ISSN 0729-0683

ISBN 0 86746 711 8

MONASH UNIVERSITY

TESTING FOR FOURTH-ORDER AUTOCORRELATION IN REGRESSION DISTURBANCES WHEN FIRST-ORDER AUTOCORRELATION IS PRESENT

Maxwell L. King

Working Paper No. 8/88

July 1988

DEPARTMENT OF ECONOMETRICS

ISSN 0729-0683

ISBN 0 86746 711 8

TESTING FOR FOURTH ORDER AUTOCORRELATION IN REGRESSION DISTURBANCES WHEN FIRST-ORDER AUTOCORRELATION IS PRESENT

Maxwell L. King

Working Paper No. 8/88

July, 1988

DEPARTMENT OF ECONOMETRICS, FACULTY OF ECONOMICS AND POLITICS

MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

TESTING FOR FOURTH-ORDER AUTOCORRELATION

IN REGRESSION DISTURBANCES WHEN

FIRST-ORDER AUTOCORRELATION IS PRESENT*

by

Maxwell L. King

Department of Econometrics

Monash University

Clayton, Victoria 3168

Australia

ABSTRACT

This paper considers the problem of testing the null hypothesis of first-order autoregressive disturbances in the linear regression model against the alternative that the disturbances follow a joint first-order, simple-fourth-order autoregressive process. The class of approximate point optimal invariant tests are discussed and rules are given for choosing an appropriate member from this class of tests. The beneficial nature of these rules are illustrated by a limited empirical power comparison which shows the recommended test has good small sample power properties.

1. INTRODUCTION

For some time now, econometricians have recognized the possibility that quarterly regression models may possess fourth-order autocorrelation because of seasonal effects. This has largely been through the work of Thomas and Wallis (1971), Wallis (1972) and others. Tests of the null hypothesis of independent disturbances against the alternative of simple fourth-order autoregressive (AR(4)) disturbances have been proposed by a number of authors. They include the fourth-order analogues of the Durbin-Watson test proposed by Wallis (1972) and Vinod (1973) and the point optimal invariant test suggested by King (1984). A recent review of these and other tests may be found in King (1987a, section 11.2).

The usual reason for including a disturbance term in the regression is to account for the effects of omitted or unobservable regressors, errors in the measurement of the dependent variable, arbitrary human behaviour and functional approximations. In the context of a quarterly regression model, it is not clear that these effects will give rise solely to fourth-order autocorrelation. For example, one would expect the omission of relevant variables with seasonal components to lead to both first-order and fourth-order effects in the disturbances. Thus the presence of first-order autocorrelation in a quarterly regression model would be a good reason to suspect additional autocorrelation of a seasonal nature. Having discovered first-order autocorrelation and decided to model it as a first-order autoregressive (AR(1)) process, it makes sense to test the resultant regression for an additional simple AR(4) component. This requires a test whose null hypothesis is a linear regression with AR(1) disturbances and whose alternative is that the disturbances are generated by a joint AR(1) and simple AR(4) process. Such a test is proposed below.

The method of test construction follows that suggested by King (1987b) who gives a method of discovering whether point optimal tests can be based on simple likelihood ratios. Point optimal tests are tests which have optimal power at a predetermined point in the alternative hypothesis parameter space. By a simple likelihood ratio, we mean the likelihood ratio of two simple hypotheses. If simple likelihood ratios fail to produce a point optimal test for a particular testing problem, King notes that they could still be used to produce "approximate" point optimal tests. One of the aims of this paper is to explore this latter option further.

After reduction through invariance, the testing problem under study becomes one of testing against a non-zero parameter (the AR(4) parameter) in the presence of a nuisance parameter (the AR(1) parameter). Simple likelihood ratios, therefore, are a function of three parameters; the AR(1) parameter, ρ_{10}, under the null hypothesis and the AR(1) and AR(4) parameters, ρ_{11} and ρ_{41}, respectively, under the alternative hypothesis. Each set of ρ_{10}, ρ_{11} and ρ_{41} values corresponds to a simple likelihood ratio, which, together with an appropriate critical value, could be used as a test. A central question is - how should values for ρ_{10}, ρ_{11} and ρ_{41} be chosen? This paper suggests the following solution. For given ρ_{11} and ρ_{41} values, ρ_1^1 and ρ_4^1 say, ρ_{10} should be chosen to make the resultant test as close to optimal at $(\rho_{11},\rho_{41})' = (\rho_1^1, \rho_4^1)'$ as possible. For a given ρ_{41} value, ρ_4^1 say, ρ_{11} is chosen to maximize the minimum power on the subspace $\rho_{41} = \rho_4^1$ in the alternative hypothesis parameter space. Finally, the ρ_{41} value is chosen so that the maximized minimum power has a value of one half. This represents a new approach in the treatment of nuisance parameters in hypothesis testing. Those that cannot be eliminated through invariance are used to advantage in the final choice of test statistic.

As we shall see, the proposed method for choosing ρ_{10}, ρ_{11} and ρ_{41} is very computer intensive. Over the past decade, econometricians have seen a rapid expansion in the availability of computer time. With further improvements in computer software and hardware expected, there is no reason why this expansion will not continue well into the next decade. It is quite possible that in the future, computational time will be a relatively minor issue for econometricians.

The approach used in this paper obviously can be applied to a number of other testing problems in which one has prior knowledge of the signs of parameters. Examples include testing regression disturbances in the presence of autocorrelation, heteroscedasticity, a random regression coefficient or an error component, for higher-order effects or combinations of these effects.

The organisation of this paper is as follows. In the following section, the testing problem is defined and Point Optimal Invariant (POI) tests are discussed. Because typically, one is unable to construct such tests for our particular testing problem using current methods, the class of "approximate" POI tests is introduced. Rules for choosing an appropriate member of this class of tests are given. In section 3, the benefits of using these rules are illustrated by a limited empirical power comparison involving three choices of test. An example of the application of the test is given in section 4 and some concluding remarks are made in the final section.

2. THEORY

Consider the linear regression model

$$y = X\beta + u, \tag{1}$$

where y is nx1, X is an nxk matrix that is assumed independent of u and of rank $k < n$, β is a kx1 vector of parameters and u is the nx1 disturbance vector. It is assumed that the model is based on n quarterly time-series observations and that, as a consequence, u is generated by the stationary seasonal-autoregressive process

$$(1 - \rho_1 L)(1 - \rho_4 L^4)\, u_t = e_t, \tag{2}$$

where

$$0 \leq \rho_4 < 1 \tag{3}$$

and

$$0 \leq \rho_1 < 1 \tag{4}$$

are unknown parameters, L is the lag operator such that $Lu_t = u_{t-1}$ and $e = (e_1,\ldots,e_n)' \sim N(0,\sigma^2 I_n)$. Note that (2) is the AR(5) process,

$$u_t = \rho_1 u_{t-1} + \rho_4 u_{t-4} - \rho_1 \rho_4 u_{t-5} + e_t.$$

Our aim is to test $H_0 : \rho_4 = 0$ against $H_a : \rho_4 > 0$.

The restriction (3) reflects the view that typically one wishes to test for extra positive correlation between the disturbances of the same quarters of adjacent years. If required, an analogous test can be constructed with the restriction $-1 < \rho_4 \leq 0$ in mind. An example of such a situation is given by Wallis (1972, p. 632). As we shall see, the smaller the range of ρ_1, the more powerful the final test. We will proceed under (4) because it is typically what is assumed. If need be, it could be replaced with $-1 < \rho_1 \leq 0$ or $-1 < \rho_1 < 1$, without loss of generality. For the purpose of test construction, we will find it convenient for the range of ρ_1 to be a closed set. Because of this, we will use $0 \leqslant \rho_1 \leqslant 0.99999$ to approximate (4) in the remainder of this paper.

Let $G(\rho,p)$ be the pxp bi-diagonal matrix

$$G(\rho,p) = \begin{bmatrix} \sqrt{1-\rho^2} & 0 & 0 & \cdots & & 0 \\ -\rho & 1 & 0 & & & 0 \\ 0 & -\rho & 1 & \ddots & & \vdots \\ \vdots & & & \ddots & & \vdots \\ & & & & 1 & 0 \\ 0 & 0 & \cdots & & -\rho & 1 \end{bmatrix} .$$

Set $G_1(\rho) = G(\rho,n)$ and, when n is an integer multiple of 4, let

$$G_4(\rho) = G(\rho,n/4) \otimes I_4 . \qquad (5)$$

If n is not an integer multiple of 4, $G_4(\rho)$ is formed by first rounding n up to the next integer multiple, applying (5) and then deleting the appropriate number of bottom rows and right-hand columns to convert back to an nxn matrix.

The stationary process (2) is observationally equivalent to

$$G_1(\rho_1)\ G_4(\rho_4)\ u = e$$

so that

$$\begin{aligned} Var(u) &= \sigma^2 G_4(\rho_4)^{-1} G_1(\rho_1)^{-1} G_1(\rho_1)'^{-1} G_4(\rho_4)'^{-1} \\ &= \sigma^2 \Omega(\rho_1,\rho_4), \end{aligned}$$

say.

Observe that this testing problem is invariant with respect to transformations of the form

$$y \to \eta_0 y + X\eta,$$

where η_0 is a positive scalar and η is a kx1 vector. The mx1 vector

$$v = Pz/(z'P'Pz)^{1/2}$$

is a maximal invariant under this group of transformations where $m = n-k$, $z = My$ is the ordinary least squares (OLS) residual vector from (1), $M = I_n - X(X'X)^{-1}X'$ and P is an mxn matrix such that $PP' = I_m$ and $P'P = M$. The probability density function of v under (1) and (2) can be shown to be (King (1980))

$$f(v, \rho_1, \rho_4)dv \tag{6}$$
$$= 1/2\ \Gamma(m/2)\ \pi^{-m/2}|P\Omega(\rho_1, \rho_4)P'|^{-1/2}(v'(P\Omega(\rho_1, \rho_4)P')^{-1}v)^{-m/2}dv$$

where dv denotes the uniform measure on the unit sphere. Note that the only unknown parameters in (6) are ρ_1 and ρ_4.

Thus our testing problem simplifies to one of testing

H_0 : v has density $f(v, \rho_1, 0)$, $0 \leq \rho_1 \leq 0.99999$

against

H_a : v has density $f(v, \rho_1, \rho_4)$, $0 \leq \rho_1 \leq 0.99999$, $0 < \rho_4 < 1$.

Consider first the problem of testing the simple null hypothesis

$$H_0^1 : (\rho_1, \rho_4)' = (\rho_{10}, 0)'$$

against the simple alternative hypothesis

$$H_a^1 : (\rho_1, \rho_4)' = (\rho_{11}, \rho_{41})',$$

where $0 \leqslant \rho_{10} \leqslant 0.99999$, $0 \leqslant \rho_{11} \leqslant 0.99999$ and $0 < \rho_{41} < 1$ are known and fixed. Let $\Omega_o = \Omega(\rho_{10}, 0)$ and $\Omega_1 = \Omega(\rho_{11}, \rho_{41})$. The Neyman-Pearson lemma implies that a Most Powerful (MP) test within the class of invariant tests can be based on critical regions of the form

$$s(\rho_{10}, \rho_{11}, \rho_{41}) = \frac{v'(P\Omega_1 P')^{-1} v}{v'(P\Omega_o P')^{-1} v} < c$$

where c is an appropriate critical value. One can show (see, for example, King (1980, lemma 2)) that $s(\rho_{10}, \rho_{11}, \rho_{41})$ can also be written as

$$s(\rho_{10}, \rho_{11}, \rho_{41}) = \frac{\hat{u}' \Omega_1^{-1} \hat{u}}{\tilde{u}' \Omega_o^{-1} \tilde{u}} \qquad (7)$$

where $\hat{u}$ and $\tilde{u}$ are the generalized least squares residual vectors assuming disturbance covariance matrices Ω_1 and Ω_o, respectively. It is worth noting that the numerator and denominator of (7) can be computed as the sum of squared ordinary least squares residuals from

$$G_1(\rho_{11})G_4(\rho_{41})y = G_1(\rho_{11})G_4(\rho_{41})X\beta + G_1(\rho_{11})G_4(\rho_{41})u$$

and

$$G_1(\rho_{10})y = G_1(\rho_{10})X\beta + G_1(\rho_{10})u,$$

respectively. The statistic $s(\rho_{10}, \rho_{11}, \rho_{41})$ can also be written as

$$s(\rho_{10}, \rho_{11}, \rho_{41}) = \frac{u'\Delta_1 u}{u'\Delta_o u}$$

where

$$\Delta_i = \Omega_i^{-1} - \Omega_i^{-1}X(X'\Omega_i^{-1}X)^{-1}X'\Omega_i^{-1}$$

for i = 0,1. It is, therefore, a ratio of quadratic forms in normal variables and hence, like for the Durbin-Watson statistic, probabilities of the form

$$\Pr[s(\rho_{10},\rho_{11},\rho_{41}) < c \mid u \sim N(0,\Sigma)]$$

$$= \Pr[u'(\Delta_1 - c\Delta_o)u < c \mid u \sim N(0,\Sigma)]$$

$$= \Pr[(u\Sigma^{-1/2})'(\Sigma^{1/2})'(\Delta_1 - c\Delta_o)\Sigma^{1/2}(\Sigma^{-1/2}u) < c \mid u \sim N(0,\Sigma)] \qquad (8)$$

can be found by computing

$$\Pr[\sum_{i=1}^{n} \lambda_i \xi_i^2 < 0] \qquad (9)$$

where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of

$$(\Sigma^{1/2})'(\Delta_1 - c\Delta_o)\Sigma^{1/2} \qquad (10)$$

and $\xi_1^2,\ldots,\xi_n^2$ are independent chi-squared random variables each with one degree of freedom. Once $\lambda_1,\ldots,\lambda_n$ have been calculated using standard computer packages[1], (9) can be evaluated using either Koerts and Abrahamse's (1969) FQUAD subroutine, Farebrother's (1980) PAN procedure or Davies' (1980) algorithm.

Because the test procedure proposed below requires repeated evaluation of (8), it is helpful to consider efficient methods of computing (8), particularly when n is large. The $O(n^3)$ arithmetic operations needed to compute the eigenvalues of the nxn matrix, (10), is the main computational bottleneck.[2] Alternative methods of computing (8) without first calculating eigenvalues have been suggested by Palm and Sneek (1984) and Shively, Ansley and Kohn (1987). Palm and Sneek's approach involves using Householder transformations to tridiagonalize (10). Shively et al. proposed the use of a modified Kalman filter to calculate (8) in O(n) operations. For large sample sizes, this should result in considerable computational savings over eigenvalue methods.

As King (1987b) notes, if the critical value c is found such that

$$\Pr[s(\rho_{10},\rho_{11},\rho_{41}) < c \mid u \sim N(0,\Omega_o)] = \alpha,$$

where α is the desired significance level, one has a test of H_o^1 against the more general alternative, H_a, that is MP invariant (MPI) in the neighbourhood of $(\rho_1,\rho_4)' = (\rho_{11},\rho_{41})'$. For our problem (of testing H_o against H_a), however, the use of $s(\rho_{10},\rho_{11},\rho_{41})$ generally will not result in a test with optimal power at $(\rho_1,\rho_4)' = (\rho_{11},\rho_{41})'$. This is because when testing H_o, the critical value is found by solving

$$\sup_{0<\rho_1<0.99999} \Pr[s(\rho_{10},\rho_{11},\rho_{41}) < c^* \mid u \sim N(0,\Omega(\rho_1,0)] = \alpha$$

for c^*. In general, $c < c^*$ so the two critical regions are different. As the range of ρ_1 is closed,

$$\Pr[s(\rho_{10},\rho_{11},\rho_{41}) < c^* \mid u \sim N(0,\Omega(\rho_1,0)] = \alpha$$

is true for at least one ρ_1 value. If ρ_{10} can be chosen to be such a value then $c = c^*$ and the test based on $s(\rho_{10},\rho_{11},\rho_{41})$ is MPI in the neighbourhood of $(\rho_1,\rho_4)' = (\rho_{11},\rho_{41})'$. It is, therefore, a POI test.

Unfortunately, there is no reason why such a ρ_{10} value should exist. Furthermore, it may exist for some combinations of X, $(\rho_{11},\rho_{41})'$ and α values but not for others. Numerical experimentation suggested that, generally, such a ρ_{10} value does not exist. Any exceptions were found to be rare and involved what might be regarded as extreme values of $(\rho_{11},\rho_{41})'$.

Therefore, we will turn our attention to the class of approximately POI tests discussed by King (1987b). For our problem, they are based on $s(\rho_{10},\rho_{11},\rho_{41})$ with c* determined by

$$\sup_{0\leq\rho_1\leq 0.99999} \Pr[s(\rho_{10},\rho_{11},\rho_{41}) < c^* \mid u \sim N(0,\Omega(\rho_1,0))] = \alpha.$$

Such a test would be close to POI if

$$\alpha - \Pr[s(\rho_{10},\rho_{11},\rho_{41}) < c^* \mid u \sim N(0,\Omega(\rho_{10},0))] \qquad (11)$$

is close to zero. This is because if we modify the H_o distributions to make (11) equal to zero, then, for the new testing problem that results, our test is POI[3]. In this sense, the test may be regarded as approximately POI, particularly if ρ_{10} is chosen to minimize (11).

Again, numerical experimentation was used to find ρ_{10} values which minimize (11). It became clear, at least for our chosen X matrices, that when $\rho_{10} = 0.99999$,

$$\Pr[s(\rho_{10},\rho_{11},\rho_{41}) < c \mid u \sim N(0,\Omega(\rho_1,0))] \qquad (12)$$

is a monotonic decreasing function of ρ_1 and hence the appropriate critical value, c^*, is determined at $\rho_1 = 0$. Consequently, (11) is relatively large for this choice of ρ_{10}. As ρ_{10} is moved closer to zero, the nature of (12) as a function of ρ_1 changes. It develops a turning point and, for values of ρ_1 close to 0.99999, is a monotonic increasing function of ρ_1. It, therefore, has local maxima at the two end points of the range of ρ_1. Provided $\rho_1 = 0$ remains the global maximum, it determines the appropriate c^* value. Our experiments showed that (11) is reduced by moving ρ_{10} closer to zero provided $\rho_1 = 0$ remains the global maximum. Once $\rho_1 = 0.99999$ becomes the global maximum, it determines the c^* values. We found that moving ρ_{10} closer to zero in such circumstances results in an increase in (11).

Our recommended choice of ρ_{10} is that value which results in global maxima at both $\rho_1 = 0$ and $\rho_1 = 0.99999$. In other words, that value of ρ_{10} that causes

$$\Pr[s(\rho_{10}, \rho_{11}, \rho_{41}) < c^* \mid u \sim N(0, \Omega(\rho_1, 0))] \tag{13}$$

to be equal to α at both $\rho_1 = 0.0$ and $\rho_1 = 0.99999$. We will use ρ_{10}^* to denote this value. Our experiments showed that any movement from ρ_{10}^* resulted in an increase in (11). On a few rare occasions, $\rho_1 = 0$ was always a global maximum, in which case setting $\rho_{10} = 0$ yields a true POI test. In such circumstances, ρ_{10}^* will denote the required ρ_{10} value.

Unfortunately, it is difficult to show analytically that choosing ρ_{10} such that (13) equals α at both $\rho_1 = 0$ and $\rho_1 = 0.99999$ always minimizes (11). A complicating factor is that for those combinations of X, ρ_{11} and ρ_{41} for which a true POI test exists, such a ρ_{10} value cannot be found.

The test statistic $s(\rho_{10},\rho_{11},\rho_{41})$ is a function of ρ_{10}, ρ_{11} and ρ_{41}. In order to use it as a test of H_o against H_a, values of these three parameters need to be specified. We recommend that ρ_{10} be set equal to $\overset{*}{\rho}_{10}$. The question then remains: what values should ρ_{11} and ρ_{41} take?

In the spirit of the point optimal testing approach (see Fraser, Guttman and Styan (1976), Franzini and Harvey (1983), King (1983, 1984, 1985a, 1987b) and Evans and King (1985)), we begin by focussing on a particular value of ρ_4, say ρ_4^1. It is assumed that high power is regarded as being important at this ρ_4 value, although ρ_4^1 need not be the only value for which this is the case. Given that $\rho_{10} = \overset{*}{\rho}_{10}$ and $\rho_{41} = \rho_4^1$, the power of our test at $\rho_4 = \rho_4^1$ is

$$\pi(\rho_{11},\rho_1,\rho_4^1) = \Pr[s(\overset{*}{\rho}_{10},\rho_{11},\rho_4^1) < c^* \mid u \sim N(0,\Omega(\rho_1,\rho_4^1))]$$

which, for any choice of ρ_{11} and ρ_4^1, is a function of ρ_1 and over the range $0 \leq \rho_1 \leq 0.99999$ has a minimum. This minimum varies with the choice of ρ_{11} value. It seems sensible to want this minimum to be as large as possible. For this reason we suggest choosing ρ_{11} to maximize

$$\min_{0\leq\rho_1\leq 0.99999} \pi(\rho_{11},\rho_1,\rho_4^1)$$

and denote this ρ_{11} value (given $\rho_{41} = \rho_4^1$) by $\overset{*}{\rho}_{11}$.

Finally, there is the choice of ρ_{41} value. This value, along with $\overset{*}{\rho}_{11}$, determines the point at which the test is attempting to optimize power. Rather than select this point directly, we will adopt an indirect approach that is finding favour in the literature (see for example, Davies

(1969), King (1985b), Nyblom (1986) and Shively (1987, 1988)). It involves deciding the level of power that would be most desirable to optimize at. There seems little point in optimizing power when it is already one or when it is close to α. Rather, it seems sensible to optimize power at some middle level. For typical values of α of 0.10 or less, we suggest optimizing power when it is 0.5. In other words, ρ_4^1 is chosen so that the maximized minimum of $\pi(\rho_{11},\rho_1,\rho_4^1)$ is 0.5 or alternatively as that value of ρ_4^1 for which

$$\Pr[s(\rho_{10}^*,\rho_{11}^*,\rho_4^1) < c^* \mid u \sim N(0,\Omega(\rho_{11}^*,\rho_4^1))] = 0.5 .$$

This choice of ρ_{41} will be denoted ρ_{41}^*.

3. EMPIRICAL POWER CALCULATIONS

The sizes and powers of three versions of the $s(\rho_{10},\rho_{11},\rho_{41})$ test were computed in order to illustrate the effects of our choice of ρ_{10}, ρ_{11} and ρ_{41} values as well as to assess the power properties of the recommended test. The three tests are based on $s(0.5,0.5,0.5)$, $s(\rho_{10}^*,0.5,0.5)$ and the recommended test statistic, $s(\rho_{10}^*,\rho_{11}^*,\rho_{41}^*)$. The $s(0.5,0.5,0.5)$ test was chosen to illustrate arbitrary but "representative" choices of ρ_{10}, ρ_{11} and ρ_{41} while the $s(\rho_{10}^*,0.5,0.5)$ test seeks to improve on this test by attempting to maximize power at the arbitrary point $(\rho_1,\rho_4)' = (0.5,0.5)'$. Sizes and powers were computed at ρ_1 = 0.0, 0.2, 0.4, 0.6, 0.8, 0.99999 when ρ_4 = 0.0, 0.2, 0.4, 0.6, 0.8 for the following two X matrices with n = 20, 60:

X1 (nx3). A constant dummy, the quarterly Australian Consumer Price Index commencing 1959(1) and the same index lagged one quarter as regressors.

X2 (nx4). A constant dummy, quarterly Australian private capital movements, Government capital movements and retail trade commencing 1968(1) as regressors.

The Australian Consumer Price Index has a small seasonal pattern while the retail trade series shows much stronger seasonal behaviour. The two capital movement series exhibit some large fluctuations and are strongly seasonal with two seasonal peaks per year.

Exact critical values at the five percent level were computed using the eigenvalue method and a modified version of Koerts and Abrahamse's (1969) FQUAD subroutine with maximum integration and truncation errors of 10^{-6} to evaluate (9). Probabilities of rejecting H_0 when $u \sim N(0,\Omega(\rho_1,\rho_4))$ using critical value c^* were computed by setting $\Sigma = \Omega(\rho_1,\rho_4)$ and $c = c^*$ in (8), (9) and (10). For any choice of ρ_{11} and ρ_{41} values, the associated ρ_{10}^* and exact critical values, c^*, were found as follows:

(i) Guess a likely value of ρ_{10}^*, say $\rho_{10}^* = 0.5$.

(ii) Use the secant method to solve

$$\Pr[s(\rho_{10}^*,\rho_{11},\rho_{41}) < c^* \mid u \sim N(0,\Omega(0,0))] = 0.05$$

for c^*.

(iii) Evaluate

$$\Pr[s(\rho_{10}^*,\rho_{11},\rho_{41}) < c^* \mid u \sim N(0,\Omega(0,0.99999))] \ . \tag{14}$$

If (14) equals 0.05, then the desired ρ_{10}^* value has been found. If (14) is less (greater) than 0.05, make ρ_{10}^* smaller (larger)[4] and repeat (ii) and (iii).

The calculated sizes and powers for X1 are presented in tables 1 and 2 while the corresponding results for X2 may be found in tables 3 and 4.

Except at $\rho_1 = 0.0$ and $\rho_1 = 0.99999$, all sizes are less than the nominal size of 0.05 and they decrease as the sample increases. This decline is most noticeable for the s(0.5,0.5,0.5) test whose size for all but one ρ_1 value is below 0.001 when n = 60. At each value of ρ_1 (excluding endpoints), the $s(\rho_{10}^*,\rho_{11}^*,\rho_{41}^*)$ size is always closest to the nominal significance level with the $s(\rho_{10}^*,0.5,0.5)$ size always being the next closest.

Because all tests have sizes less than 0.05 at interior ρ_1 values, continuity implies that their powers at some points under H_a will be less than their nominal sizes. Calculated powers below 0.05 occurred most when n = 60, particularly for the s(0.5,0.5,0.5) test at $\rho_4 =$ 0.2 and 0.4. Such low powers only occurred occasionally for the $s(\rho_{10}^*,0.5,0.5)$ test and never for the $s(\rho_{10}^*,\rho_{11}^*,\rho_{41}^*)$ test.

An unusual feature of the s(0.5,0.5,0.5) test is that for $0 \leq \rho_1 \leq 0.8$ and $0 < \rho_4 \leq 0.6$, its power declines as the sample size increases. $s(\rho_{10}^*,0.5,0.5)$ exhibits this property only at $\rho_1 = 0.6$, 0.8 and $\rho_4 = 0.2$, while the powers of the $s(\rho_{10}^*,\rho_{11}^*,\rho_{41}^*)$ test always increase as the sample size increases.

The power of the $s(\rho_{10}^*,0.5,0.5)$ test almost always dominates that of the s(0.5,0.5,0.5) test and when n = 60, the differences in power can be extremely large, ranging up to 0.799 for X1 and 0.821 for X2. The only exceptions are X2 powers at $\rho_1 = 0.99999$ when n = 60.

Because the $s(\rho_{10}^*,0.5,0.5)$ and $s(\rho_{10}^*,\rho_{11}^*,\rho_{41}^*)$ tests only differ in their choice of point at which they seek to optimize power, it is not surprising that their power curves cross with the latter test having greater power at higher ρ_1 values. For the 20x3 X1 matrix, there is little difference between the two power curves. For X2 with n = 20, the power differences in favour of the $s(\rho_{10}^*,0.5,0.5)$ test at $\rho_1 = 0, 0.2$ are small relative to those differences in favour of $s(\rho_{10}^*,\rho_{11}^*,\rho_{41}^*)$ at $\rho_1 = 0.6, 0.8, 0.99999$. When n = 60, the power advantage of the latter test is even more marked. It is only less powerful at $\rho_1 = 0$ and never by more than 0.05, while for $\rho_1 \geq 0.2$, it dominates the $s(\rho_{10}^*,0.5,0.5)$ test with power differences ranging up to 0.341 and 0.528 for X1 and X2, respectively. Somewhat surprisingly, the $s(\rho_{10}^*,\rho_{11}^*,\rho_{41}^*)$ test is more powerful than the $s(\rho_{10}^*,0.5,0.5)$ test at $(\rho_1,\rho_4)' = (0.5, 0.5)'$ which is where the latter test attempts to optimize power. This can be explained by noting that (11) is quite large for this test and hence it may be rather poor at optimizing power.

On the evidence available, it does appear that the recommended test based on $s(\rho_{10}^*,\rho_{11}^*,\rho_{41}^*)$ has better power properties than the $s(\rho_{10}^*,0.5,0.5)$ test which in turn must be preferred to the s(0.5,0.5,0.5) test. The superiority of the recommended test becomes much more obvious as the sample size increases.

Further power comparisons were made in order to investigate the sensitivity of the choice of $(\rho_1,\rho_4)'$ point at which power is approximately optimized. This is of interest because the accuracy with which ρ_{11}^* and ρ_{41}^* need to be determined, dictates the degree of computational effort required. The small differences between the power curves of the $s(\rho_{10}^*, 0.5, 0.5)$ and $s(\rho_{10}^*, \rho_{11}^*, \rho_{41}^*)$ tests for the nx3 X1 matrix when $\rho_{11}^* = 0.8010$ and $\rho_{41}^* = 0.5460$ indicates that, at least for this design matrix, ρ_{11}^* need not be determined accurately.

For the X1 design matrices with n=20 and 60, the powers of four further $s(\rho_{10}^*, \rho_{11}, \rho_{41})$ tests were computed with ρ_{11} and ρ_{41} set to the ρ_{11}^* and ρ_{41}^* values with five percent errors either added or subtracted. The calculated power curves proved to be very similar to those of the $s(\rho_{10}^*, \rho_{11}^*, \rho_{41}^*)$ test, particularly at points away from boundary values of ρ_1 and when n=60. The largest differences in power were less than 0.015 for n=20 and 0.010 for n=60 and occurred at $\rho_1 = 0.99999$. This suggests that ρ_{11}^* and ρ_{41}^* need not be calculated with great accuracy.

4. AN EXAMPLE

This section considers the application of the $s(\rho_{10}^*, \rho_{11}^*, \rho_{41}^*)$ test to a quarterly regression model reported by Woglom (1981) that explains the value of stocks in the U.S.A. This model, whose disturbances are assumed to follow an AR(1) process, was used by Kramer and Sonnberger (1986) to illustrate the use of diagnostic testing in practice. Using Woglom's notation, the model is

$$\log VST_t = \beta_1 + \beta_2 \log MB_t + \beta_3 \log(1 + RTPD_t) + \beta_4 \log(1 + RTPS_t) + \beta_5 \log XBC_t + u_t,$$

$$u_t = \rho u_{t-1} + \varepsilon_t,$$

where VST_t is the current value of stocks, MB_t denotes the adjusted monetary base, $RTPD_t$ and $RTPS_t$ represent current dollar rents on producers' durables and structures, respectively, XBC_t is productive capacity for business output and u_t is the disturbance term. Quarterly data for the period 1960(1) to 1977(3) (71 observations) are given by Kramer and Sonnberger (1986, Table A.5), although in order to reproduce their results, the values given for $RTPD_t$ and $RTPS_t$ need to be divided by 100.

It was decided to apply the test at the 5% significance level. The first task was to find ρ_{10}^*, ρ_{11}^* and ρ_{41}^*. This was done using the following hierarchical iterative process.

(i) Guess likely values[5] of ρ_{11}^* and ρ_{41}^*.

(ii) For these values of ρ_{11} and ρ_{41}, compute ρ_{10}^* and the associated critical value c^* as outlined in section 3.

(iii) By exploring the power curve of the resultant $s(\rho_{10}^*, \rho_{11}, \rho_{41})$ test at different values of ρ_1 when $\rho_4 = \rho_{41}$, find the ρ_1 value which gives minimum power.

(iv) The ρ_{11} value in (ii) is adjusted by moving it closer to this ρ_1 value with the aim of increasing the minimum power.

(v) Steps (ii), (iii) and (iv) are repeated until the minimum power cannot be increased further. At this point, ρ_{41} is adjusted to make this maximum minimum power value closer to 0.5. If it is less (greater) than 0.5, ρ_{41} is increased (decreased) and steps (ii) to (v) are repeated.

(vi) The iterative procedure is complete when the maximum minimum power value at $\rho_4 = \rho_{41}$ is equal to 0.5. The resultant ρ_{11} and ρ_{41} values are the required ρ_{11}^* and ρ_{41}^* values while the ρ_{10}^* and c^* values come from the most recent evaluation of step (ii).

For our particular regression, this process gave $\rho_{10}^* = 0.8952021$, $\rho_{11}^* = 0.93$, $\rho_{41}^* = 0.28566$ and $c^* = 0.9709164$. A by-product of the calculations is that we gain some knowledge of the power function of the chosen test. We know, for example, that the power at $\rho_1 = 0.717$ and $\rho_4 = 0.28566$ is 0.5 and that this is the minimum power value when $\rho_4 = 0.28566$. The value of our chosen test statistic, s(0.8952021,0.93,0.28566), was computed as a ratio of sums of squared ordinary least squares residuals from transformed regression equations using the TSP computer package. Its calculated value was found to be 1.13854 which is greater than c^* so that $H_0 : \rho_4 = 0$ is not rejected at the five percent significance level.

5. CONCLUDING REMARKS

This paper discusses the use of "approximate" POI tests when testing for simple fourth-order autocorrelation in linear regression disturbances when an AR(1) process is also present. The empirical power comparison shows that it is essential to have a sensible set of rules for choosing the appropriate member from this class of tests. The rules suggested above give rise to a new approach to the treatment of nuisance parameters in hypothesis testing. Those that cannot be eliminated through invariance are used to advantage in the final choice of test statistic. Unfortunately, these rules need to be applied individually to each regression model and this requires a lot of computation. With continuing improvements in computer hardware and software, computation time is rapidly becoming less of an issue in econometrics. Also, knowledge of the test's power function is a valuable by-product of these computations.

The approach used in this paper can be applied to a number of other testing problems. Obvious examples include testing regression coefficients or regression disturbances in the presence of autocorrelation, heteroscedsticity, an error component or a random regression coefficient. Some of these are the subject of current research.

REFERENCES

Bowdler, H., R.S. Martin, C. Reinsch and J.H. Wilkinson, 1968, The QR and QL algorithms for symmetric matrices, Numerische Mathematik 11, 293-306.

Davies, R.B., 1969, Beta-optimal tests and an application to the summary evaluation of experiments, Journal of the Royal Statistical Society B 31, 524-538.

Davies, R.B., 1980, Algorithm AS155: The distribution of a linear combination of χ^2 random variables, Applied Statistics 29, 323-333.

Durbin, J. and G.S. Watson, 1950, Testing for serial correlation in least squares regression I, Biometrika 37, 409-428.

Evans, M.A. and M.L. King, 1985, A point optimal test for heteroscedastic disturbances, Journal of Econometrics 27, 163-178.

Farebrother, R.W., 1980, Algorithm AS153: Pan's procedure for the tail probabilities of the Durbin-Watson statistic, Applied Statistics 29, 224-227, and 30, 189.

Franzini, L. and A.C. Harvey, 1983, Testing for deterministic trend and seasonal components in time series models, Biometrika 70, 673-682.

Fraser, D.A.S., I. Guttman and G.P.H. Styan, 1976, Serial correlation and distributions on the sphere, Communications in Statistics A5, 97-118.

King, M.L., 1980, Robust tests for spherical symmetry and their application to least squares regression, Annals of Statistics 8, 1265-1271.

King, M.L., 1983, Testing for moving average regression disturbances, Australian Journal of Statistics 25, 23-34.

King, M.L., 1984, A new test for fourth-order autoregressive disturbances, Journal of Econometrics 24, 269-277.

King, M.L., 1985a, A point optimal test for autoregressive disturbances, Journal of Econometrics 27, 21-37.

King, M.L., 1985b, A point optimal test for moving average regression disturbances, Econometric Theory 1, 211-222.

King, M.L., 1987a, Testing for autocorrelation in linear regression models: A survey, in M.L. King and D.E.A. Giles, (eds.) Specification analysis in the linear model (Routledge and Kegan Paul, London) 19-73.

King, M.L., 1987b, Towards a theory of point-optimal testing, Econometric Reviews, forthcoming.

Koerts, J. and A.P.J. Abrahamse, 1969, On the theory and application of the general linear model (Rotterdam University Press, Rotterdam).

Kramer, W. and H. Sonnberger, 1986, The linear regression model under test (Physica-Verlag : Heidelberg).

Martin, R.S., C. Reinsch and J.H. Wilkinson, 1968, Householder's tridiagonalization of a symmetric matrix, Numerische Mathematik 11, 181-195.

Nyblom, J., 1986, Testing for deterministic linear trend in time series, Journal of the American Statistical Association 81, 545-549.

Palm, F.C. and J.M. Sneek, 1984, Significance tests and spurious correlation in regression models with autocorrelated errors, Statische Hefte 25, 87-105.

Shively, T.S., 1987, An analysis of tests for regression coefficient stability, mimeo, University of Texas at Austin.

Shively, T.S., 1988, An exact test for a stochastic coefficient in a time series regression model, Journal of Time Series Analysis 9, 81-88.

Shively, T.S., C.F. Ansley and R. Kohn, 1987, Fast evaluation of the distribution of the Durbin-Watson and other invariant test statistics in regression, mimeo, University of New South Wales.

Thomas, J.J. and K.F. Wallis, 1971, Seasonal variation in regression analysis, Journal of the Royal Statistical Society A 134, 57-72.

Vinod, H.D., 1973, Generalization of the Durbin-Watson statistic for higher order autoregressive processes, Communications in Statistics 2, 115-144.

Wallis, K.F., 1972, Testing for fourth order autocorrelation in quarterly regression equations, Econometrica 40, 617-636.

Woglom, G., 1981, A reexamination of the role of stocks in the consumption function and the transmission mechanism, Journal of Money, Credit, and Banking 13, 215-220.

FOOTNOTES

* This research was supported by a grant from the Australian Research Grants Scheme. I am grateful to Phillip Wright for research assistance and to Gene Savin and two referees for helpful comments and suggestions.

1. For example, the eigenvalues for the calculations reported in the next section were found by first tridiagonalizing (10) using Householder's transformation and then applying the QL algorithm. This was done using Fortran versions of Martin, Reinsch and Wilkinson's (1968) TRED1 procedure and Bowdler et al.'s (1968) TQL1 procedure, respectively.

2. Considerable savings can be made when ρ_{10}, ρ_{11}, ρ_{41} and Σ remain fixed in (8) and only c changes. Then, each of the n-k non-zero eigenvalues changes by minus the change in c. Unfortunately, if any of ρ_{10}, ρ_{11}, ρ_{41} or Σ changes, the eigenvalues have to be completely recalculated.

3. There is nothing new about modifying an hypothesis under test so that a test with desirable properties can be constructed. Durbin and Watson (1950) followed this approach when constructing their test for AR(1) disturbances.

4. The secant method was used to determine the new ρ^*_{10} values.

5. Based on the ρ^*_{11} and ρ^*_{41} values for X1 and X2 with n=60 given in tables 2 and 4, $\rho^*_{11} = 0.96$ and $\rho^*_{41} = 0.3$ seemed like reasonable starting values.

TABLE 1

Sizes and powers of different versions of the

$s(\rho_{10},\rho_{11},\rho_{41})$ test for X1 with n=20

Test	ρ_1	Size	Power			
		ρ_4= 0.0	ρ_4= 0.2	0.4	0.6	0.8
s(0.5,0.5,0.5)	0.0	.050	.154	.367	.656	.898
$s(\rho^*_{10}$,0.5,0.5)		.050	.154	.367	.657	.899
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})^\dagger$		.050	.146	.340	.619	.877
s(0.5,0.5,0.5)	0.2	.040	.135	.348	.648	.898
$s(\rho^*_{10}$,0.5,0.5)		.042	.141	.356	.656	.902
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.042	.134	.334	.624	.884
s(0.5,0.5,0.5)	0.4	.030	.111	.310	.615	.885
$s(\rho^*_{10}$,0.5,0.5)		.034	.122	.329	.634	.894
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.035	.120	.316	.613	.882
s(0.5,0.5,0.5)	0.6	.022	.087	.260	.558	.855
$s(\rho^*_{10}$,0.5,0.5)		.029	.105	.295	.596	.874
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.030	.106	.294	.594	.874
s(0.5,0.5,0.5)	0.8	.018	.069	.211	.481	.796
$s(\rho^*_{10}$,0.5,0.5)		.031	.104	.277	.559	.841
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.031	.107	.292	.587	.865
s(0.5,0.5,0.5)	0.99999	.017	.058	.160	.346	.574
$s(\rho^*_{10}$,0.5,0.5)		.050	.135	.299	.533	.769
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.050	.146	.340	.610	.854

† ρ^*_{10} = 0.74386, ρ^*_{11} = 0.8010, ρ^*_{41} = 0.5460

TABLE 2

Sizes and powers of different versions of the $s(\rho_{10}, \rho_{11}, \rho_{41})$ test for X1 with n=60

Test	ρ_1	Size ρ_4=0.0	Power ρ_4=0.2	0.4	0.6	0.8
s(0.5,0.5,0.5)	0.0	.000	.000	.021	.321	.888
$s(\rho^*_{10}, 0.5, 0.5)$		.050	.354	.820	.986	1.000
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})^\dagger$		.050	.329	.787	.979	.999
s(0.5,0.5,0.5)	0.2	.000	.000	.014	.297	.887
$s(\rho^*_{10}, 0.5, 0.5)$		.027	.279	.781	.982	1.000
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})$		.040	.319	.795	.982	1.000
s(0.5,0.5,0.5)	0.4	.000	.000	.008	.245	.867
$s(\rho^*_{10}, 0.5, 0.5)$		.010	.174	.681	.968	1.000
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})$		.028	.280	.777	.981	1.000
s(0.5,0.5,0.5)	0.6	.000	.000	.005	.197	.834
$s(\rho^*_{10}, 0.5, 0.5)$		.003	.083	.517	.929	.998
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})$		.017	.226	.737	.977	1.000
s(0.5,0.5,0.5)	0.8	.000	.001	.031	.283	.835
$s(\rho^*_{10}, 0.5, 0.5)$		.001	.048	.390	.869	.994
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})$		.014	.209	.731	.978	1.000
s(0.5,0.5,0.5)	0.99999	.050	.168	.383	.654	.887
$s(\rho^*_{10}, 0.5, 0.5)$		.050	.214	.555	.868	.977
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})$		.050	.394	.873	.994	1.000

† $\rho^*_{10} = 0.93812$, $\rho^*_{11} = 0.9580$, $\rho^*_{41} = 0.3165$.

TABLE 3

Sizes and powers of different versions of the

$s(\rho_{10},\rho_{11},\rho_{41})$ test for X2 with n=20

Test	ρ_1	Size ρ_4=0.0	Power ρ_4=0.2	0.4	0.6	0.8
s(0.5,0.5,0.5)	0.0	.012	.043	.137	.343	.662
$s(\rho^*_{10},0.5,0.5)$		.050	.137	.310	.561	.821
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})^\dagger$		.050	.121	.262	.490	.770
s(0.5,0.5,0.5)	0.2	.009	.036	.123	.323	.646
$s(\rho^*_{10},0.5,0.5)$		.038	.113	.275	.528	.803
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.043	.111	.254	.489	.773
s(0.5,0.5,0.5)	0.4	.007	.028	.102	.286	.607
$s(\rho^*_{10},0.5,0.5)$		.026	.084	.225	.468	.762
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.034	.095	.231	.467	.760
s(0.5,0.5,0.5)	0.6	.005	.023	.083	.242	.547
$s(\rho^*_{10},0.5,0.5)$		.018	.060	.172	.389	.692
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.026	.078	.203	.435	.739
s(0.5,0.5,0.5)	0.8	.008	.028	.086	.224	.484
$s(\rho^*_{10},0.5,0.5)$		.016	.052	.142	.320	.593
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.024	.073	.195	.428	.737
s(0.5,0.5,0.5)	0.99999	.050	.117	.234	.408	.648
$s(\rho^*_{10},0.5,0.5)$		.050	.117	.237	.417	.656
$s(\rho^*_{10},\rho^*_{11},\rho^*_{41})$		.050	.140	.322	.587	.852

† $\overset{*}{\rho}_{10} = 0.85277$, $\overset{*}{\rho}_{11} = 0.8390$, $\overset{*}{\rho}_{41} = 0.6523$.

TABLE 4

Size and powers of different versions of the

$s(\rho_{10}, \rho_{11}, \rho_{41})$ test for X2 with n=60

Test	ρ_1	Size	Power			
		ρ_4=0.0	ρ_4=0.2	0.4	0.6	0.8
s(0.5,0.5,0.5)	0.0	.000	.000	.006	.180	.788
$s(\rho^*_{10}, 0.5, 0.5)$		.050	.344	.802	.981	1.000
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})^\dagger$		.050	.313	.756	.969	.999
s(0.5,0.5,0.5)	0.2	.000	.000	.004	.157	.782
$s(\rho^* , 0.5, 0.5)$		.024	.251	.740	.973	.999
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})$		.041	.304	.765	.973	.999
s(0.5,0.5,0.5)	0.4	.000	.000	.002	.120	.747
$s(\rho^*_{10}, 0.5, 0.5)$		.008	.135	.596	.941	.998
$s(\rho^*_{10}, \rho^*_{11}, \rho_{41})$		.028	.265	.742	.970	.999
s(0.5,0.5,0.5)	0.6	.000	.000	.001	.091	.695
$s(\rho^*_{10}, 0.5, 0.5)$		.002	.047	.370	.843	.990
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})$		.017	.211	.697	.964	.999
s(0.5,0.5,0.5)	0.8	.000	.000	.011	.167	.713
$s(\rho^*_{10}, 0.5, 0.5)$		.000	.013	.163	.613	.942
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})$		.013	.189	.685	.965	.999
s(0.5,0.5,0.5)	0.99999	.050	.189	.447	.738	.938
$s(\rho^*_{10}, 0.5, 0.5)$		.050	.153	.340	.591	.813
$s(\rho^*_{10}, \rho^*_{11}, \rho^*_{41})$		.050	.393	.868	.993	1.000

† ρ^*_{10} = 0.96223, ρ^*_{11} = 0.9660, ρ^*_{41} = 0.3313.

Recent Working Papers
(Available on Request)

1987

1/87 Maxwell L. King and Merran A. Evans, "Locally Optimal Properties of the Durbin-Watson Test".

2/87 R.D. Snyder, "Computational Aspects of Kalman Filtering with a Diffuse Prior Distribution".

3/87 Merran A. Evans, "The Twelfth Order Analogue of the Durbin-Watson Test".

4/87 Radhey S. Singh, "A Family of Improved Estimators in Linear Regression Models with Errors Having Multi-variate Student-t Distribution".

5/87 Radhey S. Singh, "Estimation of Error Variance in Linear Regression Models with Errors Having Multi-variate Student-t Distribution with Unknown Degrees of Freedom."

6/87 Radhey S. Singh, "Melo and Empirical Bayes Estimators for Reciprocals of Population Means and Regression Coefficients."

7/87 A.R. Tremayne, "A Synthesis of Econometrics and Time Series Methods to Model and Forecast UK Demands for Imports".

8/87 Merran A. Evans and Maxwell L. King, "A Further Class of Tests for Heteroscedasticity".

9/87 Asraul Hoque, "Size of Holding and efficiency in Bangladesh: An Application of a Random Coefficient Model".

10/87 Brett A. Inder, "Bias in the Ordinary Least Squares Estimator in the Dynamic Linear Regression Model with Autocorrelated Disturbances".

11/87 Russel J. Cooper and Keith R. McLaren, "Approaches to the Solution of Intertemporal Consumer demand Models".

12/87 Esme Preston, "On Choosing a Technical Wordprocessor".

1988

1/88 John Preston and Esme Preston, "Two Papers on Simple Word Processors for the PC".

2/88 Esme Preston, "Multilingual and Mathematical Text Processing".

3/88 Esme Preston and John Preston, "The Accountant's PC: Getting Started".

4/88 Maxwell L. King, "Towards a Theory of Point Optimal Testing".

5/88 Ralph D. Snyder, "Statistical Foundations of Exponential Smoothing".

6/88 Grant H. Hillier, "On the Interpretation of Exact Results for Structural Equation Estimators".