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Grant H. Hillier

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ON MULTIPLE DIAGNOSTIC PROCEDURES FOR THE LINEAR MODEL

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ABSTRACT

Multiple diagnostic procedures entail a partition of the sample space into a set of critical regions. When the procedure is designed in a piecemeal fashion these regions bear no particular relation to one another, and the procedure as a whole can be logically inconsistent. We use a simple example to illustrate these difficulties, and also to suggest criteria that should be satisfied by any diagnostic procedure that is to meet the needs of the practitioner.

1. INTRODUCTION

The recent surge of interest in diagnostic testing has spawned a new breed of computer package which will automatically provide the user with as many as ten different 'diagnostic tests' of the key assumptions embodied in his/her model. I assume that this proliferation of diagnostics is intended to supply two important desiderata for model-based analysis of data, namely, procedures that enable the user to (i) make, and be seen to make, a 'serious attempt' to contradict those key assumptions, and/or (ii) obtain evidence about the direction or directions in which the model needs revision.

The profession seems to be agreed that the calculation of a number of diagnostic test statistics can meet these needs, and recent theoretical work has been concentrating largely on the question of which particular statistics are appropriate (cf. other papers in this volume). In this paper I shall argue that, if it is agreed that multiple diagnostic procedures (MDPs) are indeed intended for these purposes, then the present preoccupation with *test statistics* is misplaced. Specifically, I shall argue that, just as in classical hypothesis testing, attention should be focused on the design of *critical regions*, not test statistics, and that MDPs, as currently practised, are unlikely to produce critical regions with desirable properties.

My point of departure is the view that a MDP applied in the context of a single sample of data should be viewed as a single decision-making procedure, and should be designed accordingly. There are signs of a recognition of the need for a coherent strategy in the present volume, but it is fair to say that current diagnostic practice consists of a piecemeal approach in which tests developed for single alternative hypotheses are used jointly. Most of this paper is devoted simply to pointing out - via a simple and familiar example - the logical problems inherent in this piecemeal approach. Only briefly (Section 4) do I attempt (and this still in the context of the simple example) what might be thought of as a prescription for the design of MDPs in general. The main conclusions of the paper are that the problems are difficult and much needs to be done.

Some of the points made below are closely related to a familiar objection to multiple testing procedures, namely, the non-independence of the test statistics used. Much of what follows could be viewed as an elaboration of the consequences of this fact. However, taking explicit account of the structure of the critical regions involved adds insights that are not otherwise apparent, and also helps to suggest appropriate procedures.

For expositional purposes I shall confine attention here to the classical linear model and, in addition, to MDPs that are exact in the sense that the probability of rejecting what I shall call the null model when it is correct is known (at least in principle). Section 2 gives a characterization theorem for this class of procedures that is of independent interest. Since the class of exact procedures is characterized by a particular statistic, critical regions for both conventional tests and MDPs in this class are subsets of the (non-Euclidean) sample space for this statistic. In more general contexts the critical regions would simply be subsets of the original sample space, and the arguments below apply equally well to these.

2. NULL MODEL AND CHARACTERIZATION OF EXACT PROCEDURES

Suppose that the null model for the n-dimensional data vector y is specified as:

$$H_0 \quad : \quad y \sim N(X\beta,\ \sigma^2 I_n) \qquad (1)$$

where X is $n \times k$, fixed, and of full column rank. Alternatives to H_0 are unlimited but include (i) non-normality, (ii) a more complicated structure for the mean, E(y) (e.g., non-linear in X and/or β, or, more simply, $E(y) = X\beta + Z\gamma$ for some Z and γ), (iii) a more complex structure for the covariance matrix (e.g. non-constant variances and/or non-independence), (iv) more complicated structures for both the mean and covariance matrix induced by lag relationships.

If, as seems natural, we regard any diagnostic procedure as a test of H_0, the parameters β and σ^2 are nuisance parameters. However, the statistics $\hat{\beta} = (X'X)^{-1}X'y$ and $s^2 = y'(I_n - X(X'X)^{-1}X')y$ are jointly sufficient for (β, σ^2), and are boundedly complete under H_0. Hence (see Cox and Hinkley (1974), Kendall and Stuart (1969), or Hillier (1987)) every similar test of H_0 must have constant size in the conditional distribution of y given $\hat{\beta}$ and s^2, so that the conditional distribution $pdf(y|\hat{\beta}, s^2)$ characterizes the class of similar tests of H_0 *against any alternative whatever*. Rather than working with this conditional distribution, however, we prefer to use a particular *statistic* to characterize the class of similar tests, and we next construct this statistic.

Let C ($n \times m$; $m = n - k$) be such that $CC' = I_n - X(X'X)^{-1}X'$ and $C'C = I_m$. Define $w = C'y$, ($m \times 1$), and note that the transformation $y \to (\hat{\beta}, w)$ is one-to-one and that, under H_0, w is independent of $\hat{\beta}$, and

$w \sim N(0, \sigma^2 I_m)$. Now let $v = w(w'w)^{-1/2}$ and $s^2 = w'w$. The transformation $w \to (v, s^2)$ is one-to-one almost everywhere in R^m (the exceptional point being the origin, which is clearly of measure zero), v is a point on the surface of the unit m-sphere

$$S_m = \{v;\ v \in R^m,\ v'v = 1\},$$

and $s^2 > 0$. Lebesgue measure, (dw), on R^m transforms as

$$(dw) = 2^{-1}(s^2)^{m/2-1}\ (ds^2)(v'dv)\ ,$$

where (ds^2) denotes Lebesgue measure on R^+, and $(v'dv)$ denotes invariant (Haar) measure on S_m (cf Muirhead (1982), pp.67-72). Because $w \sim N(0,\sigma^2 I_m)$ under H_0, v and s^2 are independent under H_0, $s^2/\sigma^2 \sim \chi^2(m)$, and v is uniformly distributed on S_m under H_0 ($pdf(v) = A^{-1}(m)$, where $A(m) = 2\pi^{m/2}/\Gamma(m/2)$ is the surface content of S_m).

We have thus established a transformation $y \to (\hat{\beta},\ s^2,\ v)$ which is almost surely one-to-one, and with the property that, in particular, v is independent of $\hat{\beta}$ and s^2 under H_0. The following result now follows immediately:

Theorem 1: The class of similar tests of H_0 *against any alternative whatever* is characterized by the vector v, a point on S_m, and under H_0 v is uniformly distributed on S_m. That is, a critical region for testing H_0 has size independent of β and σ^2 if and only if it is defined in terms of v alone.

Note that the vector of OLS residuals $\hat{u} = y - X\hat{\beta}$ can be expressed in terms of v, s, and C as $\hat{u} = sCv$. Any scale-invariant test statistic computed from the OLS residuals (hence, almost all commonly used diagnostics for this model) therefore yields a test that, in principle at least, is exact. This result therefore provides a theoretical basis for the otherwise intuitively reasonable practice of basing diagnostics for H_0 on scale invariant functions of the OLS residuals. Andrews (1971) comes close to stating this result, and also uses much the same kind of geometrical approach that we shall rely on below.

Let $\tilde{u} = \hat{u}/s = Cv$ denote the vector of scaled OLS residuals. If c_i' denotes the i-th row of C, we have $\tilde{u}_i = c_i'v$, $i=1,\ldots,n$. In terms of v, the critical regions for four commonly used diagnostics are:

(i) <u>F-tests for additional variables</u>

The standard F-test for $\gamma = 0$ under $y \sim N(X\beta + Z\gamma, \sigma^2 I_n)$, where Z is $n \times p$, say, rejects H_0 when

$$d_F = v'\tilde{Z}(\tilde{Z}'\tilde{Z})^{-1}\tilde{Z}'v > c_1 \qquad (2)$$

where $\tilde{Z} = C'Z$ $(m \times p)$.

(ii) <u>The Durbin-Watson Test</u> (Durbin and Watson (1950))

This rejects H_0 against positive first-order autocorrelation when

$$d_{DW} = v'Av < c_2 , \qquad (3)$$

where $A = C'A_1C$, with A_1 the usual Durbin-Watson first-differencing matrix.

(iii) <u>The Breusch-Pagan Test for Heteroscedasticity</u>

The score test of H_0 against heteroscedasticity generated by a scalar function of $(\alpha_0 + \alpha_1 W_{1i} + \ldots + \alpha_r W_{ri})$, where $(W_{1i},\ldots,W_{ri})$, $i=1,\ldots,n$, are known, rejects H_0 when

$$d_H = f'W(W'W)^{-1}W'f > c_3 \qquad (4)$$

where W $(n \times (r + 1))$ has i^{th} row $(1, W_{1i},\ldots W_{ri})$, and f $(n \times 1)$ has i^{th} element $f_i = n\tilde{u}_i^2 - 1$ (Breusch and Pagan (1980)).

(iv) The Jarque-Bera Test for Non-Normality

The score test of H_0 against non-normal alternatives in the Pearson family rejects H_0 when

$$d_N = n(\Sigma\tilde{u}_i^3)^2 + (n\ \Sigma\tilde{u}_i^4 - 3)^2/4 > c_4 \qquad (5)$$

(Jarque and Bera (1987)).

In (2)-(5) the positive constants c_1, c_2, c_3, c_4 are chosen to give critical regions of the required sizes. Since each statistic is a function of v alone, and v is uniformly distributed on S_m under H_0, c_1-c_4 can, in principle, be chosen to give exactly the size required. In practice, however, c_3 and c_4 are usually chosen on the basis of asymptotic arguments.

Each of these tests and, in general, any similar test of H_0 against a specific alternative, has as its critical region some subset of the surface of the unit sphere S_m. The same is true, of course, of any exact test of H_0 against a family of alternatives that departs from H_0 in several respects (e.g., the Bera-Jarque (1982) test of H_0 against non-normal, heteroscedastic, and serially dependent alternatives), and also of tests against a specific alternative that are 'robust' to departures from H_0 in other directions, provided only that the critical region is defined in terms of scale-invariant functions of the OLS residuals.

We shall say that a multiple diagnostic procedure is an *exact* MDP if the overall probability of rejecting H_0 when it is true is free of unknown parameters, and we shall refer to the probability content of the overall critical region when H_0 is true as the *size* of the MDP. It follows from Theorem 1 that any exact MDP for H_0 in (1) has as its critical region a collection of subsets of the surface S_m. As diagnostic procedures are currently practised these subsets are defined by inequalities (2)-(5) and, increasingly, others like them.

3. THE DESIGN OF MDPs AND REMARKS ON CURRENT PRACTICE

Suppose, for simplicity, that just two families of densities are regarded as plausible or interesting alternatives to H_0, and that the intersection of these two families is also admitted as a possibility. There are then three families of alternatives to H_0 which we denote by H_{10}, H_{02}, and H_{12}, respectively. Ideally we would like to design an overall rejection region for H_0 that meets (in some sense) the two requirements mentioned in the introduction, but before that can be done we need to define more precisely what constitutes a 'serious attempt' to discredit a model, and what sort of procedure (critical region) is capable of providing complete information about the direction of the alternative if H_0 is rejected.

Taking the second question first, it seems reasonable to suggest that the overall critical region will be fully informative about the direction of the alternative only if *it consists of three disjoint, non-empty, parts* ω_{10}, ω_{02}, and ω_{12}, say, that can be identified with the alternatives H_{10}, H_{02}, and H_{12} respectively. Within the class of exact MDPs for the linear model, these will be disjoint subsets of the surface S_m, but in general they could be thought of simply as subsets of the natural sample space. In the case of r alternatives this requirement entails a partition of the overall critical region into 2^r-1 disjoint, non-empty parts. We shall call a diagnostic procedure with this structure a *complete* diagnostic procedure of order r. If it is agreed that MDPs should be complete in this sense, and the overall critical region is given, the problem becomes one of partitioning the overall region into its constituent parts ω_{10}, ω_{02}, and ω_{12}. A natural objective here is to seek a partition that minimizes the probabilities of incorrect decisions, and this is the criterion we shall adopt in the example below.

One obvious problem with piecemeal MDPs consisting simply of a pair of tests for the separate sub-hypotheses involved is that such a procedure is evidently incomplete in this sense, because, even if the critical regions for the two tests overlap, the region of overlap cannot, in general, be interpreted as favorable to H_{12}.

Remark 1: *Piecemeal MDPs consisting simply of a set of tests for separate hypotheses are incomplete.*

Nor does the addition of an 'omnibus' test (such as those suggested by Bera and Jarque (1982), White (1980, 1987), and White and Domowitz (1984)) to the overall procedure render it complete, because failing the omnibus test suggests only that at least one of the families of alternatives of interest holds, not necessarily both (i.e., H_{12}). Omnibus tests are considered further below.

Turning to the question of how to define a 'serious attempt' to discredit the null model, it seems clear that this idea can only be made precise by reference to the families of densities for v under H_{10}, H_{02}, and H_{12}, in the same way that, in the Neyman-Pearson theory of conventional testing, a 'good test' is defined by reference to the probabilities of Type I and Type II errors. Loosely speaking, it seems reasonable to suggest that a MDP constitutes a 'serious attempt' to discredit H_0 only if the overall rejection region includes *all* points $v \in S_m$ for which pdf(v) *can* be 'large' relative to $pdf_0(v)$ (the null density of v). Unfortunately, but not surprisingly, pdf(v) can often be 'large' relative to $pdf_0(v)$ virtually anywhere on S_m, depending on the values of the parameters that index the family of alternatives. Thus, as in conventional testing, compromises must be made in opting for a particular procedure, and the compromises that are appropriate depend upon the context. In particular, as the example below will show, the problem of designing a complete diagnostic procedure may call for quite different compromises than are traditionally used to justify tests of H_0 against a single family of alternatives.

The strategy we are suggesting, then, consists of two parts: (i) a choice of the overall rejection region based on an attempt to include all points for which the non-null density *can* be 'large' relative to the null density, and then (ii) a partition of this overall region into subsets corresponding to the families of alternatives that are entertained, with the objective of minimizing the probabilities of incorrect decisions.

The remainder of this section consists mainly of some remarks on current practice, which I interpret as consisting of a set of tests for the separate hypotheses of interest, and, possibly, an 'omnibus' test for the alternative formed from the intersection of the separate hypotheses. Some of discussion to follow would need to be modified if deliberate decisions to avoid the problems discussed were made by adjusting the sizes of the tests used. However, since this is typically not done, I shall assume throughout that the separate tests are carried out at the sizes commonly used in practise.

To provide a focus for the discussion I shall consider in detail the simple and familiar case in which:

$$H_{10}: \quad y \sim N(\, X\beta + \gamma_1 z_1,\ \sigma^2 I_n), \tag{6}$$

$$H_{02}: \quad y \sim N(\, X\beta + \gamma_2 z_2,\ \sigma^2 I_n), \tag{7}$$

$$H_{12}: \quad y \sim N(\, X\beta + \gamma_1 z_1 + \gamma_2 z_2,\ \sigma^2 I_n). \tag{8}$$

That is, the problem is to decide whether one or both of the additional variables z_1 and z_2 should be included (linearly) in the model for E(y). Most of the points that arise in more complex situations can be illustrated with this problem, and it also serves to illustrate the nature, if not the complexity, of the problem of designing suitable MDPs. Also, in the light of Pagan (1984), this problem can serve as a model for more general specification tests.

It is straightforward to check that, under H_{12} in (8), the density of v is given by

$$\mathrm{pdf}_{12}(v) = A^{-1}(m)\ \exp\{-\lambda/2\} \sum_{j=0}^{\infty} a_m(j)\ (v'\tilde{Z}\gamma/\sigma)^j \tag{9}$$

where $\tilde{Z} = (\tilde{z}_1, \tilde{z}_2) = C'(z_1, z_2)$, $\gamma = (\gamma_1, \gamma_2)'$, $\lambda = \gamma'\tilde{Z}'\tilde{Z}\gamma/\sigma^2$, and $a_m(j) = [2^{j/2}\Gamma((j+m)/2)/(j!\Gamma(m/2))]$.

Under H_{10} ($\gamma_2 = 0$) the term $(v'\tilde{Z}\gamma)$ in (9) becomes $(\gamma_1 v'\tilde{z}_1)$, and under H_{02} ($\gamma_1 = 0$) it becomes $(\gamma_2 v'\tilde{z}_2)$, so that the densities $\mathrm{pdf}_{10}(v)$ and $\mathrm{pdf}_{02}(v)$ are identical to (9) except that these terms replace $(v'\tilde{Z}\gamma)$ in the power series. The (two-sided) t-tests for $\gamma_1 = 0$ in (6) and for $\gamma_2 = 0$ in (7) have critical regions:

$$\omega^*_{10} : \quad (v'\tilde{z}_1)^2/(\tilde{z}_1'\tilde{z}_1) = \cos^2\theta_1 > c_{10}, \tag{10}$$

and

$$\omega^*_{02} : \quad (v'\tilde{z}_2)^2/(\tilde{z}_2'\tilde{z}_2) = \cos^2\theta_2 > c_{02}, \tag{11}$$

respectively, where θ_i is the angle between v and the point $\xi_i = \tilde{z}_i(\tilde{z}_i'\tilde{z}_i)^{-1/2} \in S_m$, $i = 1,2$, and c_{10} and c_{02} are constants chosen to determine the sizes of ω_{10}^* and ω_{02}^*.

These two t-tests correspond to what, in a general diagnostic problem, would be the separate diagnostic statistics for the two families of alternatives H_{10} and H_{02}. Geometrically, ω_{10}^* consists of two circular 'caps' on the unit sphere S_m, one centered on ξ_1, the other on $-\xi_1$. ω_{02}^* has exactly the same structure with ξ_1 replaced by ξ_2. If the two tests have the same size ($c_{10} = c_{02}$) these four 'caps' are all the same size. The critical region (10) ((11)) is the region that maximises the *average* power among tests of H_0 against H_{10} (H_{02}) that are based on v (i.e., among similar tests), the average being taken over $\text{sign}(\gamma_1) = +1$ and $\text{sign}(\gamma_1) = -1$ ($\text{sign}(\gamma_2) = +1$ and $\text{sign}(\gamma_2) = -1$).

Now, unless ξ_1 and ξ_2 are coincident ω_{10}^* and ω_{02}^* cannot coincide, even if they are of the same size. However, the two regions certainly can, and in general do, overlap, and the extent of this overlap can be considerable. In this example this occurs for almost all values of m of practical relevence, or, if m is very small, if ξ_1 and ξ_2 are close together (see section 4 for details on this and other assertions about this example). This situation can evidently occur with the critical regions defined by any pair of diagnostic test statistics that are chosen in a piecemeal fashion, and clearly leads to difficulties in interpreting the pair of tests.

<u>Remark 2:</u> *The separate critical regions defined by several diagnostic test statistics can have substantial subregions in common. Moreover, if the tests have different sizes, it is possible for the critical region for one test to be a subset of that for another.*

The important point here is that the fact that a pair of regions may overlap is fortuitous, not intentional, and would often go unnoticed - unless, of course, the model fails both diagnostic tests, for this obviously implies that the critical regions overlap. Thus, such a region of overlap cannot, in general, be interpreted as a region favorable to the intersection of the two families of alternatives (i.e., to H_{12}), at least not without further evidence. Such a situation can arise when the separate critical regions are obtained from local arguments that ignore the global behaviour of pdf(v).

It can clearly also happen that the critical regions defined by two diagnostic tests are disjoint, although in the present example this can only occur if m is very small (< 5).

Remark 3: *The critical regions defined by several diagnostic test statistics can be disjoint. In this case it is impossible for the procedure to reject H_0 in favor of more than one alternative.*

Again, the problem is that this circumstance can arise unintentionally, rather than by design, and I doubt very much whether most applied workers would regard such a procedure as satisfactory were it drawn to their attention: such a procedure is certainly incomplete and, without detailed evidence on the properties of the families of densities, would probably not be regarded as a 'serious attempt' to discredit the null model (since only one unfavorable outcome can occur).

The analog of an 'omnibus' test in the context of (6) - (8) is, of course, the standard F-test for $\gamma = 0$ in (8), the critical region for which is:

$$\omega^*_{12} : \quad v'\tilde{Z}(\tilde{Z}'\tilde{Z})^{-1}\tilde{Z}'v \; > \; c_{12}. \qquad (12)$$

This region again maximises the *average* power among tests of H_0 against H_{12} that are based on v, but in this case the average is over values of γ for which $\gamma'\tilde{Z}'\tilde{Z}\gamma/\sigma^2$ = constant. The statistic on the left in (12) may be interpreted as the squared cosine of the angle between v and the unit vector along the orthogonal projection of v onto the plane spanned by $\tilde{z}_1$ and $\tilde{z}_2$. The region (12) is thus a 'strip' around the surface S_m that is near this plane. Since the mode of the density (9) must be somewhere on this plane, the region (12) includes all points v for which $pdf_{12}(v)$ *can* be 'large' for some γ, and it clearly intersects both of the regions ω^*_{10} and ω^*_{02}.

In this example the critical region for the 'omnibus' test includes points in the critical regions associated with the alternatives H_{10} and H_{02}, and also points that are in neither of these regions. However, in general it is not necessarily true (of a procedure consisting of an omnibus test and two tests of the separate sub-families involved) that the critical region for the omnibus test intersects the critical regions for the separate sub-hypotheses, nor is it necessarily the case that the 'omnibus' critical region contains points that are not already included in one or both of the critical regions for the separate sub-hypotheses.

Remark 4: *The critical region for an 'omnibus' test may (i) have no points in common with the critical regions for the separate families of hypotheses embedded within the more general family of alternatives, or (ii) have no points that are not also included in the critical regions for the sub-hypotheses, or (iii) intersect one or both critical regions for the sub-hypotheses, but neither wholly contain those regions nor be wholly contained within them.*

The first of these possibilities obviously renders the procedure as a whole logically inconsistent, but it can occur in finite samples if the basis for the omnibus test (which might be, for instance, an Eicker-White information matrix test) is quite different from that used to obtain the critical regions for the separate sub-hypotheses (which might be based on the local behaviour of the likelihood function (LM tests), or the local behaviour of the power function). This situation cannot occur asymptotically if all three tests are consistent, but this is of little comfort in finite samples. The second possibility arises if the omnibus critical region is a subset of the union of the critical regions for the two sub-hypotheses. This circumstance can arise if (as is usually the case in practice) the size of the omnibus test is less than the size of the union of the critical regions for the separate tests, and obviously means that rejection of the null model by the omnibus test implies rejection by at least one of the separate tests. However, if the critical region for the omnibus test is a proper subset of the union of the critical regions for the two separate tests, the converse is not true (that is, the model can fail one or both of the separate tests and yet pass the omnibus test), so that a procedure with this structure can also lead to apparently inconsistent results. Moreover, an outcome that favours H_{12} but neither H_{10} nor H_{02} cannot occur, so again such a procedure would be incomplete.

The third possibility in Remark 4 seems closest to meeting the requirements discussed earlier, but here the model can fail an individual test yet pass the omnibus test, or fail the omnibus test yet pass one or both of the separate tests, leading again to the possibility of an inconsistent outcome. Clearly, all of these difficulties arise because the procedure as a whole has not been designed in a coherent fashion. Before considering the problem of designing an appropriate procedure in this example, we shall round off the discussion of current practice by considering the analog of robust tests for this problem.

The region (11) for testing H_0 against H_{02} has size independent of β and σ^2 under H_0, but under H_{10} its size depends on γ_1 (and σ^2). The analog of a 'robust' test for $\gamma_2 = 0$ in this example would be a test whose size is free of parameters under both H_0 and H_{10}. But, the class of regions whose size is independent of β, γ_1, and σ^2 under both H_0 and H_{10} can be obtained by applying Theorem 1 in (6), and is characterized by a vector $w_1 \in S_{m-1}$ that is defined in exactly the same way for (6) as v has been defined earlier for (1). This vector is uniformly distributed on S_{m-1} under both H_0 and H_{10}, and its distribution under H_{02} is identical to its distribution under H_{12}. Averaging $\Pr\{w_1 \in \omega | H_{02}\}$ (= $\Pr\{w_1 \in \omega | H_{12}\}$) over $\text{sign}(\gamma_2) = +1$ and $\text{sign}(\gamma_2) = -1$ yields a best 'robust' region for testing $\gamma_2 = 0$, ω^r_{02} say, which is simply the critical region for the two-sided t-test of $\gamma_2 = 0$ in (8). An exactly analogous result holds for the 'robust' region for testing $\gamma_1 = 0$. We shall see shortly that, like ω^*_{10} and ω^*_{02}, the two regions ω^r_{10} and ω^r_{02} have extensive subregions in common, that they intersect the critical region ω^*_{12} for the 'omnibus' test, but also that these regions contain points that are not in ω^*_{12}, and that ω^*_{12} contains points that are in neither ω^r_{10} nor ω^r_{02}.

<u>Remark 5:</u> *All of the earlier discussion of piecemeal multiple diagnostic procedures continues to hold if the critical regions for the separate sub-hypotheses are replaced by 'robust' critical regions.*

The fact that $\text{pdf}_{02}(w_1) \equiv \text{pdf}_{12}(w_1)$ clearly implies that the class of robust procedures for testing $\gamma_2 = 0$ in this example is incapable of distinguishing between H_{02} and H_{12}, and a similar conclusion holds for robust tests for $\gamma_1 = 0$. As long as the question of interest is simply whether or not $\gamma_2 = 0$ (or $\gamma_1 = 0$), this is not a weakness, but if distinguishing between H_{10}, H_{02}, and H_{12} is part of the purpose of the procedure it most certainly is. Of course, in more complex problems this conclusion is unlikely to be so clear cut, but it seems clear that imposing the requirement of invariance to a parameter on a family of densities under a parametric constraint must, by continuity, impose something close to that invariance on the family for which the constraint does not hold, at least locally.

Remark 6: *'Robustification' of the separate tests within a MDP may make it more difficult to distinguish between H_{10}, H_{02}, and H_{12}, rather than easier.*

One motive for 'robustification' seems to be that a departure from the null model in a particular direction may be more serious, in some sense, than a departure in the other. This seems to me to imply (if $\gamma_2 \neq 0$ represents the 'serious' departure), not that the probability of wrongly concluding that $\gamma_2 \neq 0$ should be small whatever the value of γ_1, but rather that the probability of correctly concluding that $\gamma_2 \neq 0$ should be large, whatever the value of γ_1. In our example it is not difficult to show that $\Pr\{v \in \omega^r_{02}|H_{02}\}$ ($= \Pr\{v \in \omega^r_{02}|H_{12}\}$), while free of γ_1, does depend on H_{10} through $\tilde{z}_1$, and can in fact be smaller than both $\Pr\{v \in \omega^*_{02}|H_{02}\}$ and $\Pr\{v \in \omega^*_{02}|H_{12}\}$. Thus, 'robustification' for size can diminish the probability of correctly rejecting a false hypothesis, but this is an aside to our main line of argument. In fact, a properly designed diagnostic .procedure would make the (size) robustness issue irrelevant, so we turn to that question for our example now.

4. DETAILS AND DÉNOUEMENT FOR THE EXAMPLE

Before considering the problem of designing a suitable procedure for the problem (6)-(8), it is convenient at this point to introduce a set of coordinates for points $v \in S_m$ that will enable us to define critical regions in a two-dimensional Euclidean space, rather than on the surface S_m. This will also clarify some of the statements made in section 3 about the various critical regions defined there. Let $L = [\ \xi_1,\ (\xi_2 - \rho\xi_1)/(1 - \rho^2)^{1/2}]$ (mx2), where $\rho = \xi_1'\xi_2$, so that $\cos^{-1}(\rho)$ is the angle between ξ_1 and ξ_2. The columns of L are an orthonormal basis for the two dimensional space spanned by $\tilde{z}_1$ and $\tilde{z}_2$. Also, let $\bar{L}$ (mx(m-2)) be such that $\bar{L}'L = 0$ and $\bar{L}'\bar{L} = I_{m-2}$. Then we can represent v uniquely in the form

$$v = b^{1/2}Lg + (1-b)^{1/2}\bar{L}h, \qquad (13)$$

where $g \in S_2$, $h \in S_{m-2}$, and $0 < b < 1$. In this representation of v, $b = v'\tilde{Z}(\tilde{Z}'\tilde{Z})^{-1}\tilde{Z}'v = \hat{\gamma}'\tilde{Z}'\tilde{Z}\hat{\gamma}/s^2$, and $g = T\hat{\gamma}/(\hat{\gamma}'\tilde{Z}'\tilde{Z}\hat{\gamma})^{1/2}$, where $\hat{\gamma}$ is the OLS estimate of γ in (8),

$$T = \begin{bmatrix} s_1\ , & s_2\rho \\ 0\ , & s_2(1-\rho^2)^{1/2} \end{bmatrix},$$

with $s_i^2 = \tilde{z}_i'\tilde{z}_i$, i = 1,2, so that $T'T = \tilde{Z}'\tilde{Z}$, and $h \in S_{m-2}$ is defined for (8) as v is defined for (1). Thus, g is a statistic indicating the 'direction' of the vector $T\hat{\gamma}$. The invariant measure (v'dv) on S_m factorizes under (13) as

$$(v'dv) = 2^{-1}(1 - b)^{(m-2)/2-1}(db)(g'dg)(h'dh) \qquad (14)$$

where (g'dg) ((h'dh)) denotes the invariant measure on S_2 (S_{m-2}).

Next, since $g \in S_2$, we can represent g in polar coordinates: $g = (\cos\theta, \sin\theta)'$, where θ ($-\pi < \theta < \pi$) is the angle between g and (say) the first coordinate axis, e_1. The invariant measure $(g'dg)$ becomes simply $d\theta$ under this transformation. The statistics (b,θ,h) provide an alternative characterization of the class of regions whose size is free of parameters under H_0. Transforming $v \rightarrow (b,\theta,h)$, we find that h is independent of b and θ and is uniformly distributed on S_{m-2} under H_{12}, and therefore also under H_0, H_{10}, and H_{02}. Since h contains no information about the alternatives it can be ignored, and all regions of interest can be described in terms of b and θ. Under H_{12} the joint density of b and θ becomes

$$pdf_{12}(b,\theta) = c(m)(1-b)^{(m-2)/2-1}\exp\{-\lambda/2\} \sum_{j=0}^{\infty} a_m(j)[(\lambda b)^{1/2}g'\mu]^j \qquad (15)$$

where $c(m) = [2\pi\Gamma((m-2)/2)/\Gamma(m/2)]^{-1}$ and $\mu = T\gamma/(\gamma'T'T\gamma)^{1/2}$ is an indicator of the direction of $T\gamma$. Note that we have continued to use g to denote the vector $(\cos\theta, \sin\theta)'$ to simplify the notation, and that μ can also be expressed in polar coordinates as $\mu = (\cos\bar{\theta}, \sin\bar{\theta})'$, where $\bar{\theta}$ is the angle between μ and the first coordinate axis. Under H_{10}, $\lambda^{1/2}$ is replaced by $(s_1\gamma_1/\sigma)$ and $g'\mu$ by $\cos\theta$ in (15), while under H_{02}, $\lambda^{1/2}$ is replaced by $(s_2\gamma_2/\sigma)$ and $g'\mu$ by $\cos(\theta-\theta_\rho)$, where $\theta_\rho = \cos^{-1}(\rho)$ $(0 < \theta_\rho < \pi)$ is the angle between ξ_1 and ξ_2. In terms of these coordinates the regions ω^*_{10}, ω^*_{02}, and ω^*_{12} are:

$$\omega^*_{10}: \quad b\cos^2\theta > c_{10} \quad [H_0 \text{ against } H_{10}] \qquad (16)$$

$$\omega^*_{02}: \quad b\cos^2(\theta-\theta_\rho) > c_{02} \quad [H_0 \text{ against } H_{02}] \qquad (17)$$

$$\omega^*_{12}: \quad b > c_{12} \quad [H_0 \text{ against } H_{12}] \qquad (18)$$

The 'robust' tests for $\gamma_1 = 0$ and $\gamma_2 = 0$ (i.e., the two-sided t-tests for $\gamma_1 = 0$ and $\gamma_2 = 0$, respectively, in (8)) have critical regions

$$\omega^r_{10} \;:\; b(1-\cos^2(\theta-\theta_\rho))/(1-b\cos^2(\theta-\theta_\rho)) > c'_{10} \quad [H_{02} \text{ against } H_{12}] \quad (19)$$

$$\omega^r_{02} \;:\; b(1-\cos^2\theta)/(1-b\cos^2\theta) > c'_{02} \quad [H_{10} \text{ against } H_{12}] \quad (20)$$

The critical regions (16)-(20) are subsets of the rectangle

$$R = \{b,\theta;\ 0 < b < 1,\ -\pi < \theta < \pi\},$$

and are depicted in Figures 1(a) and 1(b) for $m = 22$, $\rho = .766$, and a case where each test has the same size (.05). If we append to each of the inequalities (16)-(20) the phrase "and all $h \in S_{m-2}$", each of these regions is in one-to-one correspondence with a subset of the surface S_m, the sample space for v. Hence, Figures 1(a) and 1(b) confirm the assertions made in section 3 about the relationships between these regions.

In the remainder of the paper I suggest a procedure for this problem that meets both of the requirements discussed earlier. The reader should be forewarned, however, that, just as in conventional testing, there is no 'best' procedure even in this simple problem, because the 'best' procedure depends on unknown parameters. Thus, certain aspects of the solution offered below are necessarily ad hoc and may not appeal to some readers. The argument makes both the need for, and the rôle of, the compromises that are made, obvious.

It will be helpful in what follows to write (15) as the product

$$\text{pdf}(b,\theta) \;=\; \text{pdf}(\theta|b)\text{pdf}(b) \qquad (21)$$

with

$$\text{pdf}(b) = [\Gamma(m/2)/\Gamma((m-2)/2)](1-b)^{(m-2)/2-1}\exp\{-\lambda/2\}\,{}_1F_1(m/2,\ 1;\ \lambda b/2) \tag{22}$$

and

$$\text{pdf}(\theta|b) = (2\pi)^{-1}\sum_{j=0}^{\infty} a_m(j)[(\lambda b)^{1/2}g'\mu]^j/{}_1F_1(m/2,\ 1;\ \lambda b/2). \tag{23}$$

where ${}_1F_1(a,\ b;\ x) = \Sigma_{j=0}^{\infty}x^j(a)_j/j!(b)_j$, with $(a)_j = a(a+1)\ldots(a+j-1)$, denotes the confluent hypergeometric function. Now, it is clear from (23) that, for each fixed b, the mode of $\text{pdf}(\theta|b)$ occurs at $\theta = \bar{\theta}$, and this can be anywhere in $(-\pi,\ \pi)$. Hence, the only way to ensure that the overall rejection region for H_0 contains all points (b, θ) for which $\text{pdf}(b, \theta)$ *can* be large is to include, for each b, the entire interval $-\pi < \theta < \pi$. In other words, the overall rejection region must be of the form $b > c$, the critical region for the F-test of $\gamma = 0$ in (8), if it is to satisfy our 'serious attempt to discredit' criterion. Obviously, the size of the overall rejection region is easily determined in this case.

It remains, therefore, to partition the region $b > c$ into three disjoint subsets ω_{10}, ω_{02}, and ω_{12}, corresponding to H_{10}, H_{02}, and H_{12}. Ideally, we would like to be able to choose these regions in such a way as to minimise the probabilities of incorrect decisions. As we shall see shortly, however, this cannot be done without first making a number of simplifying assumptions.

Under H_{10}, $\text{pdf}(\theta|b)$ has a mode at $\theta = 0$ and an antimode at $\theta = \pm\pi$ when $\gamma_1 > 0$, but the mode is at $\theta = \pm\pi$ and the antimode at $\theta = 0$ when $\gamma_1 < 0$. Likewise, under H_{02} the mode is at $\theta = \theta_\rho$ ($\theta = \theta_\rho - \pi$) and the antimode at $\theta = \theta_\rho - \pi$ ($\theta = \theta_\rho$) when $\gamma_2 > 0$ ($\gamma_2 < 0$). Under H_{12} the mode is at $\theta = \bar{\theta}$ and the antimode at $\theta = \bar{\theta} \pm \pi$. In the absence of information about the likely signs of γ_1 and γ_2 it seems reasonable, as before, to work with the averages of the conditional densities $pdf_{10}(\theta|b)$ and $pdf_{02}(\theta|b)$ over $sign(\gamma_1) = \pm 1$ and $sign(\gamma_2) = \pm 1$, respectively. That is, with

$$\bar{p}_{10}(\theta|b) = {}_1F_1(m/2,\ 1/2;\ \lambda_{10}b\cos^2(\theta)/2)/2\pi{}_1F_1(m/2,\ 1;\ \lambda_{10}b/2) \qquad (24)$$

$$\bar{p}_{02}(\theta|b) = {}_1F_1(m/2,\ 1/2,\ \lambda_{02}b\cos^2(\theta-\theta_\rho)/2)/2\pi{}_1F_1(m/2,\ 1;\ \lambda_{02}b/2) \qquad (25)$$

where $\lambda_{10} = s_1^2\gamma_1^2/\sigma^2$ and $\lambda_{02} = s_2^2\gamma_2^2/\sigma^2$. Similarly, we shall work with the average of $pdf_{12}(\theta|b)$ over $\pm\mu$. That is, with the average value of $pdf_{12}(\theta|b)$ over the pair of points $\bar{\theta}$ and $\bar{\theta} \pm \pi$:

$$\bar{p}_{12}(\theta|b) = {}_1F_1(m/2,\ 1/2;\ \lambda b(\cos^2(\theta-\bar{\theta}))/2)/2\pi{}_1F_1(m/2,\ 1,\ \lambda b/2). \qquad (26)$$

Note that the positions of the modes and antimodes of $\bar{p}_{10}(\theta|b)$ and $\bar{p}_{02}(\theta|b)$ are known, but the points at which they cross depend on λ_{10} and λ_{02}, and are therefore unknown (see Fig. 2). Both the position and shape of $\bar{p}_{12}(\theta|b)$, on the other hand, are unknown, the former depending on $\bar{\theta}$, the latter on λ, and it is this that makes it difficult to devise a reasonable strategy for distinguishing between H_{10}, H_{02}, and H_{12}.

The general shapes of the averaged conditional densities (24) and (25) are illustrated in Figures 2(a) and 2(b). In Figure 2(a) $\lambda_{10} = \lambda_{02}$, while in Figure 2(b) $\lambda_{10} < \lambda_{02}$. It seems clear that, whatever criteria are used to define a 'good' procedure for distinguishing between H_{10}, H_{02}, and H_{12}, the appropriate choices for ω_{10}, ω_{02}, and ω_{12} will depend on λ_{10}, λ_{02}, and λ. Thus, little progress can be made without making assumptions about these parameters, and we shall here confine ourselves to procedures that are appropriate when the alternatives are 'balanced' in the sense that $\lambda_{10} = \lambda_{02} = \lambda$. Somewhat different procedures would no doubt be appropriate under different assumptions.

Figure 2(a) relates to this 'balanced' situation, and $\bar{p}_{12}(\theta|b)$ looks exactly like the densities shown there except that the modes can be anywhere in $-\pi < \theta < \pi$. Thus, $\bar{p}_{12}(\theta|b)$ can be very close to either of the other averaged conditional densities, depending on $\bar{\theta}$. This clearly means that it is impossible to choose regions ω_{10}, ω_{02}, and ω_{12} that uniformly minimise the probabilities of incorrect decisions for all values of $\bar{\theta}$. To overcome this difficulty it seems reasonable to suggest that the regions chosen should perform well when H_{12} is, in some sense, most unlike either H_{10} or H_{02}, and Fig. 2(a) suggests that this is so when $\bar{\theta} = (\theta_\rho + \pi)/2$ if $\theta_\rho < \pi/2$, and when $\bar{\theta} = \theta_\rho/2$ if $\theta_\rho > \pi/2$. For the purposes of choosing ω_{10}, ω_{02}, and ω_{12} we shall therefore take $\bar{\theta}$ in $\bar{p}_{12}(\theta|b)$ to be equal to $(\theta_\rho + \pi)/2$ if $\rho > 0$, and equal to $\theta_\rho/2$ if $\rho < 0$. We denote the averaged density $\bar{p}_{12}(\theta|b)$ evaluated at either of these two values of $\bar{\theta}$ by $\bar{p}^*_{12}(\theta|b)$.

If, now, we revert to the averaged joint densities $\bar{p}_{10}(b,\theta)$, $\bar{p}_{02}(b,\theta)$, and $\bar{p}^*_{12}(b,\theta)$ defined by $\bar{p}_{10}(b,\theta) = \bar{p}_{10}(\theta|b)\mathrm{pdf}(b)$ etc., the problem becomes one of discriminating between three possible densities for (b,θ). But, according to a generalization of the Neyman-Pearson Lemma due to C. R. Rao (quoted, without reference, by Kendall and Stuart (1976), p. 337), the regions that minimize the probabilities of incorrect decisions are, in the case where the three candidates are equally likely *a priori,* defined by:

$$\omega_{10} \ : \ \bar{p}_{10}(b,\theta) \geq \bar{p}_{02}(b,\theta) \ \text{ and } \ \bar{p}_{10}(b,\theta) \geq \bar{p}^*_{12}(b,\theta) \tag{27}$$

$$\omega_{02} \ : \ \bar{p}_{10}(b,\theta) \leq \bar{p}_{02}(b,\theta) \ \text{ and } \ \bar{p}_{02}(b,\theta) \geq \bar{p}^*_{12}(b,\theta) \tag{28}$$

$$\omega_{12} \ : \ \bar{p}_{10}(b,\theta) \leq \bar{p}^*_{12}(b,\theta) \ \text{ and } \ \bar{p}_{02}(b,\theta) \leq \bar{p}^*_{12}(b,\theta). \tag{29}$$

Using the expressions given earlier for the averaged conditional densities, and the fact that the confluent hypergeometric function is a monotonic function of its argument, it is easy to see that (27)-(29) are equivalent to, in the case $\rho > 0$,

$$\omega_{10} : \ b\cos^2\theta \geq b\cos^2(\theta-\theta_\rho) \ \text{ and } \ b\cos^2\theta \geq b\cos^2(\theta-(\theta_\rho+\pi)/2) \tag{30}$$

$$\omega_{02} : \ b\cos^2\theta \leq b\cos^2(\theta-\theta_\rho) \ \text{ and } \ b\cos^2(\theta-\theta_\rho) \geq b\cos^2(\theta-(\theta_\rho+\pi)/2) \tag{31}$$

$$\omega_{12} : \ b\cos^2\theta \leq b\cos^2(\theta-(\theta_\rho+\pi)/2) \ \text{ and } \ b\cos^2(\theta-\theta_\rho) \leq b\cos^2(\theta-(\theta_\rho+\pi)/2). \tag{32}$$

To each of these, of course, the condition $b > c$ should be appended. In the case $\rho < 0$ $(\theta_\rho+\pi)/2$ is replaced by $\theta_\rho/2$ throughout.

Note that b can be eliminated from (30)-(32), so that these inequalities simply define a partition of the interval $-\pi < 0 < \pi$ into strips whose intersections with the region $b > c$ define the subregions ω_{10}, ω_{02}, and ω_{12}. However, the regions (30)-(32) can also be expressed in terms of much more familiar and easily calculated statistics. To do so, first note that $b\cos^2\theta = (t_1^2/m-1)/(1+t_1^2/m-1)$ and $b\cos^2(\theta-\theta_\rho) = (t_2^2/m-1)/(1+t_2^2/m-1)$, where t_1 and t_2 are the t-statistics associated with γ_1 and γ_2 in (6) and (7) respectively. In a similar way, $b\cos^2(\theta-(\theta_\rho+\pi)/2)$, and, in the case $\rho < 0$, $b\cos^2(\theta-\theta_\rho/2)$, can be expressed as functions of the t-statistics associated with the vectors $z^+ = ZT^{-1}\mu^+$ and $z^- = ZT^{-1}\mu^-$, respectively, in the regressions of y on X and z^+ and y on X and z^-. Here,

$$\mu^{+} = \begin{pmatrix} -(1-\rho)^{1/2} \\ (1+\rho)^{1/2} \end{pmatrix}, \quad \text{and} \quad \mu^{-} = \begin{pmatrix} (1+\rho)^{1/2} \\ (1-\rho)^{1/2} \end{pmatrix}. \tag{33}$$

Using these three t-statistics, and noting that the region b>c corresponds to the region $F > F_c(2,m-2)$, where F is the usual F-statistic for testing $\gamma=0$ in (8), and $F_c(2,m-2)$ $(=(m-2)c/2(1-c))$ is the critical value for F (chosen to determine the size of the procedure), the regions ω_{10}, ω_{02}, and ω_{121} are, finally:

$$\omega_{10} \quad : \quad t_1^2 - t_2^2 \geq 0, \quad t_1^2 - t_*^2 \geq 0, \quad F \geq F_c(2,m-2), \tag{34}$$

$$\omega_{02} \quad : \quad t_1^2 - t_2^2 \leq 0, \quad t_2^2 - t_*^2 \geq 0, \quad F \geq F_c(2,m-2), \tag{35}$$

$$\omega_{12} \quad : \quad t_1^2 - t_*^2 \leq 0, \quad t_2^2 - t_*^2 \leq 0, \quad F \geq F_c(2,m-2), \tag{36}$$

where t_* denotes the t-statistic based on μ^{+} or μ^{-} as appropriate.

The regions (34)-(36) provide an optimal partition of the overall critical region under the assumptions, and in the sense, discussed above. Whether or not the procedure defined by these regions would perform well generally remains a subject for further study.

5. CONCLUDING REMARKS

We have shown that a piecemeal approach to designing multiple diagnostic procedures falls far short of meeting the needs of applied researchers. On the other hand, our simple example reveals that much detailed information is needed about the densities under the families of alternatives that are entertained before one can hope to design suitable procedures. In more complicated situations - for instance, a case in which H_{10} is (8), H_{02} corresponds to the linear model with autocorrelation, and H_{12} to both additional variables in the mean and autocorrelation - it is unlikely that the arguments that were successful for the simple example used here will be enough. Clearly, much needs to be done even in relatively simple problems before the needs mentioned in the introduction can fairly be said to be met.

For problems in which the null model is given by (1), the result in section 2 should serve to both unify the treatment of more complex diagnostic problems, and to provide geometric insights that are helpful in choosing appropriate procedures. However, the relatively simple coordinates used in section 4 will not necessarily be appropriate for other problems.

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Figure 1(a): Critical Regions for t-tests

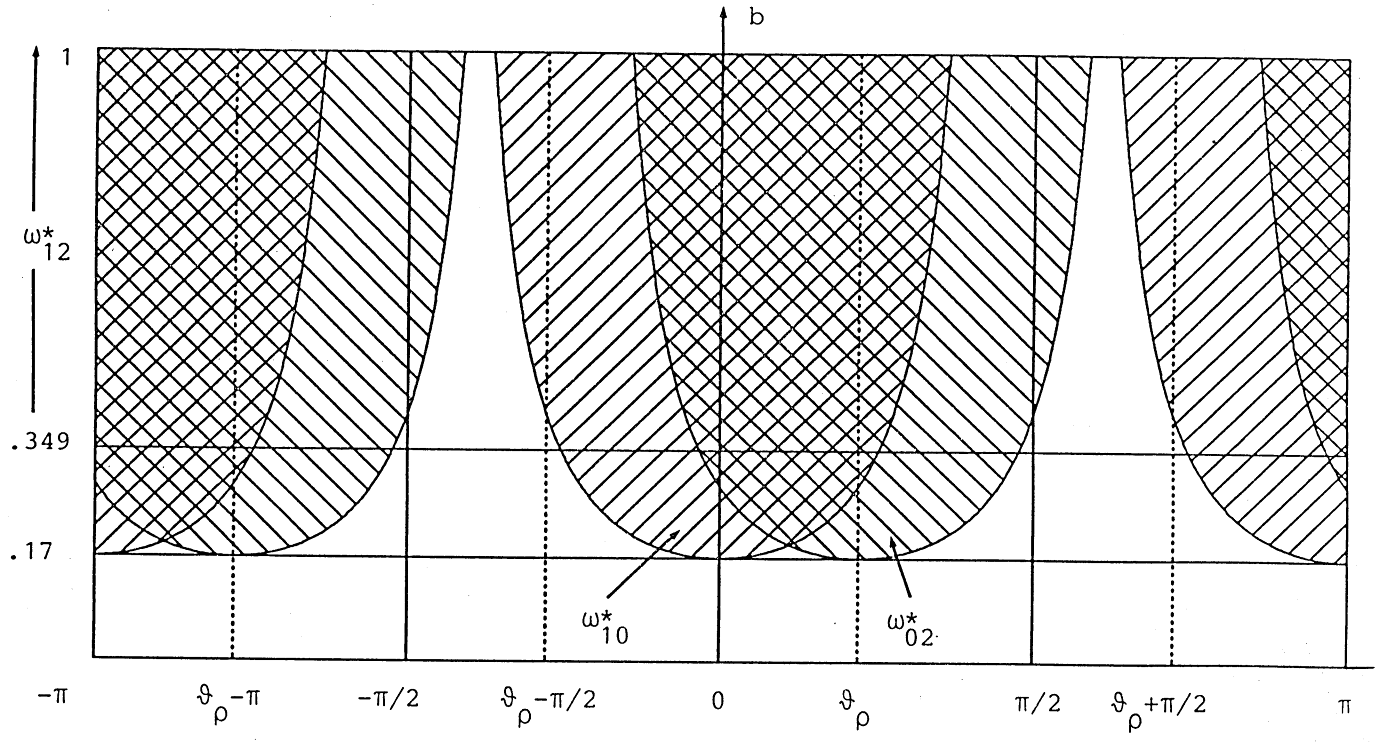


Figure 1(b): Critical Regions for 'Robust' Tests

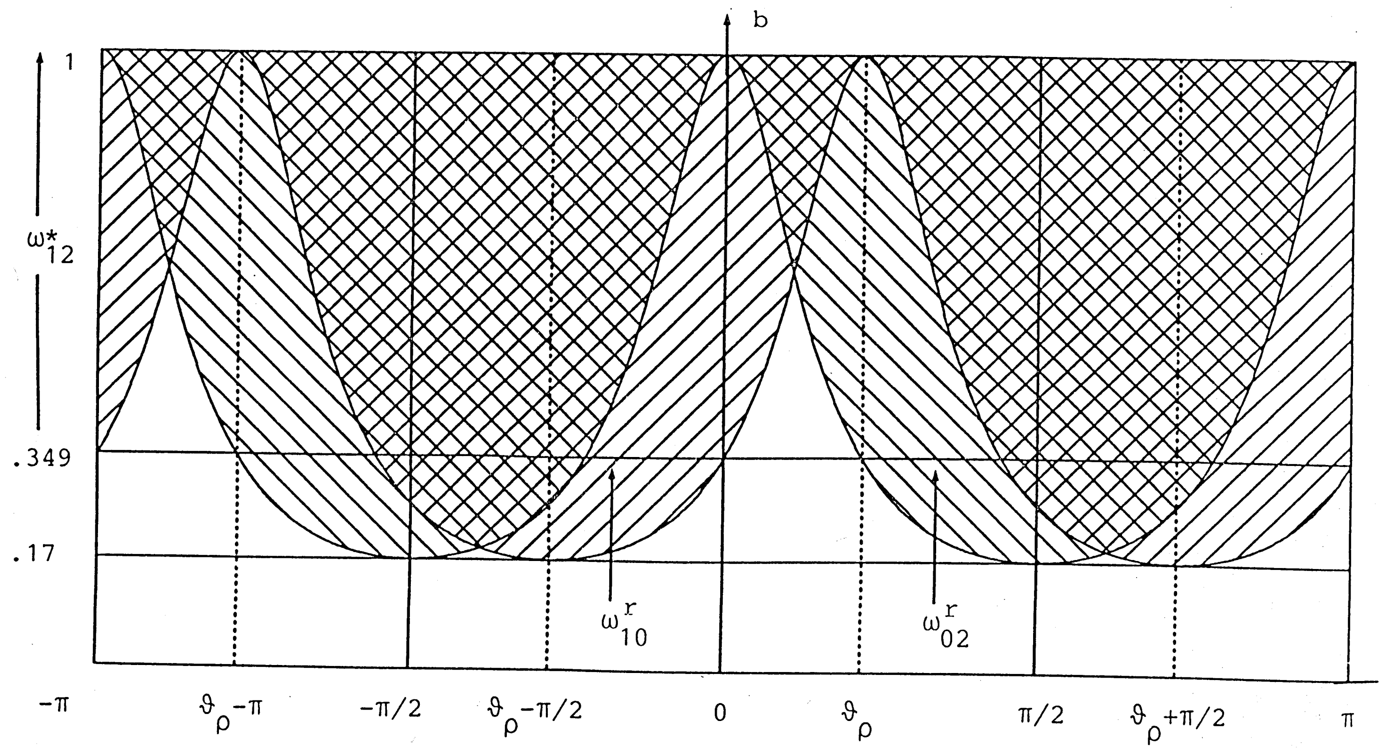


Figure 2(a): Averaged Conditional Densities of ϑ ($\lambda_{10} = \lambda_{02}$)

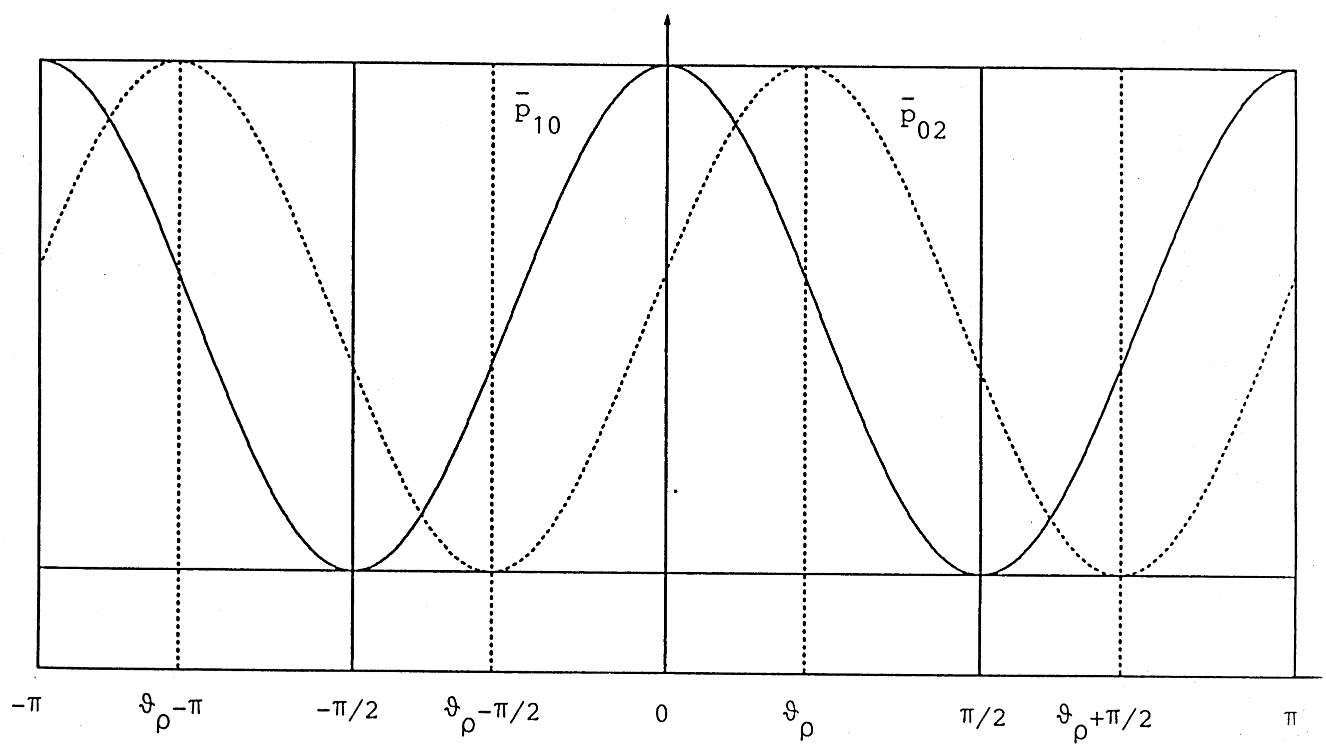
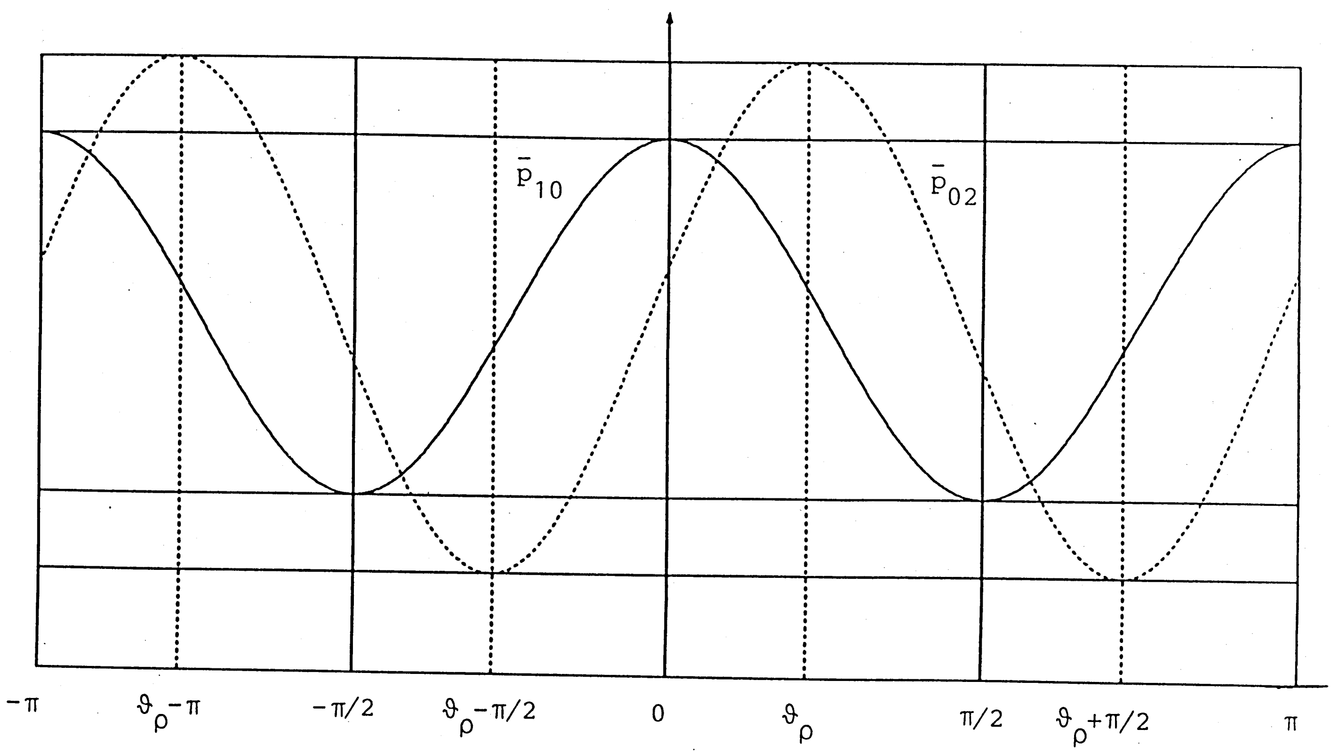


Figure 2(b): Averaged Conditional Densities of ϑ ($\lambda_{10} < \lambda_{02}$)



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