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Student Performance and School Size: A Two-stage Spatial Quantile Regression Approach to Evaluate Oklahoma High Schools

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Abstract

Debate about the size of public schools has been ongoing in the U.S. since the 1960s. However, existing studies offer conflicting results about the impact of school size on student performance. This study adds to the body of evidence on this topic using recent Oklahoma data and incorporating a modeling approach that controls for both possible endogeneity and spatial dependence. A two-stage spatial quantile regression approach is used with 424 Oklahoma high school data for the 2014-2015 school year, considering school-level grade point average and average ACT scores as dependent variables. Results suggest that school size is negatively related to both measures of school performance, with larger impacts for the top-performing quantiles. Results were found to be varied in terms of different model specifications depending on student performance measures. Smaller high schools, including those in rural areas, may have an advantage in terms of student performance by engaging parents and enhancing the efficiency of educational processes.

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Introduction

School performance in the United States is a hotly debated issue. Public schools in the U.S. have been perceived to be in a declining phase of performance since the 1960s (Marlow, 2000). In 2013, U.S. Secretary of Education Arne Duncan warned that the "educational challenge in America is not just about poor kids in poor neighborhoods, it's about many kids in many neighborhoods". He made this statement after the results of the Program on International Student Assessment (PISA) were announced, which showed the U.S. school education system in poor light in international comparison. In addition, there are wide gaps in educational quality within the U.S. (Hanushek, Peterson, and Woessmann, 2014). Due to the significant practical implications of this perceived decline, considerable research effort has been applied to analyze this issue. One major strand of this literature is determining the impacts of schooling inputs – for instance, the characteristics of students, families, teachers, and schools – on student scholastic performance measures (Rivkin et al., 2005).

The size of individual schools and/or classes is a critical input measure of an educational institution. Historically, larger schools were seen as having distinct advantages over smaller ones. In particular, lower administrative costs from economies of scale were highlighted as benefits of larger schools (Cohn, 1968; Kenny, 1982). Other factors favoring larger schools are their ability to attract students with diverse backgrounds, less "pigeonhole" effect in successive student cohorts, and greater flexibility in offering specialized courses (Leithwood and Jantzi, 2009). However, several justifications supporting larger schools have recently been seriously challenged by empirical evidence (Leithwood and Jantzi, 2009; Stevenson, 2009), with smaller schools demonstrating better academic results (Stevenson, 2009; Humlum and Smith, 2015). As noted by Kuziemko (2006), the idea behind this prescription is that smaller schools allow closer ties between teachers, students and parents, thus enabling better scholastic performance.

Smaller schools also make students feel safer, and students are less likely to get "lost in the crowd" (Harris, 2006, p.137).

However, the debate on the impact of school size on student academic performance is not yet fully resolved. According to Kuziemko (2006) even though there is no consensus among the existing studies on the impact of school size on student performance, more studies have found a negative impact than a positive impact. In comparison to magnitude and depth of economic studies on the impact of *class size* on student performance, corresponding economics literature on the impact of *school size* is relatively meager (Kuziemko, 2006; Humlum and Smith, 2015). Moreover, some previous studies have suffered from econometric modeling issues such as the omission of relevant explanatory/control variables like costs (Harris, 2006). Since the cost element associated with school size is of particular importance to administrators and policymakers, such misspecification could have serious implications. For example, Hoagland (1995) finds that when expenditures are controlled for, overall school size did not predict student performance. Additionally, the level of aggregation of data could influence the outcomes of analysis. For example, Grogger (1996) finds that when district-level measures are used, school expenditures positively impact academic performance measures but are extremely small when compared to state-level aggregate data.

Little research has been done to explore the impact of school size on student performance, specifically using Oklahoma high school data. Using data from the mid-1990s, Jacques, Brorsen and Richter (2000) found that creating larger school through consolidation results in decreased test scores. Whitacre and Taylor (2016) found that the impact of school size on student performance varies in terms of how a high school is defined as "small". They pointed out that there seems to be a threshold for the negative relationship to be hold.

In this study, we propose to estimate the impact of school size on student scholastic performance by analyzing a cross-section of Oklahoma high-schools. Our study is an attempt to empirically answer one of the important questions in economics of education: does school size really matter in determining student academic achievement? Two distinct measures, the average Grade Point Average (GPA) for the senior class and the average ACT score, are used as measures of student performance. We check the robustness of our findings by using different regression specifications: conditional mean models without (the ordinary least-squares, OLS, estimator) and with spatial dependence in student performance; and conditional quantile functions without and with spatial dependence in student performance. We also use instrumental variables to account for the possible endogeneity of school size with respect to student scholastic achievement. We use a measure of parental motivation as an instrument for school size. These conditional mean and quantile estimates can be used to make different interpretations of the impacts of a change in school size on student performance (Levin, 2001). The estimated conditional mean measures the average causal effect, i.e., the effect of a change in school size on the academic performance of the average individual in the sample. This estimate shows the impact (or overall efficiency) of a change in school size in changing the performance of the average student. In contrast, the quantile estimates show the marginal effects of a change in school size on the performance of students at different points in the conditional distribution of student performance. In addition to efficiency, the quantile estimates show the equity (distributive) implications of a change in school size. In essence, quantile estimates help better understand how much effect would be experienced by whom, for example even marginal achievers and students at risk of failing, and not just the average performer. This type of specification may be particularly relevant given the skewed distribution of public high school size across Oklahoma.

The assumption of spatial dependence is of particular relevance in our context. This stems from previous studies that have found spatial dependence exerted through school size in educational achievement measures. For examples, Angrist and Lavy (1999) find that larger schools (in terms of enrollment) tend to be located in larger cities catering to prosperous families, while schools with smaller enrollments tend to be located in rural areas catering to comparatively poorer households. Due to this, the difference in socioeconomic status of students within a school tends to be less; this will also impact the class sizes assigned to schools. Therefore, enrollment and neighborhood socioeconomic status have a positive association (Murnane and Willett, 2011). Studies on neighborhood effects on educational outcomes suggest that the affluent neighborhood's educational climate is likely to have a positive association with school performance such as high school graduation rate and grades/test scores (Crowder and South, 2011; Nieuwenhuis and Hooimeijer, 2016). Spatial spillovers may occur when families cross over school boundaries in response to salient characteristics of local schools such as the racial profile of students (Angioloni and Ames, 2015). When parents can choose school districts to send their children, schools tend to get pressure to improve to attract and retain students, and hence leads to better school performance. Therefore, in order to take the spatial interdependent effect of neighboring schools on school academic performance measures into account, a spatial model approach is more appropriate. Brasington (2007) analyzes competition between public and private schools in Ohio and estimates the relationship between outcomes from a private school and the number of public-school districts in the county. The results were sensitive to model specification; i.e., a model without spatial dependence showed competitive effects, but a model with spatial dependence mostly did not show competitive effects. As proposed by Brasington (2007), we make use of spatial dimension of the data to address spatial spillover effects of public schools on school performance and to minimize the omitted spatial variables bias.

Econometric procedure

The most basic model in this study estimates the influence of school size on student achievement as follows:

$$y_i = \alpha + \beta x_i + \delta z_i + \varepsilon_i, \quad (1)$$

where y_i is the outcome variable of interest (average senior GPA or average ACT score) at school i, x_i is a vector of control variables, for instance, characteristics of students, families, teachers, and schools, z_i is the school size measured as logarithm of total enrollment in the school, α, β, and δ are the parameters to be estimated, and ε_i is the random error term. Model (1) is initially estimated using the OLS estimator. It has been shown previously that the class size variable could be determined endogenously along with student performance (Hoxby, 2000; Levin, 2001). For instance, in equation (1), the estimate of δ is unbiased if z_i is not correlated with ε_i (i.e., $cov(z_i, \varepsilon_i) = 0$). However, if z_i is correlated with ε_i (i.e., $cov(z_i, \varepsilon_i) \neq 0$), then the estimate of δ will be biased. Suppose the school size is correlated with parents' motivation, which is not observed. In that case, the estimated effect of the school size on student performance will be biased from omitting relevant schooling inputs. For example, highly motivated parents might choose a smaller school size given the fact that the number of teachers per enrolled student is higher. These highly motivated parents might also devote more time to their children's education. Walsh (2010) showed that parental involvement decreases as the size of schools increases, though the magnitude of effect is relatively small. Previous studies examined the impact of parental involvement on children's education found a positive relationship between parental involvement and their children's academic achievement (Stevenson and Baker, 1987; Izzo et al., 1999; Fan and Chen, 2001; Jeynes, 2007; Tan and Goldberg, 2009). To reduce this endogeneity bias from omitted variables in estimates of the effect of school size on student achievement, we use instrumental variables that measure the motivation of the parents. This variable is measured as both the percentage of parents attending parent-teacher meetings and the average number of days absent. Using these instruments, the model is estimated using the two-stage least squares (2SLS) estimator as follows:

$$y_i = \alpha + \beta x_i + \delta \hat{z}_i + \varepsilon_i, \tag{2}$$

$$z_i = \gamma_0 + \gamma_1 x_i + \gamma_2 m_i + v_i, \tag{3}$$

where m_i is a vector of instrumental variables. In the first stage, equation (3) is estimated using OLS and then the fitted values of z_i are used in the second stage where equation (2) is estimated.

The next model estimated is the basic quantile regression model, which is the quantile analog of equation (1), using the least absolute deviation (LAD) estimator. That is, the influence of x_i and z_i on y_i is estimated at different points of the conditional distribution of y_i. The estimation is carried out as proposed in Koenker and Bassett (1978) by minimizing the objective function given in equation (4) as follows:

$$\min_{\beta,\delta \in R^K} \left[\sum_{i \in \{i: y_i \geq \beta x_i + \delta z_i\}} \tau |y_i - \beta x_i + \delta z_i| + \sum_{i \in \{i: y_i < \beta x_i + \delta z_i\}} (1-\tau)|y_i - \beta x_i + \delta z_i| \right], \quad \tau \in (0,1), \tag{4}$$

where K is the dimension of vector of explanatory variables, and τ is the quantile of the distribution of y_i[1]. We next estimate the models (2) and (3) using the two-stage LAD (2SLAD) estimator developed by Ameiya (1982)[2]. The 2SLAD works similar to 2SLS; in the first-stage, the OLS estimator is used to estimate model (3); and in the second stage, model (2) is estimated

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[1] Eide and Showalter (1998) use the LAD estimator to analyze the impact of school quality variables on student achievement.
[2] Levin (2001) uses this estimator to study the effect of class size on student achievement.

using the LAD estimator for given quantiles (τ)[3]. The benefit of the quantile regression model over simple OLS is that, in many cases, the non-linearity of the relationship between the dependent variable and exogenous variables may not allow an assumption of non-linear functions. Alternatively, the relationship may be adequately explained by latent moderators such as quantiles of the distribution of the dependent variable. In our case, a scatter plot of student performance against school size would describe a distribution that is asymmetric or non-identical over the levels of school size. Thus, quantile regression would reveal differences in the influence of school size on student performance at different quantiles of conditional distribution of student performance.

None of the previous models account for possible spatial dependence in factors influencing student achievement. To preliminary test whether there is special autocorrelation, the Moran's I test can be conducted on the residuals of OLS and 2SLS models for student academic performance, using a spatial weight matrix. In the next set of models, we use the spatial dimensions of the data by weighting the observations with spatial weights. First, a neighborhood contiguity object is created using the school latitude and longitude information. Next, this contiguity object is used to create a spatial weight matrix[4]. While there are a host of possible spatial models to be explore[5], we use a simple spatial lag model to capture the possible influence of nearby schools. The spatial lag model (SAR) estimated is as given:

$$Y = \rho WY + \alpha + X\beta + \varepsilon, \tag{5}$$

where Y denotes a vector of the outcome variable of interest (average GPA or average ACT

[3] Standard errors are obtained by bootstrapping equation (2). The "quantreg" package of Koenker (2016) using the R programming language (R Core Team, 2017) is used for the analysis.

[4] We followed the description in Bivand, Pebesma, and Gomez-Rubio (2013) for creating spatial weights. We used the packages "ggmap" (Kahle and Wickham, 2013), "sp" (Pebesma and Bivand, 2005), "spdep" (Bivand and Piras, 2015; Bivand, Hauke, and Kossowski, 2013), and "pgirmess" (Giraudoux, 2017) available in the R programming language. We first identified 5 nearest neighbors of spatial units using Euclidean distance. These 5 nearest neighbors list was converted into spatial weights object with row-standardized style.

[5] Elhorst (2010) reviews spatial econometric models.

score), W is the spatial weight matrix – and so WY captures the impacts of neighboring school outcomes, X denotes a vector of independent variables, ρ represents the spatial autoregressive coefficient, α and β are the parameters to be estimated, and ε is a vector of disturbance term. The model is estimated using the maximum likelihood estimator. The spatial Durbin model estimated is a mixed model with the following form:

(6) $$Y = \rho WY + \alpha + X\beta + WX\theta + \varepsilon,$$

where WX denotes the endogenous interaction effects among the independent variables and θ represents a fixed parameter to be estimated. The model contains spatially lagged independent variables along with the lagged dependent variable on the right-hand side. The benefit of this model is that it introduces spillover effects from neighboring region's independent variables, such as school expenditures. We also test a spatial error model:

(7) $$Y = \alpha + X\beta + u,$$

(8) $$u = \lambda Wu + \varepsilon,$$

where Wu denotes the interaction effects among the disturbance terms of the different spatial units and λ represents the spatial autocorrelation coefficient.

After testing each of these spatial models and assessing their results with Lagrange Multipliers, our final model is the quantile version of the spatial autoregressive model (5) developed by Kim and Muller (2004). The estimated conditional quantile model is as given:

(9) $$Y = \rho_{(\tau)}WY + X\theta_{(\tau)} + u$$

where τ is the τ^{th} quantile of the conditional distribution of average GPA for senior class or average ACT score. Due to the presence of the lagged dependent variable in (9), the conventional quantile estimates would be inconsistent. Kim and Muller (2004) two-stage quantile regression (2SQR) and Chernozhukov and Hansen (2006) instrumental variable quantile regression (IVQR) are two possible econometric techniques applicable in this situation. Both methods account for the general endogeneity problem in the quantile regression, and not the spatial lagged dependent variable specifically. However, both methods can be used to solve the endogeneity problem in the quantile regression (McMillen, 2013a). The IVQR method is applicable to smaller datasets and is computationally intensive (Kostov, 2009; Zhang and Leonard, 2014). Therefore, we used the 2SQR technique, available in the "McSpatial" package (McMillen, 2013b) in the R software (R Core Team, 2017). In the first stage, an instrumental variable is constructed for the lagged dependent variable (WY) using the predicted values from quantile regression of WY on a set of instruments. In the second stage, the predicted values of WY are used in the quantile regression of Y on X. Standard errors are obtained from bootstrap with replacement as standard deviations of the bootstrapped coefficients.

Data

The data used in this study come from the Oklahoma Education Indicators Program (OEIP) funded by the Oklahoma Office of Educational Quality and Accountability. Profile reports at the district and school levels were obtained for the 2014-2015 school year[6]. There were 517 school districts in Oklahoma during the school year 2014-2015. After removing high schools with missing or incomplete information, 424 high school data are used for this analysis. The reports include information on community characteristics, educational processes, and student performance, such as the district poverty rate, school district administrative expenditures, and the average senior GPA and the average ACT score. Variable description and summary statistics are presented in Table 1. The average GPA across the 424 high schools is 3.11 and the average ACT score is 19.83.

[6] The reports are available at www.schoolreportcard.org and the data are available for the years 1997-2016.

To represent student performance, we focus on the average GPA for senior class and average ACT score. Schooling inputs, including the characteristics of students, families, teachers, and schools are used to determine the influences on student performance. School size is measured as natural logarithm of total number of students enrolled in each high school. Student mobility[7] is measured as the percentage of new students enrolled in a school. The percentage of parents with some college education are used to represent family influence. Average years of teacher experience are used to measure as teacher quality[8]. The poverty rate at the district level is used for school district influence.

In addition to these schooling inputs, school district expenditures are used to control for the potential omitted variable in estimating the effects of school size on student academic performance. District expenditures data were divided into eight categories; (i) instructional expenditures, (ii) student support services, (iii) instructional staff support services, (iv) district administration, (v) school administration, (vi) district support services, (vii) debt service, and (viii) other services. The expenditures per category are measured as logarithm of the actual dollars spent per average daily membership (ADM). The expenditures on both instructional and student support services are used as instrument variables for school size in order to control for the potential endogeneity of school size with respect to student performance. Jacques and Brorsen (2002) examined the impact of school district expenditures on student performance using Oklahoma public school data and found that test scores are positively related to instructional expenditures, but are negatively related to student support services. Given this finding, we use only these 2 categories of expenditures (as opposed to all 8 listed above).

[7] Fowler-fin Fowler-Finn (2001) and Parke and Kanyongo (2012) found that student mobility is negatively associated with academic performance.
[8] Greenwald, Hedges, and Laine (1996) described the quality of teachers, including teacher experience is strongly related with student achievement in their meta-analysis.

Results

The preliminary results from the estimated relationships between school size and the two common measures of student performance, the average GPA and ACT scores, are illustrated in Figures 1a and 1b, respectively. There is a negative relationship between average GPA and school size with a decreasing effect as school size increases, indicating that smaller schools perform better than larger ones in terms of average GPA (Figure 1a). In contrast, average ACT shows a slightly negative relationship among smaller schools, but becomes positive as school size increases with a substantially increasing effect suggesting that larger schools outperform smaller ones for this measure (Figure 1b). These highly statistically significant non-linear relationships between school size and student performance suggest that a quantile regression approach is more appropriate for determining the effect of school size on each measure of student performance without specifying any non-linear functional forms for the model.

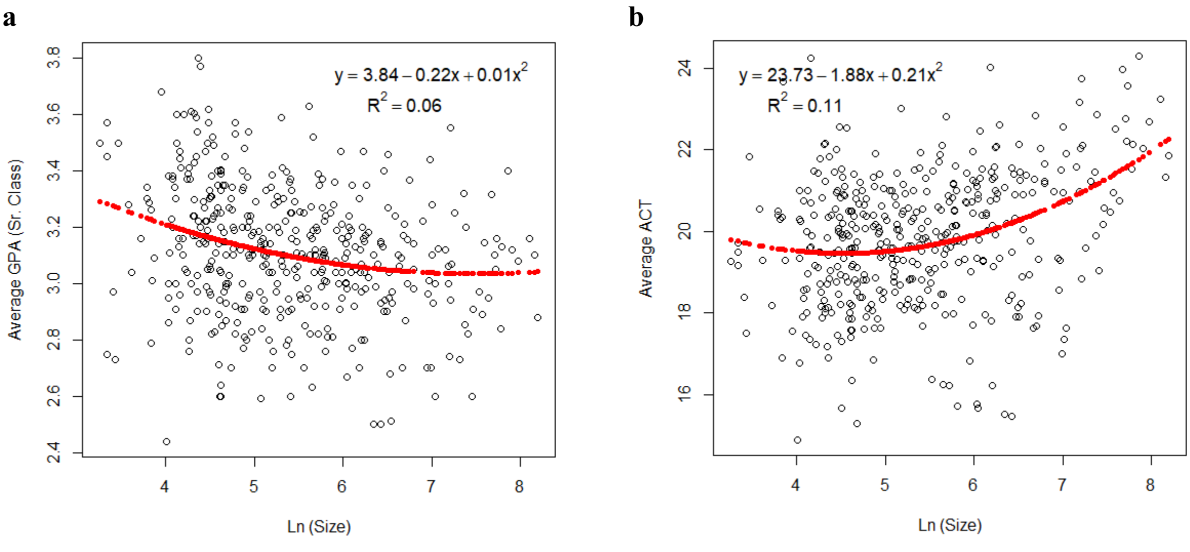

Figure 1. School size effect on average GPA and ACT scores in Oklahoma high schools

The estimation results from OLS and the five common quantile (i.e., the 10th, 25th, 50th, 75th, and 90th percentiles) regressions considering the school size variable as exogenous are presented in Table 2. The corresponding quantile regression plots are illustrated in Figures 3 and 4 for the average GPA and ACT scores, respectively. The OLS results show a significant effect of school size on both the average GPA and the average ACT score, but in opposite directions – suggesting that smaller schools are beneficial to the average GPA but that larger schools perform better on the average ACT score. These results are quite consistent with what the plots in Figures 1a and 1b have suggested based on treating school size as exogenous without imposing any non-linear functional forms. As expected, other variables such as the percentage of parents with some college education, the poverty rate at the school district level, the average years of teacher experience, and the percentage of new students enrolled, are significantly different from zero for each measure of academic performance. An increase in the proportion of parents with some college education is consistent with increased average GPA and ACT scores, indicating that the parents' educational level plays a significant role in students' academic performance. School districts with higher poverty rates and schools with greater student mobility have lower academic achievement in both the average GPA and ACT scores. Interestingly, an increase in the average years of teacher experience has little effect on the average GPA, but has a greater positive effect on the average ACT score, indicating teacher experience plays an important role in increased average ACT score.

For the quantile regression estimates, a significant effect of school size was found for both the average GPA and ACT scores. Students at the 25th and higher percentiles (not at the 10th percentile) of the conditional distribution of average GPA significantly benefit from a decrease in school size. On the other hand, students at all percentiles of the conditional distribution of average ACT substantially benefit from an increase in school size. The parents' educational level has a significant positive effect across all quantiles of each measure of student academic performance. The effect size of the parents' education level increases as it moves from the 25th to the 90th quantiles for each measure of student performance, except the 10th

percentile which was one of lower values for the average GPA and was the lowest value for the average ACT score. The school district poverty rate disadvantaged students below the 50th percentile of the conditional distribution of average GPA and students of all percentiles of the conditional distribution of average ACT score. The magnitude of the effect of the district poverty rate on average ACT score generally decreases as it moves to the higher quantiles. Interestingly, teacher experience has no significant impact on average GPA, it has only a significant effect at the conventional level although the magnitude of this effect is extremely small, but it has a significant positive effect on average ACT score across all quantiles. Student mobility has a significant negative effect across all but the 90th percentile on both the average GPA and ACT scores, and it has an insignificant effect on the 50th percentile of average ACT score. The effect size of student mobility considerably decreases as it moves from the 10th to the 75th percentile. Generally, these results reinforce that important differences do exist across the distributions of GPA and ACT scores, including the school size parameters which vary from the aggregate OLS values.

Before considering the estimates of the two-stage regression model, we first attempted to test and control for the potential endogeneity of school size with respect to student performance. If school size is endogenous, the OLS estimates are biased by correlation between school size and unobserved factors that vary with school size (i.e., factors that are difficult to quantify such as parental motivation and the efficient use of school resources). Parents with high motivation for student achievement may choose school districts with smaller school size. On the other hand, the school districts with larger school size may offer economies of size that are beneficial for students' academic performance. Table 3 presents the estimation results for the Hausman test for the school size being exogenous. School size is estimated from the reduced form equation (3) using four instrument variables: the percentage of parents attending parent-teacher conferences, the average days absent, and instructional expenditures and student support services per student enrolled. We take the residuals of the reduced form equation and include them into the structural equation (2) in order to test the statistical significance of the coefficient upon the residuals in the structural equation. The p-values for the estimates of the residuals for both the average GPA and ACT scores are less than 0.01, respectively, so we can reject the null hypothesis that these residuals are irrelevant. In other words, there is evidence that school size is endogenous with respect to both the average GPA and ACT scores.

Next, we examined the validity of instrument variables to identify the model and conduct the estimation of the school size effect within the structural equations framework. We used the first stage regression model to test whether the instruments chosen are strongly correlated to the endogenous variable of school size. The value of Wald's F statistic is 18.841 with a p-value less than 0.01. The degree of freedom is 3 (the number of instruments minus the number of endogenous variables) and the critical value at 5% level for the χ^2 distribution with 3 degrees of freedom is 7.82. Hence, we can clearly reject the null hypothesis that the instruments are irrelevant. Additionally, we implemented the Sargan test for instrument validity. We take the residuals from the second stage regression models for both the average GPA and ACT scores and use them as the dependent variables in both new regressions in which the residuals are estimated on all exogenous explanatory variables and all instruments. If the instruments selected are valid, they should be uncorrelated to these residuals. We apply the χ^2 test with 3 degrees of freedom and calculate the sample size $n * R^2$ for the test statistic. The p-values of this test are 0.103 and 0.2 for the average GPA and ACT scores, respectively, and hence we do not reject the null hypothesis of the validity of instruments.

We now turn to the structural estimation of the model. The estimation results from the

2SLS and 2SLAD models for the five common quantile regressions are presented in Table 4. The corresponding 2SLAD quantile regression plots for the average GPA and ACT scores are illustrated in Figures 5 and 6, respectively. The most striking finding from the two-stage model specification is the effect of school size on the average ACT score. The school size now has a significant negative effect on this measure of student performance, contrary to the positive effect observed in the OLS estimate. This suggests that once endogeneity is controlled for, smaller schools have an advantage due to parental motivation and the efficient use of school district administrative expenditures. The school size effect on the average GPA remains the same as the OLS estimate, although the magnitude of the coefficients increased. The mean effects of the percentage of parents with some college education, the average years of teacher experience, and the percentage of new students enrolled all remain significant and retain their original signs for both the average GPA and ACT scores. The school district poverty rate has no significant mean effect on either measure of student performance unlike the OLS estimate. It is important to point out that controlling for the endogeneity of school size through parental motivation and school district administrative cost efficiencies could suppress the influence of disadvantageous school district poverty rate on each measure of student performance.

Shifting to the 2SLAD estimates, the effects of school size predetermined by both parental motivation and the efficient use of school district educational funds are roughly similar for the average GPA and ACT scores: both are significant and negative, indicating that smaller schools outperform larger ones for each measure of student performance. While smaller schools are beneficial for the average GPA to all students, they are only beneficial for the average ACT score for students at the 10^{th} and 50^{th} percentiles. The effect of the percentage of parents with some college education is quite important for each measure of student performance, it is significantly positive across all quantiles with a strikingly larger magnitude than any of the other variables considered. Interestingly, an increase in the school district poverty rate is only disadvantageous to students in the upper quantiles of the average ACT score. Given the negative effect of school size predetermined by parental motivation and school district educational expenditures, it is possible that the influence of predetermined school size may suppress the influence of disadvantageous school district poverty rate to students across all quantiles of GPA and ACT scores with the exception of students in the upper quantiles of ACT score. Teacher experience has no significant impact on the lower quantiles and a significant but little effect upon the upper quantiles of the average GPA. For the average ACT score, teacher experience has a significantly positive effect across all the quantiles although the magnitude of this effect decreases in the higher quantiles. Student mobility has a significant negative impact across all quantiles except the 90^{th} percentile on the average GPA and only a significant negative effect on the 25^{th} and 75^{th} percentiles for the average ACT score.

To sum up the results from the OLS and the two-stage regression models, the estimated effects upon both student performance measures are consistent for the percentage of parents with some college education, teacher experience, and student mobility. The effect of school size on the average ACT score and the effect of school district poverty on the average GPA are inconsistent. The school size effect on the average ACT is significantly positive for the OLS regression, but is significantly negative for the two-stage regression. This is an important findings suggesting that controlling for endogeneity of school size can change the direction of the relationship between school size and school performance measure. The effect on the average GPA of school district poverty is significantly negative in the OLS estimate whereas there is no significance found in the two-stage model.

Finally, in order to incorporate spatial spillover effects of public school districts into our structural equations model and control for the potential omitted variable bias, we first tested whether there are spatial effects on student academic performance from neighboring school districts. If spatial effects are not considered, the estimator of the coefficients for the remaining variables will be biased and inconsistent by omitted relevant explanatory variables (Greene, 2005). The Moran's I test results on the residuals of the OLS and 2SLS models for the average GPA and ACT scores, using the five-nearest-neighbors weight matrix, find that there is evidence of spatial correlation of the average GPA between nearby schools, but not for the average ACT score. For the average GPA, the Moran's I statistics are 0.047 and 0.058 and their corresponding *p*-values are 0.045 and 0.018, respectively. Based on the results we reject the hypothesis that the OLS and 2SLS residuals are independently distributed across space, indicating that there are some unobserved characteristics causing a school's GPA to be correlated among nearby schools' GPA. But for the average ACT score, the Moran's I statistics are -0.003 and -0.005 and their corresponding *p*-values are 0.508 and 0.534, respectively, indicating the hypothesis of no spatial correlation in the OLS and 2SLS residuals cannot be rejected.

The Moran's I test results suggest that it is necessary to include spatial effects for the average GPA model. However, the Moran's I test alone is unable to demonstrate whether a spatial lag model or a spatial error model is more appropriate. Therefore, we need to estimate some spatial models to identify the exact source of spatial dependence. Table 5 presents the estimation results from each spatial econometrics model for the average GPA[9]. We found the spatial lag effect on the average GPA to be statistically significant at the 1% level, suggesting

[9] After testing each of the spatial econometrics model specified for both the average GPA and ACT scores, we found no statistically significant spatial effects on the average ACT score from neighboring school districts. Therefore, the spatial models are only estimated for the average GPA.

that a school's average GPA is positively associated with its neighboring schools' average GPA. In particular, the parameter estimate suggests that a 1-point increase in GPA by neighboring schools will raise a school's GPA by 0.175 points. The spatial error model indicates that there is insignificant spatial effect on unobserved factors in a school's average GPA from unobserved factors in its neighboring schools' average GPA. Interestingly, the spatial Durbin model found that the size of the neighboring schools has a significant effect at the 1% level on the average GPA. The average GPA of a school is negatively related to its neighboring schools' sizes. However, the estimated special lag effect of the Durbin model on the average GPA is no longer significant. Therefore, we choose the spatial lag model to estimate the spatial quantile regressions because the estimated effects of other schooling inputs on the average GPA (including the spatial lag effect) are significant, the values of AIC for both models are essentially the same, and the value of BIC for the spatial lag model is smaller. Furthermore, the Moran's I test on the residuals of the spatial lag model with the five-nearest-neighbors weight matrix confirmed this result. In particular, after controlling for the omitted spatial lag effect, the Moran's I statistic is no longer positive and significant (i.e., the statistic is -0.768 and the *p*-value for the statistic is 0.779), suggesting that there is no spatial correlation in the residuals of the spatial lag model.

Turning now to the estimates of the spatial lag and two-stage spatial lag for the five common quantile regressions. The results are presented in Table 6. In the spatial lag model, the spatial lag effect on a school's average GPA from its neighboring schools' average GPA is significantly different from zero in the interquantile range. For the two-stage spatial lag model, this effect is significant across all quantiles with the greatest magnitudes estimated at the 10^{th} and 90^{th} percentiles. Both models found a school's average GPA to be positively correlated to its neighboring schools' average GPA. The parameter estimates suggest that a 1-point increase in GPA by neighboring schools results in increases of 0.38 to 0.78 units at that school. The estimated effects of the percentage of parents with some college education, teacher experience, and student mobility, and their significance and sign, are consistent between the two models. However, the estimated effects of school size and the school district poverty rate somewhat differ across models. Generally, both models demonstrate that smaller schools are beneficial for GPA in schools above at the 75th percentile for GPA. The school district poverty effect is significant in the interquantile rage for the spatial lag model, but is insignificant across all quantiles for the two-stage spatial model. Although both models control for omitted variable bias by estimating the spatial lag effect of neighboring school districts, the two-stage spatial lag model is more robust by controlling for the endogeneity of both school size and neighboring schools' average GPA using relevant instrument variables.

Conclusion

A long standing debate in educational economy is whether school size really matters in determining student performance. Earlier studies have been concerned with this topic focused on the overall causal relationship between school size and student achievement, but did not account for possible endogeneity of school size and spatial dependence of neighboring schools' student performance. This study focuses on understanding how the effect of school size on student performance varies across different segments of the conditional distribution of student academic performance as measured by average GPA and ACT scores, using different econometric model specifications.

The results of the quantile regression show that the significant school size impact varies in direction across measures of student performance. In fact, there appears to be a negative effect of school size on average GPA, but there is a positive effect on average ACT score, which suggests that smaller schools are beneficial to average GPA but that larger schools perform better on average ACT score. The results find that the parameter estimates of school size effect differ across the distributions of average GPA and ACT scores, which are different from the OLS estimates.

The findings from the quantile approach of 2SLAD indicate that controlling for endogeneity of school size using instrumental variables can change the direction and magnitude of school size impact on school performance measures. For instance, the parameter estimate of school size effect on average ACT score is changed from positive to negative and the coefficients of school size effects on both average GPA and ACT sores are increased in magnitude. The results suggest that once endogeneity of school size is controlled for, smaller schools have an advantage due to parental motivation and school district administrative cost efficiencies.

The results from the spatial model specification test find no significant spatial effects on average ACT score from 5-nearest-neighboring schools, suggesting that average ACT score is not influenced by neighboring schools. Therefore, the spatial model is only useful for estimating the relationship between school size and average GPA. The findings of the two-stage spatial quantile regression suggest that there appears to be a strong positive spatial autocorrelation between 5-nearest-neighboring schools' average GPA. However, controlling for spatial effect on average GPA yields decreased school size impact in magnitude. In particular, the parameter estimates indicate that a one-unite increase in school size result in decreases in average GPA by 0.14 at the 90th percentile to 0.15 points at the 75th percentile.

Overall, this study finds that smaller schools may be advantageous for improving student academic performance by engaging parents and community and enhancing the efficiency of educational system. However, results are found to be varied with different model specifications depending on student academic performance measures. Therefore, different econometric techniques are needed for different measures of student performance in order to more precisely examine the causal relationship between school size and the performance of high school students.

Table 1. Descriptive Statistics for Oklahoma High Schools

Variable	Mean	Std. Dev.
Dependent		
Average GPA in senior class	3.11	0.23
Average ACT score	19.83	1.62
Independent		
Total number of students enrolled	391.75	544.42
Percentage of parents with some college education	0.18	0.08
Percentage of poverty	0.17	0.06
Average years of teacher experience	13.45	3.32
Percentage of new students enrolled	0.08	0.07
Instrument		
Percentage of parents attending parent-teacher conferences	0.54	0.24
Average days absent	10.17	4.15
Instructional expenditures ($/ADM)	258.12	142.33
Student support services ($/ADM)	565.98	207.53

Table 2. OLS and Quantile Regression Effects of the Characteristics of Students, Families, Teachers, and Schools on Average GPA and ACT

Dependent Variable		Quantile Regression					Dependent Variable		Quantile Regression				
GPA	OLS	t = 0.10	t = 0.25	t = 0.50	t = 0.75	t = 0.90	ACT	OLS	t = 0.10	t = 0.25	t = 0.50	t = 0.75	t = 0.90
Constant	3.41***	3.11***	3.26***	3.35***	3.66***	3.96***	Constant	16.59***	15.37***	15.63***	16.13***	17.40***	18.27***
	(0.08)	(0.17)	(0.10)	(0.08)	(0.12)	(0.13)		(0.49)	(1.16)	(0.95)	(0.64)	(0.41)	(0.68)
LnEnrollment	-0.07***	-0.03	-0.06***	-0.06***	-0.10***	-0.11***	LnEnrollment	0.27***	0.39***	0.28**	0.33***	0.27***	0.24***
	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)		(0.07)	(0.14)	(0.13)	(0.09)	(0.06)	(0.09)
College Education	0.69***	0.74**	0.85***	0.72***	0.66***	0.69***	College Education	6.58***	5.30**	8.35***	7.17***	6.93***	6.84***
	(0.16)	(0.34)	(0.21)	(0.15)	(0.15)	(0.22)		(1.00)	(2.72)	(1.80)	(0.86)	(1.04)	(0.91)
Poverty	-0.43***	-0.86**	-0.43**	-0.34*	-0.28	-0.69*	Poverty	-4.09***	-5.71***	-4.76***	-4.00***	-3.09***	-3.29**
	(0.17)	(0.42)	(0.20)	(0.18)	(0.21)	(0.36)		(1.08)	(2.25)	(1.70)	(1.01)	(0.97)	(1.62)
Teacher Experience	0.01**	0.00	0.00	0.01**	0.01	0.00	Teacher Experience	0.12***	0.09*	0.13***	0.12***	0.10***	0.08***
	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)		(0.02)	(0.05)	(0.04)	(0.03)	(0.02)	(0.02)
Student Mobility	-0.71***	-1.06***	-1.01***	-0.74***	-0.65**	-0.53	Student Mobility	-3.57***	-7.93***	-5.83***	-1.63	-2.36***	-1.63
	(0.14)	(0.28)	(0.23)	(0.23)	(0.28)	(0.33)		(0.91)	(2.56)	(2.21)	(1.45)	(0.70)	(1.38)

Notes: Single, double, and triple asterisks (*, **, ***) represent significance at the 10%, 5%, and 1% level. Standard errors are shown in parenthesis.

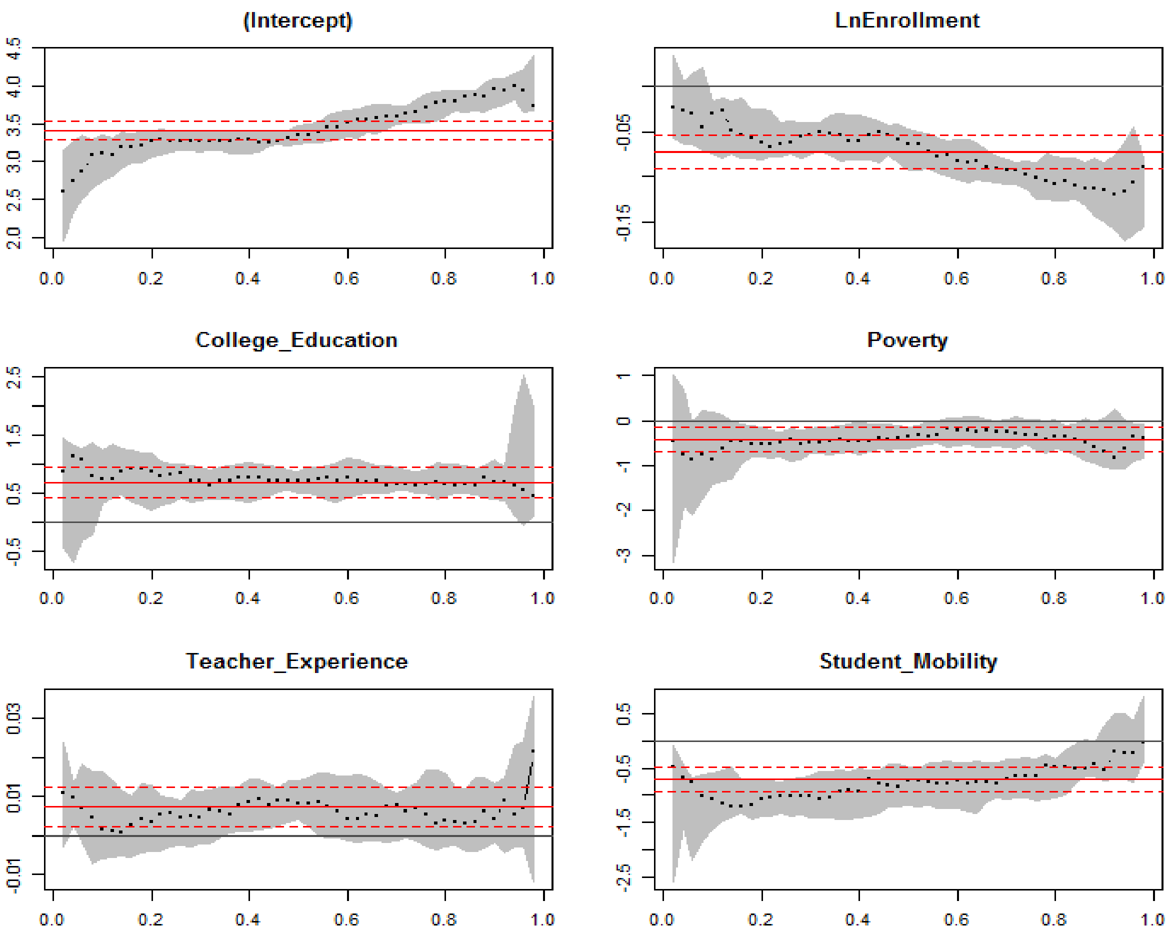

Figure 2. Quantile regression covariates effects for average GPA

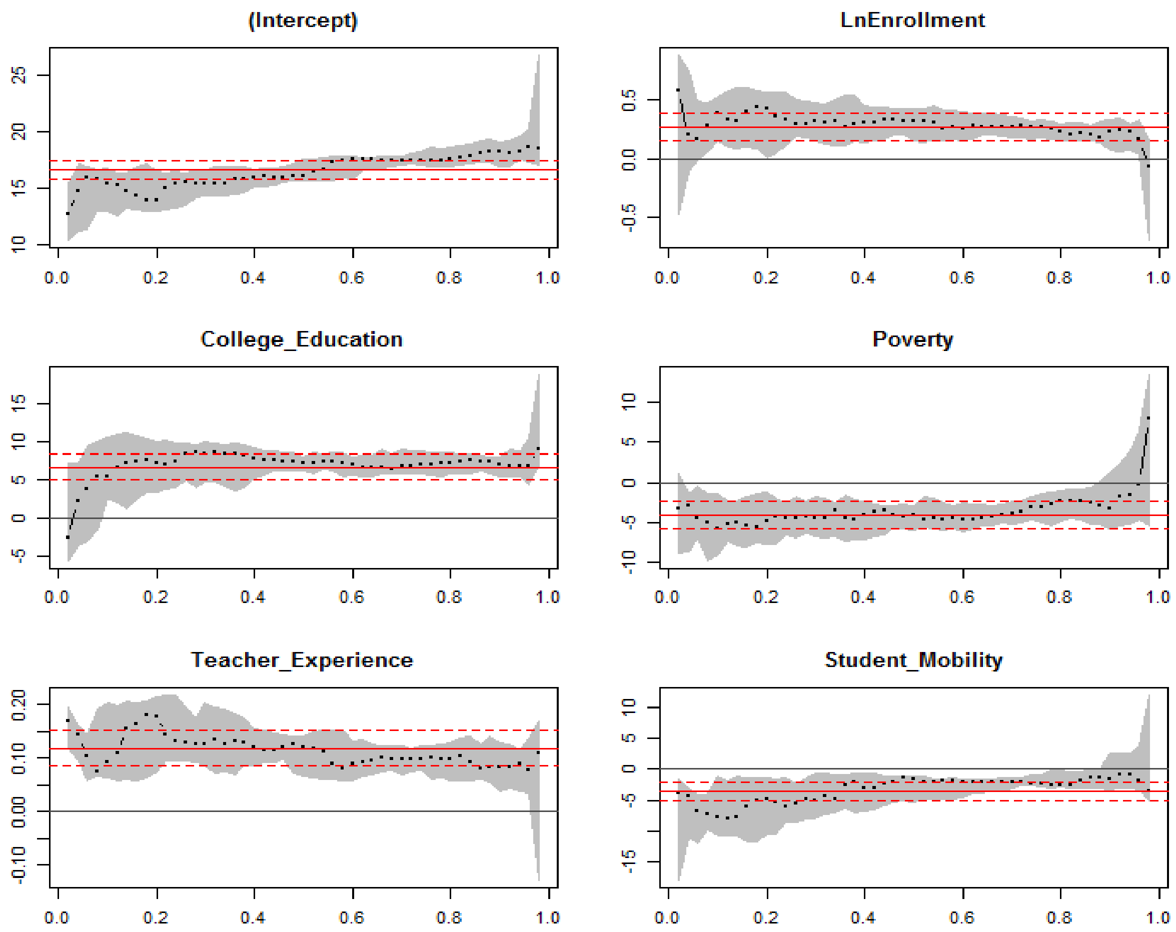

Figure 3. Quantile regression covariates effects for average ACT

Table 3. Estimation Results of the Hausman Test for Endogeneity of School Size

Dependent Variable	GPA	ACT
Constant	3.775***	19.953***
	(0.122)	(0.769)
LnEnrollment	-0.168***	-0.623***
	(0.028)	(0.176)
College Education	1.323***	12.483***
	(0.229)	(1.441)
Poverty	-0.192	-1.861*
	(0.178)	(1.119)
Teacher Experience	0.007**	0.118***
	(0.003)	(0.019)
Student Mobility	-0.700***	-3.453***
	(0.140)	(0.882)
Estimated Residual from the 1st Stage	**0.113*** **	**1.058*** **
	(0.030)	**(0.191)**
R^2	0.229	0.372
N	424	424
F	20.67	41.17

Notes: Single, double, and triple asterisks (*, **, ***) represent significance at the 10%, 5%, and 1% level. Standard errors are shown in parenthesis.

Table 4. 2SLS and 2SLAD Quantile Regression Effects of the Characteristics of Students, Families, Teachers, and Schools on Average GPA and ACT

Dependent Variable		**2SLAD Quantile Regression**					Dependent Variable		**2SLAD Quantile Regression**				
GPA	2SLS	t = 0.10	t = 0.25	t = 0.50	t = 0.75	t = 0.90	ACT	2SLS	t = 0.10	t = 0.25	t = 0.50	t = 0.75	t = 0.90
Constant	3.77***	3.71***	3.48***	3.71***	4.21***	4.28***	Constant	19.95***	19.74***	18.20***	19.99***	19.41***	18.48***
	(0.12)	(0.21)	(0.14)	(0.17)	(0.21)	(0.23)		(0.80)	(1.95)	(1.56)	(0.89)	(1.18)	(1.36)
Pred LnEnrollment	-0.17***	-0.20***	-0.13***	-0.16***	-0.22***	-0.23***	Pred LnEnrollment	-0.62***	-0.97**	-0.46	-0.57***	-0.15	0.20
	(0.03)	(0.05)	(0.03)	(0.04)	(0.04)	(0.05)		(0.18)	(0.49)	(0.34)	(0.19)	(0.28)	(0.34)
College Education	1.32***	1.88***	1.35***	1.25***	1.51***	1.44***	College Education	12.48***	10.92***	13.52***	13.02***	9.18***	7.11***
	(0.23)	(0.37)	(0.33)	(0.34)	(0.29)	(0.40)		(1.50)	(4.45)	(2.52)	(1.54)	(1.98)	(2.44)
Poverty	-0.19	-0.24	-0.31	-0.15	-0.17	-0.29	Poverty	-1.86	0.71	-2.37	-3.15***	-2.56**	-3.05*
	(0.18)	(0.32)	(0.26)	(0.21)	(0.24)	(0.40)		(1.16)	(2.68)	(1.97)	(1.28)	(1.32)	(1.66)
Teacher Experience	0.01**	0.00	0.01	0.01*	0.01*	0.01*	Teacher Experience	0.12***	0.12***	0.13***	0.11***	0.09***	0.08***
	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)		(0.02)	(0.04)	(0.04)	(0.03)	(0.02)	(0.03)
Student Mobility	-0.70***	-0.66**	-0.81***	-0.75***	-0.90***	-0.42	Student Mobility	-3.45***	-3.34	-5.20***	-1.93	-2.33***	-1.21
	(0.14)	(0.28)	(0.17)	(0.23)	(0.21)	(0.32)		(0.92)	(2.50)	(1.99)	(1.35)	(0.66)	(1.63)

Notes: Single, double, and triple asterisks (*, **, ***) represent significance at the 10%, 5%, and 1% level. Standard errors are shown in parenthesis.

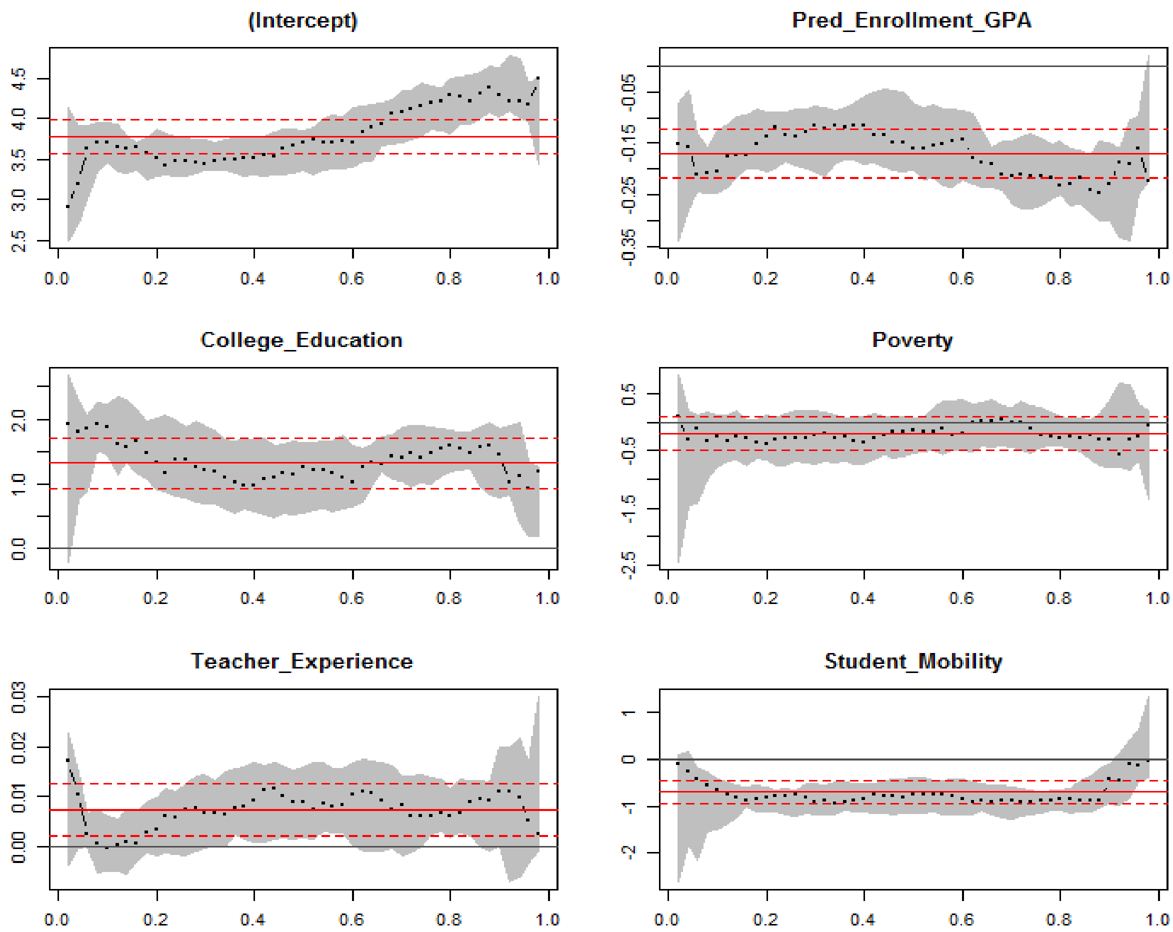

Figure 4. 2SLAD quantile regression covariates effects for average GPA

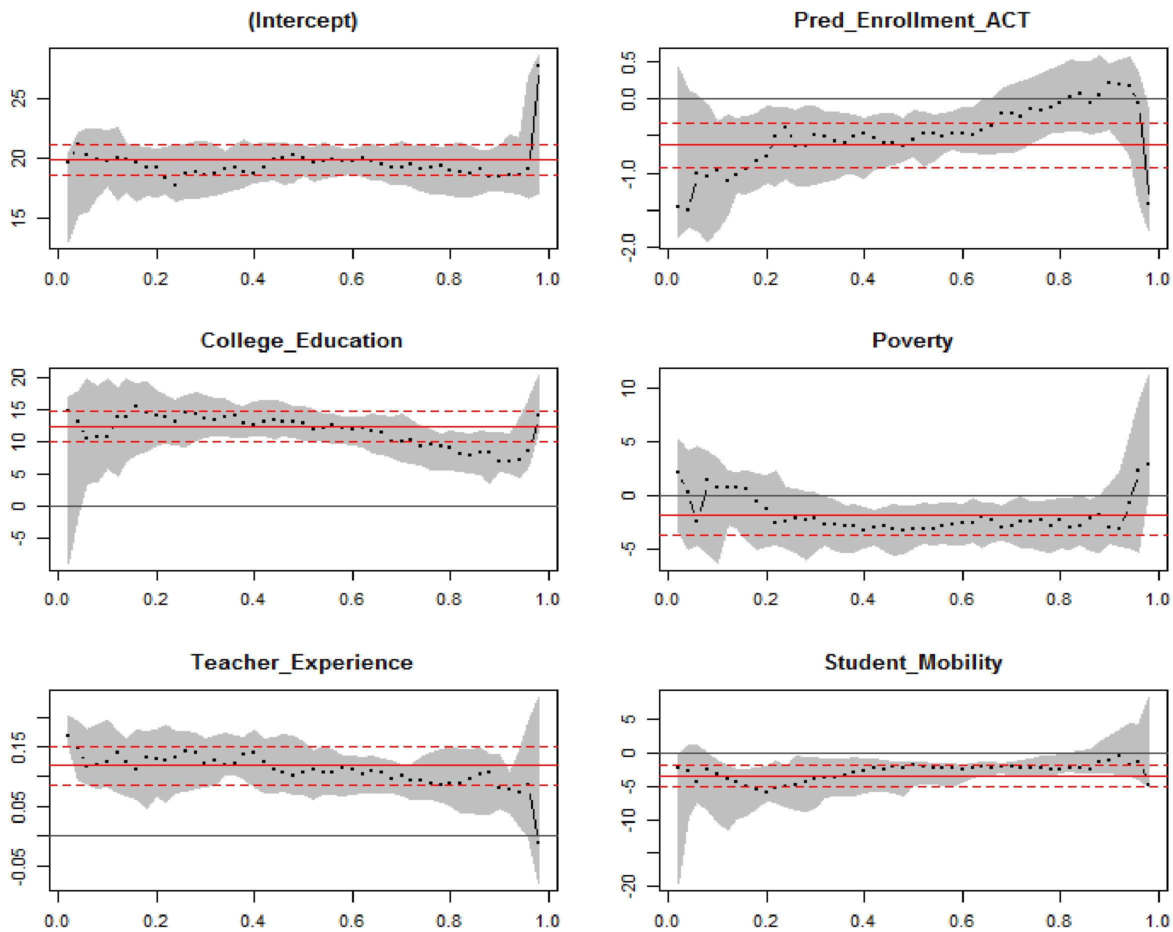

Figure 5. 2SLAD quantile regression covariates effects for average ACT

Table 5. Estimates of Spatial Models for the Characteristics of Students, Families, Teachers, and Schools on Average GPA

Dependent Variable	Spatial Lag			Spatial Error			Spatial Durbin		
GPA	Estimate	Std. Error	P-value	Estimate	Std. Error	P-value	Estimate	Std. Error	P-value
Constant	2.838	0.231	0.000	3.394	0.076	0.000	3.206	0.318	0.000
InEnrollment	-0.064	0.011	0.000	-0.066	0.011	0.000	-0.046	0.013	0.000
College Education	0.653	0.154	0.000	0.665	0.160	0.000	0.616	0.175	0.000
Poverty	-0.425	0.166	0.010	-0.444	0.171	0.009	-0.544	0.181	0.003
Teacher Experience	0.007	0.003	0.032	0.007	0.003	0.025	0.006	0.003	0.069
Student Mobility	-0.692	0.140	0.000	-0.704	0.142	0.000	-0.675	0.140	0.000
Lag.InEnrollment							-0.068	0.022	0.002
lag.College Education							0.416	0.314	0.185
lag.Poverty							0.265	0.313	0.399
lag.Teacher Experience							0.004	0.007	0.539
lag.Student Mobility							0.003	0.296	0.993
ρ	0.175	0.067	0.011				0.099	0.077	0.196
λ				0.130	0.076	0.107			
AIC	-124.489			-120.679			-124.658		
BIC	-92.091			-88.281			-72.011		

Table 6. Estimates of Spatial Lag and Two-Stage Spatial Quantile Regression on Average GPA

Dependent Variable	Spatial Lag Quantile						Two-Stage Spatial Quantile				
GPA	t = 0.10	t = 0.25	t = 0.50	t = 0.75	t = 0.90		t = 0.10	t = 0.25	t = 0.50	t = 0.75	t = 0.90
Constant	1.93**	1.38*	2.09***	2.32***	4.68***	Constant	1.61	1.79**	1.74***	1.96***	1.42*
LnEnrollment	-0.04*	-0.03	-0.04***	-0.08***	-0.13***	Predicted LnEnrollment	-0.16***	-0.09**	-0.06	-0.15***	-0.14***
College Education	0.81**	0.50**	0.64***	0.60***	0.82***	College Education	1.68***	1.03***	0.77**	1.22***	1.11***
Poverty	-0.65	-0.51***	-0.35*	-0.49**	-0.51	Poverty	-0.19	-0.27	-0.33	-0.34	-0.31
Teacher Experience	0.00	0.00	0.01*	0.01	0.01	Teacher Experience	0.00	0.00	0.01**	0.01	0.00
Student Mobility	-0.84***	-0.99***	-0.81***	-0.73***	-0.52	Student Mobility	-0.65**	-0.63***	-0.79***	-0.74***	-0.39
WY	0.40	0.58***	0.38**	0.41*	-0.24	Predicted WY	0.64*	0.51**	0.50***	0.61***	0.78***

Notes: Single, double, and triple asterisks (*, **, ***) represent significance at the 10%, 5%, and 1% level.

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