability.

Elasticities of substitution between labour and fertiliser were not significant at the 10% level for 1960-1985, while the different measures were inconsistent between land and labour and between land and fertiliser.

The long-run SES estimates (Table 3) indicate that all input pairs involving fertiliser are positive

TABLE 3 - Shadow elasticities of substitution (SES) for total agricultural production, maize production and wheat production, Republic of South Africa

Sector	Period	Input	Land	Labour	Machinery	Fertiliser	Energy
			Input				
Total agricultural production	1960-1985	Land	0	-0,302	-0,742	+1,771	-0,461
		Labour		0	-0,606	+1,337	-0,431
		Machinery			0	+2,664	-0,849
		Fertiliser				0	+1,799
		Energy					0
	1960-1972	Land	0	+1,147	+0,581	+2,003	-1,528
		Labour		0	-0,359	+0,408	-1,459
		Machinery			0	+1,425	-2,416
		Fertiliser				0	+0,245
		Energy					0
	1973-1985	Land	0	-0,571	+0,508	+0,349	-0,144
		Labour		0	+0,274	+0,009	-0,311
		Machinery			0	+0,435	-0,118
		Fertiliser				0	+0,190
		Energy					0
Maize	1973-1984	Land	0	+0,841	-0,012	-0,100	-0,441
		Labour		0	+0,708	+0,036	-0,038
		Machinery			0	-0,582	-0,743
		Fertiliser				0	-0,626
		Energy					0
Wheat	1974-1985	Land	0	+0,905	+0,665	+0,536	-0,327
		Labour		0	+0,240	+0,345	-0,624
		Machinery			0	-1,673	-0,879
		Fertiliser				0	-0,718
		Energy					0

and relatively elastic, indicating that when fertiliser forms part of an input pair agricultural production has a great deal of flexibility in dealing with input price variation.

Changes in South African agriculture, 1960-1972 to 1973-1985

The AES estimates between land (N) and labour (L) changed from + 5,250 for 1960-1972, to -0,290 for 1973-1985. The conclusion is that land and labour were substitutes during the first period, but complements during the second according to the AES. This is further confirmed by the movement of SES from + 1,147 to -0,571 and the non-symmetric MES from respectively + 0,926 and + 2,041 to -0,571 and -1,268 for the same periods. All the measures indicate that the elasticity of substitution was higher in 1960-1972 than in 1973-1985, indicating that flexibility in dealing with input price variation became smaller in the second period.

According to the AES measure land and fertiliser were substitutes during 1960-1972, but became complements for the period 1973-1985. Although only one of the non-symmetric MES estimates supports this, both the other MES and SES estimates show a marked decline from + 1,887 to + 0,584 and from + 2,003 to + 0,349 respectively for the periods 1960-1972 and 1973-1985.

Although the signs for the different estimates of elasticity of substitution between land and energy are non-consistent, AES, MES and SES estimates are all nearer to zero for 1973-1985 than for the period 1960-1972, indicating that substitution between land and energy has become more inelastic.

The signs of the AES, MES and SES estimates between labour and machinery were negative for South African agriculture during the period 1960-1972, indicating that these inputs were complementary. However, these complements for the period 1960-1972 became substitutes for the period 1973-1985. This is indicated by the positive signs for all the measures for the period 1973-1985. These results are consistent with findings of the Commission of Enquiry into South African Agriculture (1970) which concluded that mechanisation gave rise to better cultivation, bigger crops and higher labour requirements, particularly for harvesting. De Klerk (1982) found that in maize farming, mechanisation of the harvesting process later led to substitution of machines for capital. This result is also consistent with Tarr's (1975) findings.

The elasticity of substitution between machinery and energy is negative for all the measures in 1960-1972 and in 1973-1985, thus indicating that machinery and energy are complements. However, there has been a clear increase in the degree of substitutability between energy and machinery from the period 1960-1972 to the period 1973-1985. This is shown by movements in the AES from -18,058 to -0,828 for 1960-1972 and 1973-1985. The MES estimates moved respectively from -1,236 and -2,539 to -0,275 and -0,348 for the same periods, while the SES moved from -2,416 to -0,118. This tendency may at least partially be attributed to the greater emphasis on fuel-saving technology since 1970.

Estimates of the elasticity of substitution between labour and fertiliser and between labour and energy for 1973-1985, and between machinery and fertiliser for 1960-1972 are statistically insignificant at the 10% level. The AES, MES and SES estimates between land and machinery, and between fertiliser and energy are inconsistent and yield no clear indication of change from 1960-1972 to 1973-1985. These elasticities are therefore not discussed any further.

Of particular importance is the SES estimates for the periods 1960-1972 and 1973-1985, as these represent the long-run elasticity of substitution. It is interesting to note that every single SES estimate between input pairs for 1973-1985 is nearer to zero than the corresponding estimate for 1960-1972, thus indicating that the substitution between input pairs has become less elastic.

Maize and wheat production

Only three input pairs yielded consistent and significant elasticities of substitution at the 10% level for maize production, 1973-1984. Labour and machinery appear to be substitutes (see also discussion under previous heading), while labour and energy, and machinery and energy were complements.

For 1974-1985, machinery and fertiliser, and fertiliser and energy were complements in wheat production according to the AES, MES and SES measures. All the other input pairs yielded either inconsistent or insignificant results at the 10% level.

According to the long-run SES estimates (Table 3), it appears that all the estimates between input pairs for maize production, and all except the estimate between fertiliser and machinery for wheat production, are relatively inelastic. However, only three pairs of inputs are substitutes in maize production, while five pairs of inputs have a positive sign for wheat production. This shows that although both wheat and maize production have rigid production processes concerning the substitutability of inputs, maize farmers would have less flexibility than wheat farmers in dealing with variability in input prices over time.

When comparing SES estimates of wheat and maize production with that of total agricultural production in South Africa for the same period, it appears that total agricultural production, although also yielding relative inelastic estimates, has more flexibility to deal with input price variation than both wheat or maize farming.

A comparison between South African and U.S. agriculture

Aoun (1983) calculated SES estimates for U.S. agriculture for the distinct decade of 1970-1979, using the same technique outlined in this publication. The results are shown in Table 4.

Table 4 shows that the SES estimates involving machinery are relatively elastic and positive, thus indicating that U.S. agriculture has a high degree of flexibility in dealing with machinery price variations. Other input pairs, except for labour and land, are inelastic complements.

If Aoun's (1983) results for U.S. agriculture (Table 4) are compared to those obtained for South African agricultural production for the period 1973-1985 (Table 3), it is obvious that the long-run elasticities of South African agricultural production between input pairs are in general much less elastic. It appears therefore that U.S. agriculture has much more flexibility in dealing with especially machinery price variations.

Of particular interest to Aoun (1983) were estimates of the elasticity of substitution between machinery (including tractors) and energy. He concluded that there had been a clear increase in the substitutability between energy and machinery since 1950. South African agriculture experienced a similar increase in the substitutability between machinery and energy from 1960-1972 to 1973-1985 (the AES and SES estimates respectively increased from -18,058 and -2,416 to -0,828 and -0,118). However, it seems that South African agricultural production is at least 10 years behind U.S. agriculture in this respect.

CONCLUSION

South African agriculture has become markedly less elastic concerning substitution between input pairs from 1960-1972 to 1973-1985. This tendency is especially accentuated by the different SES estimates (Table 3).

When comparing SES estimates of wheat and maize production with that of total agricultural production in South Africa for the same period, it seems that total agricultural production has more flexibility to deal with input price variation than either wheat or maize production. Long-run elasticities of substitution (SES) for the U.S. indicate that it has much more flexibility in dealing with especially machinery price variations. Although South African agriculture also experienced an increase in the substitutability between energy and machinery since 1960, it seems that South African agriculture lags ten years behind U.S. agriculture.

TABLE 4 - Shadow elasticities of substitution for the distinct decade of 1970-1979, USA

Input	Land	Labour	Machinery	Fertiliser	Energy
Land	0	+0,629	+3,191	-0,380	-0,150
Labour		0	+4,278	+0,574	-1,012
Machinery			0	+1,540	+2,808
Fertiliser				0	-0,109
Energy					0

Source: Aoun (1983)

One reason for the rigidity of South African agricultural production compared to that of the U.S. probably lies in the differences in competitive structures of the two countries, both on the input and output sides. Some of the inputs of South African agricultural production are heavily protected from outside competition. This resulted in monopolies on the input side. On the output side some products, such as maize and wheat, have fixed prices that are at least partly based on production costs. There is thus little incentive for agricultural production to react to relative price changes between input pairs, resulting in a rigid production process.

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