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A BETA-OPTIMAL TEST OF THE EQUICORRELATION COEFFICIENT

by

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Summary

This paper considers the problem of testing for nonzero values of the equicorrelation coefficient of a standard symmetric multivariate normal distribution. Recently, SenGupta (1987) proposed a locally best test. We construct a beta-optimal test and present selected one and five per cent critical values. An empirical power comparison of SenGupta's test with two versions of the beta-optimal test and the power envelope shows the relative strengths of the three tests. It also allows us to assess and confirm Efron's (1975) rule of when to question the use of a locally best test, at least for this testing problem. On the basis of these results, we argue that the two beta-optimal tests can be considered as approximately uniformly most powerful tests, at least at the five per cent significance level.

Key words: beta-optimal test; locally best test; point-optimal test; power envelope; standard symmetric multivariate normal distribution; statistical (Efron) curvature.

1. INTRODUCTION

Recently, SenGupta (1987) proposed a test for nonzero values of the equicorrelation coefficient of a standard symmetric multivariate normal (SSMN) distribution. The symmetric multivariate normal (SMN) distribution is a multivariate normal distribution in which all components have equal means and equal variances and all covariances between components take the same value. These common covariances give rise to a common correlation coefficient, ρ, which is called the intraclass, equi-, uniform or familial correlation. The SSMN distribution is a SMN distribution in which the component mean and variance values are zero and unity, respectively. As Sampson (1978) notes, SSMN distributions arise naturally from multivariate normal models in which means and variances of individual variables are known, thus allowing these variables to be standardized. For example, this can occur when there are many observations on the individual variables but, because of historical, financial or practical reasons, there are comparatively few sets of joint observations. The individual observations can be used to obtain excellent estimates of means and variances which allow one to proceed as if these estimates are the true values.

SenGupta showed that in the context of the SSMN distribution, the likelihood ratio test that ρ takes a given value has a number of theoretical and practical shortcomings. He constructed the locally most powerful test of $H_0 : \rho = 0$ and found that it does not share these shortcomings while appearing to have reasonable small sample power. A locally most powerful test, often also known as a locally best (LB) test, a term we prefer (see Ferguson, 1967), is optimal in the sense that its power curve has the steepest slope at H_0 of all power curves from tests with the same size. Alternatively, a LB test can be viewed as a test which optimizes power in the neighbourhood of the null hypothesis.

This paper considers point-optimal tests (see King, 1987) which optimize power at a predetermined point under the alternative hypothesis. We find that for this testing problem, point-optimal tests are also beta-optimal tests. The concept of beta-optimality was first introduced by Davies (1969) and is based on finding the test whose power function reaches a predetermined value, p_1 say, most quickly. In contrast to LB tests, beta-optimal tests typically optimize power at a point away from the null hypothesis. The analogous tests for a SMN distribution can be constructed in a similar manner to the beta-optimal tests Davies and Harte (1987) proposed for a related testing problem.

In the next section, the model is introduced and the beta-optimal test constructed. The new test involves finding the point-optimal test which optimizes power at a predetermined level of power, say 0.5 or 0.8. Computational details are discussed and selected one and five per cent critical values are tabulated. Section 3 reports an empirical power comparison of the LB test, two versions of the beta-optimal test and the power envelope (PE) which traces out the maximum obtainable power. Among other things, the power comparison allows an empirical assessment of Efron's (1975) rule, based on statistical curvature, of when to question the use of a LB test. It also allows us to argue that the beta-optimal tests can be regarded as approximately uniformly most powerful (UMP) at least at the five per cent significance level.

2. THE MODEL AND THE TEST

Let y be a k×1 random vector with a SSMN distribution, i.e.,

$$y \sim N(0, \Sigma(\rho))$$

where $\Sigma(\rho) = (1-\rho)I_k + \rho E_k$, I_k is the k×k identity matrix and E_k is the k×k matrix of ones. Let $y_1, y_2, \ldots, y_m$ denote a random sample from this distribution and let Y denote the n×1 stacked vector of these independent random vectors where n = mk. Then

$$Y \sim N(0, \Delta(\rho))$$

where $\Delta(\rho) = (1-\rho)I_n + \rho D$, and D is the n×n block diagonal matrix,

$$D = \begin{bmatrix} E_k & 0 & & 0 \\ 0 & E_k & & 0 \\ & & \ddots & \\ 0 & 0 & & E_k \end{bmatrix} .$$

Furthermore

$$\Delta^{-1}(\rho) = \frac{1}{(1-\rho)} \left[I_n - \rho \{1 + (k-1)\rho\}^{-1} D \right] , \qquad (1)$$

provided $-1/(k-1) < \rho < 1$.

The problem of interest is one of testing

$$H_0 : \rho = 0 \text{ against } H_a : \rho > 0 .$$

To construct our test, we first consider the simpler problem of testing H_0 against the simple hypothesis $H_a' : \rho = \rho_1 > 0$ where ρ_1 is fixed and known. The Neyman-Pearson lemma implies that the critical region

$$r(\rho_1) = Y'(\Delta^{-1}(\rho_1) - I_n)Y < c_\alpha$$

is the most powerful test where c_α is an appropriate critical value. c_α

can be computed by noting that under H_0,

$$\Pr\left[\, r(\rho_1) < c_\alpha \,\right]$$

$$= \Pr\left[\, \sum_{i=1}^{n} \lambda_i \xi_i^2 < c_\alpha \,\right]$$

$$= \Pr\left[\left\{\rho_1/(1-\rho_1)\right\}\chi^2_{m(k-1)} - \left[\rho_1(k-1)/\{1+(k-1)\rho_1\}\right]\chi^2_m < c_\alpha\right] \qquad (2)$$

where $\xi = (\xi_1, \xi_2, \ldots, \xi_n)' \sim N(0, I_n)$, λ_i, $i = 1, \ldots, n$, are the eigenvalues of $\Delta^{-1}(\rho_1) - I_n$ and χ^2_j denotes a chi-squared random variable with j degrees of freedom. The second equality follows because the eigenvalues of D are k and zero with multiplicities of m and m(k-1), respectively. This together with (1) implies that the eigenvalues of $\Delta^{-1}(\rho_1) - I_n$ are $-\rho_1(k-1)/\{1+(k-1)\rho_1\}$ and $\rho_1/(1-\rho_1)$ with multiplicities m and m(k-1), respectively. In order to find c_α such that (2) equals the required significance level, α, (2) can be evaluated using either Koerts and Abrahamse's (1969) FQUAD subroutine or Davies' (1980) algorithm. Alternatively, one can note that in (2), $r(\rho_1)$ is expressed as the weighted difference of two independent chi-squared random variables so that its probability density function is given by SenGupta (1987, Theorem 3).

For the wider problem of testing H_0 against H_a, the test based on $r(\rho_1)$ is most powerful at $\rho = \rho_1$ and is therefore a point-optimal test. A central question is: how should ρ_1 be chosen? Strategies for choosing the point at which a point-optimal test optimizes power are discussed in King (1987). One approach is to choose a ρ_1 value arbitrarily. Another is to take the limit of $r(\rho_1)$ tests as ρ_1 tends to zero. This results in SenGupta's LB test. There seems little point in optimizing power when it is very low (as the LB test does) or when it is one or nearly one. We favour optimizing power at a middle power value, say 0.5. In order to do this, we need to be able to calculate readily the power of our test.

Consider the Cholesky decomposition of $\Delta(\rho)$, namely

$$\Delta(\rho) = TT'$$

where T is an n×n nonsingular, lower triangular matrix. Then under H_a

$$z = T^{-1}Y \sim N(0, I_n) .$$

Thus for any critical value c_α, the power of the critical region $r(\rho_1) < c_\alpha$ is

$$\Pr\left[r(\rho_1) < c_\alpha \mid Y \sim N(0,\Delta(\rho)) \right]$$

$$= \Pr\left[z'T'(\Delta^{-1}(\rho_1) - I_n)Tz < c_\alpha \right]$$

$$= \Pr\left[\sum_{i=1}^{n} \lambda_i \, \xi_i^2 < c_\alpha \right] , \qquad (3)$$

where λ_i, i = 1, ..., n, are the eigenvalues of $T'(\Delta^{-1}(\rho_1) - I_n)T$ and $\xi \sim N(0, I_n)$.

Note that the eigenvalues of $T'(\Delta^{-1}(\rho_1) - I_n)T$ are also the eigenvalues of

$$(\Delta^{-1}(\rho_1) - I_n)\Delta(\rho)$$

$$= \frac{\rho_1(1-\rho)}{(1-\rho_1)} I_n + \frac{\rho_1[\rho\{(\rho_1-1)(k-1) + 1\} - 1]}{(1-\rho_1)(1 + (k-1)\rho_1)} D$$

$$= aI_n + bD ,$$

say. The eigenvalues of the latter matrix are

$$a = \frac{\rho_1(1-\rho)}{1-\rho_1} \qquad (4)$$

with multiplicity of mk-m and

$$a + bk \;=\; \frac{-\rho_1(k-1)\{1 + (k-1)\rho\}}{\{1 + (k-1)\rho_1\}} \tag{5}$$

with multiplicity of m. Thus $r(\rho_1)$ in (3) can also be expressed as a weighted difference of two independent chi-squared random variables so (3) can be evaluated using any of the methods outlined above for calculating (2). In the special case in which power is being evaluated at ρ_1, (4) and (5) reduce to ρ_1 and $-(k-1)\rho_1$, respectively, so that (3) can be written as

$$\Pr[\rho_1\chi^2_{m(k-1)} - (k-1)\rho_1\chi^2_m < c_\alpha] \;=\; \Pr[\chi^2_{m(k-1)} - (k-1)\chi^2_m < c_\alpha/\rho_1].$$

Observe that both (4) and (5) decline in value as ρ increases. Given (3), this means that the test's power increases as ρ increases which implies the test is a beta-optimal test. A test is beta-optimal if its power function reaches a predetermined value, p_1, most quickly as one moves away from H_0. As noted by King (1987, pp.197-8), a point-optimal test is beta-optimal if its power function is always a monotonic non-decreasing function of the parameter under test. Davies (1969), when introducing the concept of beta-optimality, suggested p_1 should take the value 0.8.

Given the desired level of significance, α, and the level of power at which we wish to optimize power, p_1, then ρ_1 and the associated critical value, c_α, can be found as follows:

(i) Solve

$$\Pr[\chi^2_{m(k-1)} - (k-1)\chi^2_m < c_\alpha/\rho_1] \;=\; p_1$$

for c_α/ρ_1 .

(ii) Given this ratio, determine c and ρ by solving

(ii) Given this ratio, determine c_α and ρ_1 by solving

$$\Pr[r(\rho_1) < c_\alpha] = \alpha .$$

In the remainder of this paper, we will denote this as the r_{p_1} test.

Selected one and five per cent significance points, c_α, and their associated ρ_1 values for the $r_{0.5}$ (i.e., $p_1 = 0.5$) and $r_{0.8}$ (i.e., $p_1 = 0.8$) tests are tabulated in Tables 1 and 2 respectively. They were calculated using a FORTRAN version of Davies' (1980) algorithm.

3. A COMPARISON OF POWERS

Point-optimal tests can be used to trace out the maximum attainable PE for a given testing problem. In our case, this can be done by evaluating the power of the $r(\rho_1)$ test at $\rho = \rho_1$ over a range of ρ_1 values. The PE provides an obvious benchmark against which test procedures can be evaluated. If a test's power is always found to be close to the PE, it can be thought of as an approximately UMP test. An example of such a finding is given by Shively (1988).

It is also of interest to compare the power curves of SenGupta's LB test with those of the beta-optimal tests, $r_{0.5}$ and $r_{0.8}$. Is one of these tests close enough to the PE to be called an approximately UMP test?

SenGupta used Efron's (1975) criterion of statistical curvature γ_θ to gauge the small-sample performance of the LB test. Based on "very rough calculations", Efron (1975, p.1201) suggested that

$$\gamma^2_{\theta_0} \geq \frac{1}{8} \tag{6}$$

is a "large" value where θ_0 is the null value of the parameter under test, in which case it is reasonable to question the use of a LB test. For our testing problem, SenGupta found that (6) is equivalent to $mk \le 64$. How good is Efron's rule in this case?

With these thoughts in mind, we computed and compared the PE with the powers of the LB, $r_{0.5}$ and $r_{0.8}$ tests at the 5 per cent level of significance. Powers were calculated at ρ = 0.05, 0.1, 0.2, 0.3, ..., 0.9 for m = 10, 15, 25 and k = 2, 3, 4, 6, 10. Selected results of these calculations, performed using a FORTRAN version of Davies' (1980) algorithm, are given in Tables 3, 4 and 5. The values for ρ = 0.7, 0.9 have been omitted because, especially for large m and k values, they are very similar to those for ρ = 0.8.

The PE and the powers of all tests increase as k increases and m increases, ceteris paribus. As expected, of the three tests, the LB test is most powerful for ρ values associated with small PE probabilities, the $r_{0.5}$ test is most powerful for ρ values associated with middle PE probabilities and the $r_{0.8}$ test is most powerful for ρ values associated with large PE probabilities. Particularly for larger k and m values, the PE and all three power curves generally reach a value of one as ρ increases. The PE reaches this maximum value first, followed by the power curves of the $r_{0.8}$, $r_{0.5}$ and LB tests.

More importantly, for each combination of k and m values, the LB test always has the largest maximum power deviation below the PE of the three tests. For k = 2 and m = 10, 15, 25, this maximum power difference is 0.246, 0.162 and 0.095, respectively. In contrast, the largest maximum power difference for the $r_{0.5}$ test is never greater than 0.031 while that for the $r_{0.8}$ test is never greater than 0.027. On the basis of these results it can be argued that the $r_{0.5}$ and $r_{0.8}$ tests are approximately UMP, at least at the 5% level. It also seems evident that these maximum power deviations from the PE decrease as either k increases or m increases, ceteris paribus.

Efron's rule of questioning the use of a LB test when (6), or equivalently $mk \leq 64$ holds, appears to work well in this situation. Maximum deviations from the PE when $mk \leq 64$ range from 0.246 to 0.071 while for $mk > 64$, they range from 0.057 to 0.015. There does seem to be a tendency for the rule to work better for smaller m values.

Finally, there is the question of which of the r_{p_1} tests is better. While the $r_{0.5}$ test has larger maximum deviations from the PE, the $r_{0.8}$ test has larger maximum percentage deviations. This is because the $r_{0.5}$ test has increased power for lower levels of power while the $r_{0.8}$ test is relatively more powerful for higher levels. The differences between the two tests are not great. The choice of test, therefore, boils down to a choice between extra power at lower or higher values of ρ.

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TABLE 1

Selected values of ρ_1 and c_α for the $r_{0.5}$ test at the one and five percent significance levels

k	α	m = 5 ρ_1	m = 5 c_α	m = 10 ρ_1	m = 10 c_α	m = 15 ρ_1	m = 15 c_α	m = 25 ρ_1	m = 25 c_α
2	0.01	.6694	10^{-4}	.6581	10^{-5}	.5577	10^{-5}	.4451	10^{-5}
	0.05	.8329	-10^{-5}	.4973	-10^{-5}	.4124	-10^{-6}	.3233	10^{-5}
3	0.01	.6269	.3779	.4531	.2885	.3700	.2394	.2850	.1867
	0.05	.4610	.2778	.3227	.2055	.2012	.1690	.2002	.1311
4	0.01	.4996	.6146	.3461	.4448	.2779	.3618	.2107	.2770
	0.05	.3530	.4342	.2403	.3088	.1924	.2504	.1459	.1918
5	0.01	.4151	.7748	.2803	.5430	.2228	.4364	.1674	.3308
	0.05	.2864	.5346	.1917	.3714	.1525	.2988	.1149	.2271
6	0.01	.3550	.8900	.2356	.6105	.1860	.4869	.1389	.3664
	0.05	.2412	.6046	.1596	.4136	.1264	.3309	.0949	.2503
7	0.01	.3102	.9769	.2032	.6598	.1597	.5233	.1187	.3918
	0.05	.2084	.6562	.1367	.4440	.1079	.3538	.0808	.2667
8	0.01	.2754	1.045	.1787	.6973	.1399	.5507	.1037	.4109
	0.05	.1835	.6960	.1196	.4670	.0942	.3709	.0704	.2792
9	0.01	.2477	1.099	.1594	.7270	.1245	.5722	.0920	.4257
	0.05	.1639	.7275	.1063	.4849	.0836	.3843	.0623	.2884
10	0.01	.2250	1.442	.1439	.7509	.1121	.5895	.0828	.4378
	0.05	.1481	.7531	.0957	.4993	.0751	.3949	.0560	.2959

TABLE 2

Selected values of ρ_1 and c_α for the $r_{0.8}$ test at the one and five percent significance levels

		m = 5		m = 10		m = 15		m = 25	
k	α	ρ_1	c_α	ρ_1	c_α	ρ_1	c_α	ρ_1	c_α
2	0.01	.9114	3.068	.7767	3.906	.6835	4.289	.5656	4.653
	0.05	.8189	2.756	.6624	3.331	.5697	3.575	.4615	3.796
3	0.01	.7657	4.680	.5942	5.338	.4980	5.546	.3910	5.672
	0.05	.6469	3.953	.4821	4.331	.3979	4.431	.3086	4.476
4	0.01	.6584	5.784	.4833	6.212	.3940	6.263	.3008	6.211
	0.05	.5384	4.730	.3823	4.914	.3082	4.900	.2336	4.824
5	0.01	.5788	6.621	.4084	6.819	.3268	6.739	.2448	6.552
	0.05	.4633	5.299	.3178	5.306	.2523	5.203	.1883	5.039
6	0.01	.5173	7.289	.3541	7.270	.2794	7.080	.2066	6.788
	0.05	.4077	5.744	.2724	5.592	.2138	5.416	.1579	5.187
7	0.01	.4684	7.840	.3128	7.621	.2442	7.337	.1788	6.962
	0.05	.3646	6.103	.2385	5.811	.1856	5.576	.1360	5.294
8	0.01	.4284	8.305	.2803	7.901	.2170	7.539	.1577	7.095
	0.05	.3302	6.401	.2123	5.983	.1640	5.699	.1195	5.376
9	0.01	.3950	8.705	.2540	8.132	.1952	7.701	.1410	7.201
	0.05	.3019	6.654	.1913	6.124	.1470	5.797	.1065	5.441
10	0.01	.3666	9.052	.2323	8.324	.1775	7.835	.1275	7.287
	0.05	.2783	6.871	.1741	6.240	.1331	5.878	.0961	5.493

TABLE 3

Comparison of the PE with the powers of LB, $r_{0.5}$ and $r_{0.8}$ tests at the 5 percent significance level for m = 10

ρ =	.05	.1	.2	.3	.4	.5	.6	.8
Test				k=2				
PE	.069	.093	.155	.242	.357	.504	.683	.983
LB	.069	.092	.153	.213	.325	.430	.538	.737
$r_{0.5}$	.067	.088	.147	.234	.354	.504	.675	.952
$r_{0.8}$	.065	.083	.135	.215	.331	.489	.680	.978
				k=3				
PE	.088	.139	.279	.456	.650	.829	.951	1.000
LB	.088	.139	.270	.423	.572	.701	.802	.926
$r_{0.5}$	.085	.135	.275	.456	.646	.809	.920	1.000
$r_{0.8}$	.081	.125	.255	.438	.646	.829	.946	1.000
				k=4				
PE	.109	.192	.406	.636	.830	.950	.994	1.000
LB	.109	.190	.389	.582	.735	.841	.909	.974
$r_{0.5}$	.106	.188	.405	.633	.812	.922	.975	.999
$r_{0.8}$	.100	.174	.387	.631	.830	.945	.990	1.000
				k=6				
PE	.155	.305	.622	.851	.964	.997	1.000	1.000
LB	.155	.301	.589	.785	.893	.949	.976	.995
$r_{0.5}$	.153	.303	.621	.836	.941	.983	.996	1.000
$r_{0.8}$	.144	.288	.617	.851	.958	.992	.999	1.000
				k=10				
PE	.261	.521	.861	.977	.999	1.000	1.000	1.000
LB	.259	.509	.820	.936	.977	.991	.997	.999
$r_{0.5}$	.259	.521	.849	.960	.990	.998	1.000	1.000
$r_{0.8}$	.249	.513	.860	.971	.996	1.000	1.000	1.000

TABLE 4

Comparison of the PE with the powers of LB, $r_{0.5}$ and $r_{0.8}$ tests at the 5 percent significance level for m = 15

ρ =	.05	.1	.2	.3	.4	.5	.6	.8
Test				k=2				
PE	.074	.105	.191	.315	.478	.668	.852	.999
LB	.074	.104	.188	.299	.431	.569	.699	.889
$r_{0.5}$	.072	.101	.186	.312	.478	.663	.833	.993
$r_{0.8}$	.070	.096	.174	.295	.464	.665	.851	.998
				k=3				
PE	.098	.167	.359	.591	.804	.943	.994	1.000
LB	.098	.165	.346	.547	.720	.844	.922	.985
$r_{0.5}$	.096	.163	.358	.590	.792	.921	.979	1.000
$r_{0.8}$	.092	.154	.343	.585	.804	.939	.990	1.000
				k=4				
PE	.125	.237	.522	.783	.939	.992	1.000	1.000
LB	.125	.235	.499	.724	.867	.942	.977	.997
$r_{0.5}$	.123	.235	.522	.774	.920	.979	.996	1.000
$r_{0.8}$	.118	.224	.514	.783	.935	.988	.999	1.000
				k=6				
PE	.186	.388	.763	.947	.995	1.000	1.000	1.000
LB	.186	.381	.726	.901	.967	.990	.997	1.000
$r_{0.5}$	.185	.388	.757	.932	.986	.998	1.000	1.000
$r_{0.8}$	.178	.378	.763	.944	.992	.999	1.000	1.000
				k=10				
PE	.327	.650	.950	.997	1.000	1.000	1.000	1.000
LB	.325	.635	.922	.985	.997	.999	1.000	1.000
$r_{0.5}$	.326	.649	.940	.992	.999	1.000	1.000	1.000
$r_{0.8}$	.320	.649	.947	.995	1.000	1.000	1.000	1.000

TABLE 5

Comparison of the PE with the powers of LB, $r_{0.5}$ and $r_{0.8}$ tests at the 5 percent significance level for m = 25

ρ =	.05	.1	.2	.3	.4	.5	.6	.8
Test				k=2				
PE	.082	.126	.259	.449	.672	.868	.975	1.000
LB	.082	.126	.253	.424	.609	.773	.889	.986
$r_{0.5}$	.080	.123	.256	.449	.669	.854	.960	1.000
$r_{0.8}$	.078	.119	.246	.440	.670	.867	.972	1.000
				k=3				
PE	.115	.216	.499	.780	.946	.995	1.000	1.000
LB	.115	.215	.480	.731	.889	.964	.990	1.000
$r_{0.5}$	.114	.214	.499	.774	.932	.988	.999	1.000
$r_{0.8}$	.110	.207	.493	.780	.943	.993	1.000	1.000
				k=4				
PE	.154	.320	.700	.930	.994	1.000	1.000	1.000
LB	.153	.316	.670	.887	.970	.994	.999	1.000
$r_{0.5}$	.152	.319	.697	.919	.987	.999	1.000	1.000
$r_{0.8}$	.148	.311	.699	.928	.991	.999	1.000	1.000
				k=6				
PE	.243	.529	.912	.994	1.000	1.000	1.000	1.000
LB	.242	.519	.884	.981	.997	1.000	1.000	1.000
$r_{0.5}$	.242	.529	.905	.989	.999	1.000	1.000	1.000
$r_{0.8}$	.237	.526	.911	.992	1.000	1.000	1.000	1.000
				k=10				
PE	.443	.820	.994	1.000	1.000	1.000	1.000	1.000
LB	.440	.805	.987	.999	1.000	1.000	1.000	1.000
$r_{0.5}$	.443	.817	.991	1.000	1.000	1.000	1.000	1.000
$r_{0.8}$	.440	.820	.993	1.000	1.000	1.000	1.000	1.000

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