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SOME AUSTRALIAN EVIDENCE

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MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

TIME STATIONARITY OF SYSTEMATIC RISK : SOME AUSTRALIAN EVIDENCE

BY

Robert W. Faff*,
John H. H. Lee** and
Tim R. L. Fry**

ABSTRACT

While the evidence on time-varying systematic risk of U.S. assets is well documented in the literature, little work has been conducted in the Australian equity market. This paper intends to fill this gap in the literature by employing an alternative testing procedure to those used in previous studies. Moreover, a new methodology of determining the p-value of a test statistic is applied. The results of our study suggest that there is evidence of time-varying systematic risk for both individual assets and portfolios in Australia.

* Department of Accounting and Finance, Monash University.

** Department of Econometrics, Monash University.

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1. INTRODUCTION

The notion of risk and its measurement is of fundamental importance, to modern finance theory. For many years the dominant paradigm of the equilibrium risk/return tradeoff has been the capital asset pricing model (CAPM). According to the CAPM, the relevant risk measure for any asset is the asset's systematic risk (beta) because any non-systematic risk can be diversified away through holding the market portfolio. Empirically, beta is often estimated by applying ordinary least squares (OLS) to the market model. However,by using OLS, it is assumed that beta is constant through time.

The study by Blume (1971) represents the pioneering work that first subjected the assumption of beta stationarity to close empirical scrutiny. Based on an examination of seven-year estimation periods he found assets' betas to have a "regression tendency", i.e. over time the estimated betas tended to regress toward the grand mean of unity. For example, a portfolio which has an extremely high estimated beta in one period will tend to have a less extreme estimate in the successive period. Blume (1975) examined this phenomenon further and showed that the explanation conventionally given, of a statistically induced "order bias" was at best a misleading argument[1]. Instead he claimed that the regression tendency truly reflected real nonstationarities of individual firm's betas.

In recent times there has been an expanding empirical literature related to the stability of the beta in the market model. This literature provides strong evidence that beta is best described by some type of stochastic parameter model. Such research is well represented by Fabozzi and Francis (1978), Sunder (1980), Lee and Chen (1982), Ohlsen and Rosenberg (1982), Alexander and Benson (1982), Bey (1983b), Bos and Newbold (1984), Simonds, La Motte and McWhorter (1986), Collins, Ledolter and Rayburn (1987) and Rahman, Kryzanowski and Sim (1987). It is notable that all these papers examine US data. In these studies, two basic types of stochastic parameter model have been considered: (1) random and (2) sequential. The most common example of the latter is a first-order autoregressive process (AR(1)). We argue that the AR(1) process is indeed an appropriate and parsimonious model of beta variation, encapsulating the explanation and evidence of Blume (1971,1975). Hence using Australian equity return data, the aim of this paper is to test whether the betas of assets (both individual and portfolio) are constant against the alternative hypothesis that they vary according to an AR(1) process. A further aim of this paper is to apply an alternative procedure for testing this hypothesis than that used by other researchers. The majority of previous studies use standard large sample tests such as the likelihood ratio test. However, in our analysis, a locally best invariant (LBI) test is used similar to that derived by King (1987).

This paper is organised as follows. In section 2 a summary of previous empirical research of beta stationarity is given. Section 3 provides some justification for treating beta variation as a AR(1) process. In the fourth section we develop the market model with an AR(1) beta process. Section 5 outlines the application of the LBI test to our problem. In section 6 the data is described, while in the penultimate section the empirical results are presented and discussed. In the final section a summary and conclusions are provided.

2. PREVIOUS EMPIRICAL RESEARCH

The early work of Blume (1971, 1975) and others was based on an ad hoc analysis of the cross-sectional correlation between beta estimates from successive periods. A limitation of this approach is the implicit assumption that beta is stationary within each period. Consequently, these tests lacked power compared to the later work that formally modelled beta variation and in doing so, readily allowed more formal testing to be conducted.

Fabozzi and Francis (1978) applied the Hildreth-Houck random coefficient model to beta. That is

$$\beta_t = \bar{\beta} + d_t \qquad [1]$$

where β_t is the systematic risk and d_t are serially uncorrelated random error terms. Their results provided strong evidence in favour of the random coefficient model for individual stocks over a six year period. However, in a re-examination of their work, Alexander and Benson (1982) concluded that Fabozzi and Francis overstated the case for the random coefficient model.

Sunder (1980) considered two alternative models for beta variation: a random walk model and an AR(1) model. The random walk model suggests that beta varies according to

$$\beta_t = \beta_{t-1} + d_t. \qquad [2]$$

The AR(1) model may be written as

$$(\beta_t - \bar{\beta}) = \rho(\beta_{t-1} - \bar{\beta}) + d_t \qquad [3]$$

where the d_t for both [2] and [3] are serially uncorrelated random error terms. Empirically, Sunder tested only against the random walk model. He examined both individual stocks and randomly generated portfolios over periods of twenty-five and fifty years. Not surprisingly, he found stationarity was soundly rejected for these long periods, but did not find a significant incidence of nonstationarity for shorter subperiods of seventy-five months duration.

Simonds, La Motte and McWhorter (1986) re-examined the Sunder (1980) results using an exact test for the random walk specification of beta variability. In constrast to Sunder (1980), they detected considerable beta instability for eight-year subperiods which they attributed to the much stronger power of the tests they applied.

Bos and Newbold (1984) investigated the relative merits of an AR(1) beta process, a random coefficient model and a standard fixed parameter market model. They concluded that there is strong evidence in favour of nonstationary systematic risk of individual assets over a ten year period. However the evidence on which model best described the stochastic behaviour of beta is inconclusive.

Ohlsen and Rosenberg (1982) proposed a model which accommodates both autocorrelated variation and random variation in the same model. According to this model the beta coefficient varies in the following way

$$\beta_t = \bar{\beta} + u_t + \delta_t$$

where $\delta_t = \rho\delta_{t-1} + d_t$.

This model may be written as

$$(\beta_t - \bar{\beta}) + u_t = \rho((\beta_{t-1} - \bar{\beta}) - u_{t-1}) + d_t \qquad [4]$$

where u_t and d_t are two independent sequences of serially uncorrelated random error terms. This model implies an ARMA(1,1) process for the beta coefficient. Over a fifty year period, using a value-weighted index to proxy market returns, they found that the beta of an equally-weighted index revealed highly significant mixed random and autoregressive behaviour.

An extensive examination of the Ohlsen and Rosenberg model was conducted by Collins, Ledolter and Rayburn (1987). They analysed a large sample of individual securities and random portfolios of various sizes. They also reported a limited analysis of size based portfolios. In addition both monthly and weekly data were used and results for five-year and ten-year subperiods compared. Generally, their results comfirm the support for the mixed random and autoregressive model, found by Ohlsen and Rosenberg.

Generally, while there exists considerable U.S. evidence of beta nonstationarity, the findings are mixed regarding its specific nature. Following this previous research, it is apparent that several aspects of the research design are of interest e.g. individual versus portfolios assets, the portfolio dimension, the portfolio selection process and different time periods. These issues will be examined in the empirical work that follows.

3. BETA AS AN AR(1) PROCESS : SOME JUSTIFICATION

It is not the purpose of this paper to resolve the issue of which of the models discussed in the previous section is the most appropriate. Rather we are interested in testing whether the betas of assets are stationary or not using Australian equity return data. For this purpose, the alternative model which we choose to test against is the AR(1) model.

The AR(1) model is considered because it incorporates the arguments Blume (1975) provided regarding the nonstationarities of the betas over time. In particular, the AR(1) model incorporates a "memory" into the model such that deviations in the beta from its mean are serially correlated. This is consistent with Blume's (1975) "formal model"[2],

$$E(\beta_{it+1} \backslash \hat{\beta}_{it}) - 1 = \rho^{*}(\hat{\beta}_{it} - 1)$$

where $\rho^{*} = \rho(\beta_{it+1}, \beta_{it})$. This model is based on the assumption that $\hat{\beta}_{it}$ and β_{it+1} are bivariate normal random variables, with mean equal to unity.

Furthermore, the AR(1) model is more parsimonious than the other models considered in the previous section in that it encompasses the other models depending on the value of ρ. For example, in [3] when ρ is one, we will have a random walk model and when ρ is 0, we have a Hildreth-Houck random coefficient model. However, it does not encompass the ARMA(1,1) model proposed by Ohlson and Rosenberg (1982).

In this framework, the beta of any asset will have a tendency to revert towards the grand mean of unity, as implied by CAPM, which is the grand mean of all betas. This means that firms of extreme risk, either high or low, will tend to have less extreme risk characteristics over time. Blume (1975) provides two alternative explanations as to how this may be so. The first explanation is that when firms engage in any project which is risky, the risk of the project may tend to become less extreme over time. However, while this explanation seems appropriate for high risk firms, it is not applicable to low risk firms. The second explanation is based on the notion that firms tend to take on new projects which have less extreme risk characteristics than their existing projects. This might occur, for example, as the result of a relative scarcity of profitable risky projects over time. This explanation is plausible for both high risk and low risk firms.

Empirically, Blume (1975) provided evidence consistent with this argument. He found that the estimated beta coefficients of various portfolios are less extreme or closer to the market beta of one in later periods than the estimated beta coefficients in earlier periods. Therefore, based on Blume's (1975) explanation and evidence we should expect the beta of any asset to be serially correlated in the manner described in [3].

4. THE MARKET MODEL WITH AN AR(1) BETA PROCESS

The implication of the above discussion is that the beta coefficient in the market model varies according to an AR(1) process. Given this, we can write the market model as

$$R_{it} = \alpha + \beta_{it} R_{mt} + \varepsilon_{it} \qquad t = 1, \ldots, n \qquad [5]$$

where $$\beta_{it} = \rho\beta_{it-1} + (1-\rho)\bar{\beta} + d_{it}, \qquad [6]$$

R_{it} is the return on asset i in period t and R_{mt} is the return on a market index in period t. We can omit the subscript i on the $\bar{\beta}$ since $\bar{\beta}$

should be the same for all assets. Alternatively, the market model can be written as

$$R_{it} = \alpha + \bar{\beta}R_{mt} + v_{it} \qquad [7]$$

where $\quad v_{it} = (\beta_{it} - \bar{\beta})R_{mt} + \varepsilon_{it}. \qquad [8]$

Before proceeding, we need to make a number of fairly general assumptions about the error terms ε_{it} and d_{it}, and the coefficient ρ.

Assumption 1: Let ε_{it} be a independently and identically distributed random variable with $E(\varepsilon_{it}) = 0$ and $E(\varepsilon^2_{it}) = \sigma^2_{\varepsilon}$.

Assumption 2: Let d_{it} be an independently and identically distributed random variable with $E(d_{it}) = 0$ and $E(d^2_{it}) = \sigma^2_{d}$.

Assumption 3: ε_{it} and d_{it} are mutually independent, ie $Cov(\varepsilon_{it}d_{it}) = 0$.

Assumption 4: ρ satisfies the stationarity condition, ie $|\rho| < 1$. In fact, this assumption is implied by a stationary AR(1) model.

If we repeatedly substitute for the β_{it} in [6], we can rewrite [6] as

$$\beta_{it} = \bar{\beta} + \sum_{j=1}^{\infty} \rho^j d_{it-j}$$

where $E(\beta_{it}) = \bar{\beta}$, $Var(\beta_{it}) = \sigma^2_d/(1 - \rho^2)$ and $Cov(\beta_{it}\beta_{it-1}) = \rho\sigma^2_d/(1 - \rho^2)$ and so on. We can generalise this as

$$\beta \sim N[\bar{\beta}\mathit{1},\ p\Sigma(\rho)]$$

where $\quad p = \sigma^2_d/(1 - \rho^2)$

l is a n x 1 vector of ones and

$$\Sigma(\rho) = \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \rho \\ \rho^{n-1} & \cdots & & \rho & 1 \end{bmatrix}.$$

Given the distribution of β, we can determine the distribution of the error term in the market model. In matrix notation [8] can be written as

$$V = D\beta - D\bar{\beta}l + \varepsilon$$

where D is a diagonal matrix with elements R_{m1}, R_{m2}, ... , R_{mn} on the diagonal. This implies that the distribution of V is

$$V \sim N[0,\ pD\Sigma(\rho)D + \sigma^2_{\varepsilon}I_n]. \qquad \text{or}$$

$$V \sim N[0,\ \sigma^2_{d}D\Sigma(\rho)D/(1 - \rho^2) + \sigma^2_{\varepsilon}I_n] \qquad [9].$$

5. THE LBI TEST

To test whether assets' betas are constant or not against the alternative that they vary according to an AR(1) model, most of the previous studies use standard large sample tests such as the likelihood ratio test. However, Watson and Engle (1985) argued that these tests cannot be used in the usual fashion because for this application, under the null hypothesis, the autoregressive parameter (ρ) is unknown and unidentified. They suggest an alternative test procedure based on Davies' (1977) approach. Unfortunately, their test statistic has no closed form and is approximated by maximisation using a grid search. Furthermore, both its finite sample and asymptotic distribution are unknown under the null hypothesis.

King (1987) proposed an alternative test which helps overcome some of the problems associated with Watson and Engle's (1985) test procedure. This test is a locally best invariant test (LBI). An advantage of the LBI test is that it is also LBI against the hypothesis that the beta coefficient follows a random walk process[3].

In our testing problem, a LBI test similar to that derived by King (1987) can be derived. From [9], a test of $H_o : \sigma^2_d = 0$ against $H_a : \sigma^2_d > 0$, $\rho = \rho_1$ is equivalent to a test of

$$H_o : V \sim N[0, \sigma^2_\varepsilon I_n] \text{ against}$$

$$H_a : V \sim N[0, \sigma^2_d D\Sigma(\rho_1)D/(1-\rho_1^2) + \sigma^2_\varepsilon I_n].$$

This belongs to the class of problems considered by King and Hillier (1985). Their result implies that a LBI test of H_o against H_a is to reject H_o for large values of

$$S = \hat{v}'A\hat{v}/\hat{v}'\hat{v}$$

where $\hat{v}$ is the OLS residual vector from [7]. Shively (1988) showed that A in this testing problem is

$$D\Sigma(\rho_1)D/(1 - \rho_1^2).$$

Therefore, the LBI test of H_o against H_a is to reject H_o for large values of

$$S = Y'MAMY/Y'MY \qquad [10]$$

where $A = D\Sigma(\rho_1)D/(1 - \rho_1^2)$,

$M = I_n - X(X'X)^{-1}X'$ and

X = the matrix of independent variables.

However, in order to implement the LBI test, a value for the ρ must be set. Since the test is a LBI test, setting ρ to different values should not affect the size of the test but may have an effect on its power. This conjecture will be addressed in the empirical analysis.

Traditionally, when carrying out an empirical application, the critical values of a test statistic are first determined. However, a more useful approach to take is to calculate the p-value for a given value of the test statistic. This can be determined through the use of a member of the general system of distributions proposed by Burr (1942). For any distribution, the shape of the distribution can be described adequately by its first four moments. These are the mean, variance, skewness and kurtosis. This information can be used to approximate the distribution of our test statistic with one selected from Burr's family of distributions. This approach is described in detail in Evans and Fry (1990). In this case, the particular Burr distributions used are the Type XII and its "reciprocal" Type III. The appeal of these two distributions is that their distribution functions and associated inverses have simple closed forms. Consequently, this allows us to calculate the approximate p-value for a given value of the test statistic.

The distribution function of a random variable X, which follows a Burr Type XII distribution is given by

$$F(X) = 1 - (1 + X^c)^{-k} \qquad X \geq 0 \quad c, k > 0$$

$$= 0 \qquad X < 0$$

where c and k are the parameters of the distribution. Selecting the appropriate Burr distribution involves finding values for the parameters c and k, which involves solving two non-linear simultaneous equations, to match the skewness and kurtosis of the test statistic[4]. The computed c and k then identify the Burr distribution to be used as an approximation to that of the LBI test statistic.

Having found the closest approximating Burr distribution, the approximate p-value for any given value of the LBI test statistic, say $\hat{x}$, can be calculated from the distribution of interest. Suppose that random variable X has the distribution with mean μ_S and standard deviation σ_S. Then the p-value for $\hat{x}$ is given by

$$p = P(X > \hat{x}) = P(Z > (\hat{x} - \mu_S)/\sigma_S) = P(Z > \hat{z})$$

$$= [1 + (\mu_B + \sigma_B\hat{z})^c]^{-k},$$

where the values of c and k have been determined by matching the skewness and kurtosis for the distribution of interest, and μ_B and σ_B are the mean and standard deviation of the Burr distribution. In Appendix 1, the manner in which the mean, variance, skewness and kurtosis of the LBI test statistic can be calculated is shown.

One final point to note is that if the test statistic has skewness and kurtosis values close to the boundary of such values covered by the Burr distributions, this boundary defines a "limit" distribution for the Burr family. This "limit" distribution is that of the Weibull, which also has a convenient closed form for the distribution function. The p-value for $\hat{x}$ using the Weibull distribution is given by

$$p = P(X > \hat{x}) = P(Z > (\hat{x} - \mu_S)/\sigma_S) = P(Z > \hat{z})$$

$$= \exp[\ -(\mu_W + \sigma_W \hat{z})^c]$$

where μ_W and σ_W are the mean and standard deviation of the Weibull distribution respectively. The mean and standard deviation of the Burr and Weibull distributions can be calculated as shown in Appendix 2.

6. DATA

The data used in the following empirical analysis is monthly returns on ordinary Australian equities, obtained from the Price Relatives File of the Centre for Research in Finance (CRIF), at the Australian Graduate School of Management (AGSM). Two periods are examined : 1978/1 to 1982/12 and 1983/1 to 1987/9. Securities are included in each period only if they have a complete price relative history for that period. This resulted in two samples of 159 and 310 individual assets, respectively. Two different indices are used to obtain a measure of the return on the market portfolio. The first is an equally weighted index of all firms in the Price Relatives File. The second is a value weighted index supplied by CRIF.

Tests were performed on both individual and portfolio returns. Portfolios were formed according to beginning of period market capitalisation and on a random basis. The market capitalisation based portfolios were examined in three alternative dimensions : five assets, ten assets and twenty assets per portfolio. This resulted in thirty-one, fifteen and seven portfolios respectively for the 1978 to 1982 period and sixty-two, thirty-one and fifteen portfolios, respectively, for the period 1983 to 1987. In each period one hundred randomly formed portfolios were examined comprising ten securities each.

Finally, returns were measured on both a discrete and continuously compounded basis. The discrete return for firm i in period t, R_{it}, is given by

$$R_{it} = \frac{P_{it} - P_{it-1} + D_{it}}{P_{it-1}}$$

where P_{it} is the security's price at time t and D_{it} is the dividend paid, by firm i, at time t. The associated continuously compounded return in period t, R_{it}', is given by,

$$R_{it}' = \ln(1 + R_{it}).$$

7. EMPIRICAL RESULTS

As discussed earlier it is necessary to set a value of ρ in any empirical application. Three values were examined in this paper : 0.2, 0.5 and 0.8. These were chosen to be sufficiently representative of the range appropriate for a stationary AR(1) model. However, we report for the case of ρ equal to 0.5 only as the results were not greatly sensitive to the ρ value chosen[5]. Furthermore, King (1990) provided some justification for choosing ρ to be 0.5.

The basic results for the individual assets for the 1978 to 1982 and 1983 to 1987 time periods are reported in Table 1. The table shows the number of times the LBI test statistic rejects the null hypothesis

[TABLE 1 INSERT HERE]

for the sample of assets considered at three different significance levels. The number of rejections expressed as a percentage of the sample are in parentheses. The tests were conducted for both the discrete and continuous returns cases. In addition the results for both an equally-(EWMI) and value-weighted market index (VWMI) are reported.

In Table 1, for the first time period, there is a 10% - 15% rejection at the 5% significance level when the EWMI is used. For this market index, there are more rejections for discrete returns than continuous returns. But when VWMI is used, the number of rejections are substantially more than when the EWMI is used. Similarly, there are more rejections for discrete returns than continuous returns when VWMI is used.

From the same table, for the period 1983 to 1987, the difference between the EWMI and the VWMI in terms of the number of rejections is much smaller. Again for both market indices there are more rejections for discrete returns than for continuous returns. There seems to be a smaller proportion of rejections in general compared to the period 1978 to 1982 especially when VWMI is used. Nevertheless, there is some evidence that the beta of some assets is not stationary.

It is of some interest to consider whether there is any systematic relationship between the tendency toward rejection and some specific firm characteristic. Three potentially important firm characteristics are riskiness, size and industrial sector. These are analysed in turn in Tables 2 to 4. In Table 2 we examine whether there is some relationship between the estimated beta value and beta nonstationary across the samples, considered in Table 1. Firstly, each sample was

[TABLE 2 INSERT HERE]

divided into estimated betas less than 1 and into estimated betas greater than 1. Secondly, they were divided into three groups: less than 0.8; greater than 0.8 but less than 1.2; and greater than 1.2. Finally, the rank correlation coefficient between the estimated beta value and the associated LBI statistic is reported.

Generally, the analysis indicates a tendency of greater non-stationarity for higher risk firms. The percentage of nonstationary betas is between two and four times greater for assets with an estimated beta greater than unity relative to assets with estimated betas less than one. This tendency toward greater nonstationarity for high risk versus low risk firms is further accentuated in a comparison of firms with estimated betas less than 0.8 versus firms with estimated betas greater than 1.2. For example consider the EWMI, discrete return case for the period 1983 to 1987 where the comparison is a 7.1% versus a 42.2% rejection rate, respectively, at a 5% significance level. Finally, note that the rank correlations are all significantly positive further indicating a positive relationship between a tendency toward nonstationarity and the riskiness of the firm. This evidence is consistent with the notion that the risk of any project tends to become less extreme over time. This was an explanation proposed by Blume (1975) of the observed "regression tendency" of betas as discussed in an earlier section.

We also investigated whether there was any relationship between firm size and beta nonstationarity. This analysis is reported in Table 3. For both periods the sample is split up into five groups according

[TABLE 3 INSERT HERE]

to firm size denoted S1 (smallest firms), S2, S3, S4 and S5 (largest firms). The percentage rate of rejections at 5% significant level are reported for each group. There appears to be no discernible pattern of rejections across firm size. This is confirmed by the rank correlation coefficients between firm size and the associated LBI statistics. All are insignificantly different from zero.

An analysis of whether the industrial sector to which a firm belonged, influenced the likelihood of a rejection of stationarity in beta, is reported in Table 4. Firms in each time period sample are

[TABLE 4 INSERT HERE]

split into two groups, All Resources versus All Industrials. In five out of eight cases there is minimal difference between the rejection rates of the two industry sectors. However, in the other cases, there is a considerably higher rejection rate for the All Resources firms, For example in the 1978 to 1982 period, using discrete returns and the EWMI, the rejection rate is 26.2% versus 11.1%. This may be simply reflecting the higher risk of resource stocks and so be echoing the result of Table 2. Note however, that if this is so it is not consistently occuring across all cases, in contrast to the analysis in Table 2.

Tables 5 and 6 present the results for the portfolios formed by market capitalisation for the two periods considered. Similar to the case of the individual assets, there are more rejections when VWMI is used than when EWMI is used for the period 1978 to 1982. But the

[TABLES 5 AND 6 INSERT HERE]

difference is much smaller between the two market indices for the period 1983 to 1987. In terms of the dimensions of the portfolios, for both periods, there are more rejections for portfolios of increasing dimension. For example, for the period 1978 to 1982 using discrete returns, there is 100% rejection at 5% significant level when the portfolio size is twenty but only 74.2% rejection when the portfolio size is five. There are slightly more rejections using discrete returns than using continuous returns as in the case of individual assets. Similarly, the number of rejections are higher for the period 1978 to 1982 than for the period 1983 to 1987.

Table 7 presents the results of beta stationarity for portfolios grouped randomly with ten assets in each portfolio. Again, there is

[TABLE 7 INSERT HERE]

a substantial difference in the number of rejections between using VWMI and using EWMI in the period 1978 to 1982 but not the period 1983 to 1987. In fact, most of the results in this table are similar to those previously discussed.

8. CONCLUSION AND SUMMARY

In this paper we tested the hypothesis that betas of Australian equities are stationary against the alternative that they vary according to an AR(1) process. In contrast to previous studies, our analysis employed a LBI test similar to that derived by King (1987).

Generally, it was found that across all variations of our analysis a nontrivial degree of nonstationarity was evident. In the case of individual assets nonstationaity was more prevalent when discrete returns, as opposed to continuously compounded returns, were used. There was some weaker evidence of greater nonstationarity when a value-weighted market index, as opposed to an equally-weighted market index, was used to proxy general market movements. In addition, the analysis suggested that risker firms (higher betas) tended to be less stationary than low beta firms. However, no strong pattern between firm size or industry sector and nonstationarity was detected. Portfolios whether random or grouped according to market capitalisation showed increasing beta nonstationarity as the dimension of the portfolio increased. This is consistent with the results of Collins et. al. (1987). They argued that this reflected a higher ratio of beta variance to background noise in larger portfolios. As the portfolios become larger, the background noise decreases at a faster rate than the variability in beta, and so leads to more powerful tests of stationarity hypothesis[6].

The results of the current work potentially has important implications for any Australian empirical research which involves the estimation of systematic risk using equity return data. Studies that have utilised some form of fixed parameter regression to estimate risk and/or abnormal returns must be interpreted cautiously. Evidence here confirms the results of previous research that significant cases of beta variability exist, particularly for portfolios of assets. Researchers need to consider whether the degree of beta nonstationarity identified from statistical tests translates into nonstationarity that is significant in economic terms, so as to warrant explicit modelling of beta variability. While there is growing awareness of this issue, it has by no means achieved widespread recognition.

One interesting potential implication of beta nonstationarity, discussed by Collins et. al. (1987), is to the capital market anomalies literature. Their work showed some evidence of a systematic relationship between beta variability and risk-adjusted returns for size based portfolios. In particular, they suggested that in using a fixed parameter model there is greater uncertainty in estimating the beta risk of small firm portfolios[7]. This may have some potential in providing a partial explanation of the "firm size" effect.

This paper has not resolved the controversy in the empirical literature regarding which model best describes the stochastic behaviour of betas. It may be true that no one model will prevail. Some assets' betas might vary according to an AR(1) process, whereas, others might vary according to the model suggested by Ohlson and Rosenberg (1982), ie an ARMA(1,1) model. On the other hand, it may be true that none of these models adequately describe the stochastic behaviour of betas. Theoretically, beta is a measure of an asset's systematic risk which is defined as the covariance of the asset return with the market portfolio divided by the variance of the market portfolio return. Therefore, the non-stationarity of betas could be due to the time-varying covariance and/or variance of the returns. In fact, there is a large body of evidence on time-varying variance of returns, for example, Pagan and Schwert (1990) and Schwert (1989). Further research is needed to investigate the relationship between time-varying betas and time-varying variance of returns similar to that of Schwert and Seguin (1990), and Braun, Nelson and Sunier (1990).

APPENDIX 1

Given [10], the mean of the LBI test statistic can be calculated by

$$\mu_1 = \mathrm{tr}K/m,$$

the variance of the test statistic by

$$\mu_2 = 2[m\,\mathrm{tr}K^2 - (\mathrm{tr}K)^2]/[m^2(m+2)],$$

the coefficient of skewness by

$$\mu_3/\mu_2^{3/2} = \frac{8\{m^2\mathrm{tr}K^3 - 3m\,\mathrm{tr}K\,\mathrm{tr}K^2 + 2(\mathrm{tr}K)^3\}}{m^3(m+2)(m+4)\mu_2^{3/2}}, \text{ and}$$

the coefficient of kurtosis by

$$\mu_4/\mu_2^2 = 12\{m^3[4\mathrm{tr}K^4 + (\mathrm{tr}K^2)^2] - 2m^2[8\mathrm{tr}K\,\mathrm{tr}K^3 + \mathrm{tr}K^2(\mathrm{tr}K)^2] + m[24\mathrm{tr}K^2(\mathrm{tr}K)^2 + (\mathrm{tr}K)^4] - 12(\mathrm{tr}K)^4\}/\{m^4(m+2)(m+4)(m+6)\mu_2^2\}.$$

where $m = n - k$,

n = number of observations,

k = number of parameters,

$K = MA$ in [10] and

tr denotes the trace of a matrix.

APPENDIX 2

The mean and standard deviation of the Burr distribution can be calculated in the following way

$$\mu_B = \frac{\Gamma(1 + 1/c)\Gamma(k - 1/c)}{\Gamma(k)}$$

$$\sigma_B = \sqrt{\frac{\Gamma(1 + 2/c)\Gamma(k - 2/c)}{\Gamma(k)} - \mu_B^2}\ .$$

The Weibull mean and standard deviation are calculated in the following way

$$\mu_W = \Gamma[(1 + 1/c)]$$

$$\sigma_W = \sqrt{\{\Gamma(1 + 2/c) - \mu_W^2\}}.$$

FOOTNOTES

1) See Blume (1975, pp 786-788).

2) Refer to Blume (1975, pp. 788-790). The version of the "formal model" given in the text, is for the case where the order bias is zero (in Blume's equation 2) and hence it focusses on real nonstationarities in the underlying beta values.

3) See King (1987, pp. 380)

4) See Evans and Fry (1990) on how to compute the c and k values. Alternatively, Burr (1973) produced tables of c and k for various values of skewness and kurtosis of the distribution of interest.

5) Results for the case of ρ equal to 0.2 and/or 0.8 can be obtained from the authors.

6) See Collins, Ledolter and Rayburn (1987, pp. 442-443).

7) ibid.

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TABLE 1

LBI TEST RESULTS OF BETA STATIONARITY[a]:

INDIVIDUAL ASSETS USING (A) DISCRETE AND (B) CONTINUOUS RETURNS; AND (C) EQUALLY-WEIGHTED AND (D) VALUE-WEIGHTED MARKET INDEX. NUMBER OF REJECTIONS FOR THREE SIGNIFICANCE LEVELS

	Discrete Returns Significance Levels			Continuous Returns Significance Levels		
	0.01	0.05	0.10	0.01	0.05	0.10
1978/1982 (N=159)						
EWMI	9[b] (5.7%)	24 (15.1%)	33 (20.8%)	7 (4.4%)	19 (11.9%)	26 (16.4%)
VWMI	46 (28.9%)	57 (35.8%)	66 (41.5%)	24 (15.1%)	47 (29.6%)	57 (35.8%)
1983/1987 (N=310)						
EWMI	38 (12.3%)	70 (22.6%)	90 (29.0%)	17 (5.5%)	41 (13.2%)	64 (20.6%)
VWMI	36 (11.6%)	65 (21.0%)	87 (28.1%)	17 (5.5%)	40 (12.9%)	68 (21.9%)

a. Cases where stationary β is rejected against the AR(1) alternative, using the LBI test.

b. The number of rejections for the sample at the stated significance level. This is expressed as a percentage of the sample in parentheses.

TABLE 2

PERCENTAGE OF SECURITIES WITH BETA NONSTATIONARITY[a]

BY $\hat{\beta}$ AND RANK CORRELATION BETWEEN $\hat{\beta}$

AND TENDENCY TOWARD NONSTATIONARITY[b]

	1978/1982 Discrete	(N)	1978/1982 Continuous	(N)	1983/1987 Discrete	(N)	1983/1987 Continuous	(N)
EWMI								
$\hat{\beta} < 1$	9.3%	(118)	9.4%	(117)	10.1%	(178)	6.1%	(179)
$\hat{\beta} > 1$	31.7%	(41)	19.0%	(42)	39.4%	(132)	22.9%	(131)
$\hat{\beta} < 0.8$	8.7%	(104)	9.9%	(101)	7.1%	(156)	4.5%	(154)
$0.8 < \hat{\beta} < 1.2$	11.1%	(18)	10.0%	(20)	26.3%	(38)	15.4%	(39)
$\hat{\beta} > 1.2$	35.1%	(37)	18.4%	(38)	42.2%	(116)	23.9%	(117)
Rank Correlation	0.364 (0.0001)[c]		0.238 (0.0025)		0.549 (0.0001)		0.377 (0.0001)	
VWMI								
$\hat{\beta} < 1$	27.3%	(117)	21.9%	(114)	8.8%	(171)	6.3%	(176)
$\hat{\beta} > 1$	59.5%	(42)	48.9%	(45)	36.0%	(139)	21.6%	(134)
$\hat{\beta} < 0.8$	25.3%	(99)	21.2%	(99)	8.5%	(130)	5.3%	(131)
$0.8 < \hat{\beta} < 1.2$	36.4%	(22)	26.1%	(23)	15.7%	(70)	9.7%	(72)
$\hat{\beta} > 1.2$	63.2%	(38)	54.1%	(37)	39.1%	(110)	24.3%	(107)
Rank Correlation	0.447 (0.0001)		0.403 (0.0001)		0.452 (0.0001)		0.356 (0.0001)	

a. Cases where stationary β is rejected against the AR(1) alternative, using the LBI test at 5% significance level.

b. The rank correlation between $\hat{\beta}$ and the associated LBI statistic is reported.

c. The p-value for the associated rank correlation is given in parentheses.

TABLE 3

PERCENTAGE OF SECURITIES WITH BETA NONSTATIONARITY[a]

BY SIZE AND RANK CORRELATION BETWEEN SIZE AND TENDENCY TOWARD NONSTATIONARITY[b]

		1978/1982 Discrete	(N)	Continuous	1983/1987 Discrete	(N)	Continuous
EWMI	S1[c]	25.8%	(31)	12.9%	21.0%	(62)	16.1%
	S2	15.6%	(32)	15.6%	27.4%	(62)	14.5%
	S3	9.3%	(32)	6.3%	25.8%	(62)	17.7%
	S4	9.3%	(32)	12.5%	19.4%	(62)	9.7%
	S5	15.6%	(32)	12.5%	19.4%	(62)	8.1%
Rank Correlation		-0.064		-0.056	-0.017		-0.033
		(0.426)[d]		(0.487)	(0.762)		(0.568)
VWMI	S1	35.5%	(31)	25.8%	22.6%	(62)	17.7%
	S2	37.5%	(32)	34.4%	21.0%	(62)	14.5%
	S3	25.0%	(32)	21.9%	22.6%	(62)	9.7%
	S4	34.4%	(32)	25.0%	19.4%	(62)	14.5%
	S5	46.9%	(32)	40.6%	19.4%	(62)	8.1%
Rank Correlation		0.149		0.118	-0.041		-0.039
		(0.061)		(0.138)	(0.468)		(0.497)

a. Cases where stationary β is rejected against the AR(1) alternative, using the LBI test at 5% significance level.

b. The rank correlation between firm size (beginning of period market capitalisation) and the associated LBI statistic is reported.

c. Both subperiods were partitioned into five quintiles according to size. The smallest (largest) firms are represented by S1 (S5).

d. The p-value for the associated rank correlation is given in parentheses.

TABLE 4

PERCENTAGE OF SECURITIES WITH BETA NONSTATIONARITY[a]

BY INDUSTRY SECTOR: ALL RESOURCES VERSUS ALL INDUSTRIALS.

	1978/1982			1983/1987		
	Discrete	(N)	Continuous	Discrete	(N)	Continuous
EWMI						
ALL RESOURCES	26.2%	(42)	14.2%	22.2%	(135)	14.1%
ALL INDUSTRIALS	11.1%	(117)	11.1%	22.9%	(175)	12.6%
VWMI						
ALL RESOURCES	52.4%	(42)	45.2%	17.8%	(135)	11.1%
ALL INDUSTRIALS	29.9%	(117)	23.9%	23.4%	(175)	14.3%

a. Cases where stationary β is rejected against the AR(1) alternative, using the LBI test at 5% significance level.

TABLE 5

LBI TEST RESULTS OF BETA STATIONARITY :PERIOD 1978/1982.

PORTFOLIOS GROUPED ON BEGINNING OF PERIOD MARKET CAPITALISATION[a]

USING (A) DISCRETE AND (B) CONTINUOUS RETURNS; AND (C)

EQUALLY-WEIGHTED AND (D) VALUE-WEIGHTED MARKET INDEX

NUMBER OF REJECTIONS FOR THREE SIGNIFICANCE LEVELS

	Discrete Returns Significance levels			Continuous Returns Significance levels		
	0.01	0.05	0.10	0.01	0.05	0.10
N=7						
EWMI	2[b] (28.6%)	2 (28.6%)	3 (42.9%)	2 (28.6%)	2 (28.6%)	2 (28.6%)
VWMI	7 (100%)	7 (100%)	7 (100%)	6 (85.7%)	7 (100%)	7 (100%)
N=15						
EWMI	2 (13.3%)	4 (26.7%)	4 (26.7%)	2 (13.3%)	4 (26.7%)	4 (26.7%)
VWMI	12 (80%)	13 (86.7%)	14 (93.3%)	12 (80.0%)	13 (86.7%)	13 (86.7%)
N=31						
EWMI	3 (9.7%)	7 (22.6%)	9 (29.0%)	1 (3.2%)	6 (19.4%)	8 (25.8%)
VWMI	19 (61.3%)	23 (74.2%)	25 (80.6%)	11 (35.5%)	20 (64.5%)	23 (74.2%)

a. Securities are grouped into portfolios according to market capitalisation as measured at the end of December 1977.

b. The number of rejections for the sample at the stated significance level. This is expressed as a percentage of the sample in parenthesesd.

TABLE 6

LBI TEST RESULTS OF BETA STATIONARITY :PERIOD 1983/1987

PORTFOLIOS GROUPED ON BEGINNING OF PERIOD MARKET CAPITALISATION[a]

USING (A) DISCRETE AND (B) CONTINUOUS RETURNS; AND (C)

EQUALLY-WEIGHTED AND (D) VALUE-WEIGHTED MARKET INDEX.

NUMBER OF REJECTIONS FOR THREE SIGNIFICANCE LEVELS

		Discrete Returns Significance levels			Continuous Returns Significance levels		
		0.01	0.05	0.10	0.01	0.05	0.10
N=15							
	EWMI	7[b] (46.7%)	8 (53.3%)	9 (60.0%)	5 (33.3%)	7 (46.7%)	8 (53.3%)
	VWMI	6 (40.0%)	9 (60.0%)	9 (60.0%)	5 (33.3%)	8 (53.3%)	8 (53.3%)
N=31							
	EWMI	11 (35.5%)	17 (54.8%)	20 (65.5%)	4 (12.9%)	14 (45.2%)	18 (58.1%)
	VWMI	9 (29.0%)	14 (45.2%)	16 (51.6%)	6 (19.4%)	14 (45.2%)	15 (48.4%)
N=62							
	EWMI	18 (29.0%)	23 (37.1%)	24 (38.7%)	10 (16.1%)	16 (25.8%)	19 (30.6%)
	VWMI	14 (22.6%)	19 (30.6%)	25 (40.3%)	8 (12.9%)	15 (24.2%)	19 (30.6%)

a. Securities are grouped into portfolios according to market capitalisation as measured at the end of December 1982.

b. The number of rejections for the sample at the stated significance level. This is expressed as a percentage of the sample in parentheses.

TABLE 7

LBI TEST RESULTS OF BETA STATIONARITY : PORTFOLIOS[a]

GROUPED RANDOMLY (N=100) USING (A) DISCRETE AND

(B) CONTINUOUS RETURNS; AND (C) EQUALLY-WEIGHTED AND

(D) VALUE-WEIGHTED MARKET INDEX.

NUMBER OF REJECTIONS FOR THREE SIGNIFICANCE LEVELS.

	Discrete Returns Significance levels			Continuous Returns Significance levels		
	0.01	0.05	0.10	0.01	0.05	0.10
1978/1982						
EWMI	7[b]	13	20	7	11	18
VWMI	89	93	95	69	87	93
1983/1987						
EWMI	41	52	61	33	41	53
VWMI	36	48	56	30	47	51

a. Portfolios are comprised of ten securities each.

b The number of rejections for the sample at the stated significance level.

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