Exclusive production of two and four pions
in proton-proton scattering

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Abstract

We consider exclusive $pp \to pp\pi^+\pi^-$ and $pp \to pp\pi^+\pi^-\pi^+\pi^-$ reactions at high energies. The calculation is based on a tensor pomeron model and the amplitudes for the processes are formulated in an effective field-theoretic approach. In the case of $pp \to pp\pi^+\pi^-$ process we consider both diffractive and photoproduction mechanisms and we include the non-resonant $\pi^+\pi^-$ continuum and the resonance $f_0(500)$, $f_0(980)$, $f_2(1270)$, $\rho(770)$ contributions. We discuss how two pomerons couple to tensor meson $f_2(1270)$ and the interference effects of resonance and dipion continuum. We find that the relative contribution of resonances $\rho(770)$, $f_2(1270)$ and dipion continuum strongly depends on the cut on proton transverse momenta. In the case of exclusive central 4π production we include the contribution via the intermediate $\sigma\sigma$ and $\rho\rho$ states. For both processes the theoretical results have been compared with the experimental data and predictions for planned or being carried out experiments (e.g. STAR, ATLAS-ALFA) are presented.

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1   Introduction

Central production mediated by the "fusion" of two exchanged pomerons [1, 2] is an important diffractive process for the investigation of properties of dipion resonances, in particular, for search of gluonic bound states (glueballs). The experimental groups at the CERN-ISR [4], COMPASS [3], STAR [5], CDF [6], ALICE [7], and CMS [8] all show visible structures in the $\pi^+\pi^-$ invariant mass. The LHCb experiment is also well suited to measuring central exclusive production processes [9].

Some time ago two of us have formulated a Regge-type model of the dipion continuum for the exclusive reaction $pp \to pp\pi^+\pi^-$ with parameters fixed from phenomenological analysis of total and elastic NN and πN scattering [12]. The model was extended to include rescattering corrections due to pp nonperturbative interaction [13, 10]. The exclusive reaction $pp \to pp\pi^+\pi^-$ constitutes an irreducible background to the scalar $f_0(1500)$ [11] and χ_{c0} [13] mesons production. These model studies were extended to the exclusive $pp \to ppK^+K^-$ reaction [14]. The largest uncertainties in the model are due to the unknown off-shell pion form factor and the absorption effects; see Ref. [15]. Such an approach gives correct order of magnitude cross sections, however, does not include resonance contributions which interfere with the continuum contribution.

First calculations of central exclusive diffractive production of $\pi^+\pi^-$ continuum together with the dominant scalar $f_0(500)$, $f_0(980)$, and tensor $f_2(1270)$ resonances was performed in Ref. [1]. Here we use the tensor-pomeron model formulated in [16]; see also [17]. In this model pomeron exchange is effectively treated as the exchange of a rank-2 symmetric tensor. In [18] we show that the tensor pomeron is consistent with the STAR experimental data on polarised high-energy pp elastic scattering [19]. In Ref. [20] the model was applied to the diffractive production of several scalar and pseudoscalar mesons in the reaction $pp \to ppM$. The corresponding pomeron-pomeron-meson coupling constants are not known and have been fitted to existing WA102 experimental data. In most cases one has to add coherently amplitudes for two pomeron-pomeron-meson couplings with different orbital angular momentum and spin of two "pomeron particles". [1] In [21] an extensive study of the photoproduction reaction $\gamma p \to \pi^+\pi^- p$ was presented. The resonant $(\rho^0 \to \pi^+\pi^-)$ and non-resonant (Drell-Söding) photon-pomeron/reggeon $\pi^+\pi^-$ production in pp collisions was studied in [22].

The identification of glueballs can be very difficult. The studies of different decay channels in central exclusive production would be very valuable in this context. One of the possibilities is the $pp \to pp\pi^+\pi^-\pi^+\pi^-$ reaction being analysed at the RHIC and LHC. In Ref. [23] we analysed the exclusive diffractive production of four-pion via the intermediate $\sigma\sigma$ and $\rho\rho$ states within the tensor-pomeron model.

2   Sketch of the formalism

The Born-level diagrams for the continuum and resonant $\pi^+\pi^-$ production are shown in Fig. 1. The purely diffractive amplitude is a sum of continuum amplitude

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[1]We wish to emphasize that the tensorial pomeron can equally well describe the WA102 experimental data on the exclusive meson production as the less theoretically justified vectorial pomeron frequently used in the literature. The existing low-energy experimental data do not allow to clearly distinguish between the two approaches as the presence of subleading reggeon exchanges is at low energies very probable for many $pp \to ppM$ reactions.
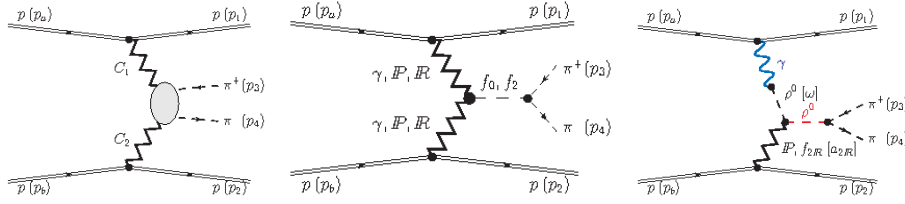
Figure 1: Generic Born-level diagrams for central exclusive production of continuum $\pi^+\pi^-$ and resonances in proton-(anti)proton collisions. Here we labelled the exchanged objects by their charge conjugation numbers C_1, $C_2 \in \{+1, -1\}$.

and the amplitudes with the s-channel scalar and tensor resonances:

$$\mathcal{M}_{pp\to pp\pi^+\pi^-} = \mathcal{M}_{pp\to pp\pi^+\pi^-}^{\pi\pi-\text{continuum}} + \mathcal{M}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\pi^+\pi^-}^{(\mathbb{P}\mathbb{P}\to f_0\to\pi^+\pi^-)} + \mathcal{M}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\pi^+\pi^-}^{(\mathbb{P}\mathbb{P}\to f_2\to\pi^+\pi^-)}. \quad (1)$$

The Born amplitude, for instance, for the process $pp \to pp(f_2 \to \pi^+\pi^-)$ can be written in the effective tensor pomeron approach as

$$\mathcal{M}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\pi^+\pi^-}^{(\mathbb{P}\mathbb{P}\to f_2\to\pi^+\pi^-)} = (-i)\,\bar{u}(p_1, \lambda_1)i\Gamma_{\mu_1\nu_1}^{(\mathbb{P}pp)}(p_1, p_a)u(p_a, \lambda_a)\,i\Delta^{(\mathbb{P})\,\mu_1\nu_1,\alpha_1\beta_1}(s_1, t_1)$$

$$\times i\Gamma_{\alpha_1\beta_1,\alpha_2\beta_2,\rho\sigma}^{(\mathbb{P}\mathbb{P}f_2)}(q_1, q_2)\,i\Delta^{(f_2)\,\rho\sigma,\alpha\beta}(p_{34})\,i\Gamma_{\alpha\beta}^{(f_2\pi\pi)}(p_3, p_4)$$

$$\times i\Delta^{(\mathbb{P})\,\alpha_2\beta_2,\mu_2\nu_2}(s_2, t_2)\,\bar{u}(p_2, \lambda_2)i\Gamma_{\mu_2\nu_2}^{(\mathbb{P}pp)}(p_2, p_b)u(p_b, \lambda_b), \quad (2)$$

where $t_1 = q_1^2 = (p_1 - p_a)^2$, $t_2 = q_2^2 = (p_2 - p_b)^2$, $s_1 = (p_a + q_2)^2 = (p_1 + p_{34})^2$, $s_2 = (p_b + q_1)^2 = (p_2 + p_{34})^2$, $p_{34} = p_3 + p_4$. $\Delta^{(\mathbb{P})}$ and $\Gamma^{(\mathbb{P}pp)}$ denote the effective pomeron propagator and proton vertex function, respectively. For the explicit expressions, see Sec. 3 of [16]. In Ref. [1] (see Appendix A) we have considered all possible tensorial structures for the $\mathbb{P}\mathbb{P}f_2$ coupling. For a more details, as form of form factors, the tensor-meson propagator $\Delta^{(f_2)}$ and the $f_2\pi\pi$ vertex, see Refs. [16, 1].

We consider also the production of $\rho(770)$ resonance and the non-resonant (Drell-Söding) $\pi^+\pi^-$ continuum produced by photon-pomeron and photon-$f_{2\mathbb{R}}$ mechanisms studied in detail in [22]. The $\mathbb{P}\rho\rho$ vertex is given in [16] by formula (3.47). The coupling parameters of Regge exchanges was fixed based on the HERA experimental data for the $\gamma p \to \rho^0 p$ reaction. In [22] we showed that the ρ^0 term interfere with the non-resonant terms producing a skewing of the ρ^0-meson line shape. Due to the photon propagators occurring in diagrams we expect these processes to be most important when at least one of the protons undergoes only a very small $|t_{1,2}|$.

3   Selected results

We start from a discussion of some dependences for the central exclusive production of the $f_2(1270)$ meson. For a detailed study of $f_2(1270)$ production see Ref. [1]. In Fig. 2 we present results for individual pomeron-pomeron-f_2 coupling terms (there are 7 possible terms [1]) at $\sqrt{s} = 200$ GeV and $|\eta_\pi| < 1$. The different predictions differ considerably which could be checked experimentally. We show that only in two cases ($j = 2$ and 5) the cross section $d\sigma/d|t_{1,2}|$ vanishes when $|t_{1,2}| \to 0$.
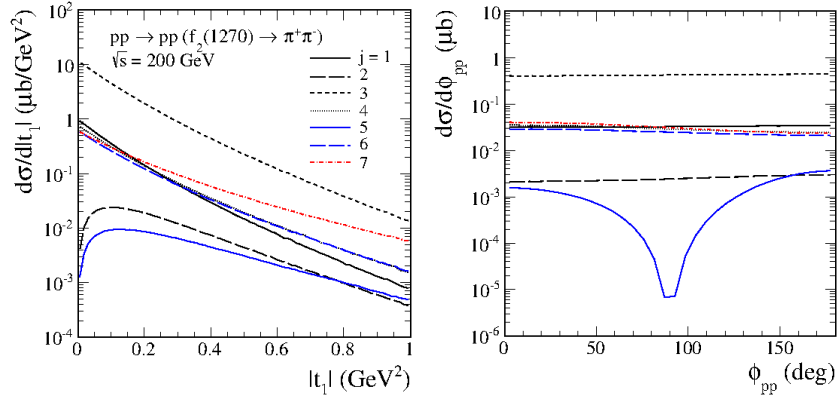
Figure 2: The (Born-level) distribution in transferred four-momentum squared between the initial and final protons (left panel) and the distribution in azimuthal angle between the outgoing protons (right panel) at $\sqrt{s} = 200$ GeV and $|\eta_\pi| < 1$. We show the individual contributions of the different pomeron-pomeron-$f_2(1270)$ couplings. For illustration the results have been obtained with coupling constants fixed at $g^{(j)}_{I\!P I\!P f_2} = 1.0$.
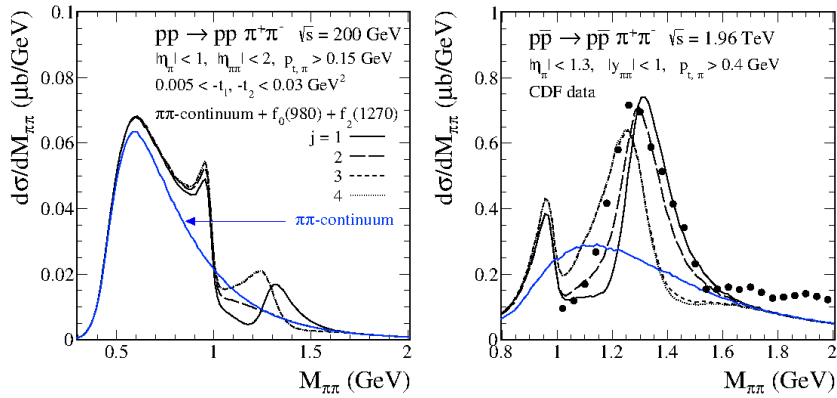

Figure 3: Two-pion invariant mass distribution for the STAR [5] (left) and CDF [6] (right) kinematics. The individual contributions of different $I\!P I\!P f_2$ couplings ($j = 1, ..., 4$) are compared with the CDF data [6]. The Born calculations for $\sqrt{s} = 200$ GeV and $\sqrt{s} = 1.96$ TeV were multiplied by the gap survival factors $\langle S^2 \rangle = 0.2$ and $\langle S^2 \rangle = 0.1$, respectively. The blue solid lines represent the non-resonant continuum contribution only ($\Lambda_{off,M} = 0.7$ GeV) while the black lines represent a coherent sum of non-resonant continuum, $f_0(980)$ and $f_2(1270)$ resonant terms.

In [1] we tried to understand whether one can approximately describe the dipion invariant mass distribution observed by different experiments assuming only one of the seven possible $I\!P I\!P f_2$ tensorial couplings. We found that the feature of the $\pi^+\pi^-$ distribution depends on the cuts used in a particular experiment (usually the t cuts are different for different experiments). As can be clearly seen from Fig. 3 different $I\!P I\!P f_2$ couplings generate different interference patterns around $M_{\pi\pi} \sim$ 1.27 GeV. A sharp drop around $M_{\pi\pi} \sim 1$ GeV is attributed to the interference of $f_0(980)$ and continuum. We can observe that the $j = 2$ coupling gives results close to those observed by the CDF Collaboration [6]. In this preliminary study we did not try to fit the existing data [6] by mixing different couplings because the CDF data are not fully exclusive (the outgoing p and $\bar{p}$ were not measured). The calculations were done at Born level and the absorption corrections were taken into account by multiplying the cross section by a common factor $\langle S^2 \rangle$ obtained from [15]. The two-pion continuum was fixed by choosing a form factor for the off-shell pion $\hat{F}_\pi(k^2) = \frac{\Lambda_{off,M}^2 - m_\pi^2}{\Lambda_{off,M}^2 - k^2}$ and $\Lambda_{off,M} = 0.7$ GeV.
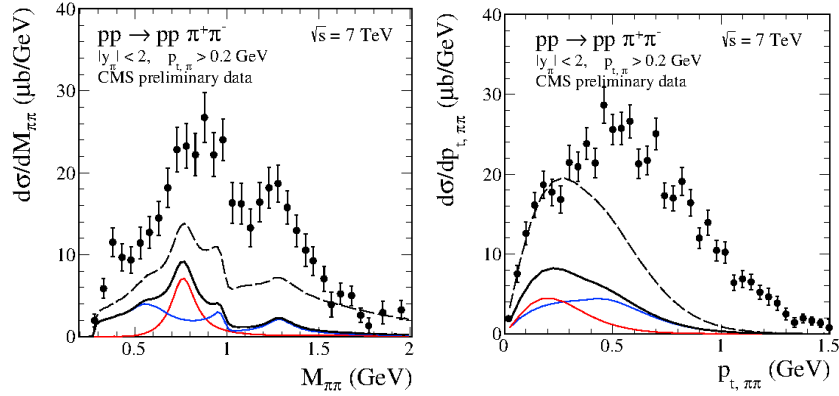

Figure 4: The distributions for two-pion invariant mass (left panel) and transverse momentum of the pion pair (right panel) for the CMS kinematics at $\sqrt{s} = 7$ TeV. Both photoproduction (red line) and purely diffractive (blue line) contributions multiplied by the factors $\langle S^2 \rangle = 0.9$ and $\langle S^2 \rangle = 0.1$, respectively, are included. The complete results correspond to the black solid line ($\Lambda_{off,M} = 0.7$ GeV) and the dashed line ($\Lambda_{off,M} = 1.2$ GeV). The CMS preliminary data scanned from [8] are shown for comparison.

In Fig. 4 we show results including in addition to the non-resonant $\pi^+\pi^-$ continuum, the $f_2(1270)$ and the $f_0(980)$ resonances, the contribution from photoproduction ($\rho^0 \to \pi^+\pi^-$, Drell-Söding mechanism), as well as the $f_0(500)$ resonant contribution. Our predictions are compared with the CMS preliminary data [8]. Here the absorption effects lead to huge damping of the cross section for the purely diffractive term (the blue lines) and relatively small reduction of the cross section for the photoproduction term (the red lines). Therefore we expect one could observe the photoproduction contribution. The CMS measurement [8] is not fully exclusive and the $M_{\pi\pi}$ and $p_{t,\pi\pi}$ spectra contain contributions associated with other processes, e.g., when one or both protons undergo dissociation. In addition, the dashed line corresponds to results with $\Lambda_{off,M} = 1.2$ GeV and better describe the preliminary CMS data. If we used the set of parameters adjusted to the CDF data [8] for the

STAR or CDF measurements our theoretical results there would be above the pre-
liminary STAR data [5] at $M_{\pi\pi} > 1$ GeV and in complete disagreement with the
CDF data from [6]. Only purely central exclusive data expected from CMS-TOTEM
and ATLAS-ALFA will allow to draw definite conclusions.

In Fig. 5 we show the four-pion invariant mass distributions for the the reaction
$pp \to pp\pi^+\pi^-\pi^+\pi^-$ proceeding via the intermediate $\sigma\sigma$ and the $\rho\rho$ states. The
results for processes with the exchange of heavy mesons (compared to pion) strongly
depend on the details of the hadronic form factors. By comparing the theoretical
results and the cross sections found in the CERN-ISR experiment [24] we fixed the
parameters of the off-shell meson form factor and the $I\!\!P\sigma\sigma$ and $f_{2I\!\!R}\sigma\sigma$ couplings.
In the case of $\sigma\sigma$ contribution we use two sets of the coupling constants; standard
(set A) and enhanced (set B) ones, see (2.11) and (2.12) of [23], respectively. [2]
In the case of $\rho\rho$ contribution the ρ meson reggeization suppresses large masses of
$M_{4\pi}$ distributions. This is also the case when the separation in rapidity between
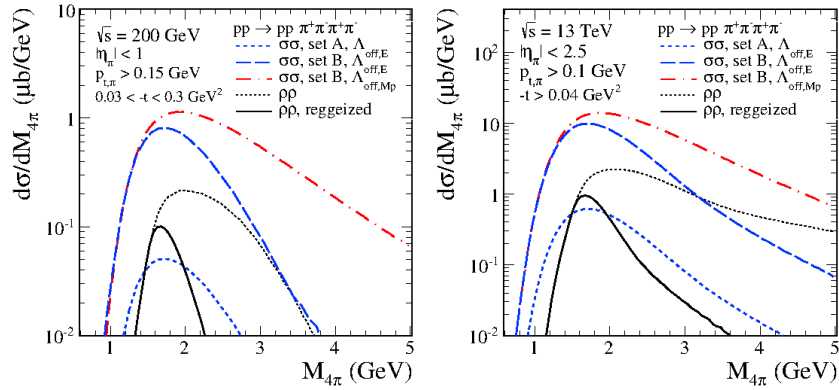the two ρ mesons increases, see Fig. 4 of [23]. [3]

Figure 5: The 4π invariant mass distributions (for different experimental cuts) multi-
plied by the factors $\langle S^2 \rangle = 0.30$ (for $\sqrt{s} = 200$ GeV) and 0.23 (for $\sqrt{s} = 13$ TeV)
estimated within the eikonal approximation (only the pp rescattering). The blue
and red lines for the $\sigma\sigma$ contribution for the exponential off-shell meson form fac-
tors ($\Lambda_{off,E} = 1.6$ GeV) and the monopole ones ($\Lambda_{off,M} = 1.6$ GeV), respectively.
The black lines represent results for the $\rho\rho$ contribution without (the dotted line) and
with (the solid line) the ρ meson reggeization.

4   Conclusions

In our recent paper [1] we have analysed the exclusive central production of dipion
continuum and resonances contributing to the $\pi^+\pi^-$ pair production in proton-
(anti)proton collisions in an effective field-theoretic approach with tensor pomerons

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[2]There is quite a good agreement between our $\sigma\sigma$ result with a monopole form factor and
the 4π ($J = 0$, phase space) data from [24]. Note that this implies that the set B of couplings,
which are larger than the corresponding pion couplings, seems to be preferred.

[3]We have found that the diffractive mechanism in pp collisions considered by us leads to
the cross section for the $\rho\rho$ final state more than three orders of magnitude larger than the
corresponding cross section for $\gamma\gamma \to \rho\rho$ and double scattering photon-pomeron (pomeron-
photon) mechanisms considered in [25].

and reggeons as proposed in [16]. We have included the scalar ($f_0(500)$, $f_0(980)$) and tensor $f_2(1270)$ resonances as well as the vector $\rho(770)$ resonance in a consistent way. In the case of $f_2(1270)$-meson production via "fusion" of two tensor pomerons we have found (see Appendix A of [1]) the seven possible $\mathbb{P}\mathbb{P}f_2$ tensorial couplings. The different couplings give different results due to different interference effects of the f_2 resonance and the dipion continuum contributions. We have shown that the resonance structures in the measured two-pion invariant mass spectra depend on the cut on proton transverse momenta and/or on four-momentum transfer squared $t_{1,2}$ used in experiment. The model parameters of the optimal $\mathbb{P}\mathbb{P}f_2$ coupling ($j = 2$) have been roughly adjusted to the recent CDF and preliminary STAR experimental data and then used for the predictions for the ALICE, and CMS experiments. We have made estimates of cross sections for both the diffractive and photoproduction contributions. We have shown some differential distributions related to produced pions as well as some observables related to final state protons, e.g., different dependence on proton transverse momenta and azimuthal angle correlations between outgoing protons could be used to separate the photoproduction term, see [1]. The absorption effects due to pp and πp interactions, discussed in [15], lead to a significant modification of the shape of the distributions in ϕ_{pp}, $p_{t,p}$, $t_{1,2}$ and it would therefore be useful to study such observables experimentally when measuring forward protons (STAR, ATLAS-ALFA, CMS-TOTEM).

To summarize: We have given a consistent treatment of the exclusive $\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-$ production in pp collisions in an effective field-theoretic approach. A measurable cross section of order of a few μb was obtained for both processes which should provide experimentalists interesting challenges to check and explore it.

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