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THE AVAILABILITY OF WATER AND LABOUR AS VARIABLE RESOURCES IN PLANNING FOR OPTIMUM ORGANISATION IN IRRIGATION FARMING

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1. INTRODUCTION

Optimum farming organisation demands the most efficient use of available resources. Under conditions of "unlimited availability" of resources optimum utilisation occurs when marginal income is equal to marginal costs. In practice, however, especially in the short-term, resources are often so limited that this "maximum profit" is not readily attainable. In this article optimum utilisation of resources under conditions of scarce, but still variable availability of resources is investigated.

Under conditions of scarce, but still variable, availability of resources "the quantity of variable input which maximizes the rate of return on investment in the variable input is perhaps as relevant as the input which maximizes profit in the sense of equating marginal costs and revenues".[2] Using this as a point of departure, the optimum utilisation of irrigation water and labour, both scarce resources, but available in variable quantities, in irrigation farming in the Hartebeespoort Dam Scheme,[3] is determined normatively.

In the following analyses it was also accepted that farm sizes may vary and models for three sizes of irrigation plots were used: 25 hectares; 43 hectares and 60 hectares.

Seasonal labour and short-term capital were also considered as variable resources and were dealt with as purchasing activities according to the methods of Heady and Chandler.[4]

2. METHOD OF ANALYSIS

The technique of parametric linear programming was used to maximise incomes from irrigation farming below the Hartbeespoort Dam under conditions of scarce, variable availability of resources.[5] In this method variable resources are incorporated into a linear programming model as parametric variables.[6]

In brief this method[7] amounts to optimum organisation being determined at various levels of availability of scarce, variable inputs. This optimum organisation is determined by investing the variable inputs in those alternative production activities with the largest marginal rate of return. Every unit of the scarce resource therefore gives the highest possible advantage. This procedure is repeated as the availability of the scarce resources changes until the marginal profit rate of these resources is equal to nought*. At this point the net income curve reaches its maximum and the greatest advantage per variable scarce resource is realised.

3. IRRIGATION WATER AS A VARIABLE RESOURCE

The annual allocation of irrigation water per farming unit by the Hartbeespoort Dam Irrigation Scheme to a great extent determines the gross margin of the main enterprises.[8] The allocation is traditionally made during October of each year and is based on the amount of water in the dam. The allocated quantity of water is used during the subsequent production year as required by the farming plan. What the situation handled by the simplex model amounts to is that the water not used in October is available for all the other months of the year. In the same way the remaining water not used in November is available for subsequent months, etc.

Mathematically the problem is presented as follows:

*E.g. d gross profit/d labour costs = 0 and gross profit is therefore at a maximum
Because variable costs were built into the model by means of deductions, this at the same time means that marginal income is equal to marginal cost.

A = annual allocation of irrigation water

October irrigation:

$A \geqslant x_1$

$A = x_1 + x_{21}$

where X_1 = water used in October;

x_{21} = water not used in October and therefore still available

November irrigation:

$x_{21} \geqslant x_2$

$0 = x_2 - x_{21} + x_{22}$

where X_2 = water used in November;

x_{22} = water not used in November and therefore still available

December irrigation:

$x_{22} \geqslant x_3$

$0 = x_3 - x_{22} + x_{23}$

where X_3 = water used in December;

X_{23} = water not used in December and therefore still available

The same procedure is followed for all the other months (x_{21}, x_{22} and x_{23} are removal activities).

The water consumption for each month is a function of the crops produced (x_{100}, x_{200}, etc.) and is presented as follows in the model:

X_1 = water used in October

$= a_{11}x_{100} + a_{12}x_{200}$

$x_1 = a_{11}x_{100} + a_{12}x_{200}$

$0 = a_{11}x_{100} + a_{12}x_{200}$

X_2 = water used in November

$= a_{21}x_{100} + a_{22}x_{200}$

$x_2 = a_{21}x_{100} + a_{22}x_{200}$

$0 = a_{21}x_{100} + a_{22}x_{200} - x_2$

etc.

where a_{ij} (i = 1,2 and j = 1,2) = coefficients of utilisation of water on production activities x_{100} and x_{200}

The simplex model is given in Table 1.

4. THE INCORPORATION OF REGULAR LABOUR

In farming both regular and seasonal labour are used. The amount of regular labour ought to be so specified that it can handle the scope of the continuous activities. In some months more regular labour is normally used than in other months. However, regular labourers cannot be dismissed or employed indiscriminately. The month in which most regular labour is used is therefore taken as the basis for employment. This means that during some months regular labour is not fully utilised. For the purposes of this investigation and for practical considerations regular labour is considered a limited resource which can be employed from production year to production year in variable quantities.[9] The optimum farming organisation would therefore depend largely on the amount of regular labour available. Regular labour has been parametrised as a variable resource. According to the technique of variable resource programming, regular labour would therefore continually be "invested" in those production activities where its marginal value product is the greatest, i.e. where an additional labour unit can make the greatest contribution to an increase in the gross profit.

The simplex model must therefore:

(i) Incorporate regular labour as a parametrised variable resource.

(ii) Keep the quantity of available regular labourers the same for different months.

(iii) Indicate the number of regular labourers used monthly.

Regular labour is parametrised by the PARARHS method in the MPS routine.[10]

Mathematically the second requirement is specified as follows:

X_0 = regular monthly labour available (quantity parametrised) and X_{10} = regular labour used in October.

$\therefore X_0 \geqslant X_{10}$

$\therefore 0 = x_{10} + x_{50} - x_0$

where x_{10} = regular labour used in October

x_{50} = amount of idle labour available in October (x_{50} is therefore the removal activity)

Therefore also:

$x_0 \geqslant x_{11}$

$0 = x_{11} + x_{51} - x_0$

where X_{11} = regular labour used in November

X_{51} = regular labour idle in November

5. WATER AND LABOUR IN THE SIMPLEX MODEL (TABLE 1)

5.1 Irrigation water

The -1 in the x_{21} column and P_2 row shows that the water not used in October can be carried over for use in November. The -1 in the x_{22} column and P_3 row shows the carry-over of water not used in November and available for December, etc. The amount of irrigation water used in October is determined by the utilisation coefficients a_{11} and a_{12}. The -1 in the x_1 column and P_{11} row and the +1 in the same column and the P_1 row ensure that the amount used does not exceed the amount available (A).

By merely changing A the result for different levels of availability of irrigation water may be obtained.

5.2 Regular labour

To ensure that an indication is given of the number of labourers active monthly the procedure set out below is followed:

regular labour utilisation October

$$= a_{11}x_{100} + a_{12}x_{200}$$
$$x_{10} = a_{11}x_{100} + a_{12}x_{200}$$
$$0 = a_{11}x_{100} + a_{12}x_{200} - x_{10}$$

where x_{100} and x_{200} represent production activities and a_{11} and a_{12} represent the utilisation coefficients of regular labour for these activities.

The regular labour used during October is now treated as an intermediary activity. x_{10} is therefore determined by the quantities of x_{100} and x_{200} produced. The production of x_{100} and x_{200} is in turn subject to B, the regular monthly labour available. The same procedure is followed for all the remaining months (see simplex table).

6. PRODUCTION ACTIVITIES AND LIMITATIONS

For the farming plan the following branches and alternatives were considered:

Summer crops	:	Early tobacco
		Late tobacco
		Cotton
		Sunflowers
Winter crops	:	Wheat
		Onions
		Potatoes
Stock	:	Dairy
		Pigs
Fodder crops	:	Maize silage
		Babala silage
		Oats
		Velvet beans

The maximisation of the farming income takes place within the framework of the following limitations:

Variable resource	Way in which dealt with in the simplex model
Water	Dependent on state of dam (3 m, 6 m, 12 m, and 18 m)
Regular labour	Parametrised
Seasonal labour	Buying activity
Short-term capital	Buying activity
Land	Various farm sizes (27 ha, 43 ha and 60 ha)
Fixed limitations	
Fluctuating cultivation limitations	: Tobacco can be planted only once every three years on the same ground
Tractive power	: Only 208,333 tractor hours available per farm

The complete simplex model consisted of 86 activities and 70 limitations. This was solved on the IBM 70 computer of the University of Pretoria with the aid of the M.P.S. routine and the accompanying PARARHS method.

7. RESULTS: OPTIMUM FARMING PLANS

A simplex model was drawn up for three farm sizes and four states of the dam (which determine the allocation of irrigation water). Regular labour was parametrised in variable quantities in each model and the other variable resources and limitations were dealt with as buying activities. In total 12 models were drawn up. Income results are given in Table 3.

TABLE 3 - Maximum gross profits on various farming units (R)*

Farm sizes (ha)	Gross incomes according to state of dam 3 m R	6 m R	12 m R	18 m R
25	660	2 085	5 580	10 004
43	1 100	3 474	8 703	15 000
60	1 541	4 821	11 638	19 425

*1973 values

In order to illustrate the results (optimum farming plans and utilisation of resources and production factors) the situation on a 43 hectare unit with a medium-full dam (state of dam 12 metres) is discussed.

Table 4 and Fig. 1 give the optimum farming plans for various levels of regular labour.

The following deserves mention:

(i) *Labour is used where its marginal return is the greatest*

Regular labour is available in variable amounts, fluctuating between 0 man-hours and 7 000 man-hours. Every basic point (A, B, C, D, E, F, G, H, I, J, K, L) indicates the maximum profit plan for the specified number of man-hours of regular labour. Under each of these basic points regular labour is "invested" in the production activity with the highest marginal return for labour. The maximum profit combination of production activities therefore changes from one basic point to the next. Table 4, for example, shows consecutive basic points.

TABLE 4 - Optimum plans at various basic points

Basic points	H	I
Maximum profit plan:		
Early tobacco	4,5 ha	5,3 ha
Late tobacco	9,8 ha	9,0 ha
Wheat	-	7,8 ha
Onions	2,4 ha	2,7 ha
Potatoes	3,7 ha	4,0 ha
Maize silage	1,6 ha	-
Babala silage	1,0 ha	-
Lucerne	2,1 ha	-
Oats	1,9 ha	-
Velvet beans	1,4 ha	-
Dairy	7,5 G.V.E.	-
Pigs	2,7 V.E.	-
Net farming income	**R5 162,35**	**R5 190,40**

Between two consecutive base points the same range of marginal rates of substitution of products applies. The maximum profit plans for levels of regular labour between base points can therefore be

TABLE 1 - The incorporation of regular labour and irrigation water in the simplex model

Target function		-k	0	0	0	0	0	0	0	0	0	0	0	0	+L	+M
Resource limitations	Levels of availability	Regular labour available per month	Regular labour used			Irrigation water used			Regular labour not used			Irrigation water not used			Production activities	
															Tobacco	Wheat
			Oct	Nov	Dec	Oct	Nov	Dec	Oct	Nov	Dec	Oct	Nov	Dec		
		XO	X10	X11	X12	X1	X2	X3	X50	X51	X52	X21	X22	X23	X100	X200
Regular labour available:																
Per month	B	+1														
October	O	- 1	+1						+1							
November	O	- 1		+1						+1						
December	O	- 1			+1						+1					
January																
Regular labour used:																
October	O		- 1												+A11	+A12
November	O			- 1											+A21	+A22
December	O				- 1										+A31	+A32
January	O															
Irrigation allocation:																
October (P_1)	A					+1						+1				
November (P_2)	O						+1					- 1	+1			
December (P_3)	O							+1					- 1	+1		
January (P_4)														- 1		
Irrigation water used:																
October	O					- 1									+A51	+A52
November	O						- 1								+A61	+A62
December	O							- 1							+A71	+A72
January																

-k = costs per man-hour; + L = gross profit per unit of tobacco; +M = gross per unit of wheat

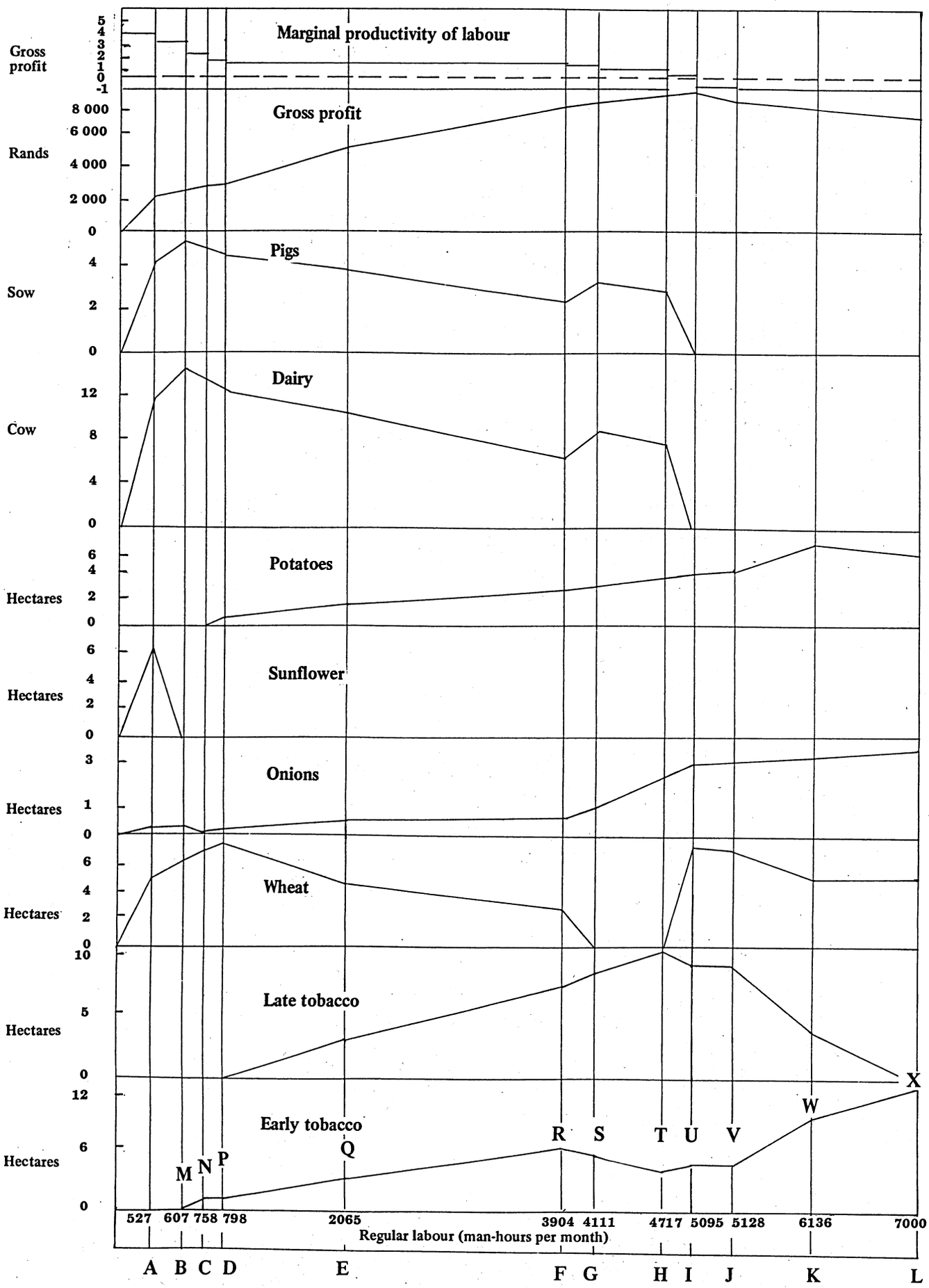

FIG. 1 - Optimum farming plans on 43 hectares: state of dam = 12 metres

TABLE 2 - Optimum farming plans on 43 hectares: state of dam = 12 metres

Item	1	2	3	4	5	6	7	8	9	10	11	12
Gross profit	R 2 134	2 415	2 792	2 870	5 176	8 324	8 447	8 675	8 703	8 689	8 162	7 373
Fixed costs	R 3 513	3 513	3 513	3 513	3 513	3 513	3 513	3 513	3 513	3 513	3 513	3 513
Net income	R-1 378	-1 096	- 720	- 642	1 663	4 811	4 934	5 162	5 190	5 176	4 649	3 860
Regular labour* hours	528	607	758	795	2 065	3 904	4 111	4 717	5 096	5 128	6 135	7 017
Cash capital	R 3 266	3 798	3 858	4 015	6 162	8 939	9 633	10 647	10 317	10 376	12 198	12 918
Tobacco: Hectares												
Early: Hectares			1,0	9,0	3,6	7,2	6,0	4,5	5,2	5,4	10,7	14,3
Late: Hectares	-	-	-	-	2,7	7,0	8,2	9,8	9,0	8,8	3,6	-
Wheat: Hectares	5,5	6,5	7,3	8,0	5,1	3,0	-	-	7,7	7,6	5,2	5,4
Cotton: Hectares	-	-	-	-	-	-	-	-	-	-	-	-
Onions: Hectares	0,3	0,3	0,1	0,1	0,5	0,8	1,1	2,3	2,6	2,8	3,0	3,3
Sunflowers: Hectares	7,3	-	-	-	-	-	-	-	-	-	-	-
Potatoes: Hectares	-	-	-	0,7	1,5	2,8	2,9	3,5	4,1	4,2	5,3	5,8
Maize***: Hectares	2,5	3,1	2,9	2,6	2,2	1,3	1,6	1,6	-	-	-	-
Babala***: Hectares	1,6	2,0	1,8	1,7	1,4	0,8	1,2	1,0	-	-	-	-
Lucerne: Hectares	3,2	4,0	3,7	3,4	2,9	1,7	2,4	2,0	-	-	-	-
Oats: Hectares	2,9	3,6	3,3	3,0	2,6	1,6	2,1	1,8	-	-	-	-
Velvet beans: Hectares	2,1	2,6	2,4	2,2	1,9	1,1	1,5	1,3	-	-	-	-
Dairy: Cow	11,6	14,6	13,5	12,4	10,6	6,5	7,5	7,5	-	-	-	-
Pigs: Sow	4,2	5,2	4,8	4,4	3,7	2,3	2,7	2,7	-	-	-	-
Utilisation of land: Hectares												
October	16	19	20	20	21	23	23	23	24	24	22	23
November	17	12	12	11	17	22	85	26	21	21	23	23
December	17	12	12	11	17	22	25	26	21	21	23	23
January	20	15	15	15	19	24	27	28	21	21	23	23
February	25	22	22	23	24	27	27	28	29	29	28	28
March	23	20	20	21	23	28	26	27	29	29	28	28
April	21	17	18	19	21	24	24	25	29	29	28	28
May	19	14	14	15	12	9	8	9	14	14	14	14
June	11	14	14	15	12	9	8	9	14	14	14	14
July	13	16	16	16	12	7	6	7	10	10	8	8
August	13	16	16	16	12	7	6	7	10	10	8	8
September	13	16	16	16	12	7	6	7	10	10	8	8
Regular labour* hours												
October	528	607	787	798	2 065	3 541	3 702	4 420	5 086	5 128	6 135	7 017
November	352	383	563	578	2 065	3 904	4 111	4 717	5 086	5 128	5 895	6 544
December	208	212	328	312	1 455	3 143	3 397	3 674	3 441	3 406	2 521	1 889
January	528	456	577	575	1 311	2 340	2 391	2 363	2 191	2 201	2 460	2 511
February	485	485	742	741	1 924	3 565	3 506	3 382	3 347	3 381	4 261	4 732
March	528	607	758	788	2 065	3 904	4 111	4 351	4 188	4 168	3 839	3 400
April	376	438	758	798	2 065	3 904	3 733	3 741	4 183	4 218	4 946	5 433
May	528	607	758	798	2 065	3 904	4 111	4 717	5 096	5 065	4 255	3 824
June	208	253	331	798	2 065	3 904	4 111	4 717	5 096	5 128	6 135	5 523
July	242	292	394	395	926	1 678	1 881	1 784	2 931	1 934	1 994	2 090
August	299	360	443	448	1 038	1 888	1 928	2 146	2 295	2 291	2 177	2 165
September	367	434	445	472	1 099	1 909	2 139	2 671	2 885	2 869	2 450	2 256

*Man-hours per month; **Short-term capital; *** Silage

TABLE 2 - (continued)

Item	1	2	3	4	5	6	7	8	9	10	11	12
Seasonal labour* hours												
October	-	-	-	-	-	-	-	-	-	-	-	-
November	6	7	136	127	431	862	711	522	624	648	648	1 265
December	47	56	139	133	717	1 543	1 619	1 734	1 727	1 722	1 722	1 596
January	33	42	291	268	1 125	2 360	2 190	1 960	2 049	2 079	2 079	2 821
February	187	237	610	563	2 141	4 410	4 288	4 078	4 073	4 101	4 101	4 803
March	102	119	541	516	2 647	5 713	5 719	5 807	5 935	5 936	5 936	6 213
April	88	99	459	439	2 628	5 758	5 924	6 237	6 282	6 275	6 275	6 075
May	162	191	338	332	1 928	4 179	4 551	5 136	5 068	5 032	5 032	4 112
June	34	36	155	152	1 060	2 350	2 433	2 716	2 727	2 716	2 431	2 258
July	-	-	129	120	796	1 793	1 507	1 825	1 816	1 814	1 755	1 714
August	68	72	165	170	919	1 911	2 021	2 285	2 403	2 406	2 487	3 535
September	153	163	60	95	637	1 203	1 636	2 466	2 617	2 594	2 007	1 702
Labour** hours												
Dairy: Cow	203	257	235	215	184	112	152	152	-	-	-	-
Pigs: Sow	39	50	45	42	36	22	29	29	-	-	-	-
Tractor use*** hours												
October	32	40	48	51	76	114	106	105	124	125	148	161
November	63	55	52	53	96	141	160	208	208	208	208	208
December	58	21	19	23	47	78	90	107	101	101	96	76
January	26	21	18	27	42	58	67	83	85	86	112	106
February	48	60	58	56	53	46	50	48	36	37	43	48
March	37	45	45	49	51	54	53	62	80	80	81	79
April	44	53	58	61	55	54	41	46	88	87	84	91
May	75	71	79	84	68	63	40	41	100	99	76	74
June	12	16	15	18	29	42	47	52	48	48	57	50
July	6	8	7	6	8	8	10	11	7	6	4	3
August	8	11	9	14	22	30	34	43	45	45	62	60
September	9	11	14	31	62	106	167	118	133	135	203	203
Irrigation use****												
October	1 528	1 942	2 147	-	2 623	3 341	3 212	2 508	1 784	1 855	3 651	4 880
November	3 728	1 684	1 677	1 535	2 491	3 750	4 221	4 369	3 373	3 336	2 422	1 796
December	948	1 204	1 246	1 141	1 694	2 383	2 551	2 456	1 844	1 845	1 873	1 892
January	2 042	1 168	1 162	1 064	1 557	2 173	2 431	2 437	1 791	1 778	1 448	1 223
February	1 370	694	635	844	1 255	1 799	2 036	2 288	2 102	2 114	2 552	2 110
March	455	573	529	563	587	562	662	679	450	458	695	634
April	518	658	602	697	791	861	933	1 047	834	849	1 289	1 176
May	3 298	4 005	4 128	4 417	3 424	2 491	1 730	2 016	4 386	4 371	3 939	3 960
June	422	516	503	507	410	268	254	325	470	468	403	431
July	1 424	1 733	1 752	1 801	1 382	668	630	776	1 596	1 585	1 267	1 337
August	1 852	2 290	2 322	2 390	1 795	1 136	801	473	2 085	2 071	1 639	1 726
September	4 543	5 687	5 457	5 231	4 164	2 526	2 644	2 284	1 444	1 428	980	1 004

*Woman-hours **Man-hours per month; ***Tractor hours; ****Units in hectares millimetres

obtained by combining the various base points of the same production activity. The straight lines MN, NP, PQ, RS, ST, TU, UV, VW and WX in Fig. 1 therefore show the path of expansion of early tobacco if the regular labour available is continually increased.

The gradients of the various extension lines also indicate the product:product ratios of the various production activities. Between base points O and A wheat, sunflowers, onions, dairy produce and pigs complement one another. Between A and B wheat and onions supplement one another, however (onions remain constant with an expansion in wheat); wheat and sunflowers are competitive (sunflowers decrease as wheat production increases); and wheat and dairy produce are still complementary (both production activities are still expanding).

(ii) The highest gross profit of maximum profit plans 1 to 12 is determined at the point where the marginal profit rate on labour drops to zero (see top part of Fig. 1). For this plan 5 096 man-hours of regular labour per month are needed. Above this point an addition in labour will result in larger additional costs than additional income, and the gross profit will therefore drop. This phenomenon is explained by the fact that other resources become scarce at this stage and have a limiting effect on any further extension in activities.

(iii) A labour force providing 5 096 man-hours per month will be fully utilised only during November, May and June and virtually throughout October. During the rest of the year the unused labourers can be used for repair work, development work, etc., and for the replacement of seasonal labour. It would also be a good idea to arrange labourers' leave in such a way that they are absent during the months when least labour is used: December to February and July to September.

(iv) *Scarce resources*

Labour would seem to be a critical resource only up to a level of 5 096 hours per month.

Although 43 ha are available, this quantity of land is not fully utilised during any month. Land therefore cannot be considered under the resource availability levels set out. Available tractor hours and irrigation water, however, limit the further extension of the farming plan.

The available tractor hours (208,3 per month) are fully utilised during November. From a second set of l.p. models in which tractor hours were doubled, it appeared that the purchase of a similar additional tractor would be economically advantageous (increased gross profit = R9 296,32).

The available irrigation water is also fully used. An increase (when the dam is full) would increase the maximum gross profit to R14 999,78.

It therefore seems that under present conditions at the Hartbeespoort Scheme water is a scarce resource; because of the water: area ratios land cannot be considered a scarce resource.

8. LONG-TERM PLANNING AND ECONOMIC FARMING UNITS

For farm planning and policies and agricultural development a knowledge of expected long-term net farming incomes is important. With the help of the l.p. models the expectations regarding net farming income for the relevant scheme can be formulated for expected levels of labour employment and water availability.

If it is accepted that as much regular labour can be employed as is necessary for a total maximum profit plan, expectations regarding availability of irrigation water would give an indication of the expected long-term net farming income. From this investigation it appeared that medium full is the most probable state of the dam. The long-term annual net income for the various units was as follows:

25 hectares : R3 389,00
43 hectares : R4 955,00
60 hectares : R6 287,00

Measured at the fixed standard of a net farm income ensuring an existence that is viable and comparable with other sectors, policy decisions concerning economic farming units can be made.

9. CONCLUSION

Although a great deal of farming data and surveys are necessary, variable resource programming offers possibilities for planning on the farm and at agricultural policy level. Short-term and long-term planning can be implemented.

By following the variable resource programming approach a great deal of economically relevant information concerning optimum product combinations, resource utilisation, farming units, scarce resources and production factors, product:product and factor:factor ratios, the creation of work opportunities, etc., can be obtained. This type of information is essential for agricultural development of farms, districts and regions.

The techniques of parametric linear programming also lend themselves to the quantification of expectations as regards changes in price, subsidising and stabilising programmes, etc.

Broadly speaking, variable resource programming would seem to be a valuable aid both at micro and macro-economic level.

REFERENCES

1. Based on "Die gebruik van proefboerderyresultate vir die beplanning van boerderye by die Hartebeespoortdambesproeiingskema." M.Sc.(Agric.) thesis by C.J. van Rooyen, University of Pretoria, 1973.
2. HEADY, E.O. & DILLON, J.L. Agricultural production functions. Iowa State University Press, Ames, Iowa, 1964, p. 46.
3. GROENEWALD, J.A. & HANCKE, H.P. The effect of resource availability on optimum

organisation in irrigation farming. Agrekon, Vol. 11, No. 3, July 1972.
4. HEADY, E.O. & CANDLER, W. Linear programming methods. Iowa State University Press, Ames, Iowa, 1966.
5. Profit maximisation is acceptable as business strategy in irrigation farming.
6. GASS, S.I. Linear programming. McGraw Hill. Book Company Inc, N.Y., 1958, p. 115.
7. HEADY, E.O. & CANDLER, W. Linear programming methods. Iowa State University Press, Ames, Iowa, 1966, p. 233.
8. VAN ROOYEN, C.J. Expectation models in farm business management Agrekon, Vol. 15, No. 3, July 1976.
9. Regular labour is not necessarily scarcer than irrigation water, but is parametrised because the number of labourers employed can be arbitrarily changed.
10. Mathematical Programming System 360 Version 2, Linear and separable programming - Users Manual, I.B.M. Company, 1969.