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ABSTRACT

While the Almost Ideal Demand System has received increasing attention in empirical studies of consumer demand, the fact that the underlying PIGLOG (and PIGL) cost function is not globally regular has often led to violation of concavity in the estimated Slutsky matrix. This violation typically occurs at many points within the actual sample. Although the estimating form of AIDS is attractive, it is also obvious that it can be at best a local approximation to a regular form since, beyond an arbitrarily restrictive regularity region, the implied AIDS shares must fall outside the (0,1) interval. This paper suggests a modification to the PIGLOG class of preferences which preserves regularity in a much wider region of expenditure-price space. The Modified AIDS, termed MAIDS, may be shown to contain AIDS as a local linear approximation. Because MAIDS is not a member of the Generalised Linear class of cost functions, it does not have the exact aggregation properties of AIDS. Instead of employing a representative individual approach to aggregation, we explicitly aggregate over individuals and parameterize on certain macro averages in order to apply the model to available macro data. The resulting MACRO MAIDS form contains all the parameter restrictions of MICRO MAIDS together with some additional terms which arise explicitly through the aggregation. We compare the estimated MACRO MAIDS with AIDS and demonstrate the improved regularity features.

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1. INTRODUCTION

The Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) has become an established model for the estimation of systems of demand equations. The AIDS model is a particular example of Muellbauer's PIGLOG, itself a special case of the price independent generalized linear (PIGL) class of preferences (Muellbauer (1975)). The PIGL class allows explicit aggregation over households. In the PIGLOG sub-class, the form implies share equations which are consistent with the empirically attractive Working-Leser model. In the AIDS sub-case of PIGLOG, Deaton and Muellbauer specify two price aggregator functions which are sufficiently general to ensure that the AIDS cost function is a locally flexible functional form. Alternative specifications of at least the first of these aggregator functions have been suggested. Chalfant (1987) suggests that the Translog aggregator function be replaced by a Fourier form to extend the region of flexibility; Diewert and Wales (1987) suggest that Translog functions in general - not specifically in the AIDS context - are better replaced by functions with enhanced regularity properties such as their Generalized McFadden function. However, it is the second aggregator function which is crucial to the empirical attractiveness of all members of the PIGL class of preferences. It is also this second function which is at the heart of regularity problems for the PIGL class. Of course, the Chalfant contribution was not addressed to the problem of regularity, and the Diewert-Wales contribution was not addressed to the peculiarities of the PIGL class of cost functions. We are merely pointing out here that the question of global regularity of AIDS-type specifications has not been fully addressed in the literature.

To be more specific, the PIGL class of cost functions admit an arbitrarily restrictive regularity region beyond which implied shares fall outside the (0,1) interval and the Slutsky matrix fails to be negative semi-definite. As Deaton and Muellbauer point out, the only member of PIGL which is globally regular is trivial. Recently, Lewbel (1987) characterised the set of fractional demand systems which (subject to positivity of parameter estimates) will satisfy the (0,1) restriction globally. However, not all members of the fractional class exhibit reasonable regularity with respect to the curvature condition.

In this paper, we discuss a particular member sub-class of fractional forms which allows the imposition of all regularity conditions over an extensive region of expenditure-price space. This sub-class is developed as a modification of the indirect utility function underlying PIGLOG preferences. Since the most well known member of PIGLOG is AIDS, and since the precise specification of the price aggregator function is to some extent a matter of taste, we refer to our suggested class of functional forms by the acronym MAIDS - Modified AIDS.

Section 2 elaborates on the regularity issue as it affects PIGLOG (and AIDS in particular), and Section 3 presents MAIDS as a regular modification to AIDS. Section 4 discusses the aggregation problem in the context of GMAIDS, a more general functional form that allows the nesting of AIDS and MAIDS as special cases. A particular empirical specification which best allows for a comparison between AIDS and MAIDS is presented in Section 5, and an empirical example is provided in Section 6, where it is found that MAIDS is indeed more regular than AIDS.

2. PIGLOG AND AIDS

Deaton and Muellbauer introduce the AIDS cost function as a special case of the PIGLOG specification:

$$\ln C(u,p) = (1-u) \ln a(p) + u \ln b(p) \tag{2.1}$$

where a(p) and b(p) are positive and homogeneous of degree one (HD1) functions in p.

In order to discuss regularity, we note firstly the standard properties of a regular cost function:

C1 C is non-negative

C2 C is HD1 in p

C3 C is non-decreasing in u

C4 C is non-decreasing in p

C5 C is concave in p.

The only regularity conditions which (2.1) obviously satisfies globally are C1 and C2. It is a trivial matter, however, to design a and b to satisfy C3, by defining ln b = ln a + (a positive function). More problematic are C4 and C5. For example, as Deaton and Muellbauer point out, a set of sufficient conditions for (2.1) to be concave in p would be that a and b be concave and $0 \leq u \leq 1$. However, on the one hand these conditions are not necessary and on the other they do not represent a very realistic set of sufficient conditions since the condition $0 \leq u \leq 1$ is in no way implied for the evaluation of the indirect utility function dual to (2.1). Consequently, even if concavity of the cost function were imposed in estimation, it could not be guaranteed in simulation work in the context, for example, of a computable general equilibrium model employing the AIDS specification.

To discuss C4 and C5 in more detail, it is useful to write (2.1) in terms of two reparameterised price aggregator functions P_1 and P_2, where $P_1 = a$ and $P_2 = \ln (b/a)$. Hence:

$$\ln C(u,p) = \ln P_1 + u P_2 \tag{2.2}$$

where: P_1 is HD1 in p,
P_2 is HD0 in p.

It is important to note that it is the aggregator function P_2 which "does the work" (in the sense of providing Working-Leser type share equations) but that this function must be HD0 to satisfy C2. For Deaton and Muellbauer's AIDS specification, P_1 is Translog and P_2 is Cobb-Douglas. Chalfant's generalization of P_1 does not affect the regularity problems, which arise primarily from P_2. It is true that the Translog specification for P_1 presents an additional source of irregularity in the case of the specific AIDS form, but that source of irregularity is not our chief concern here. Even if P_1 were specified along more regular lines following Diewert-Wales, the functional form (2.2) would exhibit regularity problems. The problem is that P_2 cannot be simultaneously HD0, non-decreasing and concave in p. Euler's theorem rules out the HD0, non-decreasing combination, and the HD0, concave combination, for any non-trivial functions. And while these conditions on P_2, together with $u \geq o$ and P_1 HD1 non-decreasing and concave, are merely a set of sufficient conditions, if any of the three suggested but incompatible conditions on P_2 were not specified, regularity of (2.2) could only be imposed at the expense of further arbitrary restriction of the range of u.

In order to motivate an inherently more regular alternative to (2.2), it is useful to outline the implications of PIGLOG irregularity for the share form of the underlying demand system. Let $q_i = Q_i(c,p)$ denote an arbitrary Marshallian demand function. Let $E_i = \partial \ln Q_i / \partial \ln c$ and $M_{ij} = \partial \ln Q_i / \partial \ln p_j$ denote the (Marshallian) expenditure and price elasticities respectively ($j = 1, \ldots, n$). Let $w_i = p_i q_i / c$ denote the i^{th} expenditure share. A standard consumer utility maximization problem subject to a linear budget constraint yields aggregation, homogeneity, symmetry and concavity restrictions which we summarise:

Aggregation	Engel:	$\Sigma_i \, w_i \, E_i = 1$
	Cournot:	$w_j + \Sigma_i \, w_i \, M_{ij} = 0$
Homogeneity	$E_i + \Sigma_j \, M_{ij} = 0$	
Symmetry	$w_i \, (M_{ij} + w_j \, E_i)$ symmetric	
Concavity	$[w_i \, (M_{ij} + w_j \, E_i)]$ a negative semi-definite matrix.	

Define $R = \ln(c/P_1)$, and for the PIGLOG specification (2.2), let

$$\varepsilon_{ki} = \partial \ln P_k \, / \, \partial \ln p_i \text{ and}$$

$$\varepsilon_{kij} = \partial^2 \ln P_k \, / \, \partial \ln p_i \, \partial \ln p_j, \; (k = 1,2), \; (i,j = 1, \ldots, n).$$

Note that, since P_1 is HD1 but P_2 is HD0, $\Sigma_i \, \varepsilon_{1i} = 1$ while $\Sigma_i \, \varepsilon_{1ij}$, $\Sigma_i \, \varepsilon_{2i}$ and $\Sigma_i \, \varepsilon_{2ij}$ all = 0. Shephard's Theorem applied to (2.2) gives:

$$w_i = \varepsilon_{1i} + \varepsilon_{2i} R, \tag{2.3}$$

and it follows that:

$$E_i = 1 + \varepsilon_{2i}/w_i \, , \tag{2.4}$$

and:

$$M_{ij} = (\varepsilon_{1ij} + \varepsilon_{2ij} R - \varepsilon_{2i} \, \varepsilon_{1j})/w_i - \delta_{ij} \tag{2.5}$$

where δ_{ij} is the Kronecker delta.

It is clear that specification of P_1 HD1 and P_2 HD0 has been sufficient to maintain Engel and Cournot aggregation, homogeneity and symmetry for the PIGLOG expenditure share system (2.3). However, it is equally apparent from (2.3) that for real expenditure sufficiently large, w_i will violate the (0,1) interval.

Moreover, maintenance of concavity is even more fragile. In terms of the previous notation, a typical term in the Slutsky matrix is:

(2.6) $$S_{ij} = (c/p_i p_j)(w_i M_{ij} + w_i w_j E_i)$$

and some manipulation leads to the revealing formulation for the PIGLOG specification:

(2.7) $$S_{ij} = (c/p_i p_j)(\zeta_{1ij} + \zeta_{2ij} R)$$

where:

$$\zeta_{1ij} = \varepsilon_{1ij} + \varepsilon_{1i} (w_j - \delta_{ij}), \text{ and}$$

$$\zeta_{2ij} = \varepsilon_{2ij} + \varepsilon_{2i} (w_j - \delta_{ij}) + \varepsilon_{2i} \varepsilon_{2j}.$$

The nature of the concavity violation problem for AIDS is now apparent. As R increases, the ζ_{2ij} terms dominate in S_{ij}. But, on the diagonal, ${\varepsilon_{2i}}^2$ is necessarily positive. For the specific Cobb-Douglas formulation of P_2 in AIDS, $\varepsilon_{2ii} = 0$. Hence $\zeta_{2ii} = \varepsilon_{2i}(w_i - 1) + \varepsilon_{2i}^2$. For $\varepsilon_{2i} > 0$, the positive ε_{2i}^2 term tends to dominate as w_i tends to unity. For $\varepsilon_{2i} < 0$, the entire ζ_{2ii} term is necessarily positive for w_i in the (0,1) interval. Thus it is clear that there is a tendency for the required non-positivity of S_{ii} to be violated as R increases, and this may occur well before w_i violates the (0,1) interval.

Both (2.3) and (2.7) indicate the one source of irregularity - the term in R. Yet this term is the essential component of the Working-Leser type specification (2.3). However, another way of looking at the problem is the intransigence of the elasticity ε_{2i} - no matter how complex this is as a function of prices it does not allow the response of the i^{th} share to growth in real expenditure to be modified for higher incomes. In the next section we suggest a modification to the PIGLOG cost function (2.2) which allows an amelioration of the effect of growth in real expenditure on shares, avoiding violation of the (0,1) interval and maintaining concavity under the modest restriction $c \geq P_1$.

3. MAIDS (MODIFIED PIGLOG AND AIDS)

As an alternative to (2.2), consider the modification

$$\ln C(u,p) = \ln P_1 + u\, P_2 \,/\, C(u,p) \tag{3.1}$$

where now both P_1 and P_2 are HD1 in p.

Since (3.1) does not have an explicit analytical representation as a cost function, we continue the discussion in terms of the dual indirect utility function. For purposes of comparison we note firstly that the indirect utility function dual to the PIGLOG cost function (2.2) would be:

$$U(c,p) = [\ln (c/P_1)] \,/\, P_2 \tag{3.2}$$

while for the implicit MAIDS cost function (3.1) the dual would be:

$$U(c,p) = [\ln (c/P_1)]\, (c/P_2). \tag{3.3}$$

Of course, the degree of homogeneity of the second price aggregator function is different in the two cases. To discuss the regularity of MAIDS consider the standard properties of a regular indirect utility function:

U1 U is HD0 in (c,p)

U2 U is non-decreasing in c

U3 U is non-increasing in p

U4 U is quasiconvex in p.

By specifying both P_1 and P_2 to be positive HD1 functions, (3.3) clearly satisfies U1 and U2. If P_1 and P_2 are further specified to be non-decreasing in prices then U3 will be satisfied over the region $c \geq P_1$. If P_1 and P_2 are both concave then, over the region $c \geq P_1$, U will be quasiconvex in p (for relevant results on quasiconvex functions see Greenberg and Pierskalla (1971)).

Applying Roy's Identity to (3.3) we obtain the MAIDS share equations:

$$(3.4) \qquad w_i = \frac{\varepsilon_{1i} + \varepsilon_{2i} R}{1 + R}$$

where the ε_{1i} and ε_{2i} are defined, as before, as the i^{th} price elasticities of P_1 and P_2 respectively, but where now both $\Sigma_i\ \varepsilon_{1i} = 1$ and $\Sigma_i\ \varepsilon_{2i} = 1$, and where $R = \ln(c/P_1)$ as before. Observe that, in the region $c \geq P_1$, we have $R \geq 0$ and so the restrictions $\varepsilon_{1i} \geq 0$ and $\varepsilon_{2i} \geq 0$ are sufficient to ensure $0 \leq w_i \leq 1$, and these restrictions are natural ones to impose on the two price aggregator functions.

In general the ε_{1i} and ε_{2i} will be functions of prices unless both P_1 and P_2 are Cobb-Douglas, but even that case provides an interesting illustration of the model in general.

To facilitate comparison with PIGLOG note that for MAIDS:

$$\partial w_i / \partial \ln c = \frac{\varepsilon_{2i} - w_i}{1 + R} \tag{3.5}$$

whereas for PIGLOG $\partial w_i / \partial \ln c = \varepsilon_{2i}$. Equation (3.5) shows how in MAIDS the response of shares to growth in real expenditure is modified both by the expenditure level itself and by the pre-existing value of the share. Equation (3.4) indicates that, for given prices, the share w_i moves monotonically from ε_{1i} for the "poor" (if we interpret the lower extreme of the regular region for real expenditure as "subsistence", following Deaton and Muellbauer), asymptoting to ε_{2i} for the "rich".

More generally, for any compatible specifications of the P_1 and P_2 functions across models, it can be shown that a PIGLOG model may be interpreted as a local approximation to a MAIDS model. Rearranging (3.4) we note that:

$$w_i = \varepsilon_{2i} + \frac{\varepsilon_{1i} - \varepsilon_{2i}}{1 + R} \tag{3.6}$$

Given $c = \Sigma_i\, p_i\, q_i$, choose units of measurement for the q_i such that the price index P_1 satisfies $P_1 = \mu\, P^*$ with scaling constant $\mu < 1$ and $P^* = c$ in the base period. Let $R^* = \ln (c / P^*)$. Then $1 + R = 1 - \ln \mu + R^* = \nu + R^*$ where $\nu > 1$ by construction. Thus (3.6) may be written as:

$$w_i = \varepsilon_{2i} + \frac{(1/\nu)\ (\varepsilon_{1i} - \varepsilon_{2i})}{1 + (1/\nu)\ R^*} \quad . \tag{3.7}$$

Now $R^* = 0$ in the base period, and there will be an interval $\max\{-1, \ln \mu\} \leq R^* < 1$ which lies within the MAIDS regular region $R \geq 0$. Within this interval, a linear approximation to (3.7) is:

(3.8) $$w_i = \varepsilon^*_{1i} + \varepsilon^*_{2i} R^*$$

where:

(3.9a) $$\varepsilon^*_{1i} = \varepsilon_{2i} + (1/\nu)\,(\varepsilon_{1i} - \varepsilon_{2i}),$$

(3.9b) $$\varepsilon^*_{2i} = (1/\nu)^2\,(\varepsilon_{2i} - \varepsilon_{1i}).$$

Note that, in (3.8), $\Sigma_i\,\varepsilon^*_{1i} = 1$ while $\Sigma_i\,\varepsilon^*_{2i} = 0$, as in AIDS.

Thus estimates generated by a PIGLOG specification may be considered to be the estimates of a linearized version of MAIDS. Of course, this approximation has only local validity, which is consistent with the fact that PIGLOG can be at best locally regular. Nevertheless, the AIDS parameter estimates ε^*_{1i}, ε^*_{2i} could provide useful starting values for estimation of the inherently more nonlinear MAIDS specification. The MAIDS starting values are generated on rearrangement of (3.9) as:

(3.10a) $$\varepsilon_{1i} = \varepsilon^*_{1i} - \nu\,(\nu - 1)\,\varepsilon^*_{2i}\,,$$

(3.10b) $$\varepsilon_{2i} = \varepsilon^*_{1i} + \nu\,\varepsilon^*_{2i}\,.$$

It should also be noted that if P_1 is treated as a Stone price index (at least for initial estimation) then MAIDS, like AIDS, could be estimated in linear form. Since this form is particularly useful for interpretation, we define:

(3.11a) $$Z = R/(1 + R)$$

so that:

(3.11b) $$w_i = \varepsilon_{1i}\,(1 - Z) + \varepsilon_{2i}\,Z,$$

giving a transparent representation of the MAIDS shares as weighted averages of the rich (ε_{2i}) and poor (ε_{1i}).

Further properties of MAIDS are readily derived. The expenditure elasticities satisfy:

$$(3.12) \qquad E_i = (\varepsilon_{2i} / w_i)(1 - Z) + Z ,$$

and (3.11b) together with (3.12) demonstrate that the expenditure elasticities range from $\varepsilon_{2i}/\varepsilon_{1i}$ for the poor towards unity for the rich. It is also evident from (3.12) that $E_i \geq Z$, so that our sufficient conditions for regularity rule out inferior goods in MAIDS. While this is something of a pity, à priori, it should be noted that the existence of inferior goods in principle in AIDS arises only from the irregularity of AIDS (cf. equation (2.4)). There may well exist a region in expenditure-price space that violates the sufficient conditions for regularity, but not the (as yet undetermined) necessary conditions for regularity. This region would allow for the à priori possibility of inferiority in MAIDS. In general, the behaviour of the expenditure elasticities is much more sensible for MAIDS than is implied for AIDS. As real spending power rises, E_i tends toward unity, rising from below in the case of a necessity or falling from above in the case of a luxury; necessities become less necessitous and luxuries become less luxurious.

Although sufficient conditions exist for MAIDS to be globally regular, in many cases it may be useful (for enhanced flexibility) to employ a locally regular flexible functional form, such as the Translog, for P_1. In this case it can be shown that MAIDS has superior local regularity properties to AIDS (even if the second price aggregator is also specified by a Translog, say, rather than a Cobb-Douglas function).

Applying (2.6) to the MAIDS specification, a typical term of the MAIDS Slutsky matrix may be written:

$$S_{ij} = (c/p_i p_j) [\xi_{1ij} (1 - Z) + \xi_{2ij} Z] \tag{3.13}$$

where:

$$\xi_{1ij} = \varepsilon_{1ij} + \varepsilon_{1i} (w_j - \delta_{ij}), \text{ and}$$

$$\xi_{2ij} = \varepsilon_{2ij} + \varepsilon_{2i} (w_j - \delta_{ij}) + (\varepsilon_{2i} - w_i) (\varepsilon_{2j} - w_j).$$

As R increases Z rises from zero to unity and the Slutsky matrix terms asymptote to ξ_{2ij}. For the diagonal terms, $\xi_{2ii} = \varepsilon_{2ii} + \varepsilon_{2i} (w_i - 1) + (\varepsilon_{2i} - w_i)^2$. While the squared term is clearly positive, it asymptotes to zero as R increases, since w_i asymptotes to ε_{2i}. If P_2 is chosen Cobb-Douglas as for AIDS, the only possible source of violation of non-positivity of S_{ii} comes from the term ξ_{1ii}. But this term has less weight as R increases. Thus, for P_1 Translog and P_2 Cobb-Douglas, MAIDS is inherently more regular than AIDS.

4. MACRO MAIDS

The purpose of this section is to provide a link between the microeconomic specification of MAIDS and a form to which aggregate time series data may be applied. Obviously, the macro estimating form will not allow all the underlying microeconomic parameters to be fully identified, except under the assumption of identical preferences. Later, we make this assumption for expository purposes and ease of interpretation of the empirical results. However, we note that this assumption is not crucial to the derivation of MACRO MAIDS. We seek a reasonable set of restrictions designed to reduce the dimensionality of the problem from something of the order of nH (where n is the number of commodities and H is the number of households) to something of the order of n.

In the aggregation of MICRO MAIDS to MACRO MAIDS, we find that a set of additional terms naturally arise. These terms are shown to depend upon the distribution of household expenditure. This creates an avenue for business cycle and demographic influences on the macro shares. Our empirical results demonstrate their importance.

In the empirical work, we intend to compare MAIDS and AIDS. To develop the macro forms of both systems and allow nested hypothesis testing, we develop the macro relationships in this section for a generalized model, termed GMAIDS, which contains both MAIDS and AIDS as special cases.

To generate the form of MACRO GMAIDS it will be useful to summarise the relevant MICRO GMAIDS relationships. Let the superscript h denote a particular household. The GMAIDS indirect utility function for household h is:

$$(4.1) \qquad U^h\,(c^h,\ p) = [\ln\,(c^h\ /\ P_1^h)]\ [(c^h)^{\eta}/\ P_2^h]$$

where P_1^h is HD1 in p and P_2^h is HDη in p. By specifying household specific price indices, we may allow for varying demographic characteristics across households in a general way. The GMAIDS parameter η is treated as constant across households for expositional convenience. (A set of sufficient conditions for the regularity of (4.1) is: $0 \le \eta \le 1$, P_1^h and P_2^h concave and non-decreasing (i.e. ε_{1i}, $\varepsilon_{2i} \ge 0$). Under these conditions $\eta = 0$ requires $P_2^h = 1$. However, since our aim is to compare MAIDS with AIDS, our main interest in this paper will be to allow for regularity rather than impose it à priori. Thus if we do not impose à priori that P_1^h and P_2^h be non-decreasing and concave, then (4.1) allows AIDS as a special case when $\eta = 0$, MAIDS when $\eta = 1$, and values of $\eta > 1$ may also be admissible.) The household specific price index elasticities are:

(4.2) $$\varepsilon_{ki}^h = \partial \ln P_k^h / \partial \ln p_i, \quad (k = 1,2), \quad (i = 1, \ldots, n)$$

with $\Sigma_i \varepsilon_{1i}^h = 1$, and $\Sigma_i \varepsilon_{2i}^h = \eta$. Define real expenditure indices:

(4.3) $$R^h = \ln (c^h / P_1^h)$$

and apply Roy's Identity to obtain the individual share equations:

(4.4) $$w_i^h = \frac{\varepsilon_{1i}^h + \varepsilon_{2i}^h R^h}{1 + \eta R^h} \quad .$$

Assume that $R^h \geq 0$. (In empirical work at the macro level, the data may be constructed to ensure satisfaction of a macro equivalent of this condition over the sample period). The ε_{ki}^h will in general be functions of prices. However, at this point we do not pre-specify them at the micro level, but merely note that $\varepsilon_{ki}^h \geq 0$ is sufficient for regularity of the micro relationships.

Suppose now that time series data are only available at the aggregate level. Define average total expenditure and average shares as:

(4.5a) $$c = \Sigma_h c^h / H ,$$

(4.5b) $$w_i = \Sigma_h c^h w_i^h / \Sigma_h c^h ,$$

multiply (4.4) through by $1 + \eta R^h$ and average over households to obtain:

$$(4.6)\qquad w_i + \eta\Sigma_h c^h w_i^h R^h / \Sigma_h c^h$$

$$= \Sigma_h c^h \varepsilon_{1i}^h / \Sigma_h c^h + \Sigma_h c^h \varepsilon_{2i}^h R^h / \Sigma_h c^h .$$

In order to write (4.6) in terms of average macro parameters, we define average macro elasticities:

$$(4.7)\qquad \varepsilon_{ki} = \Sigma_h c^h \varepsilon_{ki}^h / \Sigma_h c^h$$

and average macro (commodity specific) real expenditure indices:

$$(4.8)\qquad R_i = \Sigma_h c^h w_i^h R^h / \Sigma_h c^h w_i^h ,$$

and hence rewrite the macro shares as:

$$(4.9)\qquad w_i (1 + \eta R_i) = \varepsilon_{1i} + \varepsilon_{2i} \frac{\Sigma_h c^h \varepsilon_{2i}^h R^h}{\Sigma_h c^h \varepsilon_{2i}^h} .$$

To simplify the last term in (4.9), define household expenditure share terms:

$$(4.10)\qquad s^h = c^h / \Sigma_k c^k ,$$

an overall average real expenditure index:

$$(4.11)\qquad R = \Sigma_h s^h R^h = \Sigma_i w_i R_i ,$$

and a set of commodity-specific macro average relative expenditure terms:

$$(4.12)\qquad \theta_i = \Sigma_h (s^h \varepsilon_{2i}^h/\varepsilon_{2i}) (R^h/R)$$

(where the case $R = 0$ is handled by defining $R^h/R = 1$ in that case).

Using these definitions, the macro share system (4.9) becomes:

$$(4.13) \qquad w_i \ (1 + \eta R_i) = \varepsilon_{1i} + \varepsilon_{2i} \ \theta_i \ R \ .$$

It should be emphasised that no restrictive assumptions at all have been employed in the translation from the micro specification (4.4) to the macro form (4.13). However (4.13) clearly indicates points at which assumptions will need to be made to achieve a parsimonious parameterization. Note firstly that, by construction $\Sigma_i \ \varepsilon_{2i} \ \theta_i = \eta$, and if both the ε_{2i} and the θ_i were treated as constant parameters they could not be individually identified. In any case, inspection of (4.12) and (4.7) suggests no obvious parsimonious variable parameter specification which might aid identification. We propose to parameterize on ε_{1i} and on the product $\varepsilon_{2i} \ \theta_i$, though possibly with a variable parameter specification to facilitate comparison with the usual AIDS form.

To operationalise (4.13) we must now reduce the dimensionality problem associated with the R_i terms (c.f. equation (4.8)). One approach is as follows:

Define a macro price index P by:

$$(4.14) \qquad \ln P = \Sigma_h \ s^h \ \ln P_1^h \ .$$

Now define a set of commodity specific average household deflators:

$$(4.15) \qquad \ln K_i = \Sigma_h \ c^h \ w_i^h \ (R - R^h) \ / \ \Sigma_h \ c^h \ w_i^h \ .$$

The $\ln K_i$ average the deviations of household real spending power from the average, using household expenditure share weights. By construction we have:

(4.16) $\Sigma_i \; w_i \; \ln K_i = 0$.

Now using (4.14) - (4.16) the real expenditure indices (4.8) and (4.11) may be re-expressed in macro terms as:

(4.17a) $R_i = \ln\,(c \,/\, K_i \; P)$,

and

(4.17b) $R = \ln\,(c/P)$,

and this allows the macro share equations (4.13) to be rewritten as:

$$(4.18) \qquad w_i = \frac{\varepsilon_{1i} + w_i \; \eta \; \ell n \; K_i + \varepsilon_{2i} \; \theta_i \; R}{1 + \eta \; R}$$

The MACRO GMAIDS system (4.18) may be constrasted with MICRO MAIDS (3.4) for η=1, with MICRO AIDS (2.3) for η=0, and with MICRO GMAIDS (4.4) in general. The additional term $w_i \; \eta \; \ell n \; K_i \;/\; (1 + \eta \; R)$ in MACRO GMAIDS arises because of the lack of exact aggregation in MAIDS and GMAIDS (except under unreasonably strong assumptions).

5. EMPIRICAL SPECIFICATION

To operationalise (4.18) some suitable parameterizations must be employed for the term $w_i \; \eta \; \ell n \; K_i \;/\; (1 + \eta \; R)$, for the macro price index P (in R), and for the terms ε_{1i} and $\varepsilon_{2i}\theta_i$.

From (4.15),

$$(5.1) \qquad \frac{w_i \; \eta \; \ell n \; K_i}{1 + \eta \; R} = \frac{\eta \; \Sigma_h \; s^h \; w_i^h \; (R - R^h)}{1 + \eta R} \; .$$

In view of the sensitivity of the RHS of (5.1) to the distribution of real spending power, we parameterise (5.1) as:

$$\frac{w_i \ \eta \ \ell n \ K_i}{1 + \eta \ R} = \eta(\mu_i{}'x + v_i) \tag{5.2}$$

where x is a vector of explanators sensitive to the distribution of real spending power, μ_i is a vector of parameters satisfying $\Sigma_i \mu_i = 0$, and $\eta v_i (\equiv u_i)$ is a zero mean random variable satisfying $\Sigma_i u_i = 0$. Clearly the characteristics of u_i will be sensitive to the adequacy of the use of the x variables as proxies for the weighted average real spending power deviation. Note also that, since $\eta = 0$ for AIDS, the term (5.2) does not enter MACRO AIDS.

A second obstacle to an operational specification is the macro price index P. We consider two possibilities:

Case 1: $P \approx P^*$ where P^* is the Stone price index:

$$\ell n \ P^* = \Sigma_i w_i \ \ell n p_i \ . \tag{5.3}$$

Case 2: $\varepsilon_{1i}^{h} = \varepsilon_{1i} \ \forall \ h$, so that $P = P_1$. This generates nonlinear restrictions in (4.18).

In either case, the question of the appropriateness of the imposition of symmetry restrictions arises. Write the MACRO GMAIDS estimating form as:

$$w_i = \varepsilon_{1i}(1 - \eta Z) + \varepsilon_{2i} \ \theta_i \ Z + \eta\mu_i{}'x + u_i \tag{5.4}$$

where $Z = R \ / \ (1 + \eta R)$, and observe that:

(5.5) $$\partial \varepsilon_{1i} / \partial \ell n\, p_j = \Sigma_h s^h \varepsilon_{1ij}^h ,$$

and

(5.6) $$\partial(\varepsilon_{2i}\, \theta_i) / \partial \ell n p_j$$

$$= \Sigma_h s^h [\varepsilon_{2ij}^h \; R^h/R - \varepsilon_{2i}^h (\varepsilon_{1j}^h - \varepsilon_{1j})] .$$

While symmetry clearly applies to the ε_{1i} terms, in the case of the $\varepsilon_{2i}\, \theta_i$ terms for symmetry to apply we require either: (i) $\varepsilon_{1j}^h = \varepsilon_{1j}\ \forall$ h; (ii) $\varepsilon_{2i}^h = \varepsilon_{2i}\ \forall$ h; or (iii) $\varepsilon_{1j}^h - \varepsilon_{1j} \propto \varepsilon_{2j}^h$. In the sequel, we maintain the assumption of identical preferences. In this case $P_1^h = P_1$, $P_2^h = P_2$, $\theta_i = 1$, symmetry restrictions apply to both ε_{1i} and ε_{2i} and it is a simple matter to investigate the concavity condition on the micro equations underlying (5.4). Therefore (5.4) reduces to the MACRO GMAIDS share equations.

(5.7) $$w_i = \varepsilon_{1i}(1 - \eta Z) + \varepsilon_{2i} Z + \eta \mu_i' x + u_i$$

where the ε_{1i} and ε_{2i} now have a direct interpretation as the elasticities of P_1 and P_2. Under identical preferences (4.4) gives the MICRO GMAIDS share equations:

(5.8) $$w_i^h = \varepsilon_{1i}(1 - \eta Z^h) + \varepsilon_{2i} Z^h ,$$

where $Z^h = R^h / (1 + \eta R^h)$. Since macro Z in (5.7) is obviously also a potential micro Z^h, a necessary condition for concavity holding for w_i^h is that it hold for the first two terms in (5.7). The Slutsky matrix corresponding to (5.8) has typical term:

(5.9) $$S_{ij}^h = (c^h/p_i p_j)\, [\psi_{1ij}^h (1 - \eta Z^h) + \psi_{2ij}^h Z^h]$$

where:

$$\psi_{1ij}^{h} = \varepsilon_{1ij} + \varepsilon_{1i}(w_j^h - \delta_{ij}), \text{ and}$$

$$\psi_{2ij}^{h} = \varepsilon_{2ij} + \varepsilon_{2i}(w_j^h - \delta_{ij}) + (\varepsilon_{2i} - \eta w_i^h)\ (\varepsilon_{2j} - \eta w_j^h)\ .$$

As discussed previously, for flexible but not necessarily regular P_1 and P_2, specification (5.9) is more likely to satisfy concavity for η=1 than for η=0. Using macro data, and hence using (5.7) rather than (5.8), the "pseudo Slutsky matrix" made up of terms:

$$S_{ij} = (c/p_i p_j)\ [\psi_{1ij}(1 - \eta Z) + \psi_{2ij}\ Z] \tag{5.10}$$

where:

$$\psi_{1ij} = \varepsilon_{1ij} + \varepsilon_{1i}(w_j - \delta_{ij}), \text{ and}$$

$$\psi_{2ij} = \varepsilon_{2ij} + \varepsilon_{2i}(w_j - \delta_{ij}) + (\varepsilon_{2i} - \eta w_i)(\varepsilon_{2j} - \eta w_j),$$

may be used to check the likelihood of concavity applying at the micro level by applying (5.10) to household average w_i and c at all sample data points.

Turning now to the price aggregators, AIDS corresponds to the specification of P_1 Translog and P_2 Cobb-Douglas, i.e.

$$\ln P_1 = \kappa + \sum_i \alpha_i \ln p_i + 1/2 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j \tag{5.11a}$$

$$\ln P_2 = \sum_i \beta_i \ln p_i \tag{5.11b}$$

where $\sum_i \alpha_i = 1$, $\gamma_{ij} = \gamma_{ji}$, $\sum_j \gamma_{ij} = 0$ and $\sum_i \beta_i = 0$, leading to the (MICRO and MACRO, given exact aggregation) AIDS share equations

$$(5.12) \qquad w_i = \varepsilon_{1i} + \beta_i R$$

where $\varepsilon_{1i} = \alpha_i + \sum_j \gamma_{ij} \ln p_j$ and $R = \ln(c/P_1)$.

While it is well-known that a Translog function cannot be globally regular, and that the imposition of local (sample) regularity is a non-trivial task (see Diewert and Wales (1987)), recall from Section 2 that it is in fact the specification of P_2 that is at the heart of the regularity problem of AIDS. Thus one interesting possibility for the specification of GMAIDS would be to choose P_1 and P_2 as in (5.11), except that for GMAIDS we would specify $\Sigma\beta_i = \eta$. Regularity for P_2 would merely require $\beta_i \geq 0$. (Of course this constraint is inconsistent with η=0 except in the trivial case in which AIDS is known to be regular.) This specification would lead to the MACRO GMAIDS share equations

$$(5.13) \qquad w_i = \frac{\varepsilon_{1i} + \beta_i R}{1 + \eta R} + \eta\, \mu_i' x + u_i \ .$$

While such a specification would allow a comparison of AIDS and MAIDS as special cases, such a comparison would be a priori constrained for two reasons. Firstly, in (5.12) shares are a linear function of $\ln p_j$, whereas in (5.13) shares are a linear function of $\ln p_j/(1 + \eta R)$. Modification of the size of marginal price effects for larger real expenditure levels (similar to the modification of expenditure effects) does have some appeal, but of course neither (5.12) nor (5.13) is regular over the whole price space because of the irregularity of the Translog. To make the comparison between MAIDS and AIDS more transparent, we instead propose that for GMAIDS P_2 be modelled as

$$(5.14) \qquad \ln P_2 = \sum_i \beta_i \ln p_i + \eta/2 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j$$

where $\Sigma\beta_i = \eta$ and the γ_{ij} terms are common to P_1 and P_2. Secondly, exact aggregation in AIDS does not allow the macro explanators x to appear in (5.12). Again, to aid in the comparison of the expenditure effects, we subsume η into the parameters μ_i in (5.13), but allow the μ_i to be non-zero. These specification changes lead to the empirical GMAIDS share equations

$$(5.15) \qquad w_i = \sum_j \gamma_{ij} \ln p_j + \frac{\alpha_i + \beta_i R}{1 + \eta R} + \mu_i' x + u_i$$

so that comparison of the models now depends solely on the way in which shares depend on the level of real expenditure, and is not biased by the actual form of the empirical dependence of shares on prices and on business cycle factors.

6. AN EMPIRICAL COMPARISON OF AIDS WITH MAIDS

Variants of the MACRO GMAIDS system (5.15) were estimated using Australian annual data for the period 1955 to 1986. The data used is that of Chung and Powell (1987). Their work indicated a particular problem with the rent component, and we have excluded this category from our data. Similarly, it can be argued that durables are unlikely to be well explained by a static allocation model, and this category has also been excluded. This leaves four categories of expenditure: Food (F), Tobacco and Alcohol (T), Clothing (C) and Other (O). Such a sample of data was considered well-suited to an initial comparison of MAIDS with AIDS, with the disadvantage of a small number of categories more than outweighted by the advantages of a relatively long data period using annual data, and simplicity of presentation. All estimation was carried out using the LSQ option of TSP.

Following Deaton and Muellbauer and in line with Case 1 of Section 5, initial estimation proceeded with R defined by $\ln(c/P^*)$ where P^* is the Stone price index $\ln P^* = \sum_j w_j \ln p_j$. Normalization of P^* has no effect on AIDS, and for MAIDS we set $\ln P^* = \ln c$ at the beginning of the sample. This ensures that MAIDS is regular over the entire cone $\{(c,p);\ p \in \rho,\ c \geq c_1\}$ where ρ is the intersection of the regular regions of P_1 and P_2, and c_1 is the initial sample value of c. Both AIDS and MAIDS are then linear in parameters, the only cross equation restrictions are the symmetry of the γ_{ij}, and each has the same number of parameters, allowing the comparison of two quite simple models.

To introduce the issues raised by estimation, we begin by estimating naive models of MAIDS and AIDS. By naive we mean that no macro variables are included in the specification (i.e. $\mu_i = 0\ \forall_i$). We also begin with the case where P_1 and P_2 are Cobb-Douglas (i.e. $\gamma_{ij} = 0\ \forall i,j$). These results are presented in Table 1. In all of the tables of results, asymptotic t-values are given in parentheses, and L refers to the maximized log-likelihood value.

A number of points are worth making with regard to these results. First, had all coefficients been positive, this simple MAIDS model would itself have been an interesting regular specification, rivalling say LES as a parsimonious regular specification, without the disadvantage of constant marginal budget shares (albeit with limited price effects). Although MAIDS is not quite regular, it is certainly regular over a much wider range of expenditure-price space than is AIDS. Second, at least on an informal basis, likelihoods are comparable across models, since each model has the same endogenous variables and the same number of parameters. (Thus comparing likelihoods is equivalent to comparing the Akaike information criterion - we will consider a formal hypothesis test

Table 1A: NAIVE AIDS (Cobb-Douglas)

	1:F	2:T	3:C	4:O
α_i	0.308 (118.9)	0.129 (101.2)	0.152 (86.4)	0.411 (149.4)
β_i	-0.175 (-24.9)	-0.051 (-14.9)	-0.109 (-22.8)	0.335 (44.9)
R^2	0.952	0.877	0.944	0.985
D.W.	0.343	0.321	0.449	0.613
L	116.076	139.462	128.782	114.025

L (system) = 392.687.

Table 1M: NAIVE MAIDS (Cobb-Douglas)

	1:F	2:T	3:C	4:O
α_i	0.316 (145.8)	0.131 (81.4)	0.157 (75.8)	0.396 (140.3)
β_i	0.032 (4.7)	0.050 (10.1)	-0.018 (-2.8)	0.937 (107.5)
R^2	0.973	0.840	0.937	0.987
D.W.	0.547	0.240	0.488	0.790
L	125.265	135.181	126.84	116.584

L (system) = 397.171.

later in the section.) Third, R^2 are all quite high, given the simplicity of the models and noting that we are explaining shares. On the basis of R^2, there is little to choose between the models, but on the system likelihood MAIDS outperforms AIDS. Fourth, the Durbin-Watson statistics are quite low for all sectors. Surprisingly, introducing price effects into the naive models leads to quite high R^2 (of the order of .96 to .98) but leads to lower D.W. statistics. (These results have not been reported.) This is in contrast to the findings of Deaton and Muellbauer (1980) for British data, but consistent with the findings of Chung and Powell in the fitting of alternative models to the data being used. This finding is also consistent with the expectation that the exclusion of the term due to macro-aggregation will lead to serial correlation if income distribution has been changing systematically over the sample. To account for this, we introduce a number of variables into MAIDS to capture this "macro" effect. Those variables for which data was freely available, and which we would expect to be significant determinants of changes in income distribution are: the rate of inflation (I), the rate of unemployment (U) and the participation rate (P). While the theory of aggregation outlined in Sections 4 and 5 would suggest that these variables should not appear in AIDS (since $\eta=0$), these variables were also included in AIDS in order to simplify the comparison, and to allow concentration solely on the effects of considering a more regular formulation. Introduction of these variables has the desired "purging" effect, and raises the D.W. values to more acceptable levels. These variables were henceforth included in all estimation, in the form of deviations about their sample means, in order to preserve the interpretation of the ε_{ki}.

Estimates of the unconstrained model are given in Tables 2A and 2M for AIDS and MAIDS, respectively, with homogeneity imposed in Tables 3A and 3M, and with homogeneity and symmetry imposed in Tables 4A and 4M. A * by a parameter estimate denotes that this value has been derived by restriction, while a • indicates that the corresponding value was constrained by symmetry.

Use of the Stone price index as a deflator and the constraint that $\eta=1$ in MAIDS have the advantage that tests for homogeneity can be carried out equation by equation using an exact F test. The F values reported in Tables 3A and 3M imply that, at a 5% level of significance, homogeneity is rejected for two categories in the AIDS model, but for only one category in MAIDS. For both models, the category food is the major offender. A system test using the likelihood ratio statistic gives a calculated chi-square of 42.7 for AIDS and 27.49 for MAIDS, also implying rejection of homogeneity for both models.

In order to proceed with the comparison, we henceforth impose homogeneity as a maintained hypothesis, and test for symmetry. In neither model can symmetry be rejected at the 5% level, and hence we chose the results of Tables 4A and 4M for further comparison. On purely statistical grounds, the results of these Tables indicate that MAIDS outperforms AIDS for all four categories on both equation fit and the value of the D.W. statistic. The maximized system log-likelihood is substantially higher for MAIDS than for AIDS.

Of possibly greater importance than these statistical results are the economic properties of the models. First, we note that over the entire sample, the MAIDS estimates satisfy one of the sufficient conditions for regularity, that ε_{1i} and ε_{2i} be positive. Second, we check curvature conditions, by evaluating the eigenvalues of the Slutsky

Table 2A: MACRO AIDS (Unconstrained)

	1:F	2:T	3:C	4:O
α_i	0.098 (3.01)	0.152 (4.74)	0.198 (4.50)	0.552 (7.50)
β_i	-0.186 (-10.26)	-0.051 (-2.88)	-0.057 (-2.32)	0.294 (7.17)
γ_{iF}	0.125 (8.81)	-0.037 (-2.64)	0.031 (1.60)	-0.120 (-3.69)
γ_{iT}	-0.009 (-0.56)	0.043 (2.80)	-0.029 (-1.39)	-0.005 (-0.140)
γ_{iC}	0.016 (1.40)	0.014 (1.23)	0.056 (3.66)	(-0.085) (-3.35)
γ_{iO}	-0.098 (-4.17)	-0.023 (-1.00)	-0.063 (-2.00)	0.184 (3.47)
μ_{Ii}	-0.009 (-0.52)	0.035 (2.16)	0.543 (2.41)	-0.081 (-2.16)
μ_{Ui}	-0.187 (-2.35)	-0.058 (-0.74)	0.002 (0.020)	0.243 (1.35)
μ_{Pi}	0.026 (0.29)	0.111 (1.26)	0.077 (0.63)	-0.215 (-1.05)
R^2	0.997	0.964	0.984	0.995
D.W.	1.424	0.782	0.865	0.960
L	159.349	159.823	149.22	132.342

L (system) = 471.491.

Table 2M: MACRO MAIDS (Unconstrained)

	1:F	2:T	3:C	4:O
α_i	0.182 (6.77)	0.168 (6.69)	0.207 (6.26)	0.443 (10.23)
β_i	-0.057 (-1.33)	0.089 (2.23)	0.105 (1.97)	0.863 (12.44)
γ_{iF}	0.119 (8.51)	-0.041 (-3.16)	0.022 (1.27)	-0.100 (-4.42)
γ_{iT}	-0.003 (-0.21)	0.042 (3.01)	-0.033 (-1.78)	-0.006 (-0.25)
γ_{iC}	0.001 (0.11)	0.008 (0.75)	0.047 (3.41)	-0.057 (-3.11)
γ_{iO}	-0.097 (-4.22)	-0.015 (-0.68)	-0.044 (-1.55)	0.155 (4.20)
μ_{Ii}	-0.028 (-1.70)	0.028 (1.78)	0.043 (2.09)	-0.042 (-1.57)
μ_{Ui}	-0.151 (-1.98)	-0.063 (-0.89)	-0.023 (-0.24)	0.237 (1.92)
μ_{Pi}	-0.048 (-0.57)	0.113 (1.45)	0.108 (1.04)	-0.173 (-1.28)
R^2	0.997	0.969	0.987	0.998
D.W.	1.56	0.800	1.174	1.509
L	160.095	162.373	153.232	144.323

L (system) = 477.694.

Table 3A: MACRO AIDS (Homogeneity Imposed)

	1:F	2:T	3:C	4:O
α_i	0.315 (53.83)	0.130 (37.83)	0.159 (33.40)	0.397 (46.62)
β_i	-0.097 (-4.71)	-0.060 (-4.98)	-0.073 (-4.35)	0.231 (7.67)
γ_{iF}	0.151 (6.66)	-0.039 (-2.97)	0.026 (1.41)	-0.137 (-4.17)
γ_{iT}	0.001 (0.05)	0.042 (2.78)	-0.031 (-1.48)	-0.012 (-0.32)
γ_{iC}	0.006 (0.33)	0.015 (1.34)	0.058 (3.82)	-0.078 (-2.90)
γ_{iO}	-0.158* (-5.06)	-0.017* (-0.92)	-0.052* (-2.06)	0.227* (5.00)
μ_{Ii}	0.004 (0.13)	0.034 (2.12)	0.052 (2.34)	-0.090 (-2.25)
μ_{Ui}	0.262 (3.64)	-0.104 (-2.46)	-0.080 (-1.36)	-0.079 (-0.75)
μ_{Pi}	(-0.123) (-0.85)	0.127 (1.49)	0.104 (0.88)	-0.108 (-0.51)
R^2	0.990	0.963	0.983	0.994
D.W.	1.170	0.856	0.968	0.773
L	141.879	159.496	148.672	129.498
F	45.19	0.48	0.81	4.5

L (system) = 450.356 $\quad\quad$ $2(L^{(2)} - L^{(3)}) = 42.27$

Table 3M: MACRO MAIDS (Homogeneity Imposed)

	1:F	2:T	3:C	4:O
α_i	0.317 (65.96)	0.130 (39.74)	0.160 (37.3)	0.394 (71.09)
β_i	0.142 (5.91)	(0.033) (2.03)	0.035 (1.62)	0.790 (28.41)
γ_{iF}	0.136 (7.07)	-0.046 (-3.52)	0.016 (0.94)	-0.106 (-4.78)
γ_{iT}	-0.003 (-0.16)	0.042 (2.94)	-0.033 (-1.74)	-0.006 (-0.24)
γ_{iC}	-0.006 (-0.39)	0.010 (0.94)	0.050 (3.56)	-0.054 (-2.97)
γ_{iO}	-0.126* (-4.66)	-0.006* (-0.34)	-0.033* (-1.38)	0.166* (5.30)
μ_{Ii}	-0.016 (-0.70)	0.024 (1.54)	0.038 (1.86)	-0.046 (-1.74)
μ_{Ui}	0.186 (3.55)	-0.158 (-4.43)	-0.141 (-3.00)	0.113 (1.86)
μ_{Pi}	-0.082 (-0.69)	0.123 (1.53)	0.120 (1.14)	-0.161 (-1.18)
R^2	0.993	0.966	0.986	0.997
D.W.	1.226	0.919	1.307	1.330
L	148.131	160.848	151.872	143.429
F	25.56	2.32	2.06	1.34

L (system) = 463.949 $\quad 2(L^{(2)} - L^{(3)}) = 27.49 \quad \chi^2_3(.05) = 7.81$

Table 4A: MACRO AIDS (Homogeneity and Symmetry Imposed)

	1:F	2:T	3:C	4:O
α_i	0.317 (63.14)	0.130 (43.52)	0.157 (36.59)	0.397* (55.20)
β_i	-0.103 (-6.00)	-0.062 (-5.89)	-0.066 (-4.68)	0.231* (9.00)
γ_{iF}	0.160 (8.19)	-0.030 (-2.88)	0.007 (0.63)	-0.137* (-5.86)
γ_{iT}	•	0.035 (2.71)	0.006 (0.70)	-0.011* (-0.63)
γ_{iC}	•	•	0.065 (4.84)	-0.079* (-4.18)
γ_{iO}	•	•	•	0.227* (5.52)
μ_{Ii}	-0.012 (-0.51)	0.036 (2.57)	0.064 (3.34)	-0.089* (-2.77)
μ_{Ui}	0.285 (4.51)	-0.099 (-2.68)	-0.107 (-2.03)	-0.080* (-0.89)
μ_{Pi}	-0.070 (-0.56)	0.121 (1.62)	0.060 (0.56)	-0.110* (-0.62)
R^2	0.990	0.962	0.981	0.994
D.W.	1.143	0.918	0.887	0.771

L (system) = 446.771 $\qquad 2(L^{(3)} - L^{(4)}) = 7.17 \qquad \chi^2_3(.05) = 7.81$

Table 4M: MACRO MAIDS (Homogeneity and Symmetry Imposed)

	1:F	2:T	3:C	4:O
α_i	0.319 (75.32)	0.130 (45.29)	0.158 (40.64)	0.393* (83.11)
β_i	0.140 (7.78)	0.032 (2.51)	0.038 (2.41)	0.790* (32.77)
γ_{iF}	0.146 (8.68)	-0.035 (-3.54)	-0.004 (-0.37)	-0.107* (-6.30)
γ_{iT}	•	0.036 (2.92)	0.002 (0.19)	-0.003* (-0.18)
γ_{iC}	•	•	0.056 (4.62)	-0.054* (-3.75)
γ_{iO}	•	•	•	0.164* (5.46)
μ_{Ii}	-0.032 (-1.64)	0.027 (1.95)	0.050 (2.81)	-0.045* (-2.03)
μ_{Ui}	0.205 (4.45)	-0.153 (-4.99)	-0.162 (-3.97)	0.111* (2.13)
μ_{Pi}	-0.032 (-0.30)	0.117 (1.65)	0.081 (0.83)	-0.165* (-1.42)
R^2	0.992	0.965	0.984	0.997
D.W.	1.187	0.934	1.210	1.329

L (system) = 460.269 $\quad 2(L^{(3)} - L^{(4)}) = 7.36 \quad \chi^2_3(.05) = 7.81$

matrix over the sample period. For AIDS, negative semi-definiteness is violated over the entire sample period, with one of the three non-zero eigenvalues being consistently positive. Interestingly, P_1 itself is not concave over the sample period, so AIDS suffers from two possible sources of irregularity. For MAIDS, on the other hand, the Slutsky matrix is negative semi-definite over the <u>entire</u> sample period.

To investigate this regularity further, we analyze the properties of P_1 and P_2. Note that a sufficient condition for P_1 and P_2 to be concave would be that the α_i and β_i be positive (as they are) and that the matrix $[\gamma_{ij}]$ be negative semi-definite. In fact, however, $[\gamma_{ij}]$ turns out to be positive semi-definite. (This also occurred for AIDS, and perhaps illustrates the difficulties suggested by Diewert and Wales (1987) in using constraints on $[\gamma_{ij}]$ to impose regularity on a Translog.) The α_i are sufficiently positive, however, to outweigh this effect, giving a P_1 that is concave over the entire sample. Unfortunately, this is not so for P_2, and hence there will be some sufficiently high value of (c/P_1) at which the regularity of MAIDS will break down. Thus the inherent regularity of MAIDS as a general functional form is not sufficient in this example to outweigh the inherent non-regularity of a Translog price index.

Another interesting comparison of the economic characteristics of the two models is to compare the behaviour of their expenditure elasticities over the sample. Since these elasticities are essentially monotonic over the sample, Table 5 reports their values at the beginning (1955), middle (1970) and end (1986) values. Note how for AIDS the necessities (Food, Tobacco and Alcohol, Clothing) become more necessitous as real expenditure rises, while for MAIDS they become less necessitous. (Real expenditure increases by more than 60% over the

Table 5

Income Elasticities

	F	T	C	O
		AIDS		
1954	0.68	0.52	0.58	1.58
1970	0.59	0.44	0.40	1.44
1986	0.49	0.39	0.25	1.38
		MAIDS		
1954	0.44	0.25	0.24	2.00
1970	0.60	0.51	0.40	1.40
1986	0.64	0.61	0.46	1.27

sample period.) At the midpoint of the sample, the elasticities are very similar, but the behaviour of elasticities as expenditure differs from the midpoint value is radically different for the two models.

As a further comparison, the parameter estimates from Tables 4A and 4M were used as starting values to estimate the two models for Case 2 of Section 5 where the true index P_1 is used as a deflator. Results are given in Tables 6A and 6M, and it can be seen that parameter estimates are little changed. Elasticities and regularity properties are similar to the results already analyzed, and hence are not reproduced here.

On the evidence presented so far, it would appear that MAIDS has outperformed AIDS on both statistical and economic grounds. A formal test of the two models is simplified if they are each nested in GMAIDS. Parameter estimates for GMAIDS are presented in Table 7. With an estimated value of η of 2.01 and a t value of 6.06, the AIDS model is decisively rejected. Again, even with two Translog price indices and η outside the inteval $0 \leq \eta \leq 1$ (so that regularity is not assured a priori), this model exhibits regularity properties (ε_{1i}, $\varepsilon_{2i} \geq 0$, Slutsky matrix negative semi-definite) over the entire sample period, without the need for constrained estimation.

Table 6A: MACRO AIDS (Homogeneity, Symmetry, Translog Deflator)

	1:F	2:T	3:C	4:O
α_i	0.316 (59.98)	0.130 (43.04)	0.156 34.81	0.398* (49.72)
β_i	-0.099 (-5.29)	-0.063 (-5.65)	-0.062 (-3.99)	0.224* (7.63)
γ_{iF}	0.163 (7.97)	-0.029 (-2.75)	0.011 (0.88)	-0.144* (-5.59)
γ_{iT}	•	0.035 (2.65)	0.007 (0.86)	-0.013* (-0.71)
γ_{iC}	•	•	0.069 (4.85)	-0.087* (-4.19)
γ_{iO}	•	•	•	0.244* (5.33)
μ_{Ii}	-0.007 (-0.27)	0.039 (2.76)	0.068 (3.39)	-0.101* (-2.85)
μ_{Ui}	0.296 (4.33)	-0.087 (-2.25)	-0.102 (-1.80)	-0.107* (-1.05)
μ_{Pi}	-0.086 (-0.65)	0.123 (1.63)	0.045 (0.40)	-0.082* (-0.41)
R^2	0.989	0.961	0.979	0.993
D.W.	1.138	0.888	0.817	0.718

L (system) = 443.491.

Table 6M: MACRO MAIDS (Homogeneity, Symmetry, Translog Deflator)

	1:F	2:T	3:C	4:O
α_i	0.319 (71.82)	0.130 (45.79)	0.158 (38.48)	0.393* (71.74)
β_i	0.137 (6.67)	0.024 (1.99)	0.041 (2.28)	0.795* (21.32)
γ_{iF}	0.146 (8.05)	-0.036 (-3.63)	-0.002 (-0.16)	-0.108* (-5.44)
γ_{iT}	•	0.034 (2.75)	0.002 (0.23)	0.000* (-)
γ_{iC}	•	•	0.059 (4.55)	-0.059* (-3.58)
γ_{iO}	•	•	•	0.167* (4.80)
μ_{Ii}	-0.027 (-1.33)	0.029 (2.15)	0.055 (2.95)	-0.057* (-2.27)
μ_{Ui}	0.229 (4.62)	-0.138 (-4.41)	-0.147 (-3.31)	0.056* (0.93)
μ_{Pi}	-0.013 (-0.12)	0.136 (1.91)	0.087 (0.85)	-0.210* (-1.55)
R^2	0.992	0.966	0.983	0.997
D.W.	1.174	0.972	1.094	1.128

L (system) = 455.824.

Table 7: MACRO GMAIDS

	1:F	2:T	3:C	4:O
α_i	0.319 (77.99)	0.130 (45.61)	0.157 (40.28)	0.394* (85.61)
β_i	0.396 (4.35)	0.132 (3.39)	0.155 (3.45)	1.326* (7.79)
γ_{iF}	0.142 (8.53)	-0.036 (-3.62)	-0.007 (-0.69)	-0.098* (-5.76)
γ_{iT}	•	0.038 (3.01)	0.001 (0.117)	-0.003* (-0.17)
γ_{iC}	•	•	0.053 (4.30)	-0.047* (-3.19)
γ_{iO}	•	•	•	0.148* (4.83)
μ_{Ii}	-0.040 (-2.05)	0.024 (1.73)	0.045 (2.49)	-0.030* (-1.30)
μ_{Ui}	0.158 (3.37)	-0.178 (-5.83)	-0.200 (-4.80)	0.218* (3.34)
μ_{Pi}	-0.049 (-0.48)	0.106 (1.51)	0.061 (0.630)	-0.119* (-1.01)
R^2	0.993	0.965	0.984	0.998
D.W.	1.228	0.904	1.303	1.655

L (system) = 461.328 η = 2.010 (6.06)

7. CONCLUSION

In this paper we have presented a modification of the PIGLOG class of functional forms which we have termed MAIDS (Modified AIDS, since AIDS is the most widely applied member of PIGLOG). We have demonstrated the improved regularity properties of MAIDS and have also outlined an approach to dealing with the aggregation problem for this model. In a similar way, it is possible to modify and regularise PIGL.

In empirical work we have compared MAIDS to AIDS. An empirical application has demonstrated that the theoretical regularity of MAIDS is borne out in practice, even in the case of the use of Translog price deflators which are chosen to make MAIDS as similar to AIDS as possible. Finally, the specification of an even more general model, GMAIDS, allows MAIDS and AIDS to be tested as nested alternatives. This more general model is also found to be regular over the entire sample period.

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