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1. INTRODUCTION

In the linear regression model with a lagged dependent variable as regressor, the assumption of uncorrelated disturbances is crucial for the properties of the structural parameter estimates. If the disturbances are autocorrelated and the parameters are estimated by ordinary least squares (OLS), the estimates will be inconsistent, and inferences from hypothesis tests on the parameters are likely to be misleading. Consequently the need to test for the presence of such correlation is obvious.

Until recently the asymptotic tests of Durbin [1970] known as Durbin's h and t tests, have been widely accepted as the appropriate tests for first order autocorrelated (AR(1)) disturbances (see Kenkel [1974, 1975, 1976] and Park [1975]). However, Inder [1986b] has proposed a procedure based on the Durbin-Watson (DW) statistic (Durbin and Watson [1950, 1951]) which he shows to be significantly more powerful than Durbin's tests, and which also performs more consistently under the null hypothesis. In this paper we present a new test which is a modification of the $s(\rho_1)$ test proposed by King [1985] for the static linear model. The results that follow suggest that this test is substantially more powerful than the DW test.

Consider the model

$$(1) \qquad y_t = \alpha y_{t-1} + x_t'\beta + u_t \qquad (t = 1, \ldots, n) ,$$

where y_t is the <u>tth</u> observation on the dependent variable, x_t is a $k \times 1$ vector of observations on the exogenous variables at time t, α and

β (k × 1) are unknown parameters, and u_t is a stochastic disturbance which follows the AR(1) process

(2) $$u_t = \rho u_{t-1} + e_t \quad (t = 1, \ldots, n) ,$$

where $|\rho| < 1$ and $e_t \sim IN(0,\sigma^2)$. We are interested in testing $H_0 : \rho = 0$ against the alternative $H_1 : \rho > 0$.

The DW statistic for the regression in (1) with the last n−1 observations is given by

(3) $$d = \hat{u}'A\hat{u}/\hat{u}'\hat{u} ,$$

where A is the first differencing matrix[2] and $\hat{u}$ is the vector of OLS residuals. Inder [1986b] proposed a test procedure using this statistic, with the critical value being the exact critical value from a regression on X alone. The tabulated bounds of Durbin and Watson [1951] could be used, and if the statistic fell in the inconclusive region, the appropriate critical value could be computed or approximated by one of the many procedures available.[3]

Durbin's h statistic is defined by

(4) $$h = (1 - d/2)[(n-1)/(1-(n-1)\hat{V}(\hat{\alpha}))]^{1/2} ,$$

where $\hat{V}(\hat{\alpha})$ is the estimate of the variance of $\hat{\alpha}$. Durbin's t test is a test of the significance of the coefficient of $\hat{u}_{-1}$ in the OLS regression of $\hat{u}$ on $\hat{u}_{-1}$,

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2. A is a tridiagonal matrix whose main diagonal elements are 2 except for the top left and bottom right elements which are both 1 and whose elements in the leading off-diagonals are all −1.

3. King [1986] surveys the various techniques for computing or approximating such critical values.

y_{-1}, and X where for any vector $z = (z_2, z_3, \ldots, z_n)$, $z_{-1} = (z_1, z_2, \ldots, z_{n-1})$ and $z_a = (z_1, z_2, \ldots, z_n)$, and $X' = (x_2, x_3, \ldots, x_n)$.

The plan of this paper is as follows: the new test, called the $s(\hat{\alpha}, \rho_1)$ test, is described in section 2, and its null distribution examined. Sections 3 and 4 then describe the results of a Monte Carlo (MC) study comparing the performance of the test with the DW, h and t tests - results under the null hypothesis are in section 3, and powers in section 4. In the final section we recommend the use of the $s(\hat{\alpha}, \rho_1)$ test in preference to any of the existing tests.

2. THE $s(\hat{\alpha}, \rho_1)$ TEST

One class of test procedure that has proved quite successful in the linear model without a lagged dependent variable is those tests which are most powerful invariant at some particular value of ρ, say ρ_1. The results of Berenblut and Webb [1973] and King [1985] suggest that these tests can be quite powerful relative to the DW test. Following King [1985], the test can be regarded as a likelihood ratio test of H_0 against the simple alternative that $\rho = \rho_1$ and will be referred to in this paper as the $s(\rho_1)$ test.

For the dynamic model, we propose a modification of the $s(\rho_1)$ test which involves estimating (1) by OLS, and obtaining $\hat{\alpha}$. A new variable

(5) $$\bar{y}_t = y_t - y_{t-1}\hat{\alpha} \qquad (t = 2, \ldots, n)$$

is constructed. The $s(\hat{\alpha}, \rho_1)$ statistic is then the $s(\rho_1)$ statistic in a regression of $\bar{y}$ on X. Specifically,

(6) $$s(\hat{\alpha}, \rho_1) = SSE(\hat{\alpha}, \rho_1)/SSE(0),$$

where SSE(O) is the sum of squared errors in an OLS regression of $\bar{y}$ on X,[4] and $SSE(\hat{\alpha}, \rho_1)$ is the sum of squared errors in an OLS regression of $\bar{y}^*$ on X^* for t = 2, ...,n, where for any vector or matrix w, $w^* = Qw$, and

$$(7) \qquad Q = \begin{bmatrix} (1-\rho_1^2)^{1/2} & 0 & 0 & . & . & . & 0 \\ -\rho_1 & 1 & 0 & . & . & . & 0 \\ 0 & -\rho_1 & 1 & . & . & . & 0 \\ . & & & . & & & . \\ . & & & . & . & & . \\ . & & & . & . & . & . \\ 0 & . & & . & . & -\rho_1 & 1 \end{bmatrix}$$

Essentially the modification to the $s(\rho_1)$ statistic involves "eliminating" the dynamic component of the model by constructing $\bar{y}$, and then computing the statistic to a now pseudo-static model.

Inder (1986a) has derived the large sample asymptotic distribution of the $s(\hat{\alpha},\rho_1)$ statistic. It was found that as with the DW statistic, this distribution depends on $V(\hat{\alpha})$, the asymptotic variance of $\hat{\alpha}$. Thus to obtain critical values based on large sample asymptotics, $V(\hat{\alpha})$ would need to be estimated. This results in a modification of the $s(\hat{\alpha},\rho_1)$ statistic similar in form to Durbin's h statistic:

$$(7) \qquad h_s(\hat{\alpha}, \rho_1) = \frac{[1 + \rho_1^2 - s(\hat{\alpha}, \rho_1)]}{2\rho_1} \sqrt{\frac{n-1}{1 - (n-1)\hat{V}(\hat{\alpha})}} \overset{a}{\sim} N(0, 1).$$

However, the discussion in Inder [1986b] indicates that the performance of the h test is poor in comparison with that of the DW test - it is

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4. SSE(O) is also the sum of squared errors in the estimation of (1).

frequently undefined, it has lower power, and the use of standard normal critical values does not lead to very good properties under H_0. The $h_s(\hat{\alpha}, \rho_1)$ test is likely to suffer in the same way relative to the $s(\hat{\alpha}, \rho_1)$ test. These potential problems are confirmed in the results presented by Inder (1986a).

In the remainder of this section we will examine the small disturbance asymptotic distribution of the $s(\hat{\alpha}, \rho_1)$ statistic.

THEOREM 2: Let $\overline{P}_X = PP'$, where $P'P = I_{n-k-1}$, and let $E(y_{-1}) = m_{-1}$. Under the model given by (1), (2), with $\rho = 0$, and provided $P'm_{-1} \neq 0$, the small disturbance asymptotic distribution of $s(\hat{\alpha},\rho_1)$ (defined in (6)) is the same as the distribution of

$$(8) \qquad s_o = u'\Omega'Q'\overline{P}_{QX}Q\Omega u/u'\Omega'\overline{P}_X\Omega u ,$$

where $\Omega = I - m_{-1}m'_{-1}\overline{P}_X/m'_{-1}\overline{P}_X m_{-1}$.

PROOF: A specific expression for the statistic is given by

$$(9) \qquad s(\hat{\alpha},\rho_1) = (y - y_{-1}\hat{\alpha})'Q'\overline{P}_{QX}Q(y - y_{-1}\hat{\alpha})/(y - y_{-1}\hat{\alpha})'\overline{P}_X(y - y_{-1}\hat{\alpha}).$$

This can be rewritten

$$s(\hat{\alpha},\rho_1) = (u-y_{-1}(\hat{\alpha}-\alpha))'Q'\overline{P}_{QX}Q(u-y_{-1}(\hat{\alpha}-\alpha))/(u-y_{-1}(\hat{\alpha}-\alpha))\overline{P}_X(u-y_{-1}(\hat{\alpha}-\alpha)),$$

and $u-y_{-1}(\hat{\alpha}-\alpha) = (I-y_{-1}y'_{-1}\overline{P}_X/y'_{-1}\overline{P}_X y_{-1})u$. Since $y_{-1} = m_{-1} + 0(\sigma)$,

$$u-y_{-1}(\hat{\alpha}-\alpha) = (I-m_{-1}m'_{-1}\overline{P}_X/m'_{-1})\bar{u}\sigma + 0(\sigma^2),$$

where $\bar{u} = u/\sigma$, and $0(\sigma^i)$ means other terms of order σ^i or higher. Thus

$$\begin{aligned} s(\hat{\alpha},\rho_1) &= u'\Omega'Q'\overline{P}_{QX}Q\Omega u/u'\Omega'\overline{P}_X\Omega u + 0(\sigma) \\ &= s_o + 0(\sigma) . \end{aligned}$$

Q.E.D.

Since s_o is a ratio of quadratic forms in normal variables, its distribution function can be obtained by the Imhof [1961] procedure namely

$$\Pr(s_o < s^*) = \Pr(u'\Omega'(Q'\bar{P}_{QX}Q - s^*\bar{P}_X)\Omega u < 0).$$

However, note that Ω depends on unknown parameters α and β, so the (small disturbance) asymptotic distribution of $s(\hat{\alpha},\rho_1)$ depends on these parameters.

For practical purposes, then, it is necessary to consider an approximation to the asymptotic distribution of $s(\hat{\alpha},\rho_1)$. Assuming that the elements of m_{-1} are of similar magnitude, the matrix $m_{-1}m'_{-1}\bar{P}_X/m'_{-1}\bar{P}_X m_{-1}$ will contain very small values (of order 1/n). Hence for reasonable values of n, replacing Ω with the identity matrix may yield quite an accurate approximation. This gives $s_o \approx s_s$, where

(10) $$s_s = u'Q'\bar{P}_{QX}Qu/u'\bar{P}_X u,$$

whose distribution function does not depend on any unknown parameters.[5]

The distribution function of s_s thus provides a computable approximation to the asymptotic distribution of $s(\hat{\alpha},\rho_1)$. Using this to obtain θ-level critical values requires solving

(11) $$\Pr(u'(Q'\bar{P}_{QX}Q - s_\theta\bar{P}_X)u < 0) = \theta$$

for s_θ. Observe further that s_s is, in fact, the $s(\rho_1)$ statistic in a model whose only regressors are X, so s_θ will be the exact critical value for the $s(\rho_1)$ statistic in a regression on X alone. This result is significant in that the bounds tabulated by King [1985] can be used as bounds for s_θ. The

5. Inder [1986b] made a similar approximation for the DW statistic, and gave several other reasons why this approximation may be expected to be reasonably accurate.

critical value need only be computed when the calculated value of $s(\hat{\alpha}, \rho_1)$ falls in the appropriate inconclusive region.[6]

3. PROPERTIES OF THE $s(\hat{\alpha}, \rho_1)$ TEST UNDER THE NULL HYPOTHESIS

The adequacy of the $s(\hat{\alpha}, \rho_1)$ test as an alternative to Durbin's tests and the DW test can be assessed in terms of its performance under H_0 (probabilities of a type I error) and under H_1 (power). We resort to MC techniques to examine the test, looking at the null distribution in this section, and its power in the next section. The following data sets and parameter values were used:

Experiment 1 : X comprised of a constant and Maddala and Rao's [1973] GNP data. $\beta' = (0, 1, 1)$; n = 32 and 76; and σ = 8, 20, and 40.

Experiment 2 : X included a constant and an artificially generated series constructed by adding a random variable V_t to the GNP series, where $V_t \sim N(0, 1600)$ (McNown and Hunter [1980]). All other parameter values were identical to experiment 1.

Experiment 3 : X contained a constant, three quarterly seasonal dummy variables, and the quarterly Australian Consumer Price Index comencing 1959(1). $\beta' = (0, 1, 6, -7, 2)$; n = 30 and 60; and σ = 1, 2, and 4.

Experiment 4 : X was Durbin and Watson's [1971] consumption of spirits data (a constant, real income, and price of spirits). $\beta' = (0, 1, 1)$; n = 30 and 60; and σ = .5 and 1.

In all four experiments α took the values .2, .4, .6, and .8.

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6. The appropriate bounds would be for n-1 observations and k regressors.

The classical approach to nonsimilar tests involves finding critical values for which the probability of a type I error never exceeds the size. This provides a basis for comparing the tests under H_0: since all of the existing tests are nonsimilar, it would be preferable if they do not have probabilities of a type I error which exceed the nominal significance level. However, in previous MC studies, interest has been focused on how close the probabilities of a type I error are to the nominal size, with fluctuations above and below this level being given equal importance. We will evaluate the critical values of the $s(\hat{\alpha}, \rho_1)$ test according to both criteria, dealing with the latter first.

Tables 1 and 2 provide a selection of typical results comparing the small disturbance asymptotic critical values of the $s(\hat{\alpha}, .5)$ test,[7] and the DW, h and t tests.[8] They show that in some cases the test can perform quite consistently (for example, Experiment 2), and in other cases they are very eratic (for example, Experiment 4).

We see that in most cases the $s(\hat{\alpha}, \rho_1)$ test probabilities of a type I error are the smallest and are further from the significance level. Only occasionally does the $s(\hat{\alpha}, \rho_1)$ test perform better than the DW test. The approximation does seem to be more accurate for large values of α and small σ (which is not surprising since it is a small σ approximation), and the difference between the two tests is much smaller in these cases. All of the tests seem to perform better for the larger sample sizes, although the improvement is only marginal.

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7. The θ-level critical values for the $s(\alpha, \rho_1)$ test were obtained for $\theta = .05$ and $.01$ using a modified version of Koerts and Abrahamse's [1969] sub-routine to solve (11) for s_θ.

8. Several values of ρ_1 were considered for the $s(\hat{\alpha}, \rho_1)$ test; varying ρ_1 made little difference to size or power, so we chose to use a "middle" value, namely 0.5.

Occasionally the probabilities of a type I error are exceptionally low for the $s(\hat{\alpha}, \rho_1)$ test: for example, with the first data set and n = 76, $\sigma = 20$, $\alpha = .2$, the probability is only .002 compared to a nominal size of .05. Such cases are of particular concern not only because they are so far from the nominal size, but also because of the detrimental effect these low probabilities can have on the power of the test. However, note that in every situation where the probability is low for the $s(\hat{\alpha}, \rho_1)$ test, it is also very small for the DW test.

Experiment 4 yields some interesting results: the DW probabilities vary greatly from close to zero when $\alpha = .2$ to around .1 when $\alpha = .8$ (with a significance level of .05). Durbin's h and t tests also have very poor probabilities of a type I error for this data set. However, the probabilities for the $s(\hat{\alpha}, \rho_1)$ test show little variability - most are around .01. While these probabilities are undesirably low when one's nominal size is .05, it is perhaps better than what occurs for the other tests, where the probability of a type I error can fluctuate wildly.

If one's goal is to find exact nonsimilar critical values for the test (that is, critical values which give probabilities of a type I error which are never greater than the size), the results in Tables 1 and 2 suggest that the small disturbance asymptotic critical values for the $s(\hat{\alpha}, \rho_1)$ test may provide a reasonable approximation. With a significance level of .05, in no case does the probability of a type I error exceed this, and it only occurs a few times at the .01 level.

To examine more closely the performance of the $s(\hat{\alpha}, \rho_1)$ critical values further MC results were obtained on the probabilities of a type I error with a wider range of parameter values. In particular, α values of .001 and .999 were considered, along with some substantially larger values of σ for each experiment. The results of this extra work are encouraging for the use of the small disturbance asymptotic critical values as an approximation to nonsimilar critical values. In only a few cases do the probabilities of a type I error exceed the significance level, and the excess is always negligible.

Recognising that broad conclusions from MC studies need to be made very cautiously, it is fair to say that we have no evidence to reject the proposal that the small disturbance asymptotic critical values for the $s(\hat{\alpha}, \rho_1)$ test can provide reasonable approximations to the exact nonsimilar critical values. The approximation has the advantage of being much easier and quicker to compute than the procedure for obtaining nonsimilar critical values of the DW test[9] given in Inder [1986b].

The small disturbance asymptotic critical values of the $s(\hat{\alpha}, \rho_1)$ test appear to be adequate. They do very well at approximating nonsimilar critical values, and although the DW test can often yield probabilities of a type I error which are closer to the nominal size, the difference is not substantial. If the $s(\hat{\alpha}, \rho_1)$ test proved to have far superior power properties, one may be inclined to overlook its slightly inferior performance under H_0. Furthermore, the $s(\hat{\alpha}, \rho_1)$ test seems to be more consistent under H_0: its probabilities of a type I error do not fluctuate very much relative to those of the DW, h and t tests.

4. PROPERTIES OF THE $s(\hat{\alpha}, \rho_1)$ TEST UNDER THE ALTERNATIVE HYPOTHESIS

In this section we report some powers of the $s(\hat{\alpha}, \rho_1)$ test and compare these with the powers of the DW, h and t tests. Powers are obtained with the

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9. The procedure for finding θ-level nonsimilar critical values of the DW test is as follows: set α to .999 and β to zero, and generate at least 1000 samples under H_0. For each sample, calculate the DW statistic, and choose the critical values so that 1000 x θ of the DW statistics are less than this value.

small disturbance asymptotic critical values described in Section 2. These latter critical values can be interpreted as approximate nonsimilar or as "asymptotically similar" critical values (see Section 3).

The results in Tables 3 and 4, together with Figure 1, provide some typical power results, as well as showing the most extreme cases of the performance of the $s(\hat{\alpha}, \rho_1)$ test (the "worst" is in Figure 1a, and the best in Figure 1b). Despite the handicap of almost always having lower probabilities of a type I error, the $s(\hat{\alpha}, .5)$ test still shows a significant power advantage over the other tests. In no case does any other test have lower probabilities of both types of error than the $s(\hat{\alpha}, .5)$ test, but there are many situations where the $s(\hat{\alpha}, .5)$ test dominates the other tests in this way.

In fact, it is possible to go even further in establishing the test's superiority: in every case considered in this MC study, the $s(\hat{\alpha}, .5)$ test is at least as powerful as its competitors for moderate to large values of ρ, even though often its probability of a type I error is substantially lower. In Figure 1b, for example, the $s(\hat{\alpha}, .5)$ test has far greater power for values of ρ above .3, despite the fact that its probability of a type I error is much smaller. In most cases the $s(\hat{\alpha}, .5)$ test has higher power for ρ values of .3 and higher, although sometimes it can be slower in catching its competitors because of the vast differences in the probabilities of a type I error. Figure 1d illustrates the most unfavourable of these cases, where the probability of a type I error is very small for the $s(\hat{\alpha}, .5)$ test (.018), and for all the other tests it is unacceptably high (above .1). Despite this, the $s(\hat{\alpha}, .5)$ test is still the most powerful for ρ greater than .5.

These results show that, despite the fact that the $s(\hat{\alpha}, .5)$ test starts at a disadvantage because of its properties under H_0, its power is so great that it is still to be preferred to any of the existing alternatives. It always gives the best power for values of ρ where this is especially important - moderate to large values - often at the same time having the lowest probability of a type I error.

The results reported in this paper show clearly that the powers of the tests vary greatly with the X data and parameter values. One possible cause of this is the variation in how well the model fits the data. To verify this, a pseudo-R^2 was calculated (using known parameter values) for each combination of parameter values.[10] This revealed great variation between and within experiments "R^2" ranged from below 0.1 in experiment 4 to more than 0.99 for some cases in experiments 1 and 3. This analysis revealed that when smaller R^2 values occur, the probabilities of a type I error of all the tests deviate much more from the nominal size. The powers, however, do not seem to vary as systematically with R^2.

Powers do, however, depend on α, with generally better powers for larger α values, and less difference between the tests. There is also some small improvement in power for the smaller σ values, and larger sample sizes. The other key source of variability seems to be the nature of the X data. The data for experiments 2 and 4 are quite choppy, and in these cases the tests all have similar performance; the superiority of the $s(\hat{\alpha}, .5)$ test is more obvious in the more smoothly evolving and trending data of experiments 1 and 3. This observation could be very important: if valid it suggests that existing procedures such as Durbin's tests may be used with confidence when detrended or choppy data is used; in other cases there is a large cost involved in not using the $s(\hat{\alpha}, \rho_1)$ test.

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10. This measure of goodness of fit was obtained by comparing the actual variance of the disturbance term with the variance of the mean vector of y, with R^2 being one minus the ratio of these quantities.

As a further test of the validity of this conclusion, power results were obtained for another nontrending, choppy X series. This series was obtained by randomly "shuffling" the GNP data in experiment 1. As anticipated, there was much less variation in the powers of the tests than for experiment 1.[11]

In Section 3 it was observed that the small disturbance asymptotic critical values of the $s(\hat{\alpha}, .5)$ test could serve as approximations to the nonsimilar critical values. It would be of interest, then, to compare the power of the test with that of the DW, h and t tests when nonsimilar critical values are used. A few results for this comparison are given in Figure 2.

It is not surprising that this comparison favours the $s(\hat{\alpha}, .5)$ test even more strongly. The $s(\hat{\alpha}, .5)$ test is the most powerful test in every situation, except for a few cases with $\rho = .1$ in experiment 4 where the probabilities of a type I error for the other tests are higher (see Figure 1c). There are many occasions where the powers of the DW, h and t tests are poor, not even reaching .5 when $\rho = .9$, where the $s(\hat{\alpha}, .5)$ test power often comes very close to one (for example, see Figure 2b). To conclude, if the researcher preferred to use nonsimilar critical values, the $s(\hat{\alpha}, .5)$ test, with its approximate nonsimilar critical values, has vastly superior power to the DW, h and t tests.

5. CONCLUSIONS

In Inder [1986b] we introduced procedures for obtaining asymptotically similar critical values and nonsimilar critical values of the DW test for first order autoregressive disturbances in the dynamic linear model. It was

11. I am grateful to an anonymous referee for suggesting this procedure.

shown that this test performs better than Durbin's h and t tests, both in terms of power and consistency under the null hypothesis. It was the aim of this paper to improve on the DW test still further. In the light of this aim, the $s(\hat{\alpha}, \rho_1)$ test was introduced. In Section 2 we considered the null distribution of this new test and suggested a simple procedure by which approximate critical values could be obtained. The results of Section 3 suggest that this procedure, based on the small disturbance asymptotic distribution of the statistic, can provide both asymptotically similar critical values and approximations to the nonsimilar critical values of the test. The test procedure using the $s(\hat{\alpha}, .5)$ statistic is as easy to perform as the $s(\rho_1)$ or DW tests in the static linear model.

The power of the $s(\hat{\alpha}, .5)$ test was investigated in Section 4 by means of a MC study: this showed overwhelmingly that the test yields a vast improvement over the DW test. It is always at least as powerful for moderate to large values of ρ, even when its probability of a type I error is often substantially lower. There are many occasions when the test has lower probabilities of both types of error. The power of the $s(\hat{\alpha}, .5)$ test is often more than double that of the DW and other tests.

It appears, then, that the aim of this paper has been realised in the $s(\hat{\alpha}, .5)$ test. While the DW test is a reliable and powerful test, the $s(\hat{\alpha}, .5)$ test is considerably more powerful. We would thus conclude by recommending that the $s(\hat{\alpha}, .5)$ test with critical values based on the procedure outlined in Section 2 be used in preference to the DW, h and t tests.

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TABLE 1

PROBABILITIES OF A TYPE I ERROR FOR THE TESTS USING ASYMPTOTIC CRITICAL VALUES : NOMINAL SIZE = 5%

	Test	$s(\hat{\alpha},.5)$	DW	t	h^a
α			EXPERIMENT 1 : n = 32		
	.2	.026	.033	.030	.089(72)
σ = 8	.4	0.035	0.043	0.025	0.055(3)
	.6	0.039	0.046	0.018	0.032
	.8	0.038	0.039	0.010	0.019
	.2	0.005	0.008	0.058	0.119(1638)
σ = 20	.4	0.011	0.027	0.060	0.147(563)
	.6	0.027	0.052	0.044	0.102(68)
	.8	0.037	0.048	0.018	0.036
	.2	0.007	0.004	0.079	0.093(3057)
σ = 40	.4	0.005	0.015	0.091	0.169(1653)
	.6	0.010	0.044	0.085	0.184(474)
	.8	0.035	0.068	0.047	0.091(35)
			EXPERIMENT 2:n = 76		
	.2	0.050	0.055	0.036	0.046
σ = 8	.4	0.048	0.054	0.035	0.043
	.6	0.046	0.051	0.034	0.042
	.8	0.046	0.050	0.033	0.040
	.2	0.048	0.054	0.037	0.045
σ = 20	.4	0.048	0.052	0.035	0.045
	.6	0.047	0.051	0.035	0.042
	.8	0.045	0.050	0.033	0.040
	.2	0.047	0.048	0.038	0.048
σ = 40	.4	0.049	0.048	0.035	0.046
	.6	0.047	0.050	0.033	0.044
	.8	0.046	0.050	0.033	0.042

[a] Figures in parentheses represent the number of times h was undefined.

TABLE 2

PROBABILITIES OF A TYPE I ERROR FOR THE TESTS USING ASYMPTOTIC CRITICAL VALUES

	Test	$s(\hat{\alpha},.5)$	DW	t	h[a]
	α	EXPERIMENT 3 : n = 60, nominal size = 1%			
$\sigma = 1$	.2	0.008	0.007	0.007	0.017
	.4	0.009	0.008	0.007	0.013
	.6	0.009	0.010	0.007	0.010
	.8	0.009	0.010	0.006	0.010
$\sigma = 2$	.2	0.004	0.002	0.010	0.050 (15)
	.4	0.007	0.005	0.010	0.023
	.6	0.007	0.008	0.009	0.012
	.8	0.009	0.010	0.007	0.011
$\sigma = 4$	.2	0.000	0.000	0.013	0.096 (897)
	.4	0.001	0.001	0.014	0.072 (99)
	.6	0.006	0.006	0.013	0.028 (2)
	.8	0.007	0.010	0.010	0.013
		EXPERIMENT 4: n = 30, nominal size = 5%			
$\sigma = .5$	.2	0.016	0.009	0.084	0.041 (4205)
	.4	0.017	0.023	0.110	0.112 (3375)
	.6	0.016	0.055	0.118	0.196 (2026)
	.8	0.012	0.103	0.109	0.229 (913)
$\sigma = 1$	.2	0.017	0.009	0.085	0.041 (4236)
	.4	0.017	0.024	0.116	0.106 (3442)
	.6	0.018	0.055	0.122	0.197 (2097)
	.8	0.018	0.106	0.115	0.237 (1029)

[a] Figures in parentheses represent the number of times h was undefined.

TABLE 3

POWERS OF THE TESTS USING ASYMPTOTIC CRITICAL VALUES : NOMINAL SIZE = 5%

EXPERIMENT 1 : $n = 76$, $\sigma = 40$[a]

ρ =	0.	.1	.3	.5.	.7	.9
Test			$\alpha = .2$			
$s(\hat{\alpha}, .5)$	.003	.076	.811	.995	1.000	1.000
DW	.001	.003	.031	.137	.379	.604
h	.158 (1614)	.230 (212)	.369 (34)	.440 (2)	.532	.637
t	.074	.120	.233	.331	.476	.578
			$\alpha = .4$			
$s(\hat{\alpha}, .5)$	.001	.077	.822	.995	1.000	1.000
DW	.006	.020	.180	.522	.830	.956
h	.183 (293)	.272 (27)	.503	.722	.884	.962
t	.079	.146	.397	.650	.843	.944
			$\alpha = .6$			
$s(\hat{\alpha}, .5)$	.009	.103	.822	.996	1.000	1.000
DW	.027	.095	.502	.867	.985	.998
h	.133 (9)	.248 (2)	.650	.909	.987	.998
t	.073	.180	.590	.879	.984	.997
			$\alpha = .8$			
$s(\hat{\alpha}, .5)$	.031	.141	.810	.996	1.000	1.000
DW	.057	.180	.745	.976	.999	1.000
h	.078	.213	.764	.977	.999	1.000
t	.055	.174	.725	.968	.997	1.000

a Figures in parentheses represent the number of times h was undefined.

TABLE 4

POWERS OF THE TESTS USING ASYMPTOTIC CRITICAL VALUES : NOMINAL SIZE = 5%

EXPERIMENT 4 : n=60, σ = 1[a]

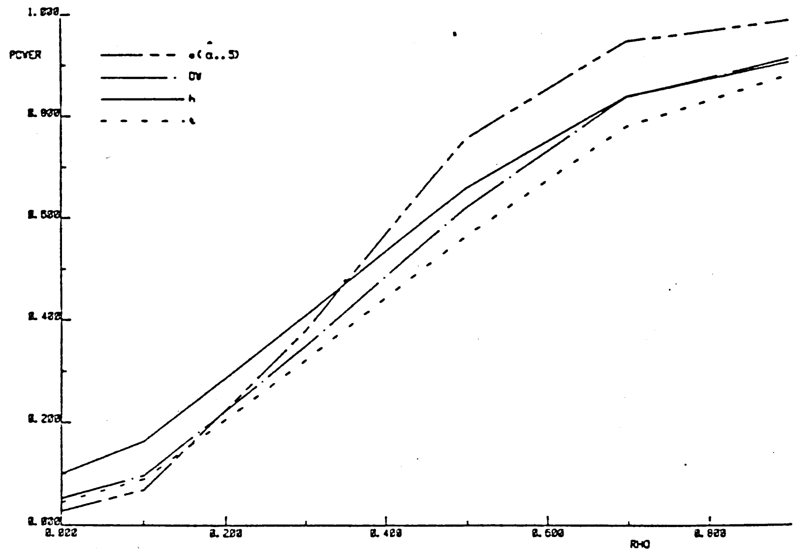
ρ =	0.		.1		.3		.5		.7	.9	
Test					α = .2						
$s(\hat{\alpha}, .5)$	.012		.034		.652		.964		.998	1.000	
DW	.001		.003		.034		.174		.348	.541	
h	.090	(3769)	.181	(573)	.373	(163)	.444	(25)	.506	.557	(1)
t	.093		.152		.224		.307		.390	.475	
					α = .4						
$s(\hat{\alpha}, .5)$	.011		.036		.656		.964		.997	1.000	
DW	.008		.027		.183		.493		.763	.911	
h	.237	(1582)	.348	(162)	.512	(24)	.685	(2)	.807	.901	
t	.118		.191		.351		.572		.754	.864	
					α = .6						
$s(\hat{\alpha}, .5)$	.010		.039		.657		.966		.997	1.000	
DW	.042		.112		.425		.832		.955	.987	
h	.253	(268)	.365	(20)	.631	(2)	.879		.956	.984	
t	.120		.216		.509		.832		.940	.979	
					α = .8						
$s(\hat{\alpha}, .5)$	.004		.049		.655		.965		.996	1.000	
DW	.103		.226		.692		.949		.990	.999	
h	.204	(26)	.342	(2)	.740		.949		.990	.999	
t	.120		.246		.664		.932		.987	.998	

[a] Figures in parentheses represent the number of times h was undefined.

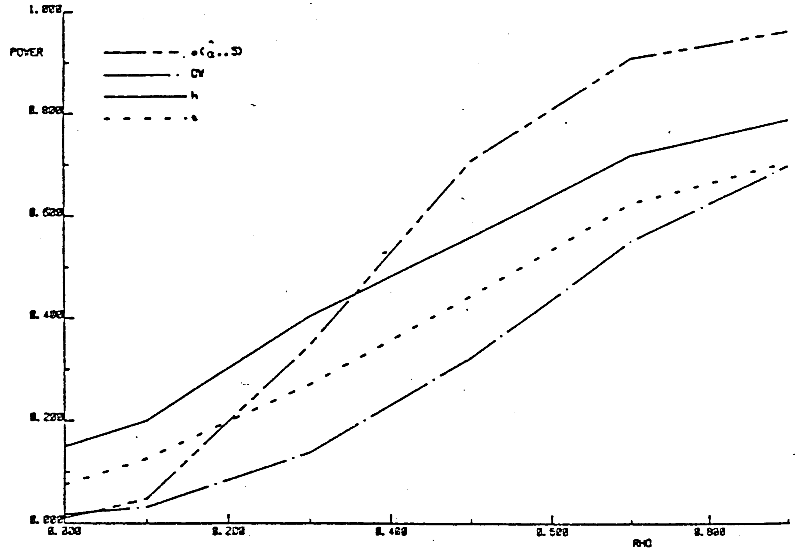
FIGURE 1

POWERS OF THE TESTS USING ASYMPTOTIC CRITICAL VALUES

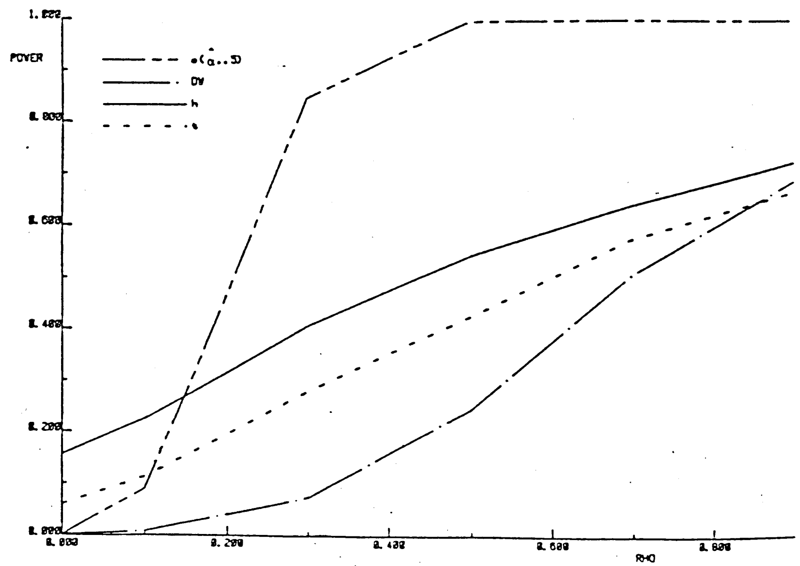
(a) EXPERIMENT 1 : n = 32; σ = 20; α = .6; SIZE = .05



(c) EXPERIMENT 3 : n = 30; σ = 4; α = .4; SIZE = .05



(b) EXPERIMENT 1 : n = 76; σ = 20; α = .2; SIZE = .05



(d) EXPERIMENT 4 : n = 30; σ = 1; α = .8; SIZE = .05

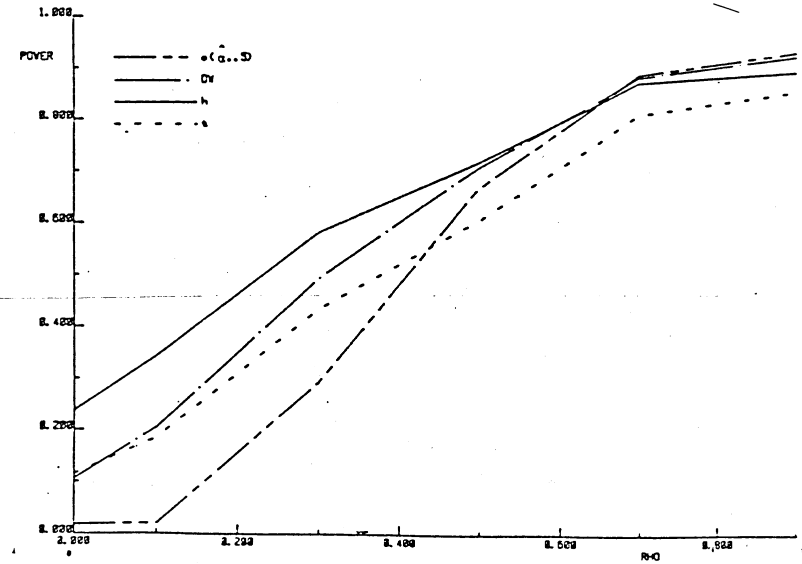
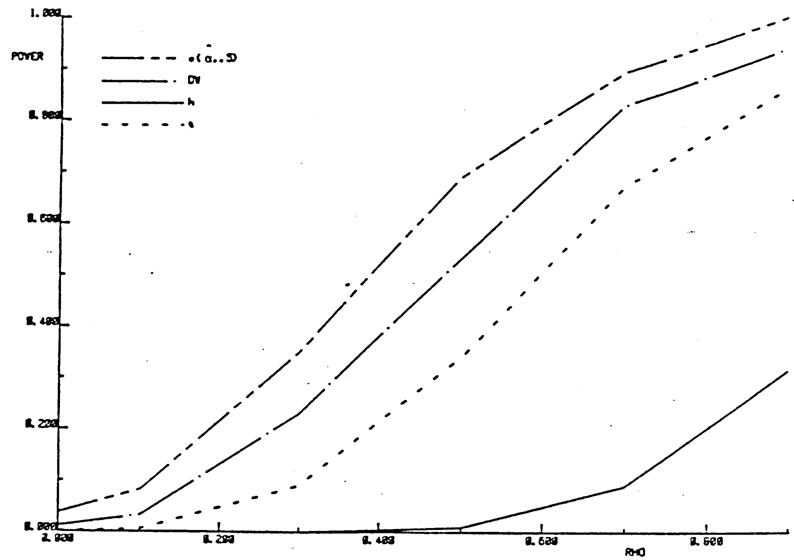


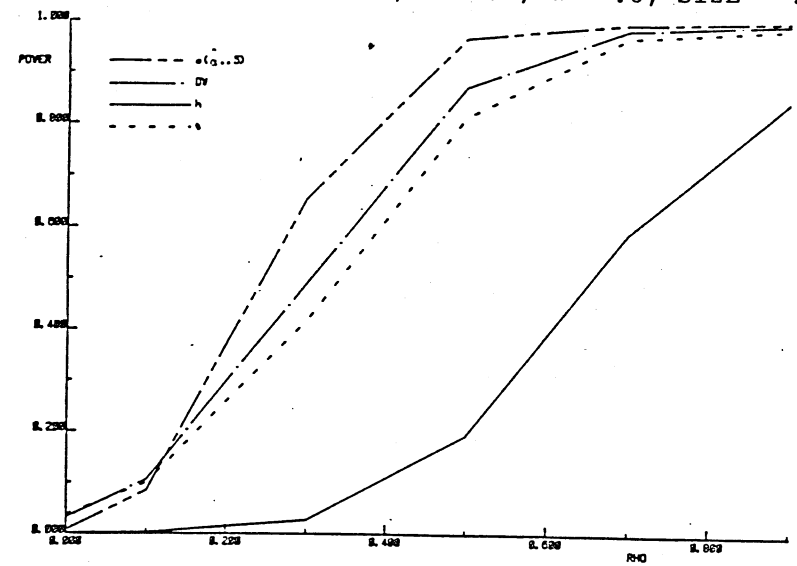
FIGURE 2

POWERS OF THE TESTS USING NONSIMILAR CRITICAL VALUES

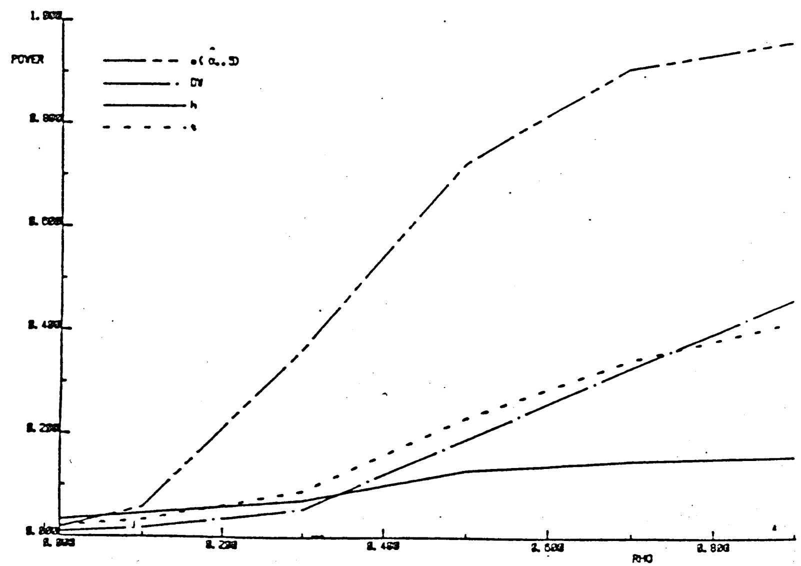
(a) EXPERIMENT 1 : n = 32; σ = 8; α = .8; SIZE = .05



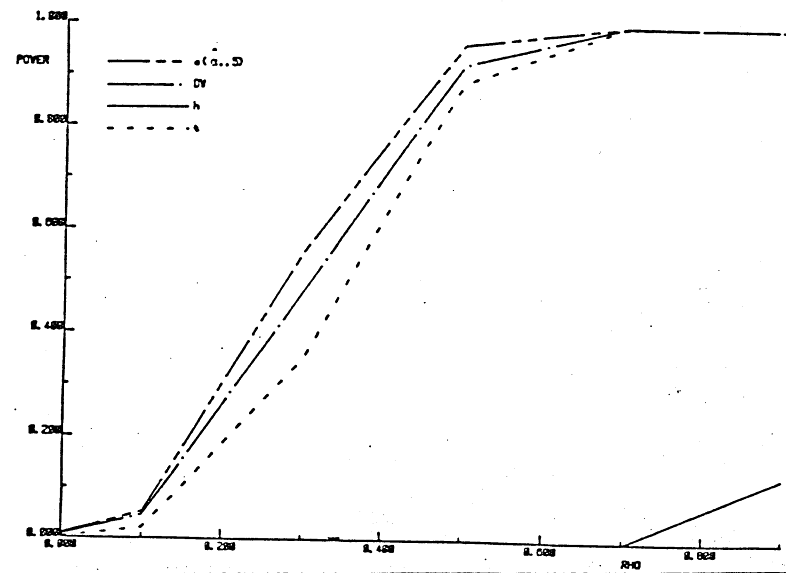
(c) EXPERIMENT 4 : n = 60; σ = .5; α = .8; SIZE = .05



(b) EXPERIMENT 3 : n = 30; σ = 2; α = .2; SIZE = .05



(d) EXPERIMENT 2 : n = 76; σ = 8; α = .6; SIZE = .01



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