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IN THE LINEAR REGRESSION MODEL

Paramsothy Silvapulle and Maxwell L. King

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TESTING MOVING AVERAGE AGAINST AUTOREGRESSIVE DISTURBANCES

IN THE LINEAR REGRESSION MODEL

by

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Abstract

This paper considers testing for MA(1) against AR(1) disturbances in the linear regression model. Tests investigated include approximate point optimal invariant (POI) tests, an asymptotic test of the second-order residual autocorrelation coefficient and a Lagrange multiplier (LM) test. A Monte Carlo experiment compares their small-sample performances. Of the asymptotic tests, the LM test has the most satisfactory sizes, while its rival has the better overall power. We find the approximate POI tests have superior size and power properties in comparison to the asymptotic tests. An approximate POI test is applied to a random walk model for Australian real interest rates.

KEY WORDS: Lagrange multiplier tests; Monte Carlo method; Point optimal tests; Power; Size.

1. INTRODUCTION

The paradigm of first-order autoregressive (AR(1)) disturbances is central to the literature on serially correlated errors in the linear regression model. However, there is also a recognition of moving average (MA) errors as an alternative model. This is because of economic reasons such as intertemporal instability or the presence of adaptive or rational expectations [see for example, Rowley and Wilton (1973), Sims (1974), and Nicholls, Pagan and Terrell (1975)] and statistical reasons such as conventional tests for AR(1) disturbances also having good power against MA(1) disturbances [see for example, King (1987a)]. This has led to an interest in testing AR disturbances against MA disturbances in the linear regression model. King (1983) proposed a test of first-order processes based on the likelihood ratio statistic for chosen AR(1) and MA(1) processes. Its small-sample properties were compared by King and McAleer (1987) with those of the Cox test, some linearized Cox-type tests and the Lagrange multiplier (LM) test of AR(1) disturbances against ARMA(1,1) disturbances. Point-optimal invariant (POI) tests were constructed and investigated by King (1987b) while Burke, Godfrey and Tremayne (1990) proposed a simple test based on ordinary least squares (OLS) residuals.

The emphasis to date has been on testing the null hypothesis of AR disturbances against an MA alternative. Obviously it is desirable that the null hypothesis be MA errors when ones preferred model has an MA disturbance term. Given the strength of the AR(1) paradigm, it would seem prudent to present the results of a diagnostic test of the proposed error structure against an AR alternative.

The aim of this paper is to investigate the problem of testing

MA(1) against AR(1) disturbances in the linear regression model. While it may seem that procedures developed for testing AR(1) against MA(1) disturbances can easily be adopted to the reverse problem, we find below that this is not necessarily the case. For example, it is not possible to construct POI tests as outlined by King (1987b) and therefore we are forced to consider King's (1987b) approximate POI tests. Do these tests have vastly superior small-sample size and power properties compared to asymptotic tests? Does the LM test perform well? How good is the analogous test to Burke *et al.*'s (1990) simple test? These are some of the questions this paper seeks to answer.

The plan is as follows. Section 2 introduces the testing problem together with the approximate POI test, an analogous test to that proposed by Burke *et al.* and an LM test. Monte Carlo experiments are outlined and their results are reported in Section 3. The use of an approximate POI is illustrated by its application to a random walk model with seasonal drift parameters for Australian real interest rates in Section 4. Some concluding remarks are made in the final section.

2. THE MODEL AND TEST PROCEDURES

Consider the linear regression model

$$y = X\beta + u, \quad (1)$$

where y is an $n \times 1$ vector, X is an $n \times k$ non-stochastic matrix of rank $k < n$ and β is a $k \times 1$ parameter vector. If the components of the $n \times 1$ disturbance vector u are generated by the MA(1) process

$$u_t = e_t + \gamma e_{t-1}, \quad t=1, \dots, n, \quad (2)$$

where $e^* = (e_0, e_1, \dots, e_n)' \sim N(0, \sigma^2 I_{n+1})$, then $u \sim N(0, \sigma^2 \Omega(\gamma))$, where $\Omega(\gamma)$ is the $n \times n$ tridiagonal matrix with $1 + \gamma^2$ as the main

diagonal elements and γ as the non-zero off-diagonal elements. If the components of u are generated by the stationary AR(1) process,

$$u_t = \rho u_{t-1} + e_t, \quad |\rho| < 1, \quad t=1, \dots, n, \quad (3)$$

where $u_0 \sim N(0, \sigma^2/(1-\rho^2))$ and $e = (e_1, \dots, e_n)' \sim N(0, \sigma^2 I_n)$, then $u \sim N(0, \sigma^2 \Sigma(\rho))$ in which $\Sigma(\rho)$ is an $n \times n$ matrix whose $(i, j)^{th}$ element is $\rho^{|i-j|}/(1-\rho^2)$.

Our main interest is in testing

$$H_0 : u \sim N(0, \sigma^2 \Omega(\gamma)), \quad 0 \leq \gamma \leq 1,$$

against

$$H_a : u \sim N(0, \sigma^2 \Sigma(\rho)), \quad 0 < \rho < 1,$$

or

$$H_0^- : u \sim N(0, \sigma^2 \Omega(\gamma)), \quad -1 \leq \gamma \leq 0, \quad (4)$$

against

$$H_a^- : u \sim N(0, \sigma^2 \Sigma(\rho)), \quad -1 < \rho < 0. \quad (5)$$

The concentration on positive or negative correlation reflects the view that if one is willing to assume error processes such as (2) or (3), one is also typically willing to state the sign of the first-order autocorrelation coefficient. We feel positive autocorrelation is most likely, but if the dependent variable has been differenced, then negative autocorrelation can be a possibility. Whatever the case, it is highly likely one will have knowledge of the parameters' signs. Such knowledge should be incorporated into the test procedure.

Although we assume throughout this paper that u is normally distributed, this can be widened to elliptical symmetry without changing

any of what follows. This is a consequence of a result proved by King (1979) which implies that the distribution of any statistic which is invariant to the scale of u , is invariant to such a widening of the normality assumption. Examples of elliptically symmetric distributions include multivariate Student's t and Cauchy distributions. All test statistics considered below are invariant to the scale of u .

2.1 Approximate Point-Optimal Invariant Tests

The problem of testing H_0 against H_a (or H_0^- against H_a^-) is invariant to transformations of the form $y \rightarrow \eta_0 y + X\eta$ where η_0 is a positive scalar and η is a $k \times 1$ vector. Following King (1987b), it may be that the critical regions of a POI test with optimal power at $\rho = \rho_0 > 0$ (or at $\rho = \rho_0 < 0$ if testing H_0^- against H_a^-) will be of the form

$$s(\rho_0, \gamma_0) = \hat{u}' \Sigma(\rho_0)^{-1} \hat{u} / \tilde{u}' \Omega(\gamma_0)^{-1} \tilde{u} < c \quad (6)$$

in which \hat{u} and \tilde{u} are the generalized least squares residual vectors from (1) assuming covariance matrices $\Sigma(\rho_0)$ and $\Omega(\gamma_0)$, respectively. The existence of a POI test of this form requires the critical value c and the parameter γ_0 to be chosen such that

$$\Pr[s(\rho_0, \gamma_0) < c \mid u \sim N(0, \Omega(\gamma_0))] = \alpha \quad (7)$$

and

$$\Pr[s(\rho_0, \gamma_0) < c \mid u \sim N(0, \Omega(\gamma)), 0 \leq \gamma \leq 1] \leq \alpha \quad (8)$$

where α is the desired level of significance. The inequality $0 \leq \gamma \leq 1$ in (8) is replaced by $-1 \leq \gamma \leq 0$ if one is testing H_0^- against H_a^- . If c and γ_0 cannot be chosen to solve (7) and (8) simultaneously, then King (1987b) recommends the use of an approximately POI test. Such tests have critical regions of the form of (6) with c chosen such that (8)

holds and γ_0 chosen to make LHS of (7) as close to α as possible.

The LHS of (8) (of which (7) is a special case) can be written as

$$\Pr[u'(\Delta_1 - c\Delta_0)u < 0 \mid u \sim N(0, \Omega(\gamma))] \quad (9)$$

in which $\Delta_0 = \Omega(\gamma_0)^{-1} - \Omega(\gamma_0)^{-1}X(X'\Omega(\gamma_0)^{-1}X)^{-1}X'\Omega(\gamma_0)^{-1}$, and $\Delta_1 = \Sigma(\rho_0)^{-1} - \Sigma(\rho_0)^{-1}X(X'\Sigma(\rho_0)^{-1}X)^{-1}X'\Sigma(\rho_0)^{-1}$. We can evaluate (9) using standard numerical methods based on Imhof's (1961) algorithm as outlined in King (1987b).

Our experience in performing the calculations reported below is that, at least for cases we considered, it is almost never possible to find c and γ_0 values that solve (7) and (8) simultaneously. This is because the LHS of (8), for a fixed choice of γ_0 , is almost always a monotonic function of γ . When γ_0 is small, the size function (LHS of (8)) is monotonic decreasing while for large γ_0 it is monotonic increasing. For a small band of γ_0 values, the size function has the shape of a flat parabola, first decreasing and then increasing with γ . For example, for design matrix X_1 defined below and $n = 20$, this occurred at $\gamma_0 = 0.406$ while the size function is monotonic decreasing for $\gamma_0 \leq 0.405$ and monotonic increasing for $\gamma_0 \geq 0.407$. It is therefore not possible to find a γ_0 value that also coincides with the maximum of the LHS of (8) thus allowing (7) and (8) to hold simultaneously. The only exception found was for an extreme case.

We therefore turn our attention to the class of approximately POI tests suggested by King (1987b) as an alternative to POI tests and found to work well for the related nested problem of testing for seasonal autocorrelation in the presence of AR(1) errors in (1) (King (1989)).

Approximate POI tests require γ_0 to be chosen so that (8) holds and

$$\alpha = \Pr[s(\rho_0, \gamma_0) < c \mid u \sim N(0, \Omega(\gamma_0))] \quad (10)$$

is minimized.

Because of the typical monotonic nature of the LHS of (8), if c is determined for fixed ρ_0 and γ_0 values so that (8) holds, one gets equality either at $\gamma = 0$ or $\gamma = 1.0$. Like King (1989), we found that choosing γ_0 to be that value which results in global maxima of the size function at both $\gamma = 0$ and $\gamma = 1.0$ minimizes (10) as required. Thus, for given values of ρ_0 and α , an approximately POI test can be constructed by the following iterative procedure:

- (i) Guess a possible value for γ_0 which is denoted γ_0^* .
- (ii) Noting that $\Omega(0) = I_n$, solve (using the secant method or otherwise) for c^* ,

$$\Pr[s(\rho_0, \gamma_0^*) < c^* \mid u \sim N(0, \Omega(0))] = \alpha.$$

- (iii) Evaluate

$$\Pr[s(\rho_0, \gamma_0^*) < c^* \mid u \sim N(0, \Omega(1.0))]. \quad (11)$$

- (iv) If (11) equals α then the desired γ_0 and c values have been found. If (11) is less (greater) than α , make γ_0^* larger (smaller) and repeat (ii) and (iii). Again the secant method can be used to determine the size of changes in γ_0^* .

2.2 A Simple Alternative Test

Burke *et al.* (1990) proposed a simple test for AR(1) against MA(1) disturbances based on testing the restriction

$$\rho_2 = (\rho_1)^2 \quad (12)$$

where ρ_1 and ρ_2 are the first- and second-order autocorrelation coefficients, respectively. (12) holds for AR(1) disturbances while the analogous restriction for MA(1) disturbances is $\rho_2 = 0$ upon which a test of MA(1) against AR(1) disturbances can be based. Denote by

$$r_i = \frac{\sum_{t=i+1}^n z_t z_{t-i}}{\sum_{t=1}^n z_t^2},$$

the i^{th} -order residual autocorrelation coefficient where $z = (z_1, \dots, z_n)'$ are the OLS residuals from (1). Based on results from Bartlett (1946, p.28) and under regularity conditions on the regressors outlined by Burke *et al.* (1990), one can show that

$$\tau = \sqrt{n} r_2 / (1 + 2r_1^2)^{1/2}$$

has an asymptotic standard normal distribution under H_0 . An asymptotic test might therefore be based on rejecting H_0 for large values of τ . In the remainder of this paper, we call this the τ test. Note that a test of H_0^- against H_a^- would also reject H_0^- for large values of τ .

2.3 The Lagrange Multiplier Test

As a test of AR(1) against MA(1) disturbances, King and McAleer (1987) concluded that the LM test of AR(1) against ARMA(1,1) disturbances has reasonable properties. We therefore consider the LM test of MA(1) against ARMA(1,1) disturbances. Write (1) as

$$y_t = x_t' \beta + u_t, \quad t = 1, 2, \dots, n,$$

where x_t' is the t^{th} row of X . Let $(\hat{\beta}', \hat{\gamma}, \hat{\sigma}^2)$ denote the maximum likelihood estimates of $(\beta', \gamma, \sigma^2)$ under H_0 . Let the corresponding residuals be denoted by

$$\hat{u}_{0,t} = y_t - x_t' \hat{\beta}, \quad t = 1, 2, \dots, n,$$

with $\hat{u}_{0,t} = 0$ for $t = 0$. The predicted values of y_t and the associated prediction errors under H_0 are

$$\hat{y}_{0,t} = x_t' \hat{\beta} + \hat{\gamma} \hat{\varepsilon}_{0,t-1}$$

and

$$\hat{\varepsilon}_{0,t} = y_t - \hat{y}_{0,t}, \quad t = 1, 2, \dots, n$$

with $\hat{\varepsilon}_{0,t} = 0$ for $t = 0$.

Let y^* and X^* denote y and X after each column has been transformed by the recursive scheme given by equations (14) in King and McAleer (1987) with $\gamma_0 = \hat{\gamma}$. For $\gamma_0 = \gamma$, this amounts to transforming (1) so that the distribution of the transformed disturbance term is $N(0, \sigma^2 I_n)$ under H_0 . Following Gill *et al.* (1986) and King and McAleer (1987), the LM test of MA(1) against ARMA(1,1) disturbances may be conducted using the auxiliary regression

$$y_t^* = x_t^{*'} \beta + \phi \hat{u}_{0,t-1} + \theta \hat{\varepsilon}_{0,t-1} + \varepsilon_t.$$

The test procedure is to estimate this regression by OLS and then test $H'_0 : \theta = 0$ based on the t-ratio which, asymptotically, is standard normal under H_0 .

3. AN EMPIRICAL COMPARISON OF SIZES AND POWERS

This section reports the results of a Monte Carlo study conducted to assess the small-sample performance of three versions of the approximate POI test, namely the $s(0.3, \gamma_0^*)$, $s(0.5, \gamma_0^*)$ and $s(0.75, \gamma_0^*)$ tests and the τ and LM tests. We included a third asymptotic test, this being analogous to the prediction test examined by King and McAleer

(1987). Its small-sample properties were found to be clearly inferior to those of the other tests and so are not reported. Because one may wish to test assuming negative autocorrelation, we also report empirical size and power comparisons of the $s(-0.3, \gamma_0^*)$, $s(-0.5, \gamma_0^*)$ and $s(-0.75, \gamma_0^*)$ tests of H_0^- against H_a^- .

3.1 Experimental Design

The first part of the experiment involved using the Monte Carlo method to estimate probabilities of a Type I error under H_0 at $\gamma = 0.0, 0.1, \dots, 0.9$, for the τ and LM tests at the five per cent nominal level. The second part involved two stages, the first being the calculation of appropriate critical values so that the comparison of powers, which constitutes the second stage, can be made at approximately the same level of significance. Critical values and γ_0^* values for the approximate POI tests were calculated as outlined in section 2.1. For the τ and LM tests, the Monte Carlo method was used to estimate exact-size critical values at $\gamma = 0.0, 0.1, \dots, 0.9$ under H_0 . From each set of ten critical values, the largest was selected, thus ensuring that, at least at the chosen points, the size of the test does not exceed five per cent. The small-sample sizes and powers were then calculated using the methodology discussed in section 2.1 for the approximate POI tests and the Monte Carlo method for the τ and LM tests. Sizes were calculated at $\gamma = -0.4, -0.2, 0.0, 0.1, 0.3, 0.5, 0.7, 0.9$ and powers were calculated at $\rho = -0.4, -0.2, 0.1, 0.3, 0.5, 0.7, 0.9$. Negative values of γ and ρ were included in order to investigate the consequences of wrongly assuming positive autocorrelation in the presence of mild negative autocorrelation. Finally, the small-sample sizes and powers of the $s(-0.3, \gamma_0^*)$, $s(-0.5, \gamma_0^*)$ and $s(-0.75, \gamma_0^*)$ tests

were calculated at γ and ρ values of -0.1, -0.3, -0.5, -0.7, -0.9.

The following design matrices were used in the comparisons:

- X1 ($n \times 3$, $n = 20, 60$). The first n observations of Durbin and Watson's (1951) consumption of spirits example.
- X2 ($n \times 3$, $n = 20, 60$). The regressors are a constant dummy, the Australian quarterly consumers' price index (CPI) commencing 1959(1) and the same index lagged one quarter.
- X3 ($n \times 5$, $n = 20, 60$). X2 augmented by adding the CPI lagged two and three quarters.
- X4 ($n \times 3$, $n = 20, 30$). The regressors are a constant, logarithms of Chow's (1957, Table 1) automobile stock per capita and personal money stock per capita variables for the United States 1921-1950.
- X5 ($n \times 3$, $n = 20, 60$). The regressors are the eigenvectors corresponding to the three smallest eigenvalues of the $n \times n$ Durbin-Watson differencing matrix A_1 . Note that the first regressor is constant.

The Monte Carlo method was applied using two thousand replications. The results of Breusch (1980) imply that the sizes and powers of each of the tests are invariant to the values taken by β and σ^2 , so for the purpose of applying the Monte Carlo method, β_i , $i = 1, \dots, k$, and σ^2 were all set to unity. Maximum likelihood estimates of β , σ^2 and γ under H_0 were computed using Ansley's (1979) approach for estimating ARMA processes applied to the linear regression. This reduces the maximization problem to a sum of squares minimization problem which was handled by the IMSL constrained minimization subroutine ZXMWd with the constraint that $|\gamma| \leq 1$ imposed. When

required, probabilities of the form of (9) were calculated using a modified version of Koerts and Abrahamse's (1969) FQUAD subroutine.

3.2 The Results

Table 1 reports the maximum and minimum estimated sizes for $\gamma = 0.0, 0.1, \dots, 0.9$ under H_0 of the τ and LM tests using asymptotic critical values at the five per cent significance level. It is noticeable that all estimated sizes of the τ test are less than the nominal size of 0.05 while those of the LM test are greater than 0.05. In all cases, the estimated sizes become closer to the nominal sizes when n increases, *ceteris paribus*. Also, for the LM test, there is a tendency for the size to increase as the number of regressors increases, *ceteris paribus*. For $n = 20$, the sizes of the τ test, which range from zero to 0.006 are completely unacceptable. Most researchers would also find the sizes of the LM test, for $n = 20$, unacceptable because they can be as high as 0.218. The sizes of the τ test, for $n = 60$, are also less than satisfactory as they are never greater than half the nominal size. The most favourable estimated sizes for $n = 60$ (and $n = 30$ for X4) are those of the LM test which range from 0.056 to 0.081.

We now discuss the comparison of tests' sizes and powers at approximately the five per cent level. Selected calculated sizes and powers for X2 are presented in Tables 2. The discussion below is based on all the results which are available from the authors upon request.

All calculated sizes vary with γ under H_0 and, because of their method of calculation, lie between zero and 0.05. The asymptotic tests have noticeably greater variation in size than the approximate POI tests which almost always have sizes closer to 0.05 than do the asymptotic

tests. While the lowest size of an approximate POI test is 0.040, the lowest sizes of the τ and LM tests are 0.003, and 0.001, respectively. For the latter tests, this variation in size tends to decrease as n increases while for the $s(\rho_0, \gamma_0^*)$ tests, there is a slight tendency for it to increase. All three $s(\rho_0, \gamma_0^*)$ tests have very similar sizes.

With two exceptions at $\rho = 0.1$, the powers of all tests increase as n increases, *ceteris paribus*. Also, powers increase as ρ increases with the exception of the τ test for small ρ values and small n . While the powers of the $s(\rho_0, \gamma_0^*)$ tests are always equal to or greater than the significance level, this is not the case for the asymptotic tests. For all sample sizes, their powers at $\rho = 0.1$ are below 0.05. When $n = 30$ or 60, the τ test is always more powerful than the LM test with power differences ranging up to 0.191 for $X_1 - X_4$ and up to 0.366 for X_5 . When $n = 20$, the τ test is typically more powerful than the LM test for $\rho \leq 0.5$ while the reverse is the case for $\rho \geq 0.7$. It seems that the τ test has the best overall power properties of the asymptotic tests, particularly for larger sample sizes.

The results clearly demonstrate that there is a distinct power advantage to be gained from using an approximate POI test. In all cases, the approximate POI tests are always more powerful than the asymptotic tests. In fact, across all design matrices, the power superiority of the $s(0.3, \gamma_0^*)$ test over the τ test ranges up to 0.287 for $n = 20$ and 0.178 for $n = 60$ while its superiority over the LM test ranges up to 0.279 for $n = 20$ and 0.492 for $n = 60$. In general, the differences between the powers of the three $s(\rho_0, \gamma_0^*)$ tests are small, usually being reflected in the third decimal place, particularly for larger sample sizes. Typically, the $s(0.3, \gamma_0^*)$ test is the most powerful

for small ρ values while the $s(0.5, \gamma_0^*)$ and $s(0.75, \gamma_0^*)$ tests are the most powerful for medium and large values of ρ , respectively. The $s(0.5, \gamma_0^*)$ test seems to have the best overall power properties, but only just.

The τ test is clearly the most powerful test when one wrongly assumes positive autocorrelation in the presence of mild negative autocorrelation. This may be partly due to its higher than nominal sizes under H_0^- , especially for $n = 20$. All approximate POI tests have sizes slightly above 0.05 under H_0^- and substantially better power than the LM test under H_a^- . The latter test has sizes well below 0.05 under H_0^- and its powers are above 0.05 only for $\rho = -0.4$ and $n > 20$.

Finally, we turn to the comparison of sizes and powers of three approximately POI tests of H_0^- against H_a^- . Selected calculated sizes and powers for X2 and X3 are presented in Table 3. The sizes of the approximate POI tests of H_0^- exhibit similar patterns to those for the analogous tests of H_0 . The major exception is the $s(-0.75, \gamma_0^*)$ test for X3 with $n = 20$ which is the one case in which a true POI test was found. While the sizes of all other tests first decrease and then increase as γ approaches zero, the sizes of this POI test always decrease. The lowest size of an approximate POI test is 0.040 and all three tests have very similar sizes. The variation in size with γ shows a slight tendency to increase with sample size.

The powers of all tests increase as $|\rho|$ increases and as n increases, *ceteris paribus*. With the exception of the true POI test, namely $s(-0.75, \gamma_0^*)$ for X3 with $n = 20$, all powers are greater or equal to the significance level. Typically, the differences between the powers of the three tests are small, particularly for larger sample sizes. The $s(-0.5, \gamma_0^*)$ test seems to have marginally better overall

power properties than its rivals. It is noticeable that the powers against H_a^- are higher than the corresponding powers against H_a , especially for large $|\rho|$ values when $n = 20$.

4. AN ILLUSTRATIVE EXAMPLE

In this section, we illustrate the application of an approximate POI test to a simple linear model for Australian real interest rates. Consider the quarterly regression model

$$\Delta r_t = \beta_1 + \beta_2 S_{1t} + \beta_3 S_{2t} + \beta_4 S_{3t} + u_t \quad (13)$$

where Δ denotes first differences, $r_t = R_t - \Pi_t$ is the *ex post* real interest rate, R_t is the nominal interest rate as measured by the 90-day bank-accepted-commercial-bill rate, Π_t is the annual inflation rate, and S_{1t} , S_{2t} and S_{3t} are quarterly seasonal dummy variables. This is a random walk model with seasonal drift. Its rationale comes from the random walk hypothesis and the work of Kinal and Lahiri (1988) on US interest rates. They reported the following model, among others, estimated over the period 1953(1) to 1979(4):

$$\begin{aligned} \hat{r}_t = & 0.973 + 0.999r_{t-1} - 1.313S_{1t} - 2.383S_{2t} - 0.195S_{3t} , \\ & (4.415) \quad (29.649) \quad (-2.495) \quad (-2.316) \quad (-0.463) \\ & R^2 = 0.251 , \end{aligned}$$

with t-ratios given in parentheses.

Using quarterly Australian data for the period 1969(2) to 1987(1), estimation of (13) by OLS results in

$$\begin{aligned} \Delta \hat{r}_t = & -2.959 + 8.283S_{1t} + 2.074S_{2t} + 2.022S_{3t} , \\ & (-2.559) \quad (4.992) \quad (1.250) \quad (1.218) \\ & R^2 = 0.299, \quad d = 2.882. \end{aligned}$$

Comparison of the DW statistic, d , with exact critical values given by King (1980) reveals significant negative first-order autocorrelation at the one per cent level; the one per cent critical value being 2.525. Kinal and Lahiri (1988) give economic reasons why the error term in (13) should follow a negative MA(1) process. An obvious alternative hypothesis is that it follows a negative AR(1) process.

We tested H_0^- given by (4) against H_a^- as in (5) by applying the approximate POI test based on $s(-0.5, \gamma_0^*)$. Using the methodology of section 2.1, we found $\gamma_0^* = -0.425$ and $c = 0.957$ jointly solved

$$\Pr[s(-0.5, \gamma_0^*) < c \mid u \sim N(0, \Omega(\gamma)), -1 \leq \gamma \leq 0] \leq \alpha$$

with equality at $\gamma = 0$ and -1 , when the significance level, α , was set at five per cent. The test statistic was calculated as

$$s(-0.5, -0.425) = \tilde{v}'\tilde{v} / v^*{}'v^* = 1.321$$

where \tilde{v} is the OLS residual vector from (1) after it has been transformed by the AR(1) Prais-Winsten transformation with $\rho = -0.5$ and v^* is the OLS residual vector from (1) after transformation by the recursive equations (14) in King and McAleer (1987) with $\gamma_0 = -0.425$. Because this calculated value is greater than the critical value, we cannot reject H_0^- at the five per cent significance level.

Finally, note that when H_a^- is tested as the null hypothesis against H_0^- using King's (1987b) POI test of AR(1) against MA(1) disturbances that optimizes power at $\gamma = -0.5$, we get $\rho_0^* = -0.4$ and the critical value $c = 0.953$, at the five per cent level. The calculated value of the test statistic, $r_\rho(-0.5) = 0.817$, suggests that we can reject H_a^- , thus reinforcing the result of the previous test.

5. CONCLUDING REMARKS

Of the asymptotic tests considered in this paper, the τ test seems the most promising, particularly for large sample sizes. While being clearly more powerful than the LM test when $n = 60$, its main drawback is that its true sizes are too small, being less than half the nominal size. It is tempting to suggest the test be applied at twice the desired significance level. However, we do not have any theoretical results to support this proposal.

This paper's main finding is that approximate POI tests have extremely desirable small-sample properties. Their true sizes are always close to the nominal size and their powers are not less than the significance level. Furthermore, at least for the cases we considered, they are always more powerful than the asymptotic tests, the extra computation required to apply an approximate POI test seems to be well rewarded. Also, it is noteworthy that the sizes and powers of the tests are largely insensitive to the choice of ρ_0 value in $s(\rho_0, \gamma_0^*)$. These positive findings are of wider interest. So far little is known about the properties of approximate POI tests, particularly in the context of non-nested testing.

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Table 1. Minimum and Maximum Values of the Estimated Size Functions of the Asymptotic Tests at the Five Per Cent Nominal Significance Level when $H_0 : u_t = \varepsilon_t + \gamma \varepsilon_{t-1}$ is Tested for $\gamma = 0.0(0.1)0.9$.

Data matrix	n	Test Statistic			
		τ		LM	
		<u>min</u>	<u>max</u>	<u>min</u>	<u>max</u>
X1	20	0.000	0.006	0.063	0.120
(n×3)	60	0.002	0.017	0.056	0.071
X2	20	0.000	0.004	0.066	0.133
(n×3)	60	0.008	0.023	0.058	0.073
X3	20	0.000	0.003	0.078	0.218
(n×5)	60	0.008	0.025	0.063	0.081
X4	20	0.000	0.004	0.067	0.149
(n×3)	30	0.001	0.011	0.059	0.068
X5	20	0.000	0.005	0.080	0.212
(n×3)	60	0.007	0.022	0.070	0.080

Table 2. Calculated Sizes and Powers for X2 when $H_0 : u_t = \varepsilon_t + \gamma\varepsilon_{t-1}$ is Tested against $H_a : u_t = \rho u_{t-1} + \varepsilon_t$ at the Five Per Cent Level.

Parameter Values	Test Statistics				
	$s(0.3, \gamma_0^*)$	$s(0.5, \gamma_0^*)$	$s(0.75, \gamma_0^*)$	τ	LM
<hr/>					
H_0	$n = 20$				
$\gamma = 0.9$	0.049	0.049	0.049	0.008	0.050
0.7	0.049	0.048	0.048	0.008	0.031
0.5	0.046	0.046	0.045	0.011	0.023
0.3	0.044	0.045	0.044	0.017	0.016
0.1	0.048	0.047	0.046	0.037	0.009
0.0	0.050	0.050	0.050	0.050	0.004
H_a					
$\rho = 0.1$	0.052	0.051	0.050	0.040	0.008
0.3	0.076	0.077	0.073	0.029	0.019
0.5	0.156	0.156	0.154	0.055	0.035
0.7	0.311	0.324	0.324	0.126	0.129
0.9	0.504	0.529	0.540	0.254	0.260
H_0^-					
$\gamma = -0.4$	0.075	0.075	0.068	0.135	0.009
-0.2	0.066	0.065	0.061	0.106	0.005
H_a^-					
$\rho = -0.2$	0.068	0.066	0.059	0.137	0.006
-0.4	0.118	0.108	0.104	0.310	0.033
<hr/>					
H_0	$n = 60$				
$\gamma = 0.9$	0.049	0.049	0.049	0.021	0.050
0.7	0.048	0.047	0.046	0.022	0.040
0.5	0.045	0.043	0.042	0.025	0.035
0.3	0.043	0.042	0.041	0.034	0.024
0.1	0.046	0.044	0.043	0.044	0.019
0.0	0.050	0.050	0.050	0.050	0.010
H_a					
$\rho = 0.1$	0.053	0.052	0.051	0.048	0.014
0.3	0.136	0.132	0.126	0.100	0.099
0.5	0.455	0.453	0.442	0.343	0.252
0.7	0.870	0.875	0.872	0.784	0.649
0.9	0.990	0.991	0.992	0.976	0.793
H_0^-					
$\gamma = -0.4$	0.089	0.088	0.080	0.086	0.030
-0.2	0.068	0.066	0.060	0.075	0.020
H_a^-					
$\rho = -0.2$	0.071	0.067	0.060	0.120	0.020
-0.4	0.246	0.246	0.232	0.346	0.110

Table 3: Calculated Sizes and Powers for X2 and X3 When

$$H_0^- : u_t = \varepsilon_t + \gamma \varepsilon_{t-1}, \gamma \leq 0 \text{ is Tested Against}$$

$$H_a^- : u_t = \rho u_{t-1} + \varepsilon_t, \rho \leq 0 \text{ at the Five Per Cent Level.}$$

Parameter Values	n = 20			n = 60		
	$s(-0.3, \gamma_0^*)$	$s(-0.5, \gamma_0^*)$	$s(-0.75, \gamma_0^*)$	$s(-0.3, \gamma_0^*)$	$s(-0.5, \gamma_0^*)$	$s(-0.75, \gamma_0^*)$
<hr/>						
H_0	X2					
$\gamma = -0.9$	0.049	0.050	0.049	0.050	0.050	0.049
-0.7	0.048	0.048	0.047	0.048	0.049	0.046
-0.5	0.045	0.045	0.045	0.044	0.045	0.042
-0.3	0.044	0.043	0.045	0.043	0.041	0.041
-0.1	0.048	0.047	0.048	0.049	0.047	0.048
H_a						
$\rho = -0.1$	0.051	0.050	0.049	0.054	0.053	0.051
-0.3	0.083	0.081	0.080	0.141	0.136	0.133
-0.5	0.215	0.217	0.214	0.505	0.508	0.494
-0.7	0.531	0.546	0.546	0.928	0.933	0.930
-0.9	0.876	0.889	0.894	0.999	0.999	0.999
<hr/>						
H_0	X3					
$\gamma = -0.9$	0.049	0.050	0.050	0.050	0.050	0.048
-0.7	0.048	0.049	0.048	0.048	0.048	0.046
-0.5	0.046	0.046	0.044	0.045	0.044	0.042
-0.3	0.046	0.045	0.041	0.043	0.043	0.047
-0.1	0.048	0.047	0.040	0.049	0.047	0.048
H_a						
$\rho = -0.1$	0.052	0.050	0.043	0.054	0.052	0.050
-0.3	0.078	0.078	0.069	0.135	0.134	0.122
-0.5	0.185	0.188	0.181	0.480	0.482	0.466
-0.7	0.460	0.484	0.487	0.913	0.919	0.918
-0.9	0.831	0.855	0.866	0.998	0.999	0.999

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