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# Multi-series Heuristics for Exponential Smoothing 

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#### Abstract

In this paper several heuristics are proposed for calculating the smoothing parameter in exponential smoothing when forecasts of many 'closely' related series are required on a regular basis. The methods are evaluated using both synthetic and real data. They not only compare favourably against several other known forecasting techniques but they are also simple and computationally efficient. Keywords: Multi-series, time series, exponential smoothing.


## 1 Introduction

The performance of exponential smoothing as a forecasting technique depends critically on the choice of the smoothing parameter. Holt(1957) treated the parameter as a constant to be selected so as to minimize the sum of squared forecast errors. Brown(1963) recommended a more flexible approach in business applications, suggesting that the choice should be left to the discretion of management who would utilize market intelligence to anticipate above normal levels of structural change and temporarily increase the parameter in such circumstances. In a quest for greater automation Trigg and Leach(1967) introduced the concept of the adaptive response rate. Since then there have been many variations of these themes (e. g. see Dennis(1978), Ekern(1981), Gardner(1985), Snyder(1988), Taylor(1981) and Whybark(1973)) but no particular approach has achieved universal acceptance. The choice of the smoothing parameter still remains an issue warranting further attention.

In practice, the above methods are frequently applied in situations where many series are forecast in parallel on a regular basis. Often the series under consideration are influenced by the same external forces and exhibit common movements. For example, in the wool industry, wool is graded into classes according to several factors such as fibre diameter and colour. In this context it is reasonable to assume that factors such as floods, droughts, wars and exchange rate fluctuations which affect one class of wool generally have a similar effect on all other classes. This is reflected in the wool price data from Table 1, some of
which after grouping have been plotted in Figure 1, where there is a clear visual evidence of common cyclical movements. Yet the above methods, with their univariate orientation, fail to exploit this cohesive behaviour.

The central theme of this paper is that information contained in closely related series can be utilized in forecasting. In this setting, multivariate methods may prove especially successful. Some progress towards this end has been made by Stevens(1964), Harrison and Stevens(1976), Enns et al. (1982) and Harvey(1986). However, little attention has been given to possibilities of adaptive response rates based on multivariate considerations. In the following such possibilities are proposed and evaluated against more traditional methods both on real data such as the wool series and synthetic data in a simulation study.

## 2 Multi-series heuristics

In this paper the one step ahead forecast $\hat{X}_{i t+1}$ of the value $X_{i t+1}$ of a series $i$ is generated by the recurrence relationship

$$
\begin{equation*}
\hat{X}_{i t+1}=\hat{X}_{i t}+\alpha_{t} e_{i t} \tag{1}
\end{equation*}
$$

where $\alpha_{t}$ is a common response rate at time $t$ and $e_{i t}$ is the forecast error given by

$$
\begin{equation*}
e_{i t}=X_{i t}-\hat{X}_{i t} . \tag{2}
\end{equation*}
$$

In Table 2 a number of heuristics are outlined. These can be used to calculate the response rate where N denotes the number of series. The formula for heuristic A1 contains the sgn function which takes the values 1 or -1 according to whether $e_{i t}$ is positive or negative. The rationale for such a formula is that when there is an unanticipated upturn(downturn) across the whole set of series, most of the forecast errors will tend to be positive(negative) and $\alpha_{t}$ will be close to one. On the other hand during periods of relative stability neither the positive nor the negative errors predominate and $\alpha_{t}$ will be close to zero. Accordingly during periods of structural instability the heuristic yields a large value for the smoothing parameter, just when it is necessary for the forecasts to adapt to the change. However, when there is structural stability, the value of the smoothing parameter is small, thus ensuring that forecasts do not alter substantially in response to the changes which are essentially random in nature.

Heuristic B1 considers the relative size of the error terms in calculating $\alpha_{t}$. This formula requires that $X_{i t}$ be non-zero for all $i$ and $t$. It should also be noted that $\alpha_{t}$ in this case is not necessarily between 0 and 1 any more. In order to restrict the value of $\alpha_{t}$ to between 0 and 1 and also take account of the relative size of error terms, heuristic C 1 is proposed.
Finally, in the formula for heuristic D1, the error terms are divided by $\hat{\sigma}_{i t-1}$, the progressive estimate of the standard deviation of the error terms instead of $X_{i t}$, where

$$
\begin{equation*}
\hat{\sigma}_{i t}^{2}=\sum_{j=2}^{t} e_{i j}^{2} /(t-1) \tag{3}
\end{equation*}
$$

Box and Jenkins(1976) when considering ARIMA( $0,1,1$ ) process, showed that exponential smoothing is stable for smoothing parameter values that are in the range 0 to 2 . Therefore
a second version of each of the above heuristics, where the formula in each case is multiplied by a factor of 2 , is also considered. They are labelled A2, B2, C2 and D2 respectively.
For all heuristics the procedure is initialised by setting $\hat{X}_{i 2}=X_{i 1}$. In the case of D1 and D2 the initialisation also involves setting $\hat{X}_{i 3}=\left(X_{i 1}+X_{i 2}\right) / 2$.

## 3 Evaluation

Although multi-series heuristics make intuitive sense it is necessary to check their performance on both simulated and real data. The statistical properties of the heuristics are too difficult to establish using analytical methods. Hence, in this study we examine their performance using naive forecasting, progressively optimized exponential smoothing (POES) and adaptive response rate smoothing (Trigg and Leach) methods, as described in appendix A, for comparison purposes.

The criteria used to evaluate the accuracy of the heuristics were:

1. the average of the mean absolute percentage errors taken over all the series according to the formula:

$$
\begin{equation*}
G M A P E=100\left(\sum_{i=1}^{N} \sum_{t=2}^{T}\left|e_{i t} / X_{i t}\right|\right) / N(T-1) \tag{4}
\end{equation*}
$$

where T is the number of observations in each series;
2. the percentage PL of series that have a lower mean squared error than progressive optimised exponential smoothing (POES).

### 3.1 Simulation

The simulation study was undertaken to determine the effectiveness of the heuristics under a range of conditions involving business cycles, step and ramp changes. For each simulation, monthly data for 50 series of six years duration were generated. Five different amplitudes for the business cycle, ten different step sizes, and ramps with ten different gradients were considered. The algorithms used to generate synthetic data are described in detail in Appendix B.

The results of the simulation are shown in Table 3 where $\rho$ is the factor which determines the amplitude of the business cycle, the size of step change, or the gradient of the ramp. It is evident that the heuristics perform reasonably well in comparison to their traditional counterparts, although no particular one is dominant throughout the entire study. Here B2, C2 and D1 have the lowest GMAPE values while A1 performs well in terms of the PL criterion.

### 3.2 Real data

The heuristics were also tested on the real data from Lehmer(1985) shown in Table 1. The results in Table 4 indicate that heuristics A2 and D2 are superior to progressively
optimized exponential smoothing and the Trigg and Leach method on this data. The results in relation to the naive method are ambiguous, depending on whether GMAPE or PL is employed as the performance measure, but they confirm the common finding that this method generally performs well on price data.

## 4 Conclusion

In this paper heuristics have been presented for determining the response rate in applications of exponential smoothing. The distinguishing feature was the exploitation of common movements in series to detect and adapt to structural change. In terms of forecast accuracy, the performance of the heuristics was shown to depend on the structure of the series. However, in almost all circumstances, at least one of the heuristics produced results which were better than progressively optimized exponential smoothing or the adaptive response rate method of Trigg and Leach(1967).

Apart from the accuracy issue the heuristics have other advantages. They are elegantly simple and easily computerized; they access only current data; they avoid the time consuming grid searches for optimal parameter values or the excessive storage requirements of progressively optimized exponential smoothing. The heuristics therefore appear to hold considerable promise and could become a significant addition to the applied statistician's forecasting toolkit.

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## A Appendix

An outline of the forecasting methods used for comparing the performance of multi-series heuristic methods is given in this appendix.

## A. 1 Naive

This is a special case of simple exponential smoothing where the smoothing parameter is set equal to a constant one. The forecast, $\hat{X}_{i t+1}$, for series $i$ in period $t+1$, is given by

$$
\begin{equation*}
\hat{X}_{i t+1}=X_{i t} \tag{5}
\end{equation*}
$$

where $X_{i t}$ is the observed value of series $i$, in period $t$.

## A. 2 Progressively optimised exponential smoothing(POES)

Twenty versions of simple exponential smoothing with varying values of the smoothing parameter are applied in parallel to each of the series. The forecast, $\hat{X}_{i j t+1}$, for series $i$ in period $t+1$, using smoothing parameter $\alpha_{j}$, is given by

$$
\begin{equation*}
\hat{X}_{i j t+1}=\hat{X}_{i j t}+\alpha_{j} e_{i j t} \tag{6}
\end{equation*}
$$

where $\alpha_{j}$ is the $j^{\text {th }}$ smoothing parameter and

$$
\begin{equation*}
e_{i j t}=X_{i t}-\hat{X}_{i j t} \tag{7}
\end{equation*}
$$

is the one step ahead forecast error for series $i$ when using the $j^{\text {th }}$ smoothing parameter. The $\alpha_{j}$ are defined by

$$
\begin{equation*}
\alpha_{j}=j / 10 \text { for } j=1 \text { to } 20 . \tag{8}
\end{equation*}
$$

A progressive total of the squared forecast error for each smoothing parameter is kept. Therefore if $S_{i j t}$ is the progressive total for series $i$ using the $j^{\text {th }}$ smoothing parameter, then after period $t$

$$
\begin{equation*}
S_{i j t}=\sum_{k=2}^{t} e_{i j k}^{2} . \tag{9}
\end{equation*}
$$

If $j_{\text {min }}$ is such that

$$
\begin{equation*}
S_{i j_{\min } t}=\min _{1 \leq j \leq 20}\left\{S_{i j t}\right\} \tag{10}
\end{equation*}
$$

then the forecast for series $i$ in period $t+1$ is

$$
\begin{equation*}
\hat{X}_{i t+1}=\hat{X}_{i j_{\min } t+1} \tag{11}
\end{equation*}
$$

The procedure is initialized by letting $\hat{X}_{i j 2}=X_{i 1}$ for every $j$.

## A. 3 Trigg and Leach's adaptive response rate smoothing

Using notation already defined,

$$
\begin{equation*}
\hat{X}_{i t+1}=\hat{X}_{i t}+\alpha_{i t} e_{i t} \tag{12}
\end{equation*}
$$

where

$$
\alpha_{i t}= \begin{cases}\left|\bar{e}_{i t}\right| / m_{i t} & \text { if } m_{i t}>0  \tag{13}\\ 1 & \text { if } m_{i t}=0\end{cases}
$$

The smoothed error, $\bar{e}_{i t}$, and the smoothed absolute error, $m_{i t}$, are calculated using the following:

$$
\begin{align*}
\bar{e}_{i t} & =(1-\gamma) \bar{e}_{i t-1}+\gamma e_{i t}  \tag{14}\\
m_{i t} & =(1-\gamma) m_{i t-1}+\gamma\left|e_{i t}\right| \tag{15}
\end{align*}
$$

where $\gamma$ is the smoothing constant for the errors. The procedure is initialized by setting $\hat{X}_{i 2}=X_{i 1}$. Three different values of $\gamma, .05, .1$ and .5 , are considered.

## B Appendix

This appendix describes the way the synthetic data was generated.

## B. 1 Time series with a business cycle

If it is assumed that the business cycle follows a simple sine curve and there are no seasonality or trend components, then the data generating function is given by

$$
\begin{equation*}
X_{i t}=A_{i}+\rho A_{i} \sin (\pi(t-1) / 18)+s_{i} N_{i t} \tag{16}
\end{equation*}
$$

where

$$
\begin{aligned}
A_{i} & =\text { mean level for the } i^{\text {th }} \text { series } \\
N_{i t}= & \text { normally distributed variate with mean zero and } \\
& \text { standard deviation one } \\
s_{i}= & \text { standard deviation of the noise component } \\
& \text { for the } i^{\text {th }} \text { series } \\
\rho= & \text { constant }
\end{aligned}
$$

The $s_{i}^{2}$ 's are determined using the following formula:

$$
\begin{equation*}
s_{i}^{2}=\sigma_{i}^{2}-\left(\rho A_{i}\right)^{2} / 2 \tag{17}
\end{equation*}
$$

where $\sigma_{i}^{2}$ is the variance of the $i^{t h}$ series and the second term on the right hand side is the variance of the business cycle. Hausman and Kirby(1970) derived empirical relations between the mean level and the variance of a series. The one that has been used in this study is

$$
\begin{equation*}
\sigma_{i}^{2}=.338 A_{i}^{900} . \tag{18}
\end{equation*}
$$

Figures 2 shows a typical series that was generated. It has a mean level of 145 and $\rho=.05$. In Figure 3 the same series is shown with $\rho=.22$. As can clearly been seen that the business cycle is more pronounced in this second case.

## B. 2 Time series with a step change

In this model a step change in the mean level of the series was assumed to occur at period 25. Using notation already defined, the data generating function is given by

$$
X_{i t}= \begin{cases}A_{i}+\sigma_{i 1} N_{i t} & \text { if } t \leq 25  \tag{19}\\ \rho A_{i}+\sigma_{i 2} N_{i t} & \text { if } t>25\end{cases}
$$

where

$$
\left.\begin{array}{rl}
\sigma_{i 1}= & \text { standard deviation of the noise component of the } \\
& i^{\text {th }} \text { series before period } 25
\end{array}\right)
$$

$\sigma_{i 1}$ and $\sigma_{i 2}$ are determined using the following formulae:

$$
\begin{align*}
\sigma_{i 1} & =.338 A_{i}^{.900}  \tag{20}\\
\sigma_{i 2} & =.338\left(\rho A_{i}\right)^{.900} \tag{21}
\end{align*}
$$

Figure 4 shows a typical series that was generated. It has an initial mean level of 145 and a jump in this level by $20 \%$ from period 25 on ( $\rho=1.2$ ). In Figure 5 a similar series with $\rho=1.5$ is shown.

## B. 3 Time series with a ramp

A linear ramp for the mean level of the series was introduced between periods 24 and 36. The data generating function is defined as

$$
X_{i t}= \begin{cases}A_{i}+\sigma_{i 1} N_{i t} & \text { if } t \leq 24  \tag{22}\\ A_{i}(3-2 \rho+(\rho-1) t / 12)+\sigma_{i t} N_{i t} & \text { if } 24<t \leq 36 \\ \rho A_{i}+\sigma_{i 2} N_{i t} & \text { if } t>36\end{cases}
$$

where

$$
\begin{aligned}
& \sigma_{i 1}=\text { standard deviation of the noise component before } \\
& \text { period } 24 \text { for the } i^{\text {th }} \text { series } \\
& \sigma_{i 2}=\text { standard deviation of the noise component after } \\
& \text { period } 36 \text { for the } i^{\text {th }} \text { series } \\
& \sigma_{i t}=\text { standard deviation of the noise component for period } \\
& t \text { for the } i^{\text {th }} \text { series where } 24<t \leq 36 \text {. }
\end{aligned}
$$

The standard deviations are calculated using the following formulae:

$$
\begin{array}{ll}
\sigma_{i 1}=.338 A_{i}^{900} & \text { if } t \leq 24 \\
\sigma_{i 2}=.338\left(\rho A_{i}\right)^{.900} & \text { if } t>36 \\
\sigma_{i t}=.338\left(A_{i}(3-2 \rho+(\rho-1) t / 12)\right)^{.900} & \text { if } 24<t \leq 36 \tag{24}
\end{array}
$$

Figure 6 shows a typical series with $\rho=1.2$. The initial level of the series is 145 and the ramp operates between periods 24 and 36 . From period 36 on the mean level of the series is $20 \%$ more than its initial value. In Figure 7 a similar series with $\rho=1.4$ is illustrated. It is obvious that the ramp is steeper in this case.

| Wool | 1978 |  | 1979 |  |  |  | 1980 |  |  |  |  |  | Quart |  |  | 1982 |  |  | 1983 |  |  |  | 1984 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Type | 376 | 373 | 400 | 491 | 513 | 555 | 589 | 619 | 3 | 575 | 545 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 |
| 60 | 371 | 369 | 383 | 478 | 501 | 555 | 589 | 619 | 585 | 575 | 545 | 572 | 564 | 564 | 578 | *** | 593 | 603 | 657 | 751 | 670 | 685 | 872 | 926 | 907 |
| 60B | 363 | 388 | 400 | 477 | 492 | 548 | 571 | 594 | 566 | 558 557 | 538 | 558 | 542 | 540 | 569 567 | 614 | 604 | 596 | 634 | 695 | 677 | 667 | 806 | 872 | 914 |
| 76 | 363 | 383 | 395 | 476 | 489 | 533 | 563 | 573 | 556 | 539 | 524 | 547 | 526 | 527 | 563 | 611 | 601 | 582 | 632 | 669 | 664 | 658 | 826 | 826 | 896 |
| 76 B | 362 | 334 | 392 | 462 | 478 | 534 | 548 | 572 | 551 | 543 | 516 | 533 | 523 | 527 | 560 | 603 | 595 | 565 | 000 | 661 | 646 | 646 | 750 | 787 | 894 |
| 157B | 329 | 333 | 352 | 407 | 400 | 432 | 481 | 471 | 456 | 472 | 473 | 475 | 479 | 487 | 492 | 498 | 484 | 487 | 512 | 525 | 532 | 544 | 739 | 833 | 838 |
| 61 | 361 | 352 | 380 | 478 | 484 | 513 | 532 | 530 | 517 | 497 | 496 | 514 | 520 | 515 | 538 | 606 | 589 | 550 | 574 | 25 | 532 | 544 | 597 | 658 | 667 |
| 61B | 360 | 360 | 395 | 468 | 482 | 519 | 529 | 527 | 514 | 498 | 494 | 511 | 518 | 514 | 540 | 602 | 581 | 546 | 573 | 600 | 609 | 605 | 623 | 672 | 688 |
| 77 | 357 | 358 | 388 | 465 | 473 | 499 | 522 | 522 | 503 | 487 | 489 | 504 | 507 | 506 | 533 | 595 | 573 | 542 | 565 | 596 | 597 | 597 | 628 | 655 | 694 |
| 77 B | 358 | 358 | 385 | 466 | 476 | 502 | 518 | 517 | 505 | 486 | 484 | 498 | 507 | 506 | 531 | 588 | 572 | 537 | 561 | 582 | 593 |  | 609 | 675 | 683 |
| 158C | 307 | 318 | 329 | 318 | 359 | 381 | 409 | 418 | 402 | 417 | 432 | 436 | 443 | 445 | 457 | 479 | 459 | 447 | 457 | 478 | 484 | 481 | 602 | 620 | 650 |
| 78 | 349 | 349 | 377 | 438 | 442 | 460 | 480 | 450 | 488 | 468 | 477 | 488 | 493 | 491 | 521 | 576 | 541 | 523 | 546 | 565 | 567 | 563 | 507 | 512 | 508 |
| 78B | 350 | 349 | 378 | 432 | 448 | 459 | 480 | 490 | 471 | 471 | 478 | 486 | 492 | 491 | 521 | 563 | 539 | 521 | 540 | 558 | 563 | 561 | 569 | 588 | 592 |
| 84B | 349 | 358 | 373 | 424 | 139 | 453 | 468 | 483 | 466 | 468 | 471 | 479 | 487 | 482 | 509 | 546 | 525 | 511 | 530 | 548 | 554 | 548 | 569 | 594 | 585 |
| 159B | 321 | 322 | 338 | 365 | 369 | 390 | 411 | 422 | 414 | 428 | 443 | 455 | 450 | 445 | 456 | 484 | 464 | 446 | 466 | 489 | 489 | 485 | 564 | 573 | 575 |
| 79 | 342 | 342 | 385 | 397 | 404 | 421 | 450 | 458 | 441 | 448 | 469 | 476 | 483 | 480 | 507 | 534 | 511 | 504 | 528 | 547 | 548 | 547 | 552 | 560 | 518 |
| 79B | 342 | 343 | 384 | 394 | 402 | 418 | 448 | 456 | 444 | 453 | 470 | 474 | 481 | 479 | 505 | 525 | 508 | 503 | 522 | 543 | 545 | 544 | 545 | 552 | 548 |
| 85B | 341 | 344 | 382 | 389 | 395 | 412 | 439 | 450 | 438 | 451 | 463 | 464 | 477 | 472 | 499 | 516 | 500 | 495 | 515 | 533 | 538 | 535 | 538 | 540 | 539 |
| 159AB | 318 | 313 | 332 | 363 | 362 | 375 | 401 | 408 | 398 | 414 | 433 | 437 | 437 | 434 | 448 | 447 | 451 | 435 | 457 | 492 | 478 | 471 | 488 | 502 | 489 |
| 80 | 335 | 333 | 352 | 371 | 370 | 336 | 422 | 419 | 413 | 430 | 456 | 457 | 466 | 466 | 491 | 484 | 487 | 488 | 507 | 528 | 527 | 527 | 529 | 533 | 527 |
| 80B | 338 | 338 | 354 | 367 | 363 | 397 | 422 | 418 | 412 | 432 | 457 | 454 | 466 | 466 | 484 | 480 | 484 | 486 | 502 | 524 | 527 | 525 | 524 | 526 | 533 |
| 80 S | 337 | 334 | 355 | 368 | 369 | 396 | 423 | 412 | 410 | 432 | 455 | 448 | 463 | 463 | 486 | 478 | 477 | 480 | 498 | 519 | 520 | 518 | 526 | 522 | 514 |
| 80 C | 337 | 333 | 354 | 357 | 363 | 391 | 403 | 395 | 403 | 432 | 454 | 445 | 462 | 469 | 468 | 459 | 475 | 476 | 495 | 513 | 518 | 518 | 511 | 509 | 505 |
| 164C | 293 | 293 | 310 | 329 | 325 | 351 | 385 | 370 | 370 | 379 | 409 | 393 | 404 | 389 | 400 | 388 | 378 | 376 | 399 | 433 | 437 | 426 | 436 | 426 | 410 |
| 81 | 330 | 326 | 3.45 | 360 | 358 | 389 | 410 | 393 | 389 | 415 | 447 | 439 | 450 | 454 | 476 | 457 | 468 | 473 | 484 | 518 | 513 | 504 | 505 | 510 | 503 |
| 81 B | 333 | 330 | 348 | 358 | 360 | 393 | 414 | 393 | 393 | 421 | 448 | 437 | 455 | 459 | 474 | 450 | 464 | 469 | 478 | 512 | 514 | 504 | 501 | 501 | 499 |
| 81C | 340 | 324 | 354 | 346 | 358 | 391 | 392 | 371 | 390 | 425 | 450 | 431 | 458 | 463 | 459 | 437 | 452 | 456 | 468 | 503 | 508 | 498 | 484 | 488 | 485 |
| 96 B | 332 | 329 | 347 | 348 | 363 | 393 | 409 | 382 | 387 | 423 | 452 | 429 | 454 | 458 | 469 | 441 | 447 | 454 | 471 | 506 | 510 | 494 | 491 | 481 | 475 |
| 160AC | 299 | 303 | 322 | 328 | 315 | 347 | 366 | 345 | 357 | 369 | 405 | 380 | 397 | 383 | 394 | 387 | 380 | 377 | 403 | 433 | 421 | 418 | 420 | 407 |  |
| 432A | 329 | 326 | 313 | 368 | 358 | 387 | 405 | 394 | 395 | 413 | 449 | 449 | 450 | 451 | 476 | 473 | 461 | 468 | 486 | 520 | 512 | 503 | 506 | 506 | 505 |
| 433 | 326 | 320 | 335 | 360 | 353 | 380 | 395 | 376 | 376 | 400 | 430 | 418 | 432 | 442 | 455 | 440 | 436 | 454 | 466 | 504 | 505 | 483 | 483 |  |  |
| 486B | 295 | 297 | 309 | 330 | 329 | 355 | 367 | 345 | 331 | 343 | 364 | 371 | 381 | 371 | 375 | 363 | 368 | 371 | 373 | 415 | 409 | 398 | 408 | 428 | 473 |
| 653 | 322 | 316 | 331 | 349 | 352 | 379 | 391 | 367 | 369 | 397 | 427 | 416 | 430 | 439 | 451 | 433 | 435 | 451 | 456 | 502 | 499 | 479 | 476 | 474 | 413 |
| 434 | 311 | 305 | 321 | 347 | 343 | 372 | 374 | 353 | 351 | 369 | 384 | 371 | 399 | 415 | 403 | 391 | 369 | 394 | 401 | 436 | 452 | 435 | 430 | 474 | 464 399 |
| 444B | 309 | 312 | 324 | 339 | 342 | 372 | 382 | 345 | 346 | 362 | 370 | 353 | 391 | 407 | 400 | 383 | 355 | 369 | 363 | 418 | 425 | 410 | 407 | 382 |  |
| 479 | 306 | 280 | 298 | 326 | 341 | 349 | 348 | 329 | 318 | 324 | 314 | 304 | 320 | 329 | 332 | 360 | 335 | 330 | 339 | 357 | 347 | 353 | 354 | 371 | 351 |
| 487B | 289 | 285 | 293 | 322 | 315 | 341 | 344 | 322 | 313 | 315 | 303 | 301 | 324 | 317 | 318 | 320 | 322 | 318 | 325 | 341 | 339 | 344 | 343 | 334 | 349 |
| 654 | 302 | 304 | 317 | 339 | 339 | 369 | 371 | 345 | 343 | 362 | 376 | 364 | 392 | 407 | 398 | 377 | 361 | 387 | 392 | 430 | 443 | 428 | 421 | 413 | 394 |
| 435 | 284 | 283 | 296 | 322 | 327. | 357 | 352 | 328 | 322 | 324 | 308 | 307 | 317 | 320 | 318 | 326 | 313 | 311 | 319 | 346 | 347 | 339 | 341 | 348 | 394 <br> 338 |
| 435B | 284 | 294 | 298 | 319 | 326 | 362 | 356 | 326 | 323 | 322 | 305 | 306 | 321 | 313 | 317 | 323 | 312 | 308 | 316 | 348 | 344 | 338 | 337 | 341 | 335 |
| 436 437 | 273 | 282 | 288 | 315 | 322 | 351 | 343 | 318 | 313 | 311 | 295 | 296 | 303 | 299 | 307 | 313 | 299 | 296 | 302 | 328 | 328 | 320 | 317 | 324 | 319 |
| 445B | 287 | 294 | 297 | 312 317 | 323 323 | 346 | 337 | 313 | 306 | 301 | 289 | 291 | 297 | 294 | 297 | 296 | 289 | 290 | 295 | 316 | 322 | 313 | 310 | 312 | 311 |
| 480 | 277 | 280 | 283 | 315 | 318 | 342 | 329 | 323 312 | 323 295 | 319 | 300 | 300 | 317 | 302 | 312 | 321 | 304 | 297 | 309 | 341 | 343 | 327 | 328 | 338 | 330 |
| 655 | 283 | 290 | 296 | 320 | 325 | 355 | 350 | 324 | 319 | 322 | 302 | 303 | 295 | 346 | 28 | 299 | 290 | 279 | 291 | 319 | 325 | 310 | 318 | 336 | 342 |
| 283C | 264 | 277 | 294 | 294 | 290 | 301 | 317 | 314 | 303 | 326 | 343 | 340 | 330 | 315 | 31 | 322 | 309 | 306 | 31 | 341 | 342 | 334 | 337 | 343 | 332 |
| 304C | 284 | 276 | 296 | 299 | 290 | 306 | 339 | 321 | 314 | 344 | 365 | 351 | 333 | 335 | 319 | 311 | 297 | 301 | 3 | 327 | 323 | 321 | 328 | 323 | 310 |
| 887C | 260 | 273 | 282 | 280 | 285 | 308 | 316 | 298 | 294 | 305 | 315 | 297 | 293 | 295 | 292 | 271 | 200 | 270 | 279 | 312 | 315 | 334 | 342 | 356 310 | 323 308 |

Table 1: Quarterly Wool Prices (Aust. cents per kg. of clean wool), 1978.3 to 1984.3. Source: AWC internal Sales Catalogue Hiṣtory Records.

| Heuristic | Formula |
| :---: | :---: |
| A1 | $\alpha_{t}=\left\|\sum_{i=1}^{N} \operatorname{sgn}\left(e_{i t}\right)\right\| / N$ |
| B1 | $\alpha_{t}=\left\|\sum_{i=1}^{N} e_{i t} / X_{i t}\right\| / N$ |
| C1 | $\alpha_{t}=\left\|\sum_{i=1}^{N} e_{i t} / X_{i t}\right\| / \sum_{i=1}^{N}\left\|e_{i t} / X_{i t}\right\|$ |
| D1 | $\alpha_{t}=\left\|\sum_{i=1}^{N} e_{i t} / \sigma_{i t-1}\right\| / \sum_{i=1}^{N}\left\|e_{i t} / \hat{\sigma}_{i t-1}\right\|$ |


| Deta characteristic | $\rho$ | Method |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A1 | A2 | B1 | B2 | C1 | C2 | D1 | D2 |  |  |  | Trigg |  |
| Business | . 05 | 19.53(56) | 20.15(26) | 19.58(44) | 19.41(42) | 20.50(42) | 18.98(76) | 19.52(46) | $\frac{\text { D2 }}{\text { 20.37(16) }}$ | Naive | POES | $\frac{.05}{19.99(44)}$ |  | 22.98(2) |
| Cycle | . 10 | 10.11(46) | 19.53(40) | 19.22(56) | 18.93(48) | 19.92(38) | 18.97(66) | 18.93(52) | 19.91(18) | 23.50 (0) | 19.32 | $19.99(44)$ $19.69(40)$ | 19.45(40) | $22.98(2)$ $21.89(2)$ |
|  | . 15 | 17.85(70) | 18.34(46) | 18.40(33) | 17.87(62) | 19.23(22) | 18.65(38) | 17.73(68) | 18.72(28) | $21.84(0)$ | 18.24 | 18.50(24) | 18.23(40) | 21.89(2) $19.99(2)$ |
|  | . 20 | 15.03(80) | 16.16(52) | 17.12(20) | 16.10(16) | 18.74(2) | 18.07(6) | 15.58(80) | 16.37(34) | 18.64(0) | 16.09 | $18.90(24)$ $17.29(4)$ | $18.23(40)$ $16.31(36)$ | $19.99(2)$ $17.05(10)$ |
|  | . 22 | 14.49(80) | 14.05(42) | 16.46(8) | 15.18(38) | 18.58(2) | 17.74(2) | 14.30(86) | 15.15(40) | 16.91(0) | 14.86 | 16.34(2) | 15.23(28) | 15.48(12) |
| $\begin{aligned} & \text { Step } \\ & \text { cisnce } \end{aligned}$ | 1.1 | 20.13 (58) | 20.85 (22) | 19.81(60) | 19.68(70) | 20.40(43) | 20.34(63) | 20.09(46) | 21.20(12) | 25.63(0) | 20.18 | 20.73(36) | 21.06(22) | 23.62(0) |
|  | 1.2 | 20.03(60) | 20.69 (24) | 19.72(54) | 19.63(70) | 20.58(32) | 19.17(54) | 20.03(50) | 21.16(14) | 25.45(0) | 20.18 | 20.70(38) | 21.00 (26) | 23.49(0) |
|  | 1.3 | 20.03(58) | 20.82(30) | 19.70(52) | 19.59(72) | 21.07(16) | 19.43(28) | 19.98(52) | 21.24(16) | 25.31 (0) | 20.23 | 20.71(38) | 20.97(28) | 23.38(0) |
|  | 1.4 | 20.01(76) | 20.62(42) | 19.70(62) | 19.55(74) | 21.65(10) | 19.68(26) | 19.90(58) | 21.14(24) | 25.20 (0) | 20.36 | 20.72(40) | 20.95(44) | 23.28(0) |
|  | 1.5 | 19.05(82) | 20.53(44) | 19.70(64) | 19.51(70) | 22.20(2) | 19.90(20) | 19.85(70) | 21.11(34) | 25.11(0) | 20.45 | 20.74(48) | 20.95(44) | 23.21 (0) |
|  | 1.6 | 19.80 (24) | 20.55(42) | 19.60(68) | 19.43(80) | 22.73(0) | 20.10(18) | 19.80(80) | 21.12(40) | 25.03(0) | 20.54 | 20.74(52) | 20.94(52) | 23.15(0) |
|  | 1.7 | 19.87(84) | 20.46 (50) | 19.68(68) | 19.46(74) | 23.32(0) | 20.27(16) | 19.75(78) | 21.15(44) | 24.96(0) | 20.59 | 20.74(62) | 20.92(56) | 23.09(0) |
|  | 1.8 1.9 | 19.82(84) | 20.47(58) | 19.66(76) | 19.48(76) | $23.67(0)$ | 20.42(12) | 19.70(82) | 21.19(46) | 24.90(0) | 20.64 | 20.73(68) | 20.90(64) | 23.03(2) |
|  | 1.9 2.0 | 19.32(83) 19.76 (83) | $20.48(60)$ $20.45(60)$ | 19.64(76) | 19.52(80) | 24.09(0) | 20.56(12) | 19.66(86) | 21.24(44) | 24.83(0) | 20.69 | 20.71(74) | 20.87(70) | 22.98(0) |
| Rempa | 1.1 | 20.79 (56) |  | 19 | 19.60(82) | 24.47(0) | 20.67(6) | 19.62(38) | 21.30(46) | 24.77(2) | 20.76 | 20.69(76) | 20.84(74) | 22.92(2) |
|  | 1.2 | 20.60 (68) | 21.00 (32) | 20.22(60) | 20.12 | 20.65(44) | 19.74(62) | 20.56(36) | 21.40(12) | 26.47 (0) | 20.68 | 21.13(40) | 21.48(22) | 24.40(2) |
|  | 1.3 | 20.45(82) | 20.25(34) | 20.23(56) | 19.98(78) | 21.73(2) | 20.01(42) |  | $21.38(8)$ $21.37(14)$ | 26.27(0) | 20.63 | 21.05(46) | 21.38(24) | 24.21(2) |
|  | 1.4 | 20.14(34) | 20.85(44) | 20.26(54) | 19.95(82) | 22.43(2) | 20.72(6) | 20.33(72) | $21.37(14)$ $21.37(16)$ | $28.10(0)$ $25.94(0)$ | 20.65 20.71 | 21.01(48) 20.99(52) | $21.27(28)$ $21.21(40)$ | $24.04(2)$ $23.89(2)$ |
|  | 1.5 | 20.41 (78) | 20.89(48) | 20.30(54) | 19.91(84) | 23.15(0) | 21.06(0) | 20.26(74) | 21.33(18) | 25.79(0) | 20.73 | 20.97(54) | 21.17(36) | 23.76(2) |
|  | 1.6 | 20.37 (82) | 20.74(62) | 20.33(52) | 19.88(88) | 23.83(0) | $21.37(0)$ | 20.19(84) | 21.29(22) | 25.66(0) | 20.76 | 20.95(66) | 21.12(42) | 23.63(2) |
|  | 1.7 | 20.31 (88) | 20.79(60) | 20.36(52) | 19.86(88) | 24.46(0) | 21.67 (0) | 20.13(86) | 21.24(24) | 25.54(0) | 20.80 | 20.93(68) | 21.08(48) | 23.52(2) |
|  | 1.8 | $20.28(86)$ | $21.01(56)$ | 20.39(52) | 19.84(90) | 25.05(0) | 21.95 (0) | 20.07(86) | 21.20(32) | 25.43(0) | 20.82 | 20.92(72) | $21.05(48)$ | 23.42(2) |
|  | 1.9 | 20.24 (80) | 20.76(64) | 20.42(48) | 19.82(c0) | 25.61 (0) | 22.22(0) | 20.02(88) | 21.17(32) | 25.33(0) | 20.84 | 20.90(74) | 21.02(56) | 23.33(2) |
|  | 2.0 | 20.17 (88) | 20.94(61) | 20.44(48) | 19.80(92) | $26.14(0)$ | $22.47(0)$ | 19.98(94) | 21.14(32) | 25.23 (0) | 20.88 | 20.89(78) | 20.99(56) | 23.24(2) |

Table 3: GMAPE and PL values (in brackets) from simulation study

| Accuracy messure | Method |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A1 | A2 | B1 | B2 | C1 | C2 | D1 | D2 | Naive | POES | . 05 | Trigg |  |
| GMAPE | 4.34 | 3.36 | 10.65 | 8.24 | 8.85 | 6.72 | 3.70 |  | 3.55 | 3.78 | . 3.97 |  | ${ }^{.} 5$ |
| PL | 18 | 76 | - | 0 | 2 | ${ }_{2}$ | 52 | 74 | ${ }_{0}$ | 3.78 | $\begin{gathered} 3.97 \\ 38 \end{gathered}$ | $\begin{gathered} 4.05 \\ 40 \\ \hline \end{gathered}$ | 3.89 40 |

Table 4: GMAPE and PL values for wool price data
——Group 1
-Group 2
_Group 3
_-Group 4
-.-. Group 5
-..- Group 6
-.-- Group 7
---- Group 8
---. Group 9
_... Group 10


Figure 1: Quarterly prices (c/kg clean)


Figure 2: Time series with business cycle $(\rho=.05)$


Figure 3: Time series with business cycle ( $\rho=.22$ )


Figure 4: Time series with a step change ( $\rho=1.2$ )


Figure 5: Time series with a step change ( $\rho=1.5$ )


Figure 6: Time series with a ramp $(\rho=1.2)$


Figure 7: Time series with a ramp $(\rho=1.4)$

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